Doubling Down on Debt: Limited Liability as a Financial Friction[‡]

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We show that limited liability and preexisting debt lead to heterogeneous investment distortions, where high leverage firms tend to overinvest in the manner of zombie firms but low leverage firms tend to underinvest. In a model with a single investment opportunity, limited liability provides equity holders with an incentive to "double-sell" cash flows in default and encourages overinvestment, provided that the firm has preexisting debt and can finance new investment with debt. With repeated investment opportunities, high leverage firms continue to overinvest, but low leverage firms tend to underinvest because debt investors anticipate equity holders' inability to commit to not double-sell cash flows. Restricting debt financed equity payouts has ambiguous efficiency implications, mitigating low leverage firms' underinvestment but exacerbating overinvestment at highly levered firms. Consistent with model predictions, we find that highly levered firms invest more than low leverage firms in Compustat data 2004-2018, despite lower profitability and investment opportunities.

Keywords: limited liability, financial friction, investment, equity payouts

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1 Introduction

Do firms invest and grow when burdened with high debt? Does the threat of bankruptcy limit firms' growth when they do not have sufficient access to equity? These age-old questions have acquired new relevance in the aftermath of the global financial crisis of 2008 and during the current pandemic.[1] While existing debt does not distort investment or encourage equity payouts in the absence of market imperfections, a large and diverse literature points to the empirical and theoretical role of financial frictions.

An essential, but often implicit, feature of most financial frictions is an interaction with limited liability.[2] For example, in models of private information (e.g., Bernanke et al. (1999) or Clementi and Hopenhayn (2006)) the limited liability of equity holders constrains the possible punishments for misreporting. Similarly, in models of inalienable human capital (Kiyotaki and Moore (1997), and Albuquerque and Hopenhayn (2004)) limited liability distorts investment because the lender cannot operate the firm in default, and the equity holder cannot commit to maintain operations. Models of risk-shifting and entrepreneurship (e.g., Jensen and Meckling (1976) and Vereshchagina and Hopenhayn (2009)) highlight the incentives for risk-taking due to the asymmetric payoff structure implied by limited liability. Finally, in models of limited enforcement (Buera et al. (2011), Moll (2014), among others) firms' can abscond with profits or capital at the time of default and limited liability prevents effective punishments.

Rather than examine a particular version of these important mechanisms, we instead show that the common element between all of these models—limited liability—alone leads to financial frictions with rich implications for real investment, leverage,

[1]See, e.g., Bloomberg, 11/17/2020, "America's Zombie Companies Rack up $2 Trillion of Debt."

[2]While it manifests in various ways, limited liability is a protection of equity holders' non-firm assets (including human capital) from creditors following a firm's bankruptcy. In all of its forms, the central friction is that limited liability leads to a commitment problem which limits possible contracts. Equity holders raising debt would otherwise promise to either (1) never default, or (2) pay debt holders personal assets outside of the firm—thereby making punishments more effective. Similar frictions, albeit in different institutional context, also exist in the sovereign debt literature, where a lack of commitment to not dilute future debt causes sovereigns to inefficiently rely on short-term debt (see Hatchondo et al. (2016) or Aguiar et al. (2019)).

and equity payouts. Our emphasis on limited liability as the main friction is driven by the observation that it is a universal institutional feature. Thus, by investigating this friction alone, we identify a common set of distortions faced by all firms in the economy rather than more elaborate models of financial frictions, which may be more individually appropriate for a subset of firms (see Lian and Ma (2021)). In our analysis we consider equity holders' financing options and allow for variable investment and equity payouts (such as dividends or equity buybacks). These ingredients prove to be crucial for our results and lead to non-monotone and heterogeneous impacts on real investment, and—unique among these models of real financial frictions—equity payouts.

To isolate the role of limited liability, we present a simple, analytically tractable model of a firm with preexisting debt and optimal investment, financing, and default decisions. We first consider a baseline model with a single investment opportunity and three types of agents. Equity holders operate the firm and are protected by limited liability, preexisting debt investors hold existing debt but are otherwise inactive, and new debt investors competitively price new debt issued by the firm. Equity holders simultaneously choose how much to invest, whether to finance this new investment with debt or equity, and how much to directly pay out to themselves. Direct equity payouts financed by debt are constrained to at most a fixed fraction of firm cash flows. Cash flows are assumed to evolve according to a continuous stochastic process and equity holders can walk away from the firm at any time, in which case debt holders claim the remaining assets. As a result, the optimal default decision is characterized by a threshold on cash flows as in Leland (1994).

With a single investment opportunity, firms have an incentive to invest more than the first-best. Our mechanism hinges on equity holders' lack of commitment to a specific default threshold and their ability to "double-sell" some cash flows previously promised to preexisting debt holders. Limited liability is central to our mechanism because it allows the firm to credibly sell claims on the firm's assets in liquidation to new bond holders. In particular, by financing new investment with debt, equity holders increase leverage and commit to defaulting earlier and at higher cash flows. This transforms coupon payments promised to existing debt holders into new bankruptcy claims, which in turn can be partially sold to new debt holders. The incentive to dilute preexisting debt holders increases equity holders' marginal benefit of debt financing, leading to overinvestment and, if permitted, direct equity payouts.

In the special case with no future investment opportunities, allowing debt financed equity payouts is unambiguously efficient, mitigating inefficient overinvestment. This is because equity payouts provide a more efficient way of diluting existing debt holders than inefficient overinvestment. In the corner case with unconstrained equity payouts we can split the equity holders' problem into two separate problems: (1) investment and (2) dilution of existing debt holders. Equity holders choose investment to maximize the net present value of the firm and then choose the level and financing of equity payouts to optimally dilute existing debt holders. Thus, as suggested by Myers (1977), restrictions on equity payouts financed by new debt tend to increase investment. However, in the extreme case of a single investment opportunity such an increase in investment is not efficient in our model because investment is already inefficiently high.

We next build on our baseline model to incorporate repeated investment opportunities, revealing the importance of dynamic considerations. We model repeated investment opportunities with a fixed stochastic arrival rate, where at each investment opportunity the equity holders face an identical problem as in the baseline model. When the investment arrival rate is nonzero a new channel emerges as debt investors anticipate equity holders' lack of commitment not to dilute them in the future, raising the cost of debt financing at the time of investment. The resulting incentive to finance investment with equity and underinvest is particularly relevant for firms with low initial leverage and a high arrival rate of investment opportunities, because these firms have the largest capacity to dilute existing debt holders and frequent opportunities to do so. We therefore obtain the prediction that low leverage and high investment opportunity firms tend to underinvest and finance their investment with equity, while high leverage firms — just as in the one-shot investment model — tend to overinvest.

With repeated investment opportunities, equity payout restrictions have new heterogeneous efficiency implications across the cross-section of firms. If equity payouts financed by new debt are prohibited this alleviates the underinvestment problem for firms that have low initial leverage and frequent investment opportunities. By contrast, for high leverage firms with infrequent investment opportunities, restricting equity payouts financed by new debt continues to exacerbate inefficient overinvestment, as in the baseline model with a single investment opportunity.

Simulating an ensemble of time series for repeated investment model shows that modeling default optimally, as opposed to an exogenous default threshold, is crucial for the leverage, bankruptcy, and real investment comparative statics with respect to equity payout restrictions. By mitigating debt investors' concerns about future dilution, equity payout restrictions lower the cost of debt finance, leading equity holders to delay bankruptcy at any given level of cash flows. A lower cost of debt finance for low leverage firms raises their investment, which however can have ambiguous efficiency implications if they switch from underinvestment to overinvestment.

Taken together, in our model highly levered firms have an incentive to overinvest because debt financed investment brings forward bankruptcy, thereby expropriating current debt holders. At the same time, our model predicts that low leverage firms with a longer time horizon tend to underinvest, as debt holders rationally require a low price to hold the debt of low leverage firms, anticipating future debt financed investment. The mechanism is strengthened when equity holders can deplete equity by making discretionary equity payouts, such as equity repurchases.

In the final part of the paper, we test the model implications using firm-level data from Compustat. We start by showing that firms in the highest leverage quartile have consistently higher capital expenditures investment than firms in the lowest leverage quartile over the sample 2004.Q1-2018.Q4. This empirical finding is robust to controlling for investment opportunities and firm profitability, and persistently present even after the financial crisis of 2008-2009. We next drill down to the firm level and use Jordà (2005) projections to show that when high-leverage firms invest, their leverage rises and profitability falls, as should be the case if these firms engage in debt financed overinvestment. By contrast, but consistent with the model predictions, we find that low-leverage firms' investment is followed by persistently higher profitability, but only a very short-lived increase in leverage that becomes statistically and economically insignificant three quarters after the investment. Empirical evidence therefore strongly supports our model mechanism, whereby highly levered zombie firms invest more than the efficient level and finance with debt, but low-leverage firms avoid debt financing and are forced to pass up profitable investment opportunities.

Literature Review —

Most broadly our paper is related to the literature that studies investment distortions arising from different types of credit market frictions (e.g. Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), Buera et al. (2011), Khan and

Thomas (2013), or Moll (2014))).[3] Relative to these strands of the literature we examine an environment with perfect information and fully competitive debt markets and focus on how—in isolation—limited liability distorts equity holders' financing and investment decisions.[4]

Our paper is also related to the debt overhang literature, see, for example, Myers (1977), Moyen (2007), and Diamond and He (2014). While this literature tends to emphasize underinvestment, we show that one single channel – limited liability – can simultaneously lead to over- and under-investment in the cross-section of firms. This difference arises because we permit new investment to be financed with debt and because investment opportunities arrive repeatedly. We also complement the "leverage ratcheting" literature (Admati et al. (2018), DeMarzo and He (2020)) by focusing on the real investment implications and the effects of restricting equity payouts when equity holders cannot commit against issuing new debt in the future. We also speak to a recent empirical and quantitative literature that investigates investment and corporate leverage at the macroeconomic level (e.g. Atkeson et al. (2017)). Recently, Crouzet and Tourre (2020) investigate how business credit programs can mitigate underinvestment and Acharya and Plantin (2019) argue in a model of agency frictions that equity payouts can lead to underinvestment.[5]

2 Model

There are three types of agents in the baseline model: equity holders that operate the firm, existing debt holders who hold debt issued in the past, and competitive outside

[3]Our mechanism is also related to debt dilution and debt overhang in the sovereign default literature (see, for example, Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2013), Hatchondo et al. (2016), and Aguiar et al. (2019)). The key difference is institutional: sovereigns have no obligation to pay their creditors anything in default; thus, they dilute the existing debt holders by defaulting earlier and re-capturing payments promised to debt holders.

[4]In contrast to risk-shifting and entrepreneurship models (e.g., Jensen and Meckling (1976) and Vereshchagina and Hopenhayn (2009)), our mechanism does not require that firms increase risk and is strongest when firms are still well away from their default threshold.

[5]We differ from both of these papers in that we emphasize heterogeneous effects across the firm leverage distribution, where overinvestment is possible and equity payout restrictions may be efficient for some firms but not for others.

creditors. Equity holders face a one-time investment opportunity at time 0, at which time they can issue new debt and equity and make direct payouts to themselves (i.e., dividends or equity buybacks). Debt is senior to equity but all debt, including newly issued debt, has equal priority (pari passu). All actions are perfectly observable and there is complete information. To keep the model analytically tractable and to highlight the underlying intuition, in our baseline model we assume that equity holders have one-time investment opportunity. We extend the model to feature repeated investment in Section 4.

2.1 Firm State, Notation, and Laws of Motion

The state of a firm is summarized by its *cash flows*, Z, and the book value of its *liabilities*, L, defined as the present discounted value (PDV) of all promised cash flows to debt holders. Equity and bond holders discount future payoffs at the same constant rate $r > 0$.[6] In the absence of new investment $Z(t)$ follows a geometric Brownian motion with risk-neutral drift μ and instantaneous volatility of $\sigma^2 > 0$

$$\mathrm{d}Z(t) = \mu Z(t)\mathrm{d}t + \sigma Z(t)\mathrm{d}\mathbb{W}(t), \quad Z(0) > 0, \tag{1}$$

where $\mathbb{W}(t)$ is a standard Brownian motion. Liabilities, L, may have a one-time jump at time 0 (i.e., at the time of investment if equity holders decide to finance some of the investment with debt) but remain constant for all $t > 0$. Thus, we do not allow equity holders to issue new debt or repurchase existing debt after time 0—and relax this assumption in Section 4.

Real Investment and Payouts to Equity At time 0 equity holders have a one-time investment opportunity that expires immediately if not executed. In particular, at time 0 equity holders can increase the initial cash flows of the firm from $Z(0)$ to $\hat{Z} \equiv (1+g)Z(0)$, where $g \geq 0$ captures equity holders' investment financed through a

[6]Due to the absence of fixed costs in this model, cash flows are equivalent to EBITDA profits and proportional to both the assets-in-place and enterprise value. The book value of liabilities L does not take into account the equity holder's option to default. For example, if the firm's liabilities consist of one unit of defaultable consol that promises coupon c every instant of time then $L = \int_0^\infty ce^{-rs}ds$.

combination of new debt and equity. After the initial jump in cash flows, cash flows follow (1). Investment is costly, with the cost function given by $q(g)/Z \equiv \frac{\zeta g^2}{2}$.

At the time of the investment, we also allow equity holders to make direct payouts to themselves, M. We interpret M as equity buybacks, dividends, or leveraged buyouts financed by issuing new debt. Thus, M captures any payout to equity holders that equity holders can use immediately for consumption. The presence of M allows us to consider proposals to limit share buybacks or dividend payments. We assume that equity holders can only consume $M \in [0, \kappa Z]$, where $\kappa \geq 0$ is the parameter capturing institutional constraints on financing equity buybacks with new debt, with $\kappa = 0$ being our baseline.[7] The assumption that equity holders have at least some ability to increase equity payouts from newly issued debt is supported by empirical evidence in Farre-Mensa et al. (2020).

Financing and Limited Liability For tractability we assume that all debt takes the form of defaultable consols, which pay one coupon until the firm defaults and represent a proportional claim to the assets of the firm in bankruptcy. Equity holders can fund the total cost of investment, $q(g)Z$, and the total equity payouts, M, with their own funds (equity financing), by issuing new debt (debt financing), or any linear combination of them. Equity holders are assumed to be deep-pocketed and hence able to finance investment or equity payouts with their own funds if they so choose. We denote the proportion of debt financing by $\psi \in [0, 1]$. If the firm issues only equity (i.e., $\psi = 0$) then the liabilities of the firm, L, have no jump at time 0. If $\psi > 0$, liabilities jump at time 0. In that case, let $\hat{L}$ denote *post-investment liabilities*—capturing the present value of all coupons.

The post-investment liabilities, $\hat{L}$, is implicitly determined by the financing choice, $\{g, \psi, M\}$, and an equilibrium price, $P(\cdot)$, for any newly issued bonds. We defer details until we can fully specify the problem given the equilibrium actions of equity holders (see Section 2.3).

Equity holders are protected by limited liability. This means that after investment and financing choices have been made, equity holders can choose to default and walk away with nothing at any time, whereupon the firm is taken over by debt holders. Deep-pocketed equity holders are assumed to have sufficient funds (i.e., can inject

[7]The restriction that $M \geq 0$ is without loss of generality since equity holders would never choose $M < 0$ (which, in the model, corresponds to buying back debt).

new equity) to keep the firm as a going concern, if they so choose, when promised debt payments exceed firm cash flows.

Equity Value At any point in time there are two states variables: current cash flows, Z and current liabilities, L. Let $V(Z, L)$ denote the post-investment value of equity (i.e., the value of operating the firm after the investment option was executed) when the current cash flows are Z and current liabilities are L. Similarly, let $V^*(Z, L)$ denote the pre-investment value of equity (i.e., the value of operating the firm to equity holders at the time they make their investment decision) when time 0 cash flows are Z and time 0 liabilities are L.[8]

It will prove useful to rescale the value of equity with cash flows. Thus, we denote the post- and pre-investment equity value relative to cash flows as $v(\cdot) \equiv V(\cdot)/Z$ and $v^*(\cdot) \equiv V^*(\cdot)/Z$, respectively. Similarly, we define current leverage as $\ell \equiv L/Z$ and the equity payouts per unit of Z as $m \equiv M/Z$.

Value in Default Upon default the firm is taken over by the debt holders who continue to operate it. However, default has real costs in the sense that immediately following default the firm's cash flows decrease from Z to $(1-\theta)Z$, where $\theta \in [0, 1]$. The parameter θ captures deadweight costs associated with bankruptcy proceedings and debt holders' lower skill in running the firm.

2.2 Equity Holders' Investment and Default Decisions

Equity holders face the following decisions. First, at time 0, they have to choose how much to invest, g, how much to pay out to themselves, M, and how to finance these choices, ψ. Having made these choices, at each instant of time they need to decide whether to keep operating the firm or default instead.

Investment Decision Given an initial state (Z, L), equity holders choose the financing mix ψ and real investment g to maximize the post-investment equity value plus direct payouts to equity, net of new equity injected into the firm. The post-investment equity value is given by $V((1+g)Z, \hat{L})$, where $\hat{L}$ are the post-investment

[8] We differentiate between pre-investment and post-investment states when discussing the investment decision, in which case we denote the post-investment states by $\hat{L}$ and $Z(1+g)$.

liabilities. Equity holders take the equilibrium price $P(\cdot)$ for newly issued bonds as given, and solve

$$V^*(Z,L) = \max_{\substack{g\geq 0\\ \psi\in[0,1]\\ 0\leq M\leq \kappa Z}} \Big\{ \overbrace{V(\underbrace{(1+g)Z}_{=\hat{Z}},\hat{L})}^{\text{Post-Investment Equity}} - \overbrace{(1-\psi)q(g)Z}^{\text{Equity Financed}} + \overbrace{M}^{\text{Payouts}} \Big\} \tag{2}$$

$$\text{s.t. } \underbrace{P(\hat{L},Z,L,g,\phi,M)}_{\text{Equilibrium Price}} \underbrace{\left(\hat{L}/r - L/r\right)}_{\text{New Bonds}} = \underbrace{\psi q(g)Z}_{\text{Debt Financed}} + M \tag{3}$$

and subject to the feasibility of the payoffs in default embedded in L and $\hat{L}$. In (3), $(\hat{L}-L)/r$ is the quantity of new bonds issued (given the constant interest rate r). Funds raised from issuing new debt at price $P(\cdot)$ can be used for equity payouts, M, or to finance a portion of investment costs, $\psi q(g)Z$. To understand how existing liabilities distort equity holders' investment choices we consider the following first-best benchmark.

Definition 1 (First-Best Investment)**.** *We define the first-best undistorted investment, g^u, as investment that maximizes the net present value of the firm. That is,*

$$g^u(Z) \equiv \arg\max_g \Big\{ \overbrace{V((1+g)Z,0)}^{\textit{Post-Investment Equity}} - \overbrace{q(g)Z}^{\textit{Equity Financed}} \Big\} \tag{4}$$

Thus, first-best investment is the level that equity holders would choose if the firm had no preexisting debt and investment had to be fully financed with equity. Since both the payoffs and costs are linear in Z, we can show that g^u is independent of Z throughout our model. The homotheticity that leads to a constant g^u is not essential, but simplifies the analysis.

Default Decision Equity holders optimally choose to default when the equity value, $V(Z,L)$, reaches 0. Note that after investment only the cash flows fluctuate, and the equity holders' *continuation* problem becomes a standard stopping problem (as in Leland (1994) with liabilities L as an additional state), which is given by

$$rV(Z,L) = Z - rL + \mu Z \boldsymbol{\partial}_Z V(Z,L) + \frac{\sigma^2}{2} Z^2 \boldsymbol{\partial}_{ZZ} V(Z,L) \tag{5}$$

$$V(\underline{Z},L) = 0 \tag{6}$$

$$\boldsymbol{\partial}_Z V(\underline{Z},L) = 0, \tag{7}$$

where $\underline{Z}$ is the endogenous default barrier. Here (6) and (7) are the standard value-matching and smooth pasting conditions, respectively. Define

$$\eta \equiv \frac{(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2} > 0 \tag{8}$$

$$\chi \equiv \left(\frac{(r-\mu)\eta}{\eta+1}\right)^{\eta} > 0 \tag{9}$$

$$s(\ell) \equiv \frac{\chi}{\eta+1}\ell^{\eta}, \tag{10}$$

where $s(\ell)$ captures the value of equity holders' option to default per unit of liabilities (see the discussion below). We now characterize the solution to equity holders' default problem (5).

Proposition 1 (Post-Investment Equity and Default)**.** *Suppose that the current state of the firms is (Z,L). Then the equity value of the firm is given $V(Z,L) = v(\ell)Z$, where $\ell \equiv L/Z$ is firm's leverage and*

$$v(\ell) = \frac{1}{r-\mu} - \ell(1 - s(\ell)) \tag{11}$$

The endogenous default threshold $\underline{Z}$ satisfies

$$\frac{\underline{Z}(L)}{r-\mu} = \frac{\eta}{1+\eta}L \tag{12}$$

and the liquidation value per unit of liabilities is given by,

$$\frac{V((1-\theta)\underline{Z}(L),0)}{L} = \frac{(1-\theta)\eta}{1+\eta} \tag{13}$$

Proof. See Appendix A.1. □

The above proposition characterizes the value of equity and equity holders optimal default decision. Note that if equity holders were not allowed to default then the value of equity would be given by $\left(\frac{1}{r-\mu} - \ell\right) Z$ as equity holders would have to repay all their liabilities. Thus, $s(\ell)L$ captures the value of equity holders' option to default.

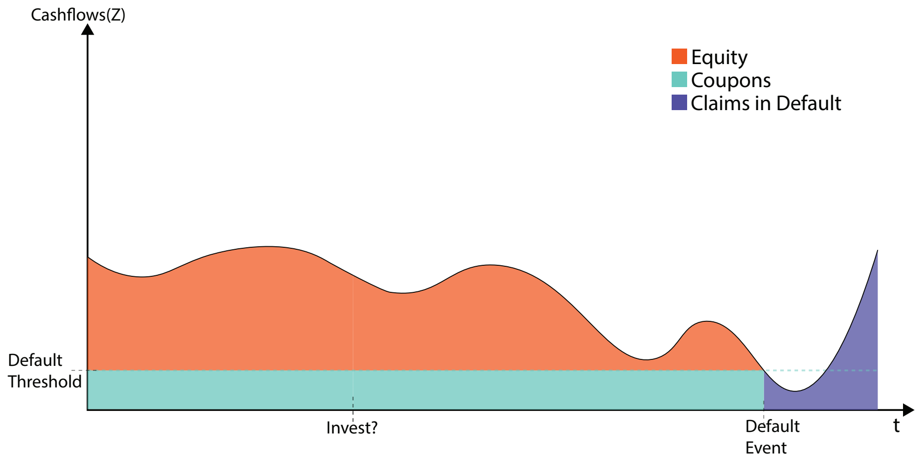

Figure 1: **The effect of limited liability on debt and equity cash flows without investment.** This figure considers the simplified case with no bankruptcy costs ($\theta = 0$).

Before moving on to how debt is priced, it is useful to visualize how firm cash flows are divided between equity and debt holders. Figure 1 depicts a possible path of Z and shows how cash flows are divided among all claimants (for simplicity we set $\theta = 0$). Default occurs at the threshold $\underline{Z}(L)$ optimally chosen by the firm, following Proposition 1. Because we are considering the corner case without bankruptcy costs ($\theta = 0$), firm cash flows are not impacted by the default event—instead they simply change claimants. Prior to default, coupons are paid and residual funds are distributed to equity holders. After default, all cash flows are owned by the default claimants. The price of any financial claim is simply the expected present discounted value of cash flows allocated to this claimant, with the expectation taken over all possible realizations of the $Z(t)$ process.

2.3 Pricing Debt Instruments

In this section, we derive how debt is priced, which then determines the equilibrium budget constraint in (3). Debt is priced by outside creditors who are risk-neutral and who anticipate equity holders' optimal default decision (as characterized in Proposition 1). Let T denote the stopping time when cash flows first reach default threshold, $\underline{Z}$, at which point equity holders choose to default.[9]

For tractability, we assume that all debt takes the form of defaultable consols following Leland (1994, 1998). A defaultable consol pays 1 in perpetuity prior to default and receives a share of the bankruptcy value of the firm in default. Conditional on the current state of the firm (Z, L) and equity holders' optimal default decision, the market price of a such bond $P(Z, L)$ equals

$$P(Z,L) \equiv \frac{p(Z,L)}{r} = \underbrace{\mathbb{E}_T\left[\int_0^T e^{-r\tau}\mathrm{d}\tau\right]}_{\substack{\text{PDV of promised coupons} \\ = P^C(Z,L)}} + \underbrace{\mathbb{E}_T\left[e^{-rT}\frac{V((1-\theta)\underline{Z}(L),0)}{rL}\right]}_{\substack{\text{PDV of claims in bankruptcy} \\ = P^B(Z,L)}}, \quad (14)$$

where $p(Z, L)$ denotes the price relative to the risk-free rate.

Equation (14) emphasizes that a defaultable consol consists of pre-bankruptcy component (i.e., the coupon payments prior to default) with market price $P^C(Z, L)$, and a component consisting of claims in bankruptcy (i.e., all debt is equal priority or pari passu) with market price $P^B(Z, L)$. We use $p^C(\cdot,\cdot)$ and $p^B(\cdot,\cdot)$ to denote these prices relative to the risk-free rate, r. In Online Appendix A.5 we use this decomposition to analyze how our results change if new debt comes with no claims in bankruptcy similarly to preferred equity.[10]

Proposition 2 establishes that leverage ($\ell \equiv L/Z$) is the relevant state for pricing debt, solves for the prices of the defaultable consol and of its pre-bankruptcy and

[9] We assume that $Z_0 > \underline{Z}$.

[10] In our baseline, all debt is equal priority (i.e., pari passu) for simplicity. This is in line with Billett et al. (2007) who report that in practice 56.2% of all bond issues are senior unsecured, which generally receive pari passu claims to the firm's remaining value in bankruptcy. While covenants might be expected to mitigate some of the inefficiencies in our model, the relevant covenants are rare or hard to implement in practice. Only 1.4% of bond issues have covenants restricting the future issue of senior debt. While negative pledge covenants ruling out future issuance of secured debt are common, they generally do not prohibit issuance of new debt that is secured and has equal priority to existing debt. Moreover, such covenants are hard to enforce for contractual and legal reasons (Billett et al. (2007), Donaldson et al. (2019)). Within our model, one reason why overly strict covenants against future pari passu debt issuance may be uncommon in practice is because such covenants would unambiguously lead to underinvestment.

bankruptcy components, and characterizes equity holders' budget constraint, (3).

Proposition 2 (Pricing Debt Instruments)**.** *The relevant state for pricing debt is leverage $\ell \equiv L/Z$. Moreover,*

$$p(\ell) = 1 - (1 + \theta\eta)s(\ell) \tag{15}$$

$$p^C(\ell) = 1 - (1 + \eta)s(\ell) \tag{16}$$

$$p^B(\ell) = (1 - \theta)\eta s(\ell) \tag{17}$$

Given these debt instruments, the budget constraint (3)*, normalized by Z, is given by*

$$p(\hat{\ell})\left((1 + g)\hat{\ell} - \ell\right) = \psi q(g) + m \tag{18}$$

$$p(\hat{\ell}) \geq p^B(\hat{\ell}) \tag{19}$$

where $\hat{\ell} \equiv \hat{L}/\hat{Z}$ is post-investment leverage.

Proof. See Appendix A.2. □

The budget constraint (18) is standard. The left hand side equals the total value of new debt issued, normalized by Z. The right hand side represents equity holders' need to raise new debt financing, and equals the debt financed portion of investment costs plus equity payouts. Equation (19) ensures the feasibility of the default claim embedded in the defaultable consol and is satisfied as long as $Z_0 > \underline{Z}$, that is, the firm is not initially in default.

In the case without bankruptcy costs (i.e., $\theta = 0$) the price of the defaultable consol relative to the risk-free rate simplifies to $p(\ell) = 1 - s(\ell)$. In this particularly simple case, $s(\ell)$ can be interpreted as the spread relative to the risk-free rate. Recall that $s(\ell)$ also appears in the expression for $v(\ell)$ (the normalized value of equity) with the opposite sign, where it captures the value of equity holders' option to default. Thus, we see that a more valuable default option increases the value of equity at the expense of bond holders.

2.4 Equity Holders' Investment Problem and the First-Best

We now restate equity holders' investment problem, normalizing by current cash flows and substituting in the budget constraint associated with defaultable consols

(18). Equity holders of a firm with pre-investment leverage ℓ choose $(g, m, \psi, \hat{\ell})$ such that

$$v^*(\ell) = \max_{\substack{g,\hat{\ell}\geq 0 \\ \psi\in[0,1] \\ 0\leq m\leq\kappa}} \Big\{ \overbrace{(1+g)v(\hat{\ell})}^{\text{Post-Investment Equity}} - \overbrace{(1-\psi)q(g)}^{\text{Equity Financed}} + \overbrace{m}^{\text{Payouts}} \Big\} \tag{20}$$

$$\text{s.t.} \quad \underbrace{p(\hat{\ell})}_{\text{Bond Price}} \underbrace{((1+g)\hat{\ell} - \ell)}_{\text{New Bonds}} = \underbrace{\psi q(g)}_{\text{Debt Financed}} + \underbrace{m}_{\text{Payouts}} \tag{21}$$

$$p(\hat{\ell}) \geq p^B(\hat{\ell}) \tag{22}$$

The first-best investment from Definition 1 solves

$$g^u \equiv \arg\max_g \Big\{ \overbrace{(1+g)v(0)}^{\text{Post-Investment Equity}} - \overbrace{q(g)}^{\text{Equity Financed}} \Big\} \tag{23}$$

The equity holders' objective function (20) is a normalization of (2). Similarly, the first-best investment in (23) is just the normalization of (4). The constraint (22) enforces the feasibility of the default payoff embedded in defaultable consols, meaning that equity holders cannot issue so much debt that the firm is strictly above its default threshold immediately after investment. While the constraint (22) never binds when $\kappa = 0$, it may bind if direct payments to equity holders are allowed ($\kappa > 0$).

3 Analysis of Investment Decision

In this section, we analyze equity holders' investment and financing decisions in our baseline model with a single investment opportunity. We first characterize the equity holders' problem relative to the first-best. This comparison allows us to identify the sources of inefficiencies in equity holders' investment decisions.

3.1 Sources of Investment Distortions

We define the function $H(\hat{\ell}) \equiv \theta\eta s(\hat{\ell})$ as the deadweight cost of default per unit of leverage, which is non-zero if a share of the firm is dissipated in default (i.e., $\theta > 0$). We then characterize the investment problem as follows.

Proposition 3. *Equity holders' investment problem can be written as*

$$v^*(\ell) = \max_{\substack{g,\hat{\ell}\geq 0 \\ \psi\in[0,1] \\ 0\leq m\leq \kappa}} \left\{\frac{1+g}{r-\mu} - q(g) - p(\hat{\ell})\ell - (1+g)H(\hat{\ell})\hat{\ell}\right\} \tag{24}$$

$$s.t.\, p(\hat{\ell})((1+g)\hat{\ell} - \ell) = \psi q(g) + m \tag{25}$$

$$p(\hat{\ell}) \geq p^B(\hat{\ell}) \tag{26}$$

The first-best investment, g^u, is the unique solution to

$$0 = \frac{1}{r-\mu} - q'(g^u) \tag{27}$$

If the investment cost is quadratic and equal to $q(g) = \zeta g^2/2$ then the optimal investment is given by $g^u = \frac{1}{\zeta(r-\mu)}$.

Proof. See Appendix A.3 for details. □

The reformulated objective function (24) shows that the post-investment value of equity equals the expected PDV of cash flows generated by the firm net of (i) the cost of investment ($q(g)$), (ii) the PDV of cash flows promised to the existing debt holders ($p(\hat{\ell})\ell$), and (iii) cash flows lost in default ($(1+g)H(\hat{\ell})\hat{\ell}$), all normalized by Z.[11] Because new debt is fairly priced equity holders bear the full cost of the investment and any change in the expected deadweight cost of default. For the same reason equity payouts m do not appear directly in (24). However, ψ and m affect the post-investment value of equity indirectly through $\hat{\ell}$.

The above characterization of the equity holders' problem emphasizes the sources of inefficient investment. Compared to (23), we see that equity holders face two distortions. The first distortion is due to existing debt, as captured by $p(\hat{\ell})\ell$. Since $p(\hat{\ell})$ is a decreasing function of post-investment leverage, $\hat{\ell}$, equity holders have an incentive to increase leverage. This is the classic conflict between equity and debt holders pointed out by Myers (1977), who however in contrast to us assumes that investment is entirely financed with equity. The second distortion is due to bankruptcy costs and is captured by $(1+g)H(\hat{\ell})\hat{\ell}$. Since $H(\cdot)$ is an increasing function, the presence of bankruptcy costs discourages equity holders from taking on additional leverage.

[11] The reformulated objective function can be obtained by substituting the budget constraint into the expression for the post-investment value of equity (the RHS of (20)).

3.2 Investment Relative to First-Best

We focus first on the case without bankruptcy costs (i.e., $\theta = 0$), so the last term in (24) vanishes, and preexisting leverage is the only source of investment distortions.[12] If in addition preexisting debt is zero (i.e., $\ell = 0$), it follows from the reformulated investment problem in Proposition 3 that equity holders' optimal investment satisfies

$$\frac{1}{r-\mu} - q'(g) = 0, \tag{28}$$

that is investment equals the first-best ($g = g^u$). Equity holders are also indifferent between equity and debt financing and whether to make equity payouts (i.e., they are indifferent over any feasible choices of ψ and m). In other words, when $\ell = 0$ the Modigliani-Miller theorem holds, and the firm value is independent of financing.

Investment deviates from this simple benchmark when the firm has preexisting debt ($\ell > 0$). Without bankruptcy costs, it is immediate from (24) that equity holders' optimal investment satisfies the following first-order condition (FOC)

$$\frac{1}{r-\mu} - q'(g) - p'(\hat{\ell})\frac{\partial \hat{\ell}}{\partial g}\ell = 0 \tag{29}$$

Equation (29) is the key equation of our model. Compared to (28), which determines equity holders' investment choice when $\ell = 0$, the FOC now includes the additional term $-p'(\hat{\ell})\frac{\partial \hat{\ell}}{\partial g}\ell$. This new term captures the marginal change in the value of existing debt due to the change in the firm's distance to default. When this term is positive at $g = g^u$ equity holders have an incentive to invest beyond the level that would maximize the total value of the firm, while the opposite is true when the term is negative. Since the value of existing debt is decreasing in leverage (i.e., $p'(\hat{\ell}) < 0$) it follows that the sign of this distortion depends on the sign of $\frac{\partial \hat{\ell}}{\partial g}$. If optimal investment is associated with an increase in leverage, that is if $\frac{\partial \hat{\ell}}{\partial g} > 0$, equity holders *overinvest* relative to first-best. If optimal investment is associated with a decrease in leverage, that is if $\frac{\partial \hat{\ell}}{\partial g} < 0$, equity holders *underinvest*.

[12] We investigate how the presence of bankruptcy costs affects investment in Section 3.5.

3.3 Dilution Mechanism and Inefficient Investment

We now present our first main result that preexisting debt encourages overinvestment. We first characterize equity holders' choices without equity payouts financed by debt (i.e., $\kappa = 0$).

Proposition 4. *Given $\kappa = 0$, denote g^* as equity holders' optimal investment*

1. *If constrained to use equity financing, equity holders underinvest ($g^* < g^u$)*
2. *If allowed to choose financing optimally, equity holders: (a) finance all their investment with debt; (b) overinvest ($g^* > g^u$)*

Proof. See Online Appendix A.2 □

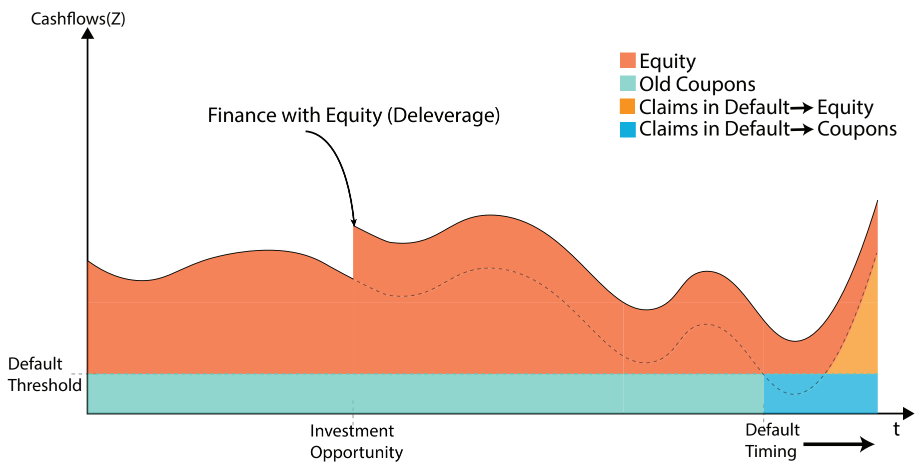

Figure 2: **Equity-financed investment, due to deleveraging, decreases the option value of default.** This figure shows debt and equity cash flows with an equity-financed investment opportunity. We show the simplified case with no bankruptcy costs ($\theta = 0$).

The first part of Proposition 4 nests the classic underinvestment result of the debt overhang literature (Myers (1977)). Thus, our model makes precise that equity financing is a condition required for this classic result. Figure 2 visualizes the intuition. Investment financed with equity leads to deleveraging, leading equity holders to pay coupons for longer. As a result, a portion of the cash flows from the new investment is allocated to existing debt holders in form of coupon payments (the portion of the "Claims in Default → Coupons" area above the dotted line). The benefit from new investment is hence partly captured by existing debt holders, implying that equity holders' benefit of investing is less than the social benefit. In terms of the key equation (29), deleveraging implies that $-p'(\hat{\ell})\frac{\partial\hat{\ell}}{\partial g}\ell < 0$, and hence a reduction of equity holder's incentive to invest. This classic argument has been used to explain the historically low investment in the aftermath of the Great Recession in Europe (see, for example, Kalemli-Ozcan et al. (2018)).[13]

By contrast, the second part of Proposition 4 shows that equity holders may want to overinvest if they can choose their financing method freely, provided that the firm has preexisting debt (i.e., $\ell > 0$). Note that the volatility of firm cash flows is assumed to remain the same before and after investment, and thus the mechanism here is distinct from the potential incentive to invest in risky projects with negative present value, so-called risk-shifting (Jensen and Meckling (1976)).

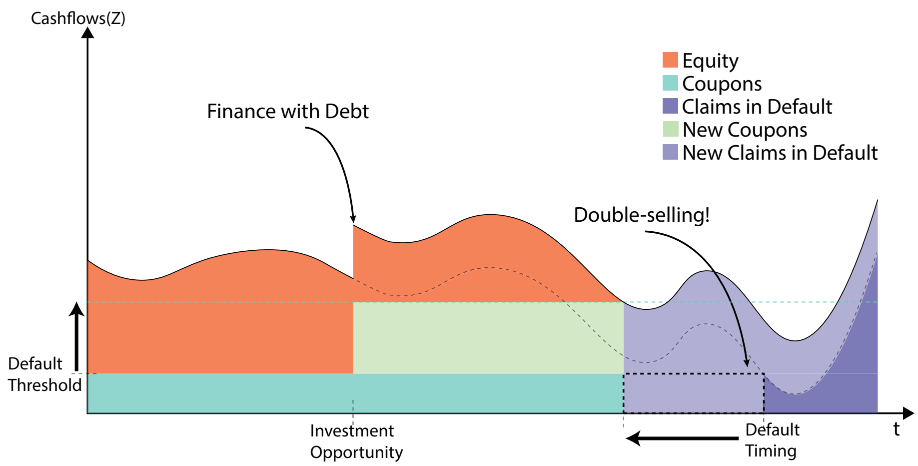

Figure 3: **Debt-financed investment, due to increased leverage, dilutes existing debt holders by double-selling some of their promised coupon payments.** This figure shows debt and equity cash flows with a debt financed investment opportunity. We show the simplified case with no bankruptcy costs ($\theta = 0$).

Why do equity holders overinvest? Figure 3 illustrates that a sufficiently large debt financed investment leads to higher leverage and earlier default, transforming a portion of the coupon payments that would have been captured by existing debt holders (the rectangular area with dashed edges) to claims in default, which have to be shared with new debt holders. Thus, by issuing new debt and increasing leverage,

[13] Jungherr and Schott (2021) show how high leverage may lead to slow recoveries from recessions.

equity holders can sell again claims to some of the cash flows that were previously promised to existing debt holders. The marginal benefit to equity holders of investing hence exceeds the social benefit, and equity holders overinvest relative to the first-best. In terms of (29), this additional benefit of financing investment with debt is captured by $-p'(\hat{\ell})\frac{\partial \hat{\ell}}{\partial g}\ell > 0$.[14]

We can equivalently think of the overinvestment incentive in terms of equity holders' option to default. When equity holders issue new debt they sell claims to their cash flows and purchase an option to default, as the default threshold increases. However, old debt holders are not compensated for this increase in equity holders' option value to default. Thus, the third term on the left-hand side of (29) can also be interpreted as the marginal change in equity holders' option to default that is not priced by the market at the time of investment. Thus, if equity payouts are restricted, equity holders have an incentive to issue more debt and use these additional funds to invest more than the first-best investment.

3.4 Equity Payouts

Having seen that the availability of debt financing can lead to overinvestment in the presence of preexisting debt, we now turn to analyzing how debt financed payouts to equity holders, such as dividends and equity buybacks, affect real investment (i.e., $\kappa > 0$). We define $\bar{m}(\psi, g)$ as the highest equity payout that satisfies the budget constraint (21) given investment, g, and financing choice, ψ. Thus, given g, ψ, equity holders' choice of m has to satisfy $m < \min\{\kappa, \bar{m}(\psi, g)\}$. As shown in Online Appendix A.3 and (A.7), $\bar{m}(\psi, g) = \frac{1+g}{r-\mu} - \frac{\eta}{1+\eta}\ell - \psi q(g)$.

[14]It is the coupon payments promised to existing debt holders that are double-sold by equity holders, not the existing debt holders' bankruptcy claims. To be precise, *before* the investment takes place, the value of existing debt holders' bankruptcy claims at the time of default is $\frac{\eta}{1+\eta}L$, the value of the firm in default (see (13)). *After* the investment takes place the value of existing debt holders' bankruptcy claims at the time of default is given by $V((1-\theta)\underline{Z}(\hat{L}), 0) \times (L/\hat{L})$ since the post-investment firm's value in default ($\frac{\eta}{1+\eta}\hat{L}$) is divided proportionally between new and old debt holders. It follows that the value of existing bankruptcy claims at the time of default is unchanged. Moreover, since after investment default happens on average earlier, the PDV of existing bankruptcy claims actually goes up.

Proposition 5. *Denote g^*, m^*, ψ^* as the equity holders' optimal choices of investment, payouts, and financing, respectively. Then there exists $\underline{\kappa} \in \mathbb{R}_+$ such that*

1. *If $\kappa < \underline{\kappa}$ then equity holders: (a) overinvest ($g^* > g^u$); (b) finance investment and equity payouts with debt ($\psi^* = 1$); (c) make payouts to themselves up to the constraint ($m^* = \kappa$); (d) continue operating firm*

2. *If $\kappa \geq \underline{\kappa}$ then equity holders: (a) invest the first-best amount($g^* = g^u$); (b) finance investment and equity payouts at least partially with debt ($\psi^* \in [\max\{\underline{\psi}_\kappa, 0\}, 1]$, where $\underline{\psi}_\kappa > 0$ is the unique solution to $\kappa = \bar{m}(g^*, \underline{\psi}_\kappa)$); (c) make payouts to themselves ($m^* = \bar{m}(g^*, \psi^*)$); (d) are indifferent between defaulting and continuing to operate the firm*

The threshold $\underline{\kappa}$ is decreasing in ℓ and r, and increasing in σ.

Proof. See Online Appendix A.3. □

Proposition 5 extends our overinvestment result to the case in which equity payouts financed with debt are permitted at the time of investment. As long as equity holders face sufficiently tight restrictions on equity payouts financed by debt ($\kappa < \underline{\kappa}$), we find that they continue to overinvest. Different from the case with $\kappa = 0$, equity holders accompany investment with direct equity payouts, further increasing post-investment leverage.

By contrast, when the constraint on direct equity payouts is lax ($\kappa \geq \underline{\kappa}$), equity holders invest the first-best amount. In this case, equity holders have a more efficient way of double-selling existing debt holders' claims without the need to resort to inefficient investment, decoupling equity holders' problem into two separate problems: (1) an investment problem and (2) a dilution of existing debt holders problem. Equity holders choose g to maximize the net present value of the firm and m to maximize the transfer from existing debt holders to themselves. The latter implies choosing the highest feasible m so that the firm defaults right after investment. Thus, when $\kappa \geq \underline{\kappa}$, equity holders essentially sell the firm to the new debt holders.[15]

Proposition 5 shows that restrictions on equity payouts can increase investment and reduce the probability of bankruptcy, in line with the intuition in Myers (1977).

[15] This suggests that in our setup, equity holders have an incentive to "collude" with new creditors in order to dilute the existing debt holders. Similar mechanism has been emphasized recently by Aguiar et al. (2019) within the context of sovereign default.

Different from Myers (1977), however, we find that such restrictions might not be desirable. We reach different conclusion since in our model investment is debt financed and tends to be inefficiently high, reminiscent of highly levered zombie firms.

The conclusion that equity payout restrictions lead to inefficiently high investment needs to be tempered by the observation that this one-shot investment problem does not account for potential future debt dilutions. We will see in Section 4 that dynamic considerations lead to more nuanced conclusions. However, the overinvestment incentive highlighted in the one-shot model tends to remain particularly for firms with relatively high leverage.

One may wonder if covenants could be used to eliminate the inefficiencies characterized in Propositions 4 and 5. However, as has been first pointed by Myers (1977), it is difficult to write and enforce debt contracts that require equity holders to invest first-best amount. For example, covenants that prohibit equity holders from issuing senior or equal priority debt would eliminate over-investment but would lead to severe underinvestment because of a large drop in the p^B for new debt—though this issue does not occur with debt directly secured by a specific asset of the firm. The same applies to covenants that limit firms' leverage or negative pledge covenants. In addition, as we discussed in Section 2.3, in practice few covenants prohibit firms from issuing new debt with equal priority to the existing debt.

3.5 Bankruptcy Costs

In this section, we analyze how the results in the baseline model are affected by the presence of bankruptcy costs. We find that realistic bankruptcy costs moderate but do not eliminate incentives for overinvestment. Throughout the bankruptcy cost analysis, we assume that there are no equity payouts (i.e., $\kappa = 0$) for simplicity. In the presence of default costs (i.e., $\theta \in (0, 1]$), debt financing is associated with the following trade-off. On the one hand, as before, issuing new debt allows equity holders to resell some of existing debt holders' claims, which we have seen encourages overinvestment. On the other hand, issuing new debt increases the deadweight cost of default and hence the cost of debt financing, encouraging underinvestment.[16]

[16] When $\theta > 0$ the Modigliani–Miller theorem no longer holds even in the absence of preexisting debt, as in that case equity holders would invest first-best amount but finance it with equity.

Proposition 6. *Suppose that $\theta > 0$. If equity holders are*

1. *constrained to use equity financing, they underinvest ($g^* < g^u$)*
2. *allowed to choose financing optimally then for each ℓ there exists $\underline{\theta}(\ell) > 0$ such that for all $\theta \in [0, \underline{\theta}(\ell)]$ they overinvest ($g^* > g^u$)*

The first part of Proposition 6 shows that the result that equity holders underinvest with equity financing extends to $\theta > 0$. The second part shows that equity holders continue to overinvest with optimal financing even in the presence of bankruptcy costs, as long as those costs are not too large. This is because for $\theta < \underline{\theta}(\ell)$ the marginal benefit associated with issuing additional debt dominates the marginal increase in deadweight cost.

Figure 4 depicts the optimal investment (relative to the first-best investment) for empirically relevant values of θ.[17] We see that these implications are robust to empirically plausible bankruptcy costs. Overinvestment initially increases with leverage, as equity holders can dilute more claims. However, as leverage increases further existing claims primarily represent claims to the bankruptcy value of the firm, limiting equity holders' ability to bring bankruptcy forward through new debt financed investment.

[17]These costs are typically estimated to be between 2% and 20% (see, e.g., Bris et al. (2006)).

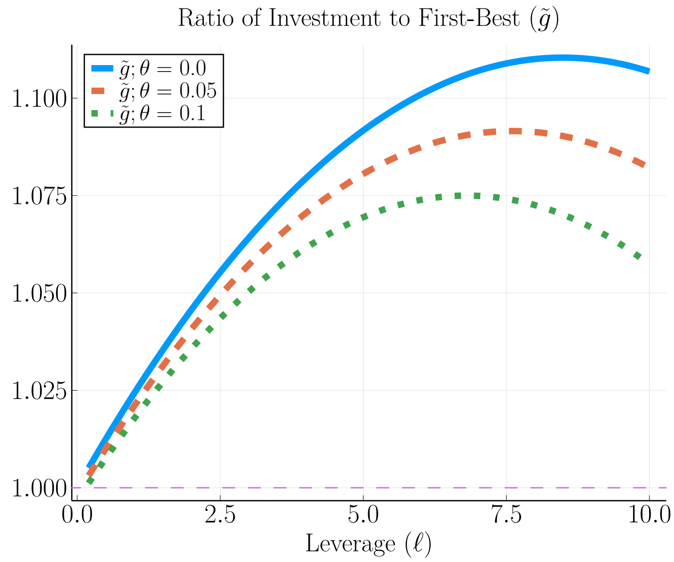

Figure 4: **Investment relative to first-best ($\tilde{g}$) against preexisting leverage (ℓ) for the baseline model with a single investment opportunity.** Each line corresponds to a different level of deadweight bankruptcy costs (θ). The case $\theta = 0$ corresponds to no bankruptcy costs. Model parameters are discussed in Online Appendix B.3 for more details.

4 Repeated Investments

So far, we have analyzed a baseline model with a single investment opportunity. In that setting, we have shown that equity holders have an incentive to "conspire" with new debt holders to dilute preexisting debt holders, thereby leading to debt financed overinvestment. In this section, we build on this model and allow for repeated investment opportunities. We show that repeated investment opportunities have non-trivial consequences as buyers of new debt price in the likelihood of future dilution, thereby increasing the cost of debt financing. We find that dynamic considerations can lead to underinvestment especially among low leverage firms with frequent investment opportunities, and that direct equity payouts (i.e., equity buybacks and dividends) financed out of debt further exacerbate underinvestment among these firms.

4.1 Model with Repeated Investment

We consider the same setup as described in Section 2, but assume that investment opportunities arrive at a constant Poisson rate λ. As above, the state of the firm at any given point in time is $\{Z, L\}$. Upon arrival of an investment opportunity, equity holders have the choice to increase current cash flows from Z to $Z(1+g)$ at cost $Zq(g)$. It follows that cash flows follow a jump diffusion

$$\mathrm{d}Z(t) = \mu Z(t)\mathrm{d}t + \sigma Z(t)\mathrm{d}\mathbb{W}(t) + g(Z(t^-), L(t^-))Z(t^-)d\mathbb{N}(t), \quad Z(0) > 0, \tag{30}$$

where $\mathbb{N}(t)$ is a Poisson process with intensity $\lambda \geq 0$ and $g(Z(t^-), L(t^-))$ is equity holders' investment at time t conditional on the state of the firm $\{Z, L\}$ and the arrival of an investment opportunity. Note that when $\lambda = 0$ we are back to the baseline model with a single investment opportunity.

Investment can be financed by issuing defaultable consols via competitive debt markets (as described in Section 2.3) or equity. This implies that, in contrast to the model of Section 2, liabilities are no longer constant. Rather, $L(t)$ is now a pure jump process, $\mathrm{d}L(t) = (\hat{L}(t) - L(t^-))\mathrm{d}\mathbb{N}(t)$, where $\hat{L}(t)$—as defined in Section 2.1—denotes the value of liabilities immediately after investment implied by equity holders' investment and financing decisions.

4.2 Optimal Investment Problem

Conditional on the arrival of an investment opportunity, the firm solves the natural analogue of the one-shot problem, except that both the $v(\cdot)$ and $p(\cdot)$ functions account for the possible arrival of future investment opportunities. The following proposition describes the equity and debt holders' problems with repeated investment

Proposition 7 (Repeated Investment)**.** *A solution consists of a value of equity $v(\ell)$, price $p(\ell)$, policies $\{g(\ell), m(\ell), \psi(\ell), \hat{\ell}(\ell)\}$, and default threshold, $\bar{\ell}$, such that*

1. *Given $v(\ell)$ and $p(\ell)$:(a) the policies $\{g(\ell), m(\ell), \psi(\ell), \hat{\ell}(\ell)\}$ solve the firm's problem in* (20)*; (b) $v(\ell)$ satisfies the differential variational equation (DVI)*

$$0 = \min\{(r-\mu)v(\ell) + \mu\ell v'(\ell) - \frac{\sigma^2}{2}\ell^2 v''(\ell) - \lambda\left(v(\hat{\ell}(\ell)) - v(\ell)\right) - (1 - r\ell), v(\ell)\} \tag{31}$$

2. *The default threshold $\bar{\ell}$ is optimal and is determined by the indifference in* (31)

3. Given $v(\ell)$ and the equity holders' policies, the price $p(\ell)$ solves

$$rp(\ell) = r + (\sigma^2 - \mu)\ell p'(\ell) + \frac{\sigma^2}{2}\ell^2 p''(\ell) + \lambda\left(p(\hat{\ell}(\ell)) - p(\ell)\right) \tag{32}$$

$$p(\bar{\ell}) = \frac{(1-\theta)v(0)}{\bar{\ell}} \tag{33}$$

Furthermore, the first-best investment choice as defined in Definition 1 is

$$g^u = \frac{1}{\zeta(r-\mu)\left(\frac{1}{2}\left(\sqrt{1 - \frac{2\lambda}{\zeta(r-\mu)^2}} - 1\right) + 1\right)} \tag{34}$$

Proof. See Appendix B. □

Unlike in the one-shot case, we do not have closed-form solutions when $\lambda > 0$. Thus, we need to solve the model numerically using upwind finite difference methods. To do so, we add artificial reflecting barriers to the stochastic process, $v'(\ell_{\min}) = 0, v'(\ell_{\max}) = 0$, and $p'(\ell_{\min}) = 0$. The absorbing boundary condition for $p(\cdot)$ comes from (33) (i.e. the liquidation value of the firm at the time of default).

4.3 Analysis

Figure 5 plots investment relative to first-best ($\tilde{g} \equiv g/g^u$) against firm leverage when $\lambda = 0.2$ (left panel) and $\lambda = 0.3$ (right panel), for different values of κ. It shows that the repeated arrival of investment opportunities generates heterogeneous investment distortions, with low leverage firms tending to underinvest and high leverage firms tending to overinvest.[18] This heterogeneous effect of limited liability on equity holders' investment decisions is our key finding.

Comparing the left panel with the right panel, for a given κ, we see that more frequent investment opportunities (i.e., higher λ) discourage overinvestment. This is because new debt investors expect their claims to be diluted sooner and require to be compensated for that, which increases the cost of debt finance to equity holders. Debt investors' anticipation of future dilution is more severe for low leverage firms,

[18]To discipline parameters, we calibrate to moments from the firm dynamics (Pugsley et al. (2020)) and investment spikes (Gourio and Kashyap (2007)) literature. The baseline parameters are $r = 0.0765, \sigma = 0.1534, \mu = -0.0514$, and $\zeta = 50.036$. See Online Appendix B.3 for more details.

so these firms' equity holders face the strongest increase in the cost of debt finance, and the strongest tendency to underinvest.

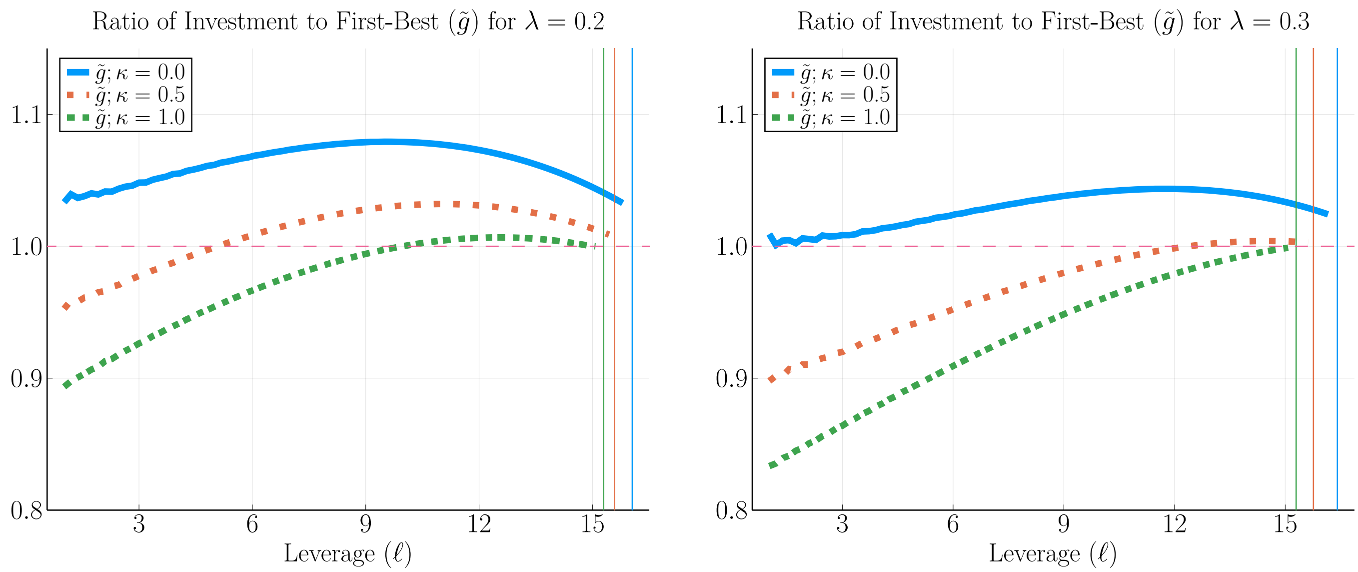

Figure 5: **The effect of equity payout restrictions on the investment-leverage relationship.** This figure shows investment relative to first-best ($\tilde{g}$) against preexisting leverage (ℓ). Each line corresponds to a different value for the constraint on equity payouts κ. The case $\kappa = 0$ is the baseline case of no equity payouts from debt. The two panels show this for an arrival rate of new investment opportunities $\lambda = 0.2$ (left) and $\lambda = 0.3$ (right). Bankruptcy costs are set to zero ($\theta = 0$). Vertical lines indicate the default thresholds for each value of κ. For a discussion of model parameter values see Appendix B.3.

Figure 5 also shows that allowing equity holders to make direct payouts to themselves (i.e., dividends and equity buybacks) strengthens low leverage firms' tendency to underinvest in the repeated model. In contrast to the model with one-shot investment, allowing equity payouts financed with debt (i.e., $\kappa > 0$) need not improve the efficiency of firm investment and may even induce equity holders to switch from overinvestment to underinvestment.[19] Comparing the left and right panels of Figure 5 shows that the ability to make direct payouts to equity holders ($\kappa > 0$) depresses

[19] Grullon and Michaely (2002) show that payouts to equity holders (dividends plus equity repurchases) are around 50% of earnings. Because not all payouts in practice are financed by new debt issuance we consider this to be an upper bound on average direct equity payouts (given by $\lambda\kappa$ in our model), leading us to consider values for κ between 0 and 1.

investment more when investment opportunities are frequent (i.e., λ is high). Intuitively, debt holders' concerns about the future dilution of their claims are exacerbated when equity holders can make direct payouts to themselves financed with debt.

For the highest levels of leverage, Figure 5 shows that investment converges to the first-best as κ increases, similarly to the one-shot model. This is because for high enough ℓ and κ equity holders are able to issue so much debt that they optimally choose to default immediately after investment, which implies that debt holders immediately take over the firm and equity holders have no opportunities to dilute new debt holders' claims. Thus, as long as there are no deadweight bankruptcy costs, equity holders' and new debt holders' incentives are again aligned, just as in the model with one-shot investment.

4.4 Dynamic Leverage, Investment, and Bankruptcy

Figure 6 compares the paths of leverage, real investment, and cumulative bankruptcy rates across equity payout restrictions. This figure shows simulated paths (averaged over 1000 paths) for two firms, one with high leverage and a low interest coverage ratio – the zombie firm – and a second one with lower leverage. The left panels assume that equity payouts are prohibited ($\kappa = 0$), while the right panels permit equity buybacks ($\kappa = 0.5$). The top panels show simulated leverage and the bottom panels simulated real investment.

Most strikingly, comparing the two bottom panels shows that permitting equity payouts shifts real investment downward, and widens the investment gap between high and low leverage firms. We see that permitting equity payouts is efficient for the high leverage firm. However, permitting direct equity payouts from new debt leads the low leverage firm to switch from overinvesting to actually underinvesting. The investment gap between firms with different initial leverage narrows on average as time passes, as low leverage firms have an incentive to lever up but high leverage firms' leverage is capped above by default.

The top panels show that permitting equity payouts leads to increases in leverage for both types of firms, as firms optimally take on more debt in order to finance direct payouts to equity holders. In addition, the default threshold is lower in the top right panel than the top left panel. The default threshold is lower when equity payouts are permitted due to dynamic considerations, as buyers of new debt price in

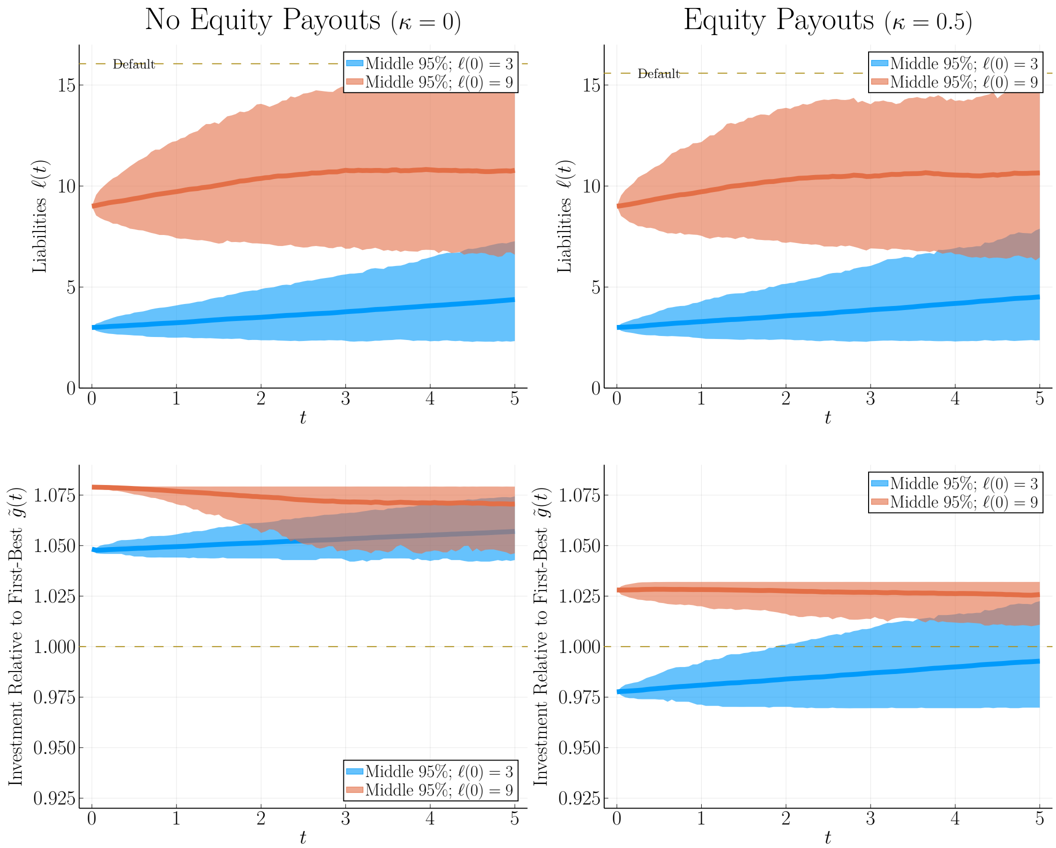

Figure 6: **Simulation of an ensemble of** 1000 **paths of leverage** ℓ **(top), investment relative to first-best** $\tilde{g}$ **(bottom) with and without equity payouts.** Each panel shows the ensemble starting from $\ell(0) \in \{3, 9\}$ corresponding to interest coverage ratios of 4 and 1.5 respectively. The left panels use $\kappa = 0$ (no equity payouts) and the right panels use $\kappa = 0.5$ (constrained equity payouts from new debt). The investment arrival rate is set to $\lambda = 0.2$. The central line is the mean, the red shaded area shows the 2.5th and 97.5th percentiles. Moments are based on non-defaulted firms.

future anticipated dilution, and thereby reduce equity holders' incentive to keep the firm as a going concern at any level of cash flows. While the changes in leverage and default thresholds from permitting equity payouts appear visually small, bankruptcy is a tail event and therefore affected by these changes. Within these simulations, the cumulative five-year bankruptcy rate for firms with initially high leverage increases when equity payouts are permitted (from 28.5% for the case of $\kappa = 0$ to 32.0% when $\kappa = 0.5$). The increase in bankruptcy rates arises through a combination of the incentive to lever up and the lower optimal leverage at which equity holders walk away from the firm, as captured by the lower default threshold.

5 Empirical Evidence

We now turn to the empirical implications of the model. A central model implication is that high leverage firms tend to overinvest and finance with debt, whereas low leverage firms tend to underinvest. Intuitively, for highly levered firms the incentive to expropriate existing debt holders immediately dominates, while for low-leverage firms debt financed investment is unattractive because debt holders' concerns about future dilution are priced in.

5.1 Investment of High- and Low-Leverage Firms in the Data

Consistent with our model, Figure 7 shows that highly levered firms invested more, not less, than low-leverage firms in Compustat data since 2004. Contrary to what one would expect if highly-levered firms' investment was simply driven by lower investment opportunities, we see that these firms had lower profitability and lower investment opportunities, as measured by Tobin's Q. This cross-sectional finding is in line with long-term historical data that high corporate leverage need not anticipate lower economy-wide investment (Jordà et al. (2020)).

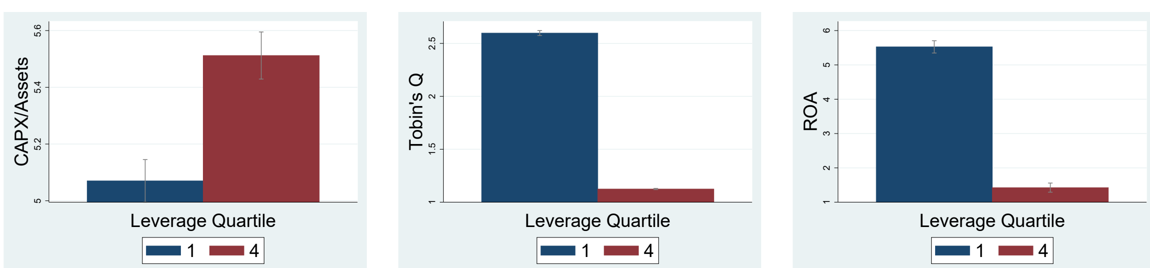

Figure 7: **Investment, Tobin's Q, and Profitability for High- vs. Low-Leverage Firms from Compustat 2004.Q1-2018.Q1.** This figure shows quarterly capital expenditures investment to lagged total assets (annualized %), Tobin's Q, and return on assets (in %) for firms in the top vs. bottom quartile of prior-quarter lagged leverage. For each bar, the figure reports a 95% confidence interval around the mean.

Our sample is based on publicly traded firms from Compustat over the period 2004.Q1-2018.Q1. It is well-known that equity repurchases are significantly less sticky than dividends, and have increased substantially over the past 20 years with a particularly large increase after 2003 (Farre-Mensa et al. (2020)).[20] Firms' ability to use equity purchases therefore plausibly corresponds to a significant increase in the equity payout parameter κ, which in the model steepens and sharpens the investment-leverage relationship. For this reason we start our sample in 2004, when our model predictions are most relevant. For a detailed data description see Online Appendix B.1.

We confirm the visual evidence in Figure 7 through panel regressions in Table 1. Column (1) starts with a simple quarterly panel regression of investment onto a dummy indicating whether the firm's previous-quarter leverage was in the top quartile. Column (2) shows that once we control for investment opportunities and profitability, the relationship between leverage and capital expenditures becomes even stronger. Column (3) controls for the firm's contemporaneous gross equity payouts to allow for the possibility that the ability to make direct equity payoffs (κ in the model) might vary across firms. Column (4) excludes the period of the financial crisis 2008-09 and its aftermath, starting the sample in 2012, and shows that the higher investment of high leverage firms not an artifact of the financial constraints during the crisis but instead a persistent pattern in the data.[21]

Columns (5) and (6) control for the possibility that the difference between high-leverage and low-leverage investment is driven by incentives to risk-shift or gamble for resurrection. Classic models of risk-shifting – where firms make risky bets to gamble for resurrection – would predict that investment increases with macroeconomic uncertainty. Column (5) controls for the price of volatile stocks (PVS) from Pflueger et al. (2020) to proxy for financial market risk perceptions and column (6) uses the news-based economic uncertainty index of Baker et al. (2016). Finally, column (7) shows that the relationship between high leverage and real investment is robust to quarter fixed-effects to flexibly control for time-varying factors, such as real interest rates or economic uncertainty.

[20] The conspicuous practice of equity buybacks, consistent with a high parameter κ, has gotten substantial media attention. See e.g. "U.S. Airlines Spent 96% of Free Cash Flow on Buybacks", Wall Street Journal, March 16, 2020.

[21] Our empirical evidence is therefore complementary to Kahle and Stulz (2013) and Xiao (2019), who previously documented puzzlingly high real investment of highly levered firms during the acute phase of the 2008-09 financial crisis.

Table 1: Capital Expenditures and Leverage Regressions

$$CAPX/Assets_{i,t} = \alpha + \beta HighLev_{i,t-1} + \gamma X_{i,t} + \varepsilon_{i,t}$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
HighLev$_{i,t-1}$	0.442*	1.173***	1.132***	1.173***	1.198***	1.175***	1.137***
	(1.93)	(4.06)	(3.89)	(3.63)	(4.02)	(4.06)	(3.92)
Tobin's $Q_{i,t-1}$		0.343***	0.373***	0.337***	0.354***	0.344***	0.335***
		(3.59)	(3.90)	(2.93)	(3.52)	(3.61)	(3.52)
ROA$_{i,t-1}$		0.0476***	0.0505***	0.0404***	0.0461***	0.0476***	0.0440***
		(5.25)	(5.40)	(3.90)	(4.72)	(5.25)	(4.67)
Gross equity payout$_{i,t}$			-0.0882**				
			(-2.03)				
PVS_{t-4}					0.409**		
					(2.23)		
$\Delta \log(EPU)_t$						-0.141	
						(-0.82)	
Constant	5.070***	4.019***	4.053***	4.036***	4.103***	4.018***	
	(30.99)	(13.71)	(13.64)	(11.38)	(13.05)	(13.71)	
N	52638	36023	36023	36023	36023	36023	36023
Time FE	No	No	No	No	No	No	Yes

Quarterly firm-level regression of capital expenditures onto top quartile leverage dummy, controls, and time fixed effects 2004.Q1-2018.Q1. The variable of interest is $HighLev_{i,t}$, an indicator variable taking the value one if $Lev_{i,t}$ is in the top quartile of all time t firms. The the time t sample consists of all firms with $Lev_{i,t-1}$ either in the top or bottom quartiles. PVS_{t-4} is the four-quarter lagged price of volatile stocks as a measure of financial market risk perceptions from Pflueger et al. (2020). $\Delta \log(EPU)_t$ is the change in the log news-based economic and policy uncertainty index from Baker et al. (2016). T-statistics based on double-clustered standard errors by firm and quarter in parentheses. Significance levels are indicated by * p<0.10, ** p<0.05, ***<0.01. For summary statistics and detailed variable definitions see Online Appendix B.

5.2 The Effect of Investment on Profitability and Debt Issuance

Having shown that investment rates across high- and low-leverage firms line up with our model predictions on average, we next show that investment, debt issuance, and profitability are consistent with our model predictions at the firm-level.

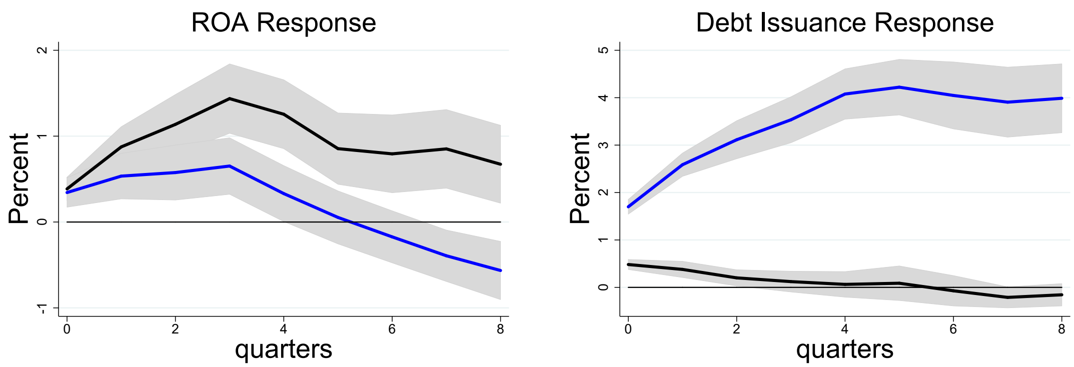

Figure 8: **Empirical impulse responses of firm profitability and debt issuance to investment for high (blue) and low (black) leverage firms.** This figure plots the impulse responses and associated 90% confidence interval bands of return on assets (ROA) and debt issuance to a one standard deviation increase in *CAPX/Assets* of firm i at time t. We compute Jordà (2005) local projections by running regressions of the following form: $y_{i,t+h} = \alpha_i + \alpha_t + \beta_0 CAPX/Assets_{i,t} \times HighLev_{i,t-1} + \beta_1 CAPX/Assets_{i,t} + \beta_2 HighLev_{i,t-1} + \gamma y_{i,t-1} + \varepsilon_{i,t}$. The variable $\text{HighLev}_{i,t}$ takes a value of one if $Lev_{i,t}$ is in the top quartile, and zero otherwise. The figure shows the coefficients $\beta_0 + \beta_1$ (high leverage) and β_1 (low leverage). Our sample consists of a quarterly panel 2004.Q1-2018.Q1 and excludes the middle 50% of firms by $t-1$ leverage. When forecasting ROA, y_{t+h} is the 4-quarter return on assets (in %) ending in quarter $t+h$. When forecasting debt issuance, y_{t+h} is the cumulative change in debt-to-assets from time t to $t+h$ (in %).

Section 5.2 shows impulse responses of firm profitability and debt issuance to a one-standard deviation increase in investment, estimated via local projections (Jordà (2005)). We separately show the responses for firms in the top quartile of pre-investment leverage (blue) and bottom quartile of pre-investment leverage (black), together with 90% confidence intervals. The impulse responses for the low leverage firms are as expected, if these firms face profitable investment opportunities and only a weak incentive to finance with debt. Following a one-standard deviation increase in the CAPX/Assets ratio, low leverage firms' return on assets increases by about

1.5 percentage point three quarters after the investment. This magnitude is not only statistically but also economically significant, compared to median return on assets of 5.99% in our sample. Debt issuance for low leverage firms shows a small increase but becomes statistically indistinguishable from zero within three quarters after the investment.

The impulse responses for high-leverage firms contrast with those of low leverage firms, consistent with the model prediction that high-leverage firms have an incentive to debt finance their investment and invest more than the first-best. Following a one-standard deviation increase in investment, high-leverage firms' profitability initially increases. However, by quarter eight after the investment profitability is significantly lower than prior to the investment. At the same time, the debt issuance response for high leverage firms is significantly larger and more persistent for high leverage firms. The regressions corresponding to Section 5.2 are in the Online Appendix.

6 Conclusion

In this paper, we show that limited liability inefficiently distorts investment away from low leverage firms and towards highly levered firms. We find that highly levered firms have an incentive to overinvest, in contrast to the traditional debt overhang channel as emphasized (Myers (1977)). At the same time, our mechanism predicts that low-leverage firms with frequent investment opportunities tend to underinvest. These distortions arise because of equity holders' ability to "double sell" some of the cash flows promised to existing debt holders. We also provide empirical evidence consistent with the model predictions for the period 2004-2018. Most strikingly, we find that highly levered firms had persistently higher investment rates despite worse investment opportunities and lower profitability.

The analysis in this paper has important policy implications—especially during times with enormous government support for corporations, as in the response to the Covid-19 crisis and the financial crisis of 2008-2009. Our model emphasizes that government programs that increase firms' debt burden may have undesired consequences, as higher leverage induces debt financed overinvestment among highly levered firms. Another lesson from our analysis is that restrictions on equity payouts are no cure-all to excessive leverage. While they reduce bankruptcy and raise investment towards the first-best for firms with low leverage, they also encouraging inefficient overinvestment among the most highly levered firms.

To emphasize the role of limited liability, we abstracted from other related mechanism and focused on decisions of a single firm. The model can be extended to allow for information frictions in the quality of collateral (Gorton and Ordoñez (2014)). If collateral quality is cyclical, debt financing would likely be further distorted relative to our benchmark. The channel emphasized in this paper likely has potentially important aggregate implications, both through investment distortions across firms (e.g., Khan and Thomas (2013), Moll (2014), and Buera et al. (2011)), and the cyclicality of the costs of financial distress (e.g. Atkeson et al. (2017)). We believe that investigating these general equilibrium consequences of limited liability will be fruitful.

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Appendix A Proofs for Section 2

A.1 Proof of Proposition 1

Proof of Proposition 1. To find the post-investment value of equity $V(Z, L)$ that solves equity holders' default problem as described by (5)- (7) we use the method of undetermined coefficients with the guess

$$V(Z, L) = \frac{1}{r - \mu}\left(Z + \frac{\omega}{\eta}Z^{-\eta}\right) - L \tag{A.1}$$

Using this guess in (5) and equating undetermined coefficients we arrive at the equation $2(r + \eta\mu) = \eta(1 + \eta)\sigma^2$. We solve this quadratic equation for η and note that the smaller of the two roots is explosive and, hence, it violates the transversality condition. Therefore,

$$\eta = \frac{(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2} \tag{A.2}$$

Next, we substitute the guess (A.1) into (7) to find that $\omega = \underline{Z}^{\eta+1}$. Then we use (A.1) and the expression for ω in (6) to find that the default threshold is given by

$$\underline{Z} = \frac{(r - \mu)\eta}{\eta + 1}L \tag{A.3}$$

(A.3) defines the default threshold and completes derivations of $V(Z, L)$. Since the default threshold depends on equity holders' liabilities L we donate it by $\underline{Z}(L)$.

Next, we show that the value of equity can be expressed as $V(Z, L) = v(\ell)Z$ and derive the expression for $v(\cdot)$. First, we note that

$$\frac{\underline{Z}(L)}{Z} = \frac{(r - \mu)\eta}{\eta + 1}\frac{L}{Z} = \frac{(r - \mu)\eta}{\eta + 1}\ell, \tag{A.4}$$

where $\ell \equiv L/Z$. Next, we use the expressions for ω and for $\underline{Z}/Z$ found above in (A.1) to find

$$V(Z, L)/Z = \frac{1}{r - \mu} + \frac{\chi}{\eta + 1}\ell^{\eta+1} - \ell, \tag{A.5}$$

where

$$\chi \equiv \left(\frac{(r-\mu)\eta}{\eta+1}\right)^{\eta} \tag{A.6}$$

Finally, we set $v(\ell) = \frac{1}{r-\mu} - \ell(1 - s(\ell))$, were $s(\ell) = \frac{\chi}{\eta+1}\ell^{\eta}$, which implies that $V(Z,L)/Z = v(\ell)$.

To find the liquidation value of the firm we note that equity holders walk away when cash flows Z reach the default threshold $\underline{Z}(L)$. At that time debt holders take over the firm so its liabilities are reset to $L = 0$ but the firm loses fraction $\theta \in [0,1]$ of its value. It follows that the liquidation value of the firm, from creditors' perspective, is given by

$$V((1-\theta)\underline{Z}(L), 0) = \frac{(1-\theta)\underline{Z}(L)}{r-\mu} \tag{A.7}$$

Thus, the liquidation value per unit of liabilities is given by

$$\frac{V((1-\theta)\underline{Z}(L), 0)}{L} = \frac{(1-\theta)\eta}{\eta+1}, \tag{A.8}$$

where we used the definition of $\underline{Z}(L)$ (see (A.3).

□

A.2 Proof of Proposition 2

Proof of Proposition 2. **(Derivations of debt prices)** In Section 2.3 we have defined Let T be the first-time cash flows, Z, reach the default threshold $\underline{Z}(L)$. Since Z follows a geometric Brownian motion (see (1)) we have

$$\mathbb{E}_T\left[e^{-rT}\right] = \exp\left(\frac{-(\mu-\sigma^2/2) - \sqrt{(\mu-\sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2}\left(\log Z - \log \underline{Z}(L)\right)\right) \tag{A.9}$$

as shown in Jeanblanc et al. (2009). Using the definition of η and χ (see (A.2) and (A.6), respectively) and the expression obtained in (A.3) we conclude that

$$\mathbb{E}_T\left[e^{-rT}\right] = (Z/\underline{Z}(L))^{-\eta} = \chi\ell^{\eta} \tag{A.10}$$

Using (A.10) we see that

$$P^C(Z,L) \equiv \frac{p^C(Z,L)}{r} = \frac{1}{r}\left[1-\chi\ell^{\eta}\right] = \frac{1}{r}\left[1-(1+\eta)s(\ell)\right], \tag{A.11}$$

where $s(\ell) = \chi/(1+\eta)\ell^{\eta}$ and $\ell = L/Z$. Note that the above equation implies that the relevant state variable is ℓ. Hence, we can express the price of the coupon claims of a defaultable consol as

$$P^C(\ell) \equiv \frac{p^C(\ell)}{r} = \frac{1}{r}\left[1-(1+\eta)s(\ell)\right] \tag{A.12}$$

Next, we consider the price of a bankruptcy claim $P^B(Z,L)$. Note that

$$P^B(Z,L) \equiv \frac{p^B(Z,L)}{r} = \frac{V((1-\theta)\underline{Z}(L),0)}{rL}\mathbb{E}_T\left[e^{-rT}\right] \tag{A.13}$$

Using (A.8)), (A.9), and the definition of $s(\ell)$ we obtain

$$P^B(Z,L) \equiv \frac{p^B(Z,L)}{r} = \frac{1}{r}(1-\theta)\eta s(\ell), \tag{A.14}$$

We see again that the relevant state variable is ℓ and, thus, we can write the price of claims bankruptcy as $P^B(\ell) \equiv \frac{p^B(\ell)}{r}$.

From the above discussion it follows that the leverage ℓ is the relevant state for pricing defaultable consols so that we can write $P(Z,L) = P(\ell)$ and $p(Z,L) = p(\ell)$. Putting together (A.12) and (A.14) we obtain

$$P(\ell) = \frac{p(\ell)}{r} = \frac{1}{r}\left[1-(1-\theta\eta)s(\ell)\right] \tag{A.15}$$

(The Budget Constraint) Equity holders issue debt to finance their equity payouts, M, and a fraction ψ of the investment cost $Zq(g)$. Let K denote the quantity of new bonds issued by equity holders to finance $\psi Zq(g) + M$. Then, K has to satisfy the following budget constraint

$$P(\hat{\ell})K = \psi Zq(g) + M, \tag{A.16}$$

where $\hat{\ell}$ is the post-investment leverage. Next, we relate K to the change in leverage $\hat{L} - L$. Recall that L is defined as the present discounted value (PDV) of liabilities.

Since each unit of debt promises a payment of a constant coupon of 1 and agents discount these payments at a rate r it follows the PDV of the cash flows promised to new debt holders is given by K/r. Therefore, the post-investment liabilities are given by $\hat{L} = L + \frac{K}{r}$. It follows that

$$K = r(\hat{L} - L) \tag{A.17}$$

Let $\hat{\ell} = \hat{L}/(Z(1+g))$. Substituting the above expression for K into the budget constraint (A.16), dividing both sides of the resulting equation by Z, setting and using the definition of $p(\cdot)$ (see (A.15)) we obtain

$$p(\hat{\ell})\left(\hat{\ell}(1+g) - \ell\right) = \psi q(g) + m, \tag{A.18}$$

which corresponds to (18) in the text. □

A.3 Proof of Proposition 3

Proof of Proposition 3. To derive (24) consider equity holders' objective function (20) and substitute the budget constraint ((A.18)) to eliminate $\psi q(g) + m$ and obtain

$$(1+g)v(\hat{\ell}) - q(g) + p(\hat{\ell})(\hat{\ell}(1+g) - \ell) \tag{A.19}$$

Using the expression we found for $v(\hat{\ell})$ (Proposition 1), (A.19) can be written as

$$\frac{1+g}{r-\mu} - (1+g)\hat{\ell}\left(1 - s(\hat{\ell})\right) - q(g) + p(\hat{\ell})\left(\hat{\ell}(1+g) - \ell\right) \tag{A.20}$$

Using the observation $p(\hat{\ell}) = 1 - (1+\theta\eta)s(\hat{\ell})$ and simplifying (A.20) we obtain

$$\frac{1+g}{r-\mu} - p(\hat{\ell})\ell - q(g) - \theta\eta s(\hat{\ell})\hat{\ell}(1+g) \tag{A.21}$$

Defining $H(\hat{\ell}) = \theta\eta s(\hat{\ell})$ and using this definition in (A.21) we obtain the equity holders' objective function (24) in Proposition 3.

To obtain (27), note that $v(0) = \frac{1}{r-\mu}$. Thus, (23) implies that g^u is a unique solution to the F.O.C. given by $0 = \frac{1}{r-\mu} - q'(g^u)$.

□

Appendix B Repeated Investment Derivations

This section derives the ODEs for a repeated investment decisions, which introduces a controlled jump-process.

Assume that upon an arrival of an investment opportunity, the state jumps to a deterministic function of the current state, $\hat{\ell}(\ell)$. Define the jump size as $\tilde{g}(\ell) \equiv \hat{\ell}(\ell) - \ell$. Then the SDE for ℓ is

$$d\ell_t = (\sigma^2 - \mu)\ell_t \mathrm{d}t + \sigma \ell_t d\mathbb{W}_t + \left(\hat{\ell}(\ell) - \ell\right) d\mathbb{N}_t \tag{B.1}$$

where $\mathbb{N}_t$ is a homogeneous Poisson process with arrival rate $\lambda \geq 0$.

Firm's HJBE First, we will derive the HJBE in ℓ-space without the jumps, and add them. Set $V(Z, L) = Zv(L/Z)$ and differentiate w.r.t. Z

$$\boldsymbol{\partial}_Z V(Z, L) = v(L/Z) - \frac{L}{Z}\boldsymbol{\partial}_\ell v(L/Z) = v(\ell) - \ell \boldsymbol{\partial}_\ell v(\ell) \tag{B.2}$$

$$\boldsymbol{\partial}_{ZZ} V(Z, L) = \frac{L^2}{Z^3}\boldsymbol{\partial}_{\ell\ell} v(L/Z) = \frac{1}{Z}\ell^2 \boldsymbol{\partial}_{\ell\ell} v(\ell) \tag{B.3}$$

Use the ODE in (5), divide by Z, and use the above derivatives to obtain

$$(r - \mu)v(\ell) = 1 - r\ell - \mu\ell\boldsymbol{\partial}_\ell v(\ell) + \frac{\sigma^2}{2}\ell^2 \boldsymbol{\partial}_{\ell\ell} v(\ell) \tag{B.4}$$

Default Decision The notation denotes $\cdot|_\ell$ as the evaluation of a function at ℓ.

For the firm's boundary conditions, add in artificial reflecting barriers at some $\ell_{\min}$ and $\ell_{\max}$. We will ensure that the equilibrium $\ell_{\min} < \bar{\ell}$ so it is never binding in the solution, the $l_{\max}$ will be chosen large enough to not effect the solution.

Then, for the firm, we can write the DVI for their stopping problem as

$$u(c) \equiv 1 - r\ell \tag{B.5}$$

$$\mathcal{L}_v \equiv r - \mu + \mu\ell\boldsymbol{\partial}_\ell - \frac{\sigma^2}{2}\ell^2 \boldsymbol{\partial}_{\ell\ell} - \lambda\left(\cdot|_{\ell+\tilde{g}(\ell)} - \cdot|_\ell\right) \tag{B.6}$$

$$0 = \min\{\mathcal{L}_\ell v(\ell) - u(\ell), v(\ell)\} \tag{B.7}$$

$$\boldsymbol{\partial}_\ell v(l_{\min}) = 0 \tag{B.8}$$

$$\boldsymbol{\partial}_\ell v(l_{\max}) = 0 \tag{B.9}$$

We would numerically find a $\bar{\ell}$ which fulfills the indifference point, and then find the value of liquidation per unit of PV of liabilities is

$$v^{\text{liq}} \equiv (1-\theta)\frac{\lim_{\ell \to 0} v(\ell)}{\bar{\ell}} \tag{B.10}$$

Bond Pricing The price of a bond, $P(Z, L)$ pays 1 unit until default. The ODE in the continuation region without jumps is

$$rP(Z,L) = 1 + \mu Z \boldsymbol{\partial}_Z P(Z,L) + \frac{\sigma^2}{2} Z^2 \boldsymbol{\partial}_{ZZ} P(Z,L) \tag{B.11}$$

Take the defintion $rP(Z, L) \equiv p(L/Z)$ and differentiate with respect to Z

$$r\boldsymbol{\partial}_Z P(Z,L) = -\frac{1}{Z}\ell \boldsymbol{\partial}_\ell p(\ell) \tag{B.12}$$

$$r\boldsymbol{\partial}_{ZZ} P(Z,L) = \frac{1}{Z^2}\left(2\ell \boldsymbol{\partial}_\ell p(\ell) + \ell^2 \boldsymbol{\partial}_{\ell\ell} p(\ell)\right) \tag{B.13}$$

Multiply by r and substitute the derivatives into (B.11)

$$rp(\ell) = r + (\sigma^2 - \mu)\ell \boldsymbol{\partial}_\ell p(\ell) + \frac{\sigma^2}{2}\ell^2 \boldsymbol{\partial}_{\ell\ell} p(\ell) \tag{B.14}$$

In default, the bond is entitled to a share of rL units of the liquidation value $V((1-\theta)\underline{Z}(L), 0)$, hence $P(\bar{\ell}) = \frac{V((1-\theta)\underline{Z}(L),0)}{rL}$. Divide by r and use the definitions of v^{liq} and $p(\cdot) = P(\cdot)/r$ to find that the boundary condition is $p(\bar{\ell}) = v^{\text{liq}}$.

Summarizing, bond pricers take v^{liq} and $\bar{\ell}$ as given, and then solve

$$\mathcal{L}_p \equiv r - (\sigma^2 - \mu)\ell \boldsymbol{\partial}_\ell - \frac{\sigma^2}{2}\ell^2 \boldsymbol{\partial}_{\ell\ell} - \lambda\left(\cdot|_{\ell+\tilde{g}(\ell)} - \cdot|_\ell\right) \tag{B.15}$$

$$\mathcal{L}_p p(\ell) = r \tag{B.16}$$

$$\boldsymbol{\partial}_\ell p(\ell_{\min}) = 0 \tag{B.17}$$

$$p(\bar{\ell}) = v^{\text{liq}} \tag{B.18}$$

where the lower boundary is an artificial reflecting barrier and the upper boundary is the liquidation absorbing barrier.

Investment Finally, the objective function of the firm at every arrival point λ remains to maximize the equity value. Given an equilibrium $p(\ell)$ and $v(\ell)$ functions—

consistent with the optimal jump process, the agent solves

First-Best The first-best is derived through a guess-and-verify approach. First, guess that the user would choose a constant g due to the homotheticity of the problem. With that, the unnormalized Bellman equation (with jumps) is

$$rV(Z) = Z + \mu Z V'(Z) + \frac{\sigma^2}{2} Z^2 V''(Z) + \lambda \max_g \left\{ V((1+g)Z) - V(Z) - \zeta \frac{g^2}{2} \right\} \tag{B.19}$$

Take the first-order condition

$$\zeta g Z = Z V'((1+g)Z) \tag{B.20}$$

Guess the solution to the problem is $V(Z) = AZ$ for an undetermined Z, and subtitute into the (B.19) and solve for A to find,

$$A = \frac{1 - \frac{1}{2}\zeta g^2 \lambda}{-g\lambda - \mu + r} \tag{B.21}$$

Similarly, substitute the guess into (B.20) to find $g = \frac{A}{\zeta}$. Use this expression to eliminate A in (B.21), solve the quadratic for g, and choose the positive root to find,

$$g^u = \frac{1}{\zeta(r-\mu)\left(\frac{1}{2}\left(\sqrt{1 - \frac{2\lambda}{\zeta(r-\mu)^2}} - 1\right) + 1\right)} = \frac{2}{\sqrt{\zeta\left(\zeta(r-\mu)^2 - 2\lambda\right)} + \zeta(r-\mu)} \tag{B.22}$$

In addition, from the $V(Z) = AZ$ guess, and noting that $V(Z, 0)$ is the first-best if unconstrained in using equity investment $v(0) = \frac{V(Z,0)}{Z} = A$. Consequently, for a default threshold $\bar{\ell}$, the liquidation value per unit of defaultable console in (B.10) is,

$$v^{\text{liq}} = \frac{1-\theta}{\bar{\ell}} \frac{1 - \frac{1}{2}\zeta(g^u)^2\lambda}{r - \mu - g^u\lambda} \tag{B.23}$$

When $\lambda = 0$, these all nest the $g^u = \frac{1}{\zeta(r-\mu)}$ case.