A Model of Product Awareness and Industry Life Cycles*

Jesse Perla

University of British Columbia

Draft Date: July 5, 2019

[Download Newest Version]
[Technical Appendix]
[Code Repository]
[Interactive Notebooks]

This paper proposes a model where consumption bundle heterogeneity is derived from a network of connections between consumers and firms. In it, consumers slowly become "aware" of differentiated products, adding connections and expanding their choice sets. Increasing network connectivity over an industry life cycle both decreases market power and increases the degree of sorting between consumers and firms. When aggregated, this information friction creates a wedge in an otherwise standard neoclassical growth model where higher product obsolescence leads to stronger distortions. Endogenizing the stochastic process for awareness using a model of directed advertising, I find two countervailing effects: (1) If firms are better able to target good matches, then there are benefits of sorting for consumers and the economy as a whole; and (2) a side effect of the directed advertising can be smaller choice sets, and hence greater market power.

1   Introduction

Detailed scanner-level data has given economists a new perspective on the high degree of consumption bundle heterogeneity, as well as the rapid churn and obsolescence of products.[1]

In most models of firm heterogeneity—such as a baseline model of monopolistic competition with a representative consumer—the age distribution and churn of products in the consumption bundle has no direct impact. The aggregate consumption function smooths out consumer heterogeneity, and firms' profit margins and markups under monopolistic competition are constant and independent of the product's age. [2] This paper shows that the crucial assumption leading to these results is the assumption of fully connected choice sets between consumers and firms. To explore the consequences of relaxing the perfect information baseline, this paper provides a new model of demand to help understand the impact of this heterogeneity on firms' decisions, consumer welfare, and factor shares.

The core idea is to reinterpret demand functions as arising from a network of connections between firms and consumers, where the properties of the network determine consumption bundle heterogeneity and the degree of market power in an industry. This takes models of customer capital to the next logical step: Instead of customer capital being an accumulated demand shifter (e.g., built through advertising, market access, or with consumers captured through search and switching costs), I model consumers as having relatively standard preferences, but with incomplete choice sets derived from the network of connections.[3]

Following the empirical literature on this topic—such as Goeree (2008) and De Los Santos et al. (2012)—I call the limited information/choice sets for consumers within the network "awareness," with which I capture frictions such as a consumer having no: knowledge of a product's existence, no knowledge of the idiosyncratic match to her preferences, no information on the particular location and method for purchasing, or a lack of geographical proximity to a distributor of the product.[4]

---

[1]For example, Broda and Weinstein (2010) documents that "40 percent of household expenditures are on goods that were created in the last 4 years, and 20 percent of expenditures are in goods that disappear in the next 4 years," while in the Adobe Digital Economy project Goolsbee and Klenow (2018) and Argente et al. (2018) find that the obsolescence rate may be even higher. Other literature discussing the high degree of product switching includes Bernard et al. (2010) and Bernard and Okubo (2016)

[2]Heterogeneity in consumer bundles and price dispersion are documented in Kaplan and Schulhofer-Wohl (2017), Kaplan et al. (2019) and Faber and Fally (2017), and Michelacci et al. (2019) documents considerable heterogeneity in consumer bundles and price indices. Finally, De Loecker and Eeckhout (2017) and Mongey (2018) discuss the aggregate implications of market power.

[3]Empirical facts on the slow expansion of customer capital has spawned a recent and influential literature on demand. See Gourio and Rudanko (2014a), in particular, for motivation on the role of customer capital in forming intangible capital. My paper builds on these motivations by showing that it is not just accumulated customers that matter, but also the features of the interconnected network that aggregate to form the customer capital.

[4]Studies able to connect individuals to choice sets consistently show that the effective choice sets are very small. Using online browsing data logging every website visited by a large number of consumers, De Los Santos et al. (2012) shows that close to 35% of consumers *visited* a *single* online bookstore in an 18-

Considering the evidence on rapid product obsolescence and churn of consumer bundles, it makes sense to consider the role of network dynamics. Product and industry maturity is tightly connected to this network formation, as choice sets are determined by the firms to which a consumer is connected. Early in an industry or product's life cycle, the network of connections is sparse. Later, because of advertising, word of mouth, direct sales, and other forces, the network may become more dense.

For a firm, the expansion of product awareness causes two countervailing effects on profits. First, increased awareness among consumers increases the level of competition, and hence, decreases market power and prices. Intuitively, if all of a firm's customers know only of that firm, it would have monopoly pricing power over them. But if some of those customers are aware of multiple firms, then a firm needs to lower its prices to compete. Hence it is the time-varying *effective number* of competitors for each consumer, rather than the *total number* of competitors for all consumers, that determines market power and the implications of this demand friction at the industry or aggregate level. Second, increased awareness among consumers gives them more choice and allows for better matches, on average, between firm and consumer. Over time, consumers sort into their preferred products, which further increases demand. These competing forces drive industry profits and valuations. At the aggregate level, the strength of these effects depends on the level of awareness maturity of different product categories and vintages, which may change as a result of industry composition and technological innovations.

The approach and structure of the paper is as follows: Section 2 introduces incomplete consumer choice sets (and hence, frictions in the underlying network of relationships between consumers and firms), but keeps the rest of the model as close as possible to a neoclassical growth model aggregated from monopolistic competition. The process for "awareness" is left exogenous and general. Section 3 takes the network of consumer-to-firm relationships to derive firms' decisions, aggregate across firms to derive an industry life cycle, and aggregate across industries to form the economy as a whole. The relatively simple change in the standard framework leads to a variety of rich effects but ultimately leads to a familiar neoclassical growth model with a representative consumer and an awareness wedge. Section 4 then examines the role of targeted advertising and endogeneity of the awareness process.

Related Literature This paper fits primarily into the literature on customer capital and demand in macro and international economics, such as Luttmer (2006), Arkolakis (2010,

---

month period. Another example is, looking at direct networks of buyers and sellers using Colombian export data, Eaton et al. (2014) finds very small networks, where "the average exporter sold to around 1.5 buyers while the average buyer had around 4 sellers." Finally, Goeree (2008) uses a similar (albeit largely static) concept of awareness as limited information sets. The author estimates, exploiting variation in advertising exposure, that median markups in the PC industry are 15% in the case of limited choice sets, as opposed to 5% under full information.

2016), Dinlersoz and Yorukoglu (2012), Drozd and Nosal (2012), and Gourio and Rudanko (2014a,b) while using a nested demand structure in the spirit of papers like Atkeson and Burstein (2008).[5] By concentrating on information frictions and heterogeneous choice sets, I am able to provide a novel perspective on demand to complement those models. This paper also fits into the related literature emphasizing price dynamics in models with consumer markets, such as in Klemperer (1995), Bergemann and Välimäki (2006), Rudanko (2017), Gilbukh and Roldan (2017), and Paciello et al. (2019).

2 Model

This section summarizes the model and introduces the notation in Section 2.1, describes general processes for awareness evolution in Section 2.2, and then solves for the key decisions of the network of consumers and firms in Sections 2.3 to 2.5. The results of Section 2 are then aggregated into a generalization of the neoclassical growth model in Section 3.[6]

2.1 Summary and Notation

There are two types of agents in the economy: a continuum of infinitely lived consumers, and firms organized by a continuum of product categories. Time, t, is continuous.

Products and Firms A continuum of product categories is indexed by $m \in [0, M(t)]$, where the mass of product categories available in the economy at time t is $M(t)$.

Within each product category, the finite set of firms producing each variety is arbitrarily indexed by i, such that (i, m) uniquely denotes a firm and its product. The set of indices of firms producing in product category m is denoted by $\mathcal{I}_m$.[7]

---

[5]To compare a few to my paper: (1) Arkolakis (2010, 2016) investigates the market access margin in trade and firm growth models, and relates it to advertising expenditures; (2) Drozd and Nosal (2012) has a notion of investment in market capital, interpreted very similarly to to the accumulated awareness here, and relates it to frictions in international price elasticities; and (3) Gourio and Rudanko (2014b,b) models customer capital as a two-sided search and matching friction, and discusses the aggregate implications of customer capital wedges. Finally, while I emphasize information frictions on profitability, the role of within- vs. between-industry heterogeneity and the creation/obsolescence of new products/innovations complements papers such as Atkeson and Burstein (2008).

[6]Throughout, the baseline to compare the model against is monopolistic competition with CES preferences for a representative consumer, inelastic labor supply, and endogenous capital and variety creation. The key features of that equilibrium, once aggregated to a neoclassical growth model, are that markups are constant and the market power has no distortionary effects on allocations (unless the labor supply is elastic or nominal frictions are introduced to create markup dispersion).

[7]For interpreting the model and taking to the data, I *do not* consider each (i, m) as representing *a single firm* in the data. Many firms produce a wide range of varieties, often in different product categories and with different vintages. This interpretation disconnects the average age of a firm in the economy from the average age of a product category or variety.

This set provides the maximum number of firms in category m that a consumer could choose from in a model without information frictions. When nesting monopolistic competition, $\mathcal{I}_m$ contains a single index for the monopolistic firm in that category, while larger sets of indices represent oligopolies.

Preferences and Choice Sets Demand is modeled as a network of choice sets, and could be written as an undirected bipartite graph of all connections. Consumers, labeled by $j \in [0,1]$, have permanent heterogeneous preferences for each product and are aware of an evolving subset of the firms in the economy (i.e., can purchase from only some of the firms in $\mathcal{I}_m$, because of frictions in access and information). This leads to the only sources of heterogeneity in the model: a permanent quality of the idiosyncratic match between each product and each consumer, $\xi_{imj} > 0$; and a consumer-specific subset of firms in the awareness set, $A_{mj}(a) \subseteq \mathcal{I}_m$, which varies with the age, a, of the product category.[8] I first analyze the evolution of awareness as an exogenous process, and then endogenize it through firm investment in Section 4.

Symmetrically, firms are heterogeneous over the set of consumers who are aware of them (i.e., are included in only a subset of consumer choice sets), and make choices given an expected evolution of that distribution. In the baseline, to make a minimal deviation from monopolistic competition, firms have no other sources of heterogeneity.

Equilibrium Firms compete by simultaneously choosing a price (i.e., repeated Bertrand pricing) taking the network of awareness as given. Over time, consumers become aware of additional products through a stochastic process, leading to a denser network of choice sets.

When aggregating in Section 3, consumers rent labor and capital to firms and invest in new product categories to license to operating firms—thereby creating a new industry. The physical and entrepreneurial investment to create more capital and product categories is kept as standard as possible to enable aggregation to a standard neoclassical growth model with an endogenous number of varieties.

For notational clarity in this section, I fix a particular product category m and drop t where possible.

---

[8]Denoting the sets, which have finite cardinality for a given consumer and industry, is more convenient than directly working with an undirected graph. If the model had a finite number of both consumers and firms per industry, then working directly with the graph may be more suitable. While the continuum of consumers here is convenient for aggregation, the mechanism would be present in models with connections between a discrete number of consumers and producers (e.g., Eaton et al. (2014)). If the network were extended to include links between consumers in a social network, then the expansion of awareness could also be modeled as information diffusion over a network. A simple version of this is given in the word-of-mouth contagion process of Example 1. This is a different mechanism than in Rob and Fishman (2005), where information about product quality is spread by word-of-mouth.

2.2 Awareness (i.e., Incomplete Choice Sets)

Limited consumer choice sets (which I have labeled "awareness") are the model's only deviation from a traditional model such as monopolistic competition or oligopolistic competition with nested preferences. For simplicity, assume that N firms enter at the birth of the product category at age $a = 0$, and operate for the life of the product category.[9]

Recall the notation that $A_j(a) \subseteq \mathcal{I}$ denotes the subset of operating firms in a product category of age a that are in consumer j's choice set. While I will use the term "awareness", incomplete choice/consideration sets arising from geography, market access, inattention, collusion, or other factors would be isomorphic.
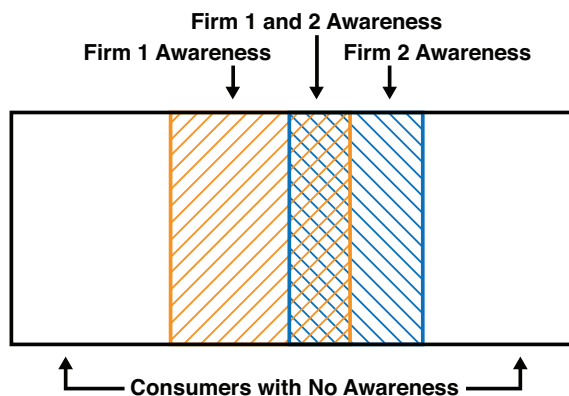

Figure 1: Example Venn Diagram of Awareness in a Duopoly

Static Analysis of Choice Set Heterogeneity   With two firms, the network of awareness links between firms and consumers can be visualized as a Venn diagram, as in Figure 1. For example, the intersection of the two sets is the mass of customers who are aware of both firms (i.e., j such that $A_j = \{1, 2\} = \mathcal{I}$), and those in the uncolored regions are aware of neither firm (i.e., j such that $A_j = \emptyset$).

This diagram provides intuition on the key forces that come out of incomplete choice sets. From the perspective of a firm making a pricing decision, a more sparse network has more *market power*. In particular, it is the *effective number* and type of competitors for each consumer, rather than the *total number* of competitors, that matters.[10] For example, if $A_j = \{1, 2\}$ or $\emptyset$ for all j, then the effective number of competitors is two—and firms would behave as if they were in a duopoly. Similarly, if the orange and blue were disjoint (i.e., all j have $A_j = \{1\}, \{2\}$, or $\emptyset$), then each firm has monopoly pricing power over all consumers who have the firm in their awareness set, and the effective number of competitors is one.

---

[9]As there are no fixed costs, firms would never endogenously choose to exit. Furthermore, we will examine cases where the returns to entry are decreasing in industry age. The case of firms entering at different times is discussed in Technical Appendices C and D.

[10]There is a tight connection between this concept and the effective number of competitors in papers such as Fajgelbaum et al. (2011) and Gabaix et al. (2016).

For a consumer, for any given set of prices the advantage of having a larger choice set is that they are more likely to find a better match. That is, the distribution of $\max\{\xi_{1j}, \xi_{2j}\}$ stochastically dominates that of ξ_{1j} or ξ_{2j} individually. This leads to the second force in the model, *consumer sorting*, where the average match quality is a function of the effective number of competitors since consumers can always choose the best price-adjusted match quality within their choice set. For an individual consumer, if they suddenly become aware of an additional firm—taking prices as symmetric and fixed—they weakly prefer additional choices as it could mean a better match.

Dynamics of Awareness Evolution   The discussion above demonstrates the static consequences when demand is modeled as a network of choice sets. Dynamics in this model of demand is simply the creation or breaking of links between consumers and firms.

Depending on the nature of the process generating dynamics of the network, it will lead to changes in the effects of the market power and consumer sorting forces discussed above, which are especially acute if the connections are asymmetric (e.g., if a new connections have different impacts on incumbents than on entrants—a case that's covered in the extensions in Technical Appendix C).

While most of this paper leaves the stochastic process generating dynamic connections flexible, assume for now that, on average, the mass of consumers aware of each firm increases over time. As seen in Figure 1, this also suggests that the intersection could become larger as well. It is the growth in the average number of firms in the awareness set—conditional on there being at least one—that drives the interesting dynamics of this model.

Figure 2 provides a visualization of the dynamics by drawing bipartite graphs for two different industry ages. Here, the solid left-hand side of each graph represents the continuum of consumers, which is sorted by connection type for clarity.
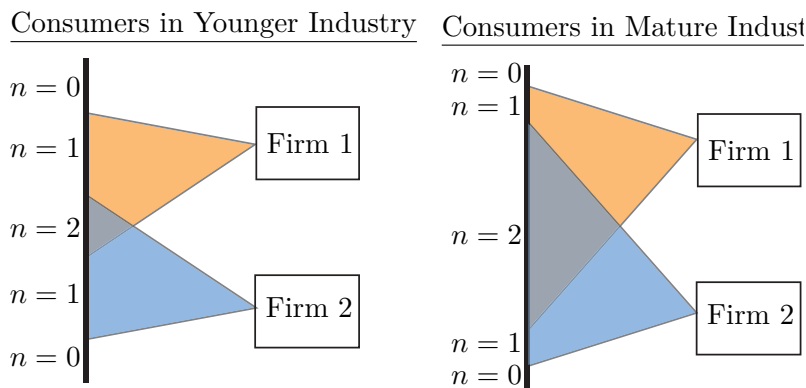

Figure 2: Awareness Sets as an Expanding Network

In our hypothetical process, the bipartite graph becomes more dense over time. This leads to several effects in the life cycle of industries as they age: (1) Fewer consumers have

an empty awareness set (i.e., mechanical industry growth); (2) both firms know that their customers tend to have larger choice sets, which leads them to compete more intensely (i.e., decreasing market power); and (3) an average consumer has more products to choose from (i.e., increasing sorting).

Stochastic Process for Awareness Evolution   In order to isolate the effects of expanding awareness from the properties of a particular awareness process, this model will derive most of its results for a general law of motion. For now, assume that the evolution of awareness is an exogenously given stochastic process. In Section 4, I solve an example where parameters for the stochastic process are an endogenous choice.

Since I do not have heterogeneity in cohorts or quality in this baseline version, the distribution of the count of firms in the consumers' awareness sets, with cardinality $N + 1$, is a sufficient statistic to use in calculating firm profits and prices.

Define the proportion of consumers aware of $n \in \{0, \ldots N\}$ firms as the probability mass function (pmf) $f_n(a)$, stacking them as $f(a) \in \mathbb{R}^{N+1}$, where $\sum_{n=0}^{N} f_n(a) = 1$.[11]

As there are a discrete number of states, the evolution of the count is a continuous-time Markov chain. Denote the intensity matrix (or infinitesimal generator) of the process by $\mathbb{Q}$, which should be thought of as a flexible parametric form. With this Markov chain (and denoting the partial derivative with respect to a by the operator $\boldsymbol{\partial}_a$), the evolution of the distribution follows a system of $N + 1$ ordinary differential equations

$$\boldsymbol{\partial}_a f(a) = f(a) \cdot \mathbb{Q}(a, f), \quad \text{given initial condition } f(0) \in \mathbb{R}^{N+1} \tag{1}$$

While the paper is written for a general function $\mathbb{Q}(a, f)$, Example 1 fixes a baseline awareness process to use in examples and comparative statics.

Example 1 (Baseline Awareness Process). *Assume each consumer has (1) an intensity $\theta > 0$ of becoming aware of a firm in a product category; (2)a probability of becoming aware of a particular firm (including the potential of repeating a meeting with an existing firm in her information set) which is the same for all firms; (3) a chance of losing links to an existing firm at rate $\mu \geq 0$;[12] and (4) word-of-mouth diffusion about the product category spreading*

---

[11]The evolution of awareness for firms entering at different times, and with different quality, is generalized in Technical Appendix C. Fortunately, if firms have some symmetry in the time of entry or in the intrinsic quality (or productivity), a simpler state-space than tracking all subsets $\mathcal{I}$ is sufficient for firms to calculate profits and make optimal pricing decisions. See Appendix C for more details on mapping measures over the graphs A_j to the counts $f_n(a)$.

[12]In queuing theory the $\theta_d = 0$ case is called an M/M/1/K with customer balking (alternatively, "discouraged arrivals"—see Kleinrock (1975), Section 3.3). Eaton et al. (2014) calibrate a 27% yearly separation rate for distributors in a related search model, which suggests that a positive μ is reasonable.

with an intensity $\theta_d \geq 0$.[13] *This creates the state-varying intensity matrix*

$$\mathbb{Q}(a,f) = \begin{bmatrix} -(\theta+\theta_d(1-f_0(a))) & \theta+\theta_d(1-f_0(a)) & 0 & \dots & & \dots & 0 \\ \mu & -\mu-\frac{N-1}{N}\theta & \frac{N-1}{N}\theta & 0 & \dots & & \dots & 0 \\ \vdots & & & & & & & \vdots \\ 0 & 0 & & 0 & 0 & \dots & \mu & -\mu-\frac{1}{N}\theta & \frac{1}{N}\theta \\ 0 & 0 & & 0 & 0 & \dots & 0 & \mu & -\mu \end{bmatrix} \in \mathbb{R}^{(N+1)\times(N+1)} \tag{2}$$

2.3 Consumers

Beyond the embedding of awareness in the consumer preferences, I keep the model and aggregation as close as possible to the benchmark model (i.e., the neoclassical growth model with CES preferences and monopolistic competition). In particular, when $N = 1$ this will nest a model of monopolistic competition with an endogenous amount of capital and varieties.

Throughout, keep in mind that the time-invariant preferences ξ_{imj} and the time-varying $A_{mj}(t)$ are the idiosyncratic state of consumer j.

Preferences and Budget Constraints The future is discounted at rate $\rho > 0$, and consumers have a constant relative risk aversion (CRRA) of $\gamma > 1$. Furthermore, all consumers have a real income $\Omega(t)$ and a bundle price $P(t)$—both derived in general equilibrium in Section 3. Preferences over goods are constant elasticity of substitution (CES) between product categories with an elasticity of $\kappa \equiv \frac{1}{1-\varsigma} > 1$. Nested within the standard Dixit–Stiglitz aggregation of product categories, goods within a product category are perfectly substitutable after adjusting for price and quality.[14]

Given awareness sets $A_{mj}(t)$, idiosyncratic preferences ξ_{imj}, and nominal prices $\hat{p}_m(t)$ for all product categories and products, the consumer chooses their demand per good, $c_{imj}(t)$,

---

[13]A typical S-curve of new product adoption, as in Mahajan et al. (1990), occurs when $\theta_d > 0$. For clarity, this example simplifies a more general setup by assuming that word-of-mouth diffusion is about adoption of the product category itself and not about customers of a particular firm (i.e., the additional θ_d arrivals are only to consumers with $n = 0$)

[14]While perfectly substitutable goods are atypical for a model aggregating to a representative agent, consumption decisions here are more similar to a discrete-choice model—which typically has each consumer purchasing a single good from each category to match the data (i.e., a consumer with full awareness of the ketchup product category would be unlikely to purchase both Heinz and Hunt's Ketchup during any period without relative price changes).

to maximize the welfare,[15]

$$\int_0^\infty e^{-\rho t} \frac{1}{1-\gamma} \left[\overbrace{\left(\int_0^{M(t)} \underbrace{\left(\sum_{i \in A_{mj}(t)} e^{\sigma \xi_{imj}} c_{imj}(t) \right)^{\varsigma}}_{\text{Industry } m \text{ demand}} dm \right)^{1/\varsigma}}^{\equiv C_j(t), \text{ a "Composite Good"}} \right]^{1-\gamma} dt \tag{3}$$

To compare the preferences to standard CES and discrete-choice utility specifications: (1) Unlike typical discrete-choice preferences, the idiosyncratic quantity ξ_{imj} enters the term multiplicative rather than additive (economically, this represents a consumer's intensive demand increasing as she meets better matches and is needed for homothetic aggregate preferences); (2) the parameter $\sigma > 0$ determines the degree of differentiation within a product category and has a direct analogy to the variance of the random utility shocks in discrete-choice theory and, (3) the consumer can purchase only from firms in the set $A_{mj}(a)$ rather than all operating firms.

For the budget constraint, I follow neoclassical growth models by decoupling the quantity purchased, y_{imj}, from the quantity directly consumed, c_{imj}. The income of consumers includes (1) rental of their inelastic supply of one unit of labor, at real wage $w(t)$; (2) rental of their capital stock $k(t)$ at real rate $r(t)$; and (3) real profits from ownership of the firms, $\Pi(t)$. The budget constraint for all purchases with intensive demand y_{imj} is

$$\int_0^{M(t)} \left[\sum_{i \in A_{mj}(t)} \hat{p}_{im}(t) y_{imj}(t) \right] dm \le P(t)\Omega(t) \equiv P(t)\left(w(t) + r(t)k(t) + \Pi(t) \right) \tag{4}$$

Static Decisions As consumers' choices do not affect their future awareness (e.g., they have no habits directly in preferences), demand can be solved as a static optimization problem. For notational clarity, I drop the index t and use the age a of a given product category. Define the real price as $p_i(a) \equiv \hat{p}_i(a)/P$, and solve for the demand functions:

Proposition 1 (Intensive Demand). *Given real prices $p(a)$ and real income Ω, a consumer purchases product i and no others if and only if*

$$\log p_{i'}(a) - \log p_i(a) > \sigma \left(\xi_{i'j} - \xi_{ij} \right) \quad \forall i' \in A_j(a) \setminus \{i\} \tag{5}$$

---

[15]Within the trade and macro literature, the preference specification here is similar to preferences in Atkeson and Burstein (2007) and Fajgelbaum et al. (2011). The closest specification of discrete-choice preferences for a single industry with heterogeneous choice sets is that of Goeree (2008)—although I have added multiplicative idiosyncratic preferences, an intensive margin, and choice set dynamics. As discussed earlier, the perfect quality- and price-adjusted substitutability between goods within the category is consistent with each m being a narrowly defined product category but is not essential to the mechanism.

The intensive demand for product i is

$$y_{ij}(a, \xi_{ij}) = \bar{\Gamma}^{\kappa-1} e^{\sigma(\kappa-1)\xi_{ij}} \underbrace{p_i(a)^{-\kappa}\Omega}_{CES}, \tag{6}$$

where $\bar{\Gamma} \equiv \Gamma(1 - \sigma(\kappa - 1))^{1/(1-\kappa)}$ is a normalizing constant to adjust for $\mathbb{E}\left[e^{\sigma(\kappa-1)\xi}\right] \neq 1$, with $\Gamma(\cdot)$ the Gamma function.

Proof. A special case of the fully differentiated proposition in Technical Appendix A.2. □

To connect Proposition 1 with the forces in the model: As $A_j(a)$ grows, consumers have more choices in (5) to find the best price-adjusted match, which will undermine market power when I aggregate demand in Sections 2.4 and 2.5. Furthermore, the demand for the chosen product in (6) has an intensive demand component as a function of ξ_{ij}—in addition to the standard CES component nesting monopolistic competition. If $\sigma > 0$, then larger $A_j(a)$ *may* lead to better matches and higher demand if consumers change the i in (5).

I can generalize a typical CES price index to be consistent with Proposition 1 by including the idiosyncratic state of the consumer, where the i for each m is that which maximizes (5):

$$P_j(a) \equiv \bar{\Gamma}^{-1} \left(\int_{|\mathcal{I}_{jm}(a)|>0} e^{\sigma(\kappa-1)\xi_{imj}} \hat{p}_{im}(a)^{1-\kappa} \mathrm{d}m \right)^{\frac{1}{1-\kappa}} \tag{7}$$

An implication of the dependence on ξ_{imj} in (7) (and, indirectly, of A_{jm} in determining the appropriate i per category) is that consumers can have different real incomes, $\Omega_j(a)$, even if they have identical nominal incomes—a key point in Faber and Fally (2017). However, given a continuum of product categories and independence of ξ from awareness evolution, infinitely lived consumers are shown to have a common bundle price P for aggregation in Section 3.

Dynamic Decisions   The dynamic decisions are kept as simple and standard as possible in order to nest a neoclassical growth model with an endogenous stock of both varieties and capital. To do this, I use a variation on the usual assumption that capital goods are created using the same technology and intermediates as consumption goods. Hence some of y_{imj} purchased through (4) is used for direct consumption c_{imj} in (3), while all of the rest will produce homogeneous capital goods or create new product categories using the same aggregated elasticities as the consumer's problem.[16]

---

[16]Following models of monopolistic competition with capital, these types of innocuous assumptions will save on notation, but are qualitatively unimportant since a competitive final goods aggregator is isomorphic to CES aggregation within the preferences. See Section 3.2 for more details and Technical Appendix B.3 for the aggregation proof.

Since there are no frictions in the production of capital or new varieties, I can rely on the aggregation results in Section 3 to have an equivalent constrained planning problem for the choice of $M(t), k(t)$, and $C(t)$. As in a standard model aggregating monopolistic competition, the "constraint" is to use the oligopolistic market for the intermediates (i, m), which is constructed to be isolated in the aggregation of the real output $Y(t)$.[17]

The constrained planner then takes the aggregate production function as given, and directly chooses aggregate consumption, $C(t)$, capital investment to increase $k(t)$, and entrepreneurial investment to increase $M(t)$.[18]

Given aggregated output $Y(t)$, the resource constraint given consumption $C(t)$, investment in capital, $i_k(t)$, and investment in new product categories, $i_M(t)$, is

$$Y(t) = C(t) + i_k(t) + i_M(t) \tag{8}$$

Within the differentiated products under oligopolistic competition, the consumer rents the capital and inelastic labor to firms, and owns the licensing rights for each product category. When a new product category is created (i.e., inventing an m), the household licenses the blueprints to the N firms starting up production in the industry. As the consumer owns a perfectly diversified portfolio of both the entrepreneurial licensing and the operating firms, the particular split of the surplus between the licensor and licensee does not matter for aggregates.

Capital depreciates at rate $\delta_K > 0$, product categories effectively depreciate at rate $\delta_M > 0$, and the relative productivity of inventing new product categories is $z_M(t)$—all of which are described in more detail in Technical Appendix B.5:

$$\partial_t k(t) = -\delta_K k(t) + i_k(t) \tag{9}$$

$$\partial_t M(t) = -\delta_M M(t) + z_M(t) i_M(t) \tag{10}$$

Further analysis of the dynamic decisions is left until the model is aggregated to a representative agent in Section 3.2.

2.4 Product and Industry Demand

This section takes Proposition 1 and aggregates over the consumers to form the demand function faced by firm i in industry m. Given a price vector $\vec{p}$, integrate (6) over all consumers

---

[17]See Technical Appendix A.6 for more on how a planner constrained by the same awareness sets would choose a different composition—and hence aggregation—than the oligopolistic competition described in Section 2.5. The key result is that a planner would want to engineer constant markups over the marginal cost, which would lead to undistorted allocations in a model with inelastic labor supply.

[18]An alternative approach would be to assume free entry of entrepreneurs, markets for the production of capital and new industries, and decentralization through the competitive rental rates in (26).

$j \in [0,1]$, where the choice of product i comes from (5).

Definition 1 (Total Demand). *Given the distribution over $A_j(a)$ and ξ_j, the total demand for firm i as a function of the price vector $\vec{p}$ is*

$$y_i(a, \vec{p}) \equiv \int_{[0,1]} y_{ij}(\vec{p}, \xi_{ij}) \mathbb{1} \left\{ Choose\ i\ from\ A_j(a)\ given\ \vec{p}\ and\ \xi_j \right\} dj \tag{11}$$

To calculate this integral analytically, assume that the evolution of awareness is independent of the idiosyncratic preferences, and that the ξ_{imj} are drawn from a Gumbel distribution upon firm entry.[19] Assume that $0 < \sigma < \frac{1}{\kappa-1}$, similar to the comparison of within- and between-industry substitutability in Atkeson and Burstein (2008).[20]

By the intuition in Section 2.2, an important determinant of market power is the sizes of the choice sets (conditional on there being at least one product) for individual consumers. The notation can be simplified considerably by defining a random variable $\hat{n}$ (i.e., $n \mid n > 0$), representing the awareness set size of a particular consumer. For any function $g(n) : \mathbb{N}_+ \to \mathbb{R}$, the expectation of the (0 truncated) awareness distribution is

$$\mathbb{E}_a[g(\hat{n})] \equiv \sum_{n=1}^{N} \frac{f_n(a)}{1 - f_0(a)} g(n) \tag{12}$$

For example, if $\mathbb{E}_a[\hat{n}] = 2$ for a particular a, then, on average, firms would know they are competing as duopolists for an average consumer who has them in their choice set. While these expectations provide summary statistics for the firms' decisions, keep in mind that both $f_0(a)$ and $\mathbb{E}_a[g(\hat{n})]$ are determined from the underlying process $\mathbb{Q}$. Integrate according to Definition 1 to find the demand function, taking every of the other firms' prices as symmetric.

Proposition 2 (Total Demand for N Symmetric Firms). *Given $\sigma(\kappa - 1) < 1$, the independence of ξ_j and $A_j(a)$, and that every firm $i' \neq i$ chooses price p, the demand curve from faced by firm i choosing p_i is*

$$y(a, p_i, p) = (1 - f_0(a)) p_i^{-\kappa} \Omega \frac{1}{N} \mathbb{E}_a \left[\hat{n} \left(1 + (\hat{n} - 1) \left(\frac{p}{p_i} \right)^{-1/\sigma} \right)^{\sigma(\kappa-1)-1} \right] \tag{13}$$

---

[19]A key property that enables analytic solutions for the demand system is that the difference between two Gumbel random variables has a Logistic distribution. Furthermore, the max-stability of the Gumbel distribution leads to convenient order-statistics for the aggregation in Section 3, where an individual consumer has a distribution of awareness set sizes across product categories. Without the Gumbel or max-stable distributions for ξ, numerical integrals might be required. A further assumption, maintained throughout, is that the distribution of A_{mj} is independent of ξ_{im}—as derived formally in Technical Appendix A.1.

[20]The σ parameter of the Gumbel distribution changes the cross-product elasticity, as in typical discrete-choice models. However, unlike typical discrete-choice models, if elasticity were measured from market shares, it would show time variation due to dynamic choice set sizes. More mature industries would tend to have higher elasticities.

If an equilibrium exists where $p_i = p$, then

$$\underbrace{Ny(a,p)}_{\substack{Industry \\ Output}} = \underbrace{(1 - f_0(a))}_{\substack{Limited \\ Awareness}} \underbrace{p^{-\kappa}\Omega}_{\substack{Typical \\ CES}} \underbrace{\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right]}_{\substack{From\ Sorting}} \tag{14}$$

Proof. See Technical Appendix Proposition 2 for a derivations with both symmetric and fully asymmetric firms and awareness sets. $\square$

Analysis of the Demand Function   We can revisit the static intuition of Figure 1 using the formalization of Proposition 2. The factor $1 - f_0(a)$ in (13) is the mechanical and uninteresting component that represents the lack of awareness, on the part of some consumers, of the product category as a whole. In fact, the factor $(1 - f_0(a))p_i^{-\kappa}\Omega$ is the CES demand function for monopolistic competition adjusted such that a fraction $f_0(a)$ of consumers are unable to purchase the product—whatever the reason.

The factor $\mathbb{E}_a[\cdot]$ in (13) contains the intuition for the role of market power and sorting in this model. To analyze this, consider splitting out the proportion of customers aware of firm i by the sizes of their awareness sets, i.e., $\hat{n} = 1$ or 2 with the duopoly example.

First, consider the set of consumers with $\hat{n} = 1$—which must be firm i for this function.[21] For these consumers, the expression inside of the expectation drops out, and there is no impact of prices or of product match dispersion (i.e., no σ) on the demand. Intuitively, since the firm has monopoly pricing power, these consumers only respond to the price through the CES factor only. Furthermore, there is no sorting, and the match quality is simply the unconditional mean (normalized to 1).

Next, consider the set of consumers with $\hat{n} = 2$, including firm i. In this case, there are countervailing effects of the larger awareness sets on demand and profits. On one hand, the firm knows they compete as duopolists for this segment of consumers, and hence consumers will be more price sensitive (that is, the factor $(\hat{n} - 1)(p/p_i)^{-1/\sigma}$), undermining market power. On the other hand, *conditional* on choosing to consume from firm i rather than the competitor, the match quality is higher. For example, when $p = p_i$, (5) shows that firm 1 will sell only to consumers with $\xi_1 \geq \xi_2$. This sorting leads to higher intensive demand on average, because of the factor $e^{\sigma\xi_{ij}}$ in (6), where σ determines the magnitude of the effects from sorting.

An important requirement for aggregation is that, with equilibrium prices, the total product category demand, $Ny(a,p)$, in (14) is independent of the number of firms N—except through its indirect effect on $\mathbb{Q}$. Conditional on a price, the only difference between demand under oligopolistic competition and demand under monopolistic competition is the additional factor $\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right]$, which summarizes the effective quality growth from the sorting

---

[21]Calculated from the distribution, the mass of this is $f_1(a)/N$ in this symmetric case.

of consumers into better matches. These industry aggregation properties are used throughout Section 3.

2.5 Firms

Firms are left as simple as possible, and their only role is to play an oligopolistic pricing game given the joint distribution of consumer awareness and matches, A_j and ξ. Each firm operates a constant-returns-to-scale production function with identical marginal costs, mc, as derived in Section 3.[22]

Firms in an industry compete to maximize profits by playing a repeated Bertrand competition, choosing a price function $p_i(\cdot)$ given the equilibrium pricing decisions of the other firms in the industry.[23] As there are no dynamic incentives for this pricing game when $A_j(a)$ evolves independently of price, I solve only for the case of repetition of a pure-strategy Nash equilibrium.[24]

Definition 2 (Bertrand–Nash Equilibrium (BNE)).
A BNE is a pure-strategy Nash equilibrium for each stage game at each period a.[25] That is, $p(a) \in \mathbb{R}^N$ such that for every $i \in \mathcal{I}$, $p_i(a) = \arg\max_{\tilde{p} \geq 0} \left\{ (\tilde{p} - mc) y_i(a, \{\tilde{p}, p_{i'}(a)\}_{i' \neq i}) \right\}$.

With this standard problem, the intuition for the pricing incentives is encapsulated in the analysis of the function $y_i(a, p_i, p)$ (13), as discussed in Section 2.4. The price elasticity depends on the average awareness set size in the industry of age a and the degree of product differentiation.

When a pure-strategy equilibrium exists, define a measure of the age-dependent average quality of matches, $q(a)$, and markup over marginal cost, $\Upsilon(a)$:

$$q(a) \equiv \mathbb{E}_a \left[\hat{n}^{\sigma(\kappa-1)} \right] \tag{15}$$

$$\Upsilon(a) \equiv 1 + \sigma \left[1 - (1 - \sigma(\kappa - 1)) \frac{\mathbb{E}_a \left[\hat{n}^{\sigma(\kappa-1)-1} \right]}{\mathbb{E}_a \left[\hat{n}^{\sigma(\kappa-1)} \right]} \right]^{-1} \in [1 + \sigma, \frac{\kappa}{\kappa - 1}] \tag{16}$$

---

[22]Technical Appendix A derives a version of the model with idiosyncratic quality, which—as in monopolistic competition—is isomorphic to differences in marginal cost in this type of model in the absence of quantity data to construct the TFP.

[23]Atkeson and Burstein (2007, 2008), Edmond et al. (2015) and Peters (2018) investigate markup heterogeneity across firms and industries. In Arkolakis (2010) and Eaton et al. (2014), consumers are heterogeneous over which firms they meet in every period, but not over their preferences for a particular good.

[24]This is in contrast to papers such as Burdett and Coles (1997) and Bergemann and Välimäki (2006), in which firms have an incentive to lower initial prices to build customer habits or induce experimentation. Technical Appendices C and D.1 provide an explanation for why entrants to an established industry would have lower prices because of awareness set asymmetry alone, and without dynamic pricing incentives.

[25]It is known that the conditions for uniqueness and existence of pure-strategy equilbria in this class of Bertrand pricing games are complicated and not generalizable, and outside the scope of this simple model. However, for the parameter values of interest, symmetric pure-strategy equilibria exist.

Then for the case of symmetric firms, the BNE from Definition 2 can be solved as follows:

Proposition 3 (Symmetric Bertrand–Nash Equilibrium). *If a symmetric pure-strategy equilibrium exists for N firms according to Definition 2, then*

$$Y(a) \equiv Ny(a) = (1 - f_0(a))p(a)^{-\kappa}q(a)\Omega \tag{17}$$

$$\Pi(a) \equiv N\pi(a) = (1 - f_0(a))(p(a) - mc)p(a)^{-\kappa}q(a)\Omega \tag{18}$$

$$p(a) = \Upsilon(a)mc \tag{19}$$

Proof. See Technical Appendix A.5. □

The solutions in Proposition 3 hold for any law of motion as parameterized by $\mathbb{Q}$—which, in turn, determines the mass of unaware consumers, $f_0(a)$, and the stochastic process $\hat{n}$ for awareness set sizes.[26]

The equilibrium of a particular industry, formally defined in Technical Appendix B.1, is fully parameterized by the awareness evolution process $\mathbb{Q}$ and the preference parameters σ and κ, with an aggregate state—exogenous to the decisions of a particular industry—of real income Ω and real marginal cost mc.

Bertrand Competition and Bounds on Markups   Keep in mind that in the canonical model of Bertrand competition with undifferentiated products, symmetric firms, and full choice sets, prices are driven to marginal costs with only a duopoly. Here, (16) shows that if a pure strategy exists, that does not occur.

The first reason for this is relatively standard: There is inherent market power in idiosyncratic matches as long as $\sigma > 0$. The intuition can be seen in (16) by taking the limit as the expected awareness set size diverges, at which point $\Upsilon(a) = 1 + \sigma$. Hence in the "competitive" limit, prices are driven to reflect only the market power inherent in the product differentiation. Furthermore, note that if $\sigma = 0$ directly, then if there were any pure-strategy equilibria, prices would have to be at marginal costs as soon as $N > 1$.

The second reason, more central to this model, for why prices are not immediately driven to marginal costs is the dispersion in the information sets. As with standard Bertrand competition, consider possible mixed-strategy equilibria when $\sigma = 0$. Firms always have the option to extract monopoly profits as long as there is a positive mass of consumers with only one firm in their information set. That is, as long as $f_1(a) > 0$ there is a profitable deviation

---

[26]The first-order approximation to (19) is a simple function of the expected awareness set size,

$$p(a) \approx \left(1 + \sigma\left(1 - (1 - \sigma(\kappa - 1))/\mathbb{E}_a\left[\hat{n}\right]\right)^{-1}\right)mc \tag{20}$$

Furthermore, if $\mathbb{Q}$ follows our process for an asymptotically large N and with $\mu = 0$, then $\mathbb{E}_a\left[\hat{n}\right] = \theta a$, and the markup converges to $1 + \sigma$ at rate $1/a$. This result is consistent with the general results in Gabaix et al. (2016).

from marginal cost pricing. The deviation, charging the monopolistically competitive price $p(a)/mc = \kappa/(\kappa - 1)$, is evident in (16).[27]|

2.6 Industry Equilibrium Examples

The interpretation in Sections 2.4 and 2.5 has been independent of the process $\mathbb{Q}$ and largely static. Taking a distribution of awareness sets as given, I examined how the choice set heterogeneity affects industry demand and the firms' pricing strategies. In this section, I simulate the process in Example 1 with a set of illustrative parameters to explore how the time-variation of the awareness sets changes industry dynamics.[28]
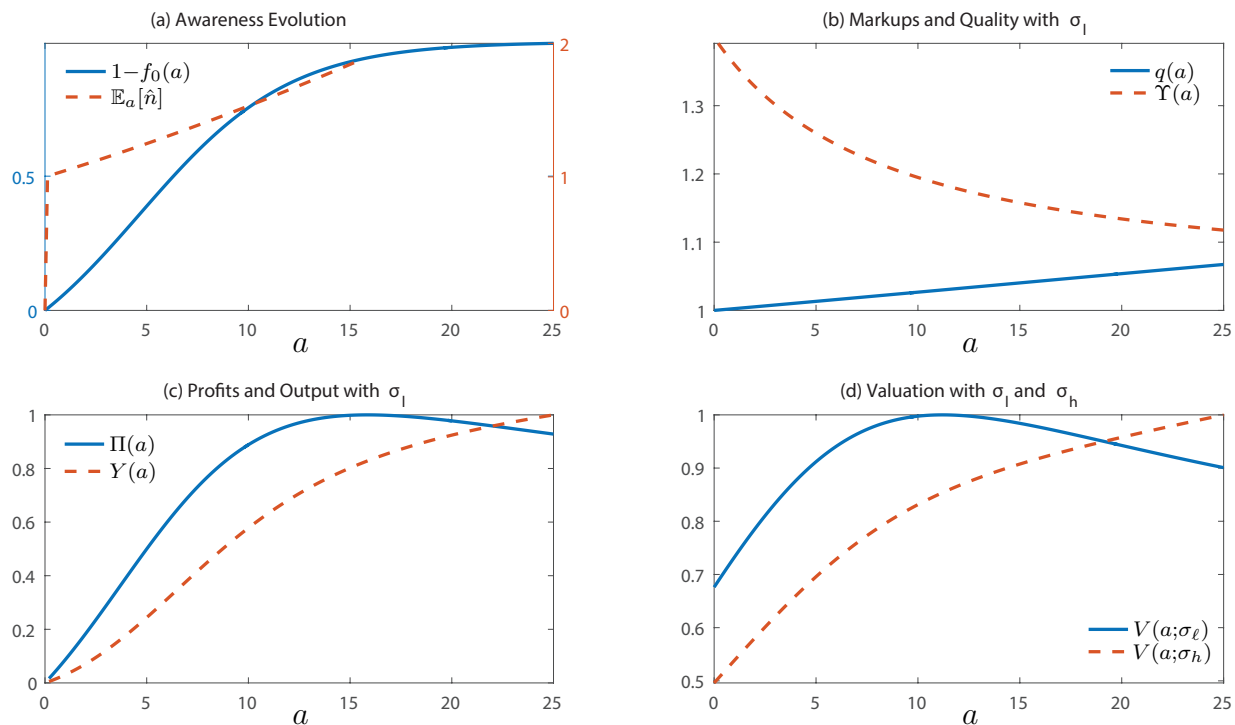

Figure 3: Example of Industry and Awareness Evolution for $\sigma_\ell < \sigma_h$

Figure 3 provides a simulation of an industry with the $\mathbb{Q}$ from Example 1. In order to highlight the role of within-product-category differentiation, the figure shows valuations for both σ_ℓ and σ_h. The markups, quality, profits, and output are normalized and shown for σ_ℓ.

---

[27]This force is a more extreme version of this paper's smoother pure-strategy version for sufficiently large σ, and is similar to Burdett and Judd (1983). As in that paper, with σ very small, there may be price dispersion caused by a mixed strategy between charging the marginal cost and monopolistic markup over marginal cost. The ratio of these would change as awareness grew.

[28]While I attempt to discipline the parameters from industry data to ensure they are within plausible ranges, these are intended to be qualitative rather than quantitative illustrations. See Table 2 in Technical Appendix E for a summary of the parameters and their crude "calibration" from the data.

The properties of this particular $\mathbb{Q}$ are evident in panel (a), which is independent of any firm decisions. Here, we see an S-shaped diffusion curve through an inflection point in the mass of customers aware of at least one firm, $1 - f_0(a)$. The shape and curvature is driven by the word-of-mouth diffusion parameter, θ_d. The growth in $\mathbb{E}_a[\hat{n}]$ is shown as well, where at the beginning of the life cycle, the average $\hat{n}$ is close to 1, which will lead to a monopolistic pricing strategy. The timescale for a in this example is somewhat arbitrary (except for the PDV of profits calculation), and can be rescaled simply by changing the parameters θ and θ_d.

The analysis of market power and sorting in Section 2.5 and Proposition 3 shows up in panel (b). For that, we see the countervailing effects on markups and average match quality. First, notice the fairly large drop-off in the markups, $\Upsilon(a)$, from the monopolistically competitive level at $a = 0$. The asymptote of $\Upsilon(a)$ is relatively low for the small σ_ℓ case (as shown in Proposition 2), since all long-run market power must come from differentiation of the product itself. In the case of a high-differentiation version, σ_h, markups would start at the monopolistically competitive level and fall both slower and by a smaller amount. The quality growth for σ_ℓ is modest because quality growth from consumer sorting is a function of the degree of product differentiation.

Continuing with the σ_ℓ example, panel (c) shows the profits and output. The output function has the familiar monotonically increasing S-shaped diffusion curve, in part from the mechanical growth of awareness in panel (a). There is also a contribution to output from the increase in intensive demand which is due to quality growth, but in panel (b) it is insufficient to have a large effect. Curiously, profits peak and then decrease in absolute terms, which can be found in some industries and is difficult to reconcile with models of monopolistic competition. This effect occurs because of intensification of competition as choice sets become larger leads to large drops in markups, as in panel (b), and because there is insufficient countervailing quality growth to make up for the loss of market power.

Finally, panel (d) shows a simulation of the valuation for the two levels of product differentiation, both normalized to their maximum level. The σ_ℓ example is non-monotone because of the non-monotone profits in panel (c), reflecting that profits and valuations may peak before competition intensifies, while the σ_h example shows that profits can be monotonically increasing if quality growth can offset decreasing market power.

3 Model Aggregation

This section examines the aggregate implications of the industry equilibrium solved in Section 2.5. The goal is to qualitatively understand where the choice set heterogeneity would manifest at the aggregate level, and to provide insights for further quantitative studies of

product market frictions. For easy comparison to standard models, I will derive a nearly standard neoclassical growth model with a single productivity wedge due to incomplete awareness, and clean distortions on the profit share, both of which remain independent of any particular process $\mathbb{Q}$.

Decomposing Aggregate and Industry States A key tool in the calculations is the separation of aggregate from industry states. By Proposition 3, the aggregate state necessary for firm decisions is summarized by the real income $\Omega(t)$ and the marginal costs $mc(t)$, while the state of a product category is summarized entirely by its age. The product category age is used, in turn, to calculate the summary statistics of awareness set sizes for each product category.[29] For simplicity, assume a common awareness process $\mathbb{Q}$ across all product categories.

Production Technology All firms have standard, identical, constant returns-to-scale Cobb–Douglas production functions in labor ℓ_i and capital K_i, with identical TFP z and output elasticity of capital $\alpha \in (0, 1)$. Labor and capital are rented from competitive markets at real factor prices $w(t)$ and $r(t)$, respectively. The cost minimization problem to produce y goods is $\min_{\ell, K} \{rK + w\ell\}$ subject to $y = zK^\alpha \ell^{1-\alpha}$. From standard producer theory, the optimal capital/labor ratio is $k = \frac{\alpha}{1-\alpha} \frac{w}{r}$, and the marginal cost mc as a function of aggregates z, k, and w is $mc \equiv \frac{1}{1-\alpha} z^{-1} k^{-\alpha} w$.

3.1 Consumption Goods Aggregation

Recall that product categories can be born with age $a = 0$ at time t (at which point they are added to the mass $M(t)$) and become obsolescent at a rate δ_M. Denote the product category age distribution of the mass of product categories, $M(t)$, by $\Phi(t, a)$, where $\Phi(t, \infty) = 1$ for all t.[30] If either $M(t)$ or $\Phi(t, a)$ changes over time, then the price index will be time dependent. In Technical Appendix B.3, it is shown that the price index in (3) under the maintained assumptions is identical for all consumers and is

$$P(t) \equiv \left(\underbrace{M(t)}_{\substack{\text{Variety} \\ \text{Effect}}} \int_0^\infty \underbrace{q(a)\hat{p}(t,a)^{1-\kappa}}_{\text{Quality adjusted price}} \underbrace{(1 - f_0(a))}_{\substack{\text{Proportion} \\ \text{Aware}}} \underbrace{d\Phi(t,a)}_{\substack{\text{Age} \\ \text{Distribution}}} \right)^{\frac{1}{1-\kappa}} \tag{21}$$

---

[29]Furthermore, by Proposition 3, the industry profits, demand, and prices are independent of the number of firms in the industry, N (except through any dependence of $\mathbb{Q}$ on N). The lack of sensitivity to the number of firms in the calibrations and solutions is especially important for aggregation, and makes the assumption that the same number of firms exist in each industry relatively harmless. As discussed in Section 2.2, the key is the effective number of competitors as summarized by moments of $\hat{n}$, rather than the actual number of competitors, N.

[30]See Technical Appendix B.5 for the full dynamics of the aggregate distribution.

In (21), the factor $1 - f(a, 0)$ represents the age-dependent limited awareness product categories. Because there is a continuum of product categories, this can be calculated by the probability that a consumer is aware of one or more firms in a product category through $\mathbb{Q}$.

Define the aggregate TFP, $Z(t)$, the factor share distortion, $B(t)$, and the cost of living quality adjustment, $Q(t)$, as weighted averages over the age distribution of product categories (their role, and reasons for the naming, will become evident in Propositions 4 and 5, where they provide distortions to standard aggregated neoclassical growth models based on the age distribution of product categories):

$$Q(t) \equiv \left[\int_0^\infty (1 - f_0(a)) \Upsilon(a)^{1-\kappa} q(a) \mathrm{d}\Phi(t, a) \right]^{\frac{1}{\kappa-1}} \tag{22}$$

$$B(t) \equiv \frac{\int_0^\infty (1 - f_0(a)) \Upsilon(a)^{-\kappa} q(a) \mathrm{d}\Phi(t, a)}{\int_0^\infty (1 - f_0(a)) \Upsilon(a)^{1-\kappa} q(a) \mathrm{d}\Phi(t, a)} \tag{23}$$

$$\underbrace{Z(t)}_{\substack{\text{Aggregate} \\ \text{TFP}}} \equiv \underbrace{z(t)}_{\substack{\text{Physical} \\ \text{TFP}}} \underbrace{M(t)^{\frac{1}{\kappa-1}}}_{\text{Varieties}} \underbrace{Q(t)}_{\substack{\text{Quality} \\ \text{Distortion}}} \underbrace{B(t)^{-1}}_{\substack{\text{Factor Share} \\ \text{Distortion}}} \tag{24}$$

With these definitions, the effects of limited choice sets are fully summarized by the distortions $B(t)$ and $Q(t)$ on otherwise standard aggregate variables.

Proposition 4 (Time-Varying Price Index, TFP, and Real Wages). *As functions of the aggregate state, $z(t), k(t), \Phi(t, a)$, and $M(t)$, the real marginal cost and wages are*

$$mc(t) = M(t)^{\frac{1}{\kappa-1}} Q(t) = \frac{w(t)}{(1-\alpha)z(t)} k^{-\alpha} \tag{25}$$

$$w(t) = \underbrace{(1-\alpha)B(t)}_{\textit{Factor Share}} Z(t)k(t)^\alpha \tag{26}$$

"Composite" good production aggregates to a function of aggregate TFP and is identical to the real income,

$$Y(t) = Z(t)k(t)^\alpha = \Omega(t), \tag{27}$$

where total production of the composite good is $Y(t) \equiv \int_0^\infty Ny(t, a)\mathrm{d}\Phi(t, a)$, and $Z(t)$ is defined by (24).

Proof. See Technical Appendix B.3. □

Written in terms of $w(t)$, the marginal cost of (25) is completely standard for a neoclassical growth model (with or without monopolistic competition of intermediate goods). The distortions from the market power enter the wages in (26) through the $B(t)$ term, which is further analyzed in the factor shares in Proposition 6. The key results of (27) are (1) that the model aggregates to a representative consumer in the same sense as monopolistically

competitive models with a competitive final goods sector; and (2) and that all consumers gain the same utility per unit of expenditure.

Distortions to TFP   The reason for interpreting the expression for $Z(t)$ in (24) as a measure of aggregate TFP is how it enters output of the composite good (27). In a representative good model, the factor $Z(t)$ in (27) directly reflects aggregate TFP as the residual of a growth regression. In a model with multiple intermediate goods, by contrast (24) decomposes the expression into effects from physical TFP and an effect from the number of varieties.

In the case of $N > 1$, the factor $Q(t)B(t)^{-1}$ is a measure of average quality adjusted for awareness and markup distortions. The wedge in productivity, factor shares, etc. enters through $Q(t)$ and $B(t)$ as calculated from the age distribution, $\Phi(t, a)$. A hypothetical TFP growth regression using (27) with a misspecified expression for $Z(t)$ in (24) that accounts only for variety effects might wrongly attribute changes in $B(t)$ and $Q(t)$ from changes in the industry age distribution to residual (physical) TFP.

Note that in the case of monopolistic competition with full awareness (i.e., $f_0(a) = 0$ and $N = 1$ with constant markups), $Q(t)B(t)^{-1} = 1$ for all t, and hence there is no distortionary term. This is well understood in models of monopolistic competition: Constant markups do not distort aggregate allocations when there is inelastic labor supply.

3.2   Consumers' Problem with Investment

With the static equilibrium fully aggregated to a "composite" good in Section 3.1, I will complete the model by nesting the dynamic decisions described in Section 2.3.[31]

Given the aggregation, this section is almost entirely standard. There are two stocks for the planner to choose: (1) $M(t)$, which depreciates at rate δ_M and is produced with relative productivity $z_M(t)$; and (2) $k(t)$, which depreciates at rate δ_k and is produced with relative productivity $z(t)$.

Proposition 5 (Problem of the Representative Agent). *Given initial conditions* $k(0), M(0),$

---

[31]As discussed in more detail in Section 2.3 and Technical Appendix A.6, this is a constrained planning problem, where the planner is subject to the market structure of the differentiated goods but can choose the allocations of capital and variety investment to represent the consumers' preferences.

and $\Phi(0, a)$, the representative consumer solves

$$\max_{C(t), i_k(t), i_M(t) \geq 0} \left\{ \int_0^\infty e^{-\rho t} \frac{1}{1 - \gamma} C(t)^{1-\gamma} \right\} \tag{28}$$

$$s.t. \, \boldsymbol{\partial}_t k(t) = -\delta_K k(t) + i_k(t) \tag{29}$$

$$\boldsymbol{\partial}_t M(t) = -\delta_M M(t) + z_M(t) i_M(t) \tag{30}$$

$$C(t) \equiv z(t) \underbrace{Q(t) B(t)^{-1}}_{\substack{Awareness \\ Wedge}} M(t)^{\frac{1}{\kappa-1}} k(t)^\alpha - i_k(t) - i_M(t), \tag{31}$$

where $\Phi(t, a)$ evolves according to the PDE in Technical Appendix B.5, and, in turn, determines $Q(t) B(t)^{-1}$ through (22) and (23).

Proof. The laws of motion are (9) and (10) with irreversibility constraints. The resource constraint is (8), using aggregate TFP and output from (24) and (27). The preferences are (3), with a substitution for the composite good aggregation from Section 3.1. See Proposition 7 in Technical Appendix B for the derivation and full solution with dynamics. $\qquad \square$

Other than the factor $Q(t) B(t)^{-1}$, Proposition 5 is isomorphic to a textbook neoclassical growth model with two stock variables (e.g., physical and human capital).[32] This shows that all of the effects of the product market friction on aggregate allocations are isolated in the wedge, which depends on the evolving age distribution of firms, $\Phi(t, a)$, and hence on the underlying awareness process $\mathbb{Q}$. The size of this distortion thus depends strongly on the age distribution of industries in the economy and the degree of connectedness of choice sets for industries of a given age.

Comparison to the Monopolistic Competition Baseline   Consider a variation on the monopolistic competition baseline, that is, $N = 1$, where we leave in the possible incomplete awareness wedge for that firm. With (22) and (23), note that the factor share distortion, $B(t) = (\kappa - 1)/\kappa$, is the inverse markup, and is independent of the distribution $\Phi(t, a)$ for any t. This is the standard distortionary effect on factor shares coming from the pure profits in monopolistic competition. The full wedge in that case is then $Q(t) B(t)^{-1} = \int_0^\infty (1 - f_0(a)) d\Phi(t, a)$. That is, a proportion $f_0(a)$ of the varieties of each age have been only incompletely assimilated into consumers' consumption bundles. In a sense, this wedge is not a distortion on allocations, as a planner constrained by the information structure would choose the same allocations (i.e., constant markups are constrained efficient, as discussed

---

[32]In particular, the neoclassical growth model with endogenous human capital accumulation, as in Acemoglu (2009), Proposition 10.1. Here, the product categories $M(t)$ are analogous to the level of human capital. As in the neoclassical model with human capital, $M(t)$ and $k(t)$ are maintained as a statically determined ratio to whatever extent reversibility in capital and product categories is allowed. The main difference comes out of the time variation of the awareness wedge, as captured in the factor $Q(t)^{-1} B(t)$.

in Technical Appendix A.6). With full awareness, the problem becomes undistorted and $Q(t)B(t)^{-1} = 1$ for all t.

3.3 Stationary Equilibrium with Aggregate Investment

The stationary equilibrium of Proposition 5 is parameterized by $\rho, \delta_k, \delta_M, \alpha, \kappa, \sigma$, and $\mathbb{Q}$ (which, in turn, encapsulating all awareness-related parameters). Additionally, z and z_M are required but only determine the scale, so I normalize to $z = 1$.

Proposition 6 (Stationary Equilibrium). *The stationary equilibrium is a steady-state mass of product categories, M, and capital, k, such that*

$$\delta_M - \delta_k = QB^{-1}k^\alpha M^{\frac{1}{\kappa-1}}\left(\frac{z_M}{\kappa-1}M^{-1} - \alpha k^{-1}\right) \tag{32}$$

$$\rho + \delta_k = \alpha QB^{-1}M^{\frac{1}{\kappa-1}}k^{\alpha-1} \tag{33}$$

Given k and M, the equilibrium C is

$$C = \underbrace{QB^{-1}}_{Wedge} M^{\frac{1}{\kappa-1}}k^\alpha - \delta_k k - \delta_M M/z_M, \tag{34}$$

where $\Phi(a) = 1 - e^{-\delta_M a}$, and $\Upsilon(a), q(a), Q$, and B are given by (15), (16), (22) and (23)— and parameterized by any $\mathbb{Q}$. The capital share, labor share, and profit share of output are $\alpha B, (1-\alpha)B$, and $(1-B)$, respectively.

Proof. See Technical Appendix B for the full proof and derivation. $\square$

As in Proposition 5, when $N = 1$ all distortions on the standard problem with aggregated monopolistic competition come in as the factor QB^{-1}, and disappear completely with full awareness. Hence the only distortions come through the incomplete diffusion of varieties to consumers' bundles—the same sort of variety effect common to trade models with market access or fixed export costs—and the market power has no allocative distortion. In all cases, the profit share with $N = 1$ is equal to $1 - B = \frac{1}{\kappa}$.

Illustrative Examples Figure 4 provides some comparative statics to analyze the forces in the model.[33] To get a sense for the role of the distortions, comparative statics are plotted for (1) the obsolescence rate δ_M, which governs the age distribution of product categories; (2) the differentiation σ, which determines the strength of sorting and drops in market power; and (3) θ, which determines both the growth rate of an industry and the degree of overlap of choice sets.

---

[33]As stated earlier, these are intended to be qualitative rather than quantitative exercises. See Table 2 in Technical Appendix E for a summary of how they are disciplined.
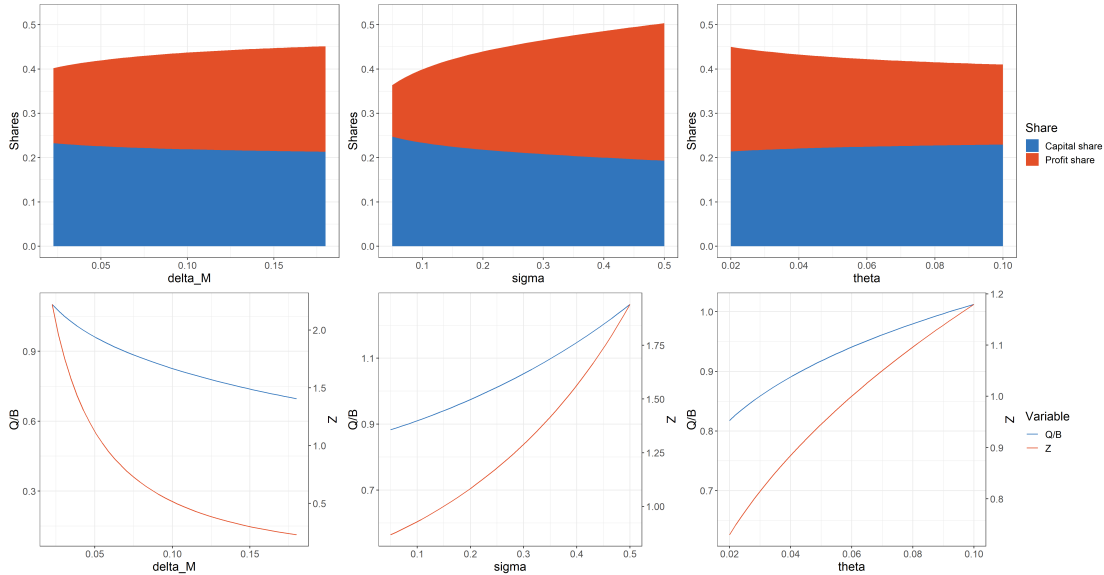
Figure 4: Comparative Statics with Exogenous Awareness

The δ_M panels in Figure 4 show the effects of the product age distribution on profit shares and distortions. As obsolescence rates rise, the profit share increases because of the higher average market power in the economy. Furthermore, the awareness wedge QB^{-1} becomes a drain on TFP and consumption, because of the allocation distortions from the markups and having fewer varieties , on average, in consumer choice sets. Of course, if newer cohorts had a higher TFP or quality than older ones, that would offset these losses, but it suggests caution in assuming that creative destruction has unambiguously positive results.

When differentiation is increased, as in the σ panels, we see the effects of market power and sorting. In particular, the profit share, TFP, and the value of QB^{-1} increase. The key is that high differentiation leads to more distortions from oligopolistic competition but also more room for the benefits of sorting.

Finally, increasing the exogenous rate θ in the process $\mathbb{Q}$ has unambiguous effects by rapidly undermining market power—which leads to a dropping profit share—and by increasing the benefits of sorting. While this might lead us to think that with the invention of better awareness/choice set technologies, we should see declining profit shares from these forces, I will show that this is not necessarily the case with the endogenous effort of Section 4.

4   Investment in Advertising Capital

The previous section suggests that, with an exogenous process for $\mathbb{Q}$, awareness and choice sets should grow faster and become larger with better advertising technology (radio, TV, sponsored Google ads, Facebook, etc.).

23

To explore these topics and understand the tension between sorting and market power, this section sketches a simple model of an advertising technology where firms can partially endogenize the awareness sets and match distributions of consumers. The goal for this section is a simple and tractable approximation as an extension of the core model, rather than a deep micro-foundation of the endogeneity.[34]

4.1 Sales and Marketing Technology

Investment in advertising capital here has two results: (1) It determines the rate at which consumers become aware of firm's products (parameterized by θ); and (2) it distorts the match type consumers who become aware of the firms' products (parameterized by μ). These distinct goals of advertising will be modeled separately to affect the choice set network orthogonally (i.e., the match distribution will be distorted conditioning on a particular set of awareness set sizes, and vice versa), but will interact in both firms' objectives and the costs.

I will concentrate on symmetric industry equilibria for the composition of advertising capital, (θ, μ), assuming a large number of firms where no individual firm has a profitable unilateral deviation. This requires specifying off-equilibrium profit and pricing functions in partial equilibrium, taking marginal costs and aggregate income as fixed.[35]

Control of Awareness Growth If all firms symmetrically choose the same θ, then the parameterized generator of the awareness process is denoted by $\mathbb{Q}(\theta)$. For easy comparison, I will simulate using the same numerical example as that of Example 1 from Section 3.3.

The "large number of firms" assumption ensures that the aggregate evolution of the awareness count is unaffected by individual deviations from the symmetric strategy. Off-equilibrium, however, if firm i chooses a larger awareness growth rate relative to the other firms, that is, $\theta_i/\theta > 1$, it skews the probability that firm i is added to awareness sets relative to the others.[36] With this approximation, I show in Technical Appendix B.8 that the demand function for a firm that chooses deviation θ_i from the symmetric strategy θ is

---

[34]In particular, I will not derive a bottom-up and micro-founded model of how the advertising technology would manifest in these functional forms for factoring the distribution of awareness and matches. That said, *given* the functional forms for the evolution of awareness/matching, the modeling of demand is done exactly, with no approximations.

[35]While I will evaluate the aggregate consequences for the awareness wedge as if every industry were in that equilibrium, I do not close the model in full general equilibrium with a connection between advertising investment and aggregate marginal costs/income. This is done, in part, to simplify the discussion, but also because the strong complementarities between advertising, output, and marginal costs lead to issues with multiplicity—a topic which is beyond the scope of this simple model. See Technical Appendix B.8 for more of a discussion on saddle-point issues when in full general equilibrium.

[36]If all N firms chose the same θ, then the probability that an individual firm would be added to a consumer's choice set (conditional on an arrival) is simply $1/N$, as reflected in Proposition 3. See Technical Appendix B.8, where I assume that deviations for asymmetric choices of θ can be modeled as an urn with a different "weighting" of the draws, that is, it becomes a Fisher's non-central hypergeometric distribution.

simply $\frac{\theta_i}{\theta} y(\cdot)$, where $y(\cdot)$ is the demand calculated with the symmetric θ.

Directed Advertising The other component of the advertising capital is the degree to which it targets some consumers at the expense of others. In particular, since all match heterogeneity is embedded in the distributions ξ_{ij}, firm i can pay to distort the awareness process so that it is more likely to be added to the choice sets of consumers with large ξ_{ij} matches and, consequently, less likely to be in those with poor matches.[37]

Let firm i have a distortion with parameter μ_i toward getting better matches in the choice sets, with μ_{-i} the associated distortion of other firms toward firm i's customers (i.e., $\mu_i > 1$ means that firm i has a better match toward its customers, whereas $\mu_{-i} < 1$ would mean that firms $i' \neq i$ have worse matches to i's customers). The demand for firm i, which is derived in detail in Technical Appendix A.2, is

$$y_i(a, p_i, p, \mu_i, \mu_{-i}) = (1 - f_0(a)) p_i^{-\kappa} \Omega \frac{1}{N} \mu_i^{\sigma(\kappa-1)} \mathbb{E}_a \left[\hat{n} \left(1 + \frac{\mu_{-i}}{\mu_i}(\hat{n} - 1) \left(\frac{p}{p_i} \right)^{-1/\sigma} \right)^{\sigma(\kappa-1)-1} \right] \quad (35)$$

The expression for the demand in (35) nests the symmetric version in (13) for $\mu_i = \mu_{-i} = 1$ and otherwise has only two differences: (1) scaling up of the average intensive demand by $\mu_i^{\sigma(\kappa-1)}$, which is the benefit of having better matches to consumers; and (2) the inclusion of the multiplier $\frac{\mu_{-i}}{\mu_i}$ on the awareness set counts, which affects market power. Intuitively, if a firm is able to skew the match distributions toward consumers with a better match, it gains market power and can increase its prices. This makes targeted advertising especially lucrative, even if at the expense of smaller awareness set sizes.

Cost and Timing of Advertising Capital Investment For simplicity, I assume that the firm makes its investment decision *at entry* by purchasing consumption goods to create "advertising capital." This capital provides advertising throughout the lifetime of the product, is irreversible, and does not depreciate (consequently, firms would never choose to scrap and rebuild the advertising capital in a stationary environment).[38] I solve for symmetric equilibria, in which all firms choose the same advertising capital size and composition, and with a sufficiently large number of firms per industry, N, to eliminate strategic considerations on those investment decisions.

The cost of building the technology is $d(\theta, \mu)/N$ consumption goods per firm, where the

---

[37]To model this, I distort the draw distribution for firm i so that it draws from a Gumbel distribution centered at $\log \mu_i$ rather than at $\log 1 = 0$. See Technical Appendix A.2 for more details.

[38]A more complicated model with control at every time period is possible, but this one-time decision simplifies the analysis of stationary equilibria. With aggregate shocks, such as in Gourio and Rudanko (2014a,b) and Drozd and Nosal (2012), this assumption would not be innocuous, as cyclical investment in advertising is central to their mechanisms.

scaling by $1/N$ is to ensure homotheticity when aggregated.

4.2 Industry Equilibrium

Each firm chooses (θ_i, μ_i) at $a = 0$ to maximize the PDV of profits that are discounted at rate $\rho + \delta_M$.

Proposition 7 (Symmetric Industry Equilibrium with Controlled Awareness). *Given aggregates Ω and mc, a symmetric industry equilibrium is a triple $(\theta, \mu, p(\cdot))$ such that for all i and a,*

$$(\theta, \mu) = \arg\max_{\theta_i, \mu_i} \left\{ \int_0^\infty e^{-(\rho+\delta_M)a} \frac{\theta_i}{\theta} (p_i(a|\mu_i) - mc) \underbrace{y_i(a, p_i, p, \mu_i, \mu_{-i})}_{\text{Use Deviation}} \, da - \frac{d(\theta_i, \mu_i)}{N} \right\} \quad (36)$$

$$p_i(a|\mu_i) = \arg\max_{\tilde{p} \geq 0} \left\{ (\tilde{p} - mc) y_i(a, \tilde{p}, p, \mu_i, \mu_{-i}) \right\} = p(a) \quad (37)$$

where $\boldsymbol{\partial}_a f(a) = f(a) \cdot \mathbb{Q}(a, f \,|\, \theta)$, and the baseline version has $\mu_{-i}(\mu) = 1$.

Proof. See the full specification and derivation in Technical Appendix B.8. The profits in the case of unilateral deviations are given in Technical Appendix B.8. □

Both the equilibrium structure and its computation are broken into three parts: First, given a symmetric industry θ, $\boldsymbol{\partial}_a f(a) = f(a) \cdot \mathbb{Q}(a, f \,|\, \theta)$ calculates the evolution of awareness. As discussed, because of the assumption of a large number of firms, I assume that a unilateral $\theta_i \neq \theta$ does not affect evolution of industry awareness. Next, (37) is the BNE in Definition 2, except with the demand function distorted by the choice μ_i, that is, using (35). This is necessary because of the change in market power with unilateral deviations μ_i.[39] The symmetric industry equilibrium requires that $p_i = p$ for all firms. Finally, (36) is the optimization problem upon entry of a firm, where the aggregate Ω and mc, together with the symmetric industry strategies μ, θ, p are taken as given (but μ_i is allowed to be used to drive unilateral deviations in the pricing game in (37)).

Analysis Equation (36) is a fixed point to ensure that the aggregate (θ, μ) is the optimal choice on entry for every firm. I will start by analyzing the choices of μ_i and θ_i individually, and then describe the complementarity.

First, consider the incentives to invest in θ_i if μ were fixed and no unilateral deviations were allowed. In that case, changes in θ_i do not affect the pricing strategy of (37), so the tradeoff in (36) is simply between the cost of deviating from the prevailing θ, which multiplies

---

[39]Given the decisions (θ_i, μ_i) of all firms, however, a static pricing game is still played at each a as the industry evolves (i.e., given the fixed policies at $a = 0$, there still are no dynamic pricing incentives, even if an individual firm may have an asymmetric p_i in the BNE for a particular off-equilibrium choice $\mu_i \neq \mu$).

the PDV of profits by θ_i/θ, and the cost of the policy. Given convexity in the function $d(\cdot)$, there may be an interior solution which is the optimal θ. In that case, technologies which make investment in θ easier have unambiguously positive benefits for consumers: (1) They become aware of both the first good, and better matches in their choice sets, faster; and (2) spillovers from overlapping awareness sets of other firms lead to declining market power of firms as the average awareness set size increases.

Second, consider the firm's incentives to invest in μ_i with a fixed θ. By (35), there is a clear benefit in higher demand stemming from skewing the sorting of customers, which is captured in the factor $\mu_i^{\sigma(\kappa-1)}$. The more subtle effect is that a unilateral deviation of μ_i changes the market power, increasing prices in the solution to (37). From the consumers' perspective then, the investment in μ has counteracting effects. They gain utility from having a higher expected matches but are hurt by the higher distortionary markups.

The effects of these forces will strongly depend on the specification of function $d(\theta_i, \mu_i)$ and the $\mu_{-i}(\mu)$. I will use a simple CES structure with quadratic returns to scale and $\mu_{-i}(\mu) = 1$:

$$d(\theta_i, \mu_i) = \left[\left(\frac{1}{\nu} \theta_i^\phi + \frac{1}{\eta} (\mu_i - 1)^\phi \right)^{\frac{1}{\phi}} \right]^2 \tag{38}$$

Here, $\nu > 0$ and $\eta > 0$ are the productivity of spreading awareness and directed advertising respectively, and $\phi > 0$ determines the substitutability in the production function.

4.3 Illustrative Comparative Statics

From the analysis above, it is clear that private investment in expanding awareness sets has beneficial spillovers by reducing market power in the economy. However, firms may prefer targeted advertising to broad advertising that reaches many. This leads to consumers that are happy with their matches, but also a socially undesirable lack of choice. This analysis hints at important policy questions relating privacy and market power.

Figure 5 provides an analysis of comparative statics of the parameters ν and η.[40] The figure demonstrates the key tradeoff, if the productivity of directed advertising, η, increases, then firms may substitute to invest in μ, leading to a decrease in θ can decrease. That is, innovations in directed advertising (e.g., Facebook) can lead to an equilibrium with better matches, and smaller choice sets.

---

[40]Here we could also calculate the wedge QB^{-1} and the profit share—assuming all industries follow the symmetric θ, μ strategy. These aggregates effectively average over the distribution $\Phi(a)$. The mc and Ω are also calculated with these parameters, and in fact are used to set the marginal costs for these figures, but since this is partial equilibrium and the investment $d(\cdot)$ is not included in any resource constraints, I cannot comment on welfare. See Technical Appendix B.8 for a description of why general equilibrium in this model is difficult because of strong complementarities. As the particular features of the comparative statics depend strongly on the cost function, see Technical Appendix B.9 for alternatives.
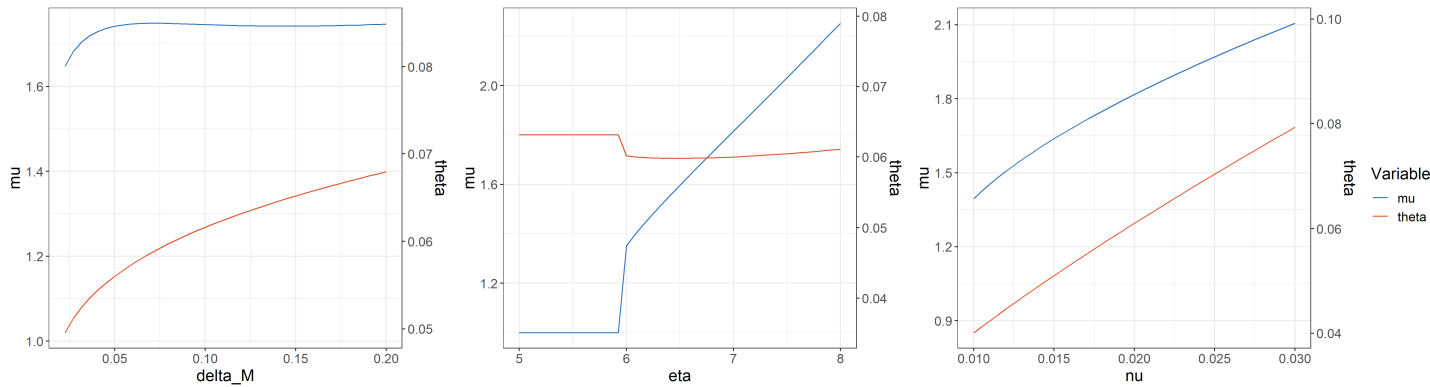
Figure 5: Comparative Statics with Endogenous Awareness

5 Conclusion

The primary contribution of this paper is a tractable theory of demand and information heterogeneity where demand functions are derived from a network of awareness sets (isomorphically, choice or consideration sets) between consumers and firms. When this network is less than fully dense, two frictions end up aggregated in demand functions: (1) firms exploiting market power inherent in incomplete choice sets; and (2) consumers sorting into their preferred products. In a simple extension with targeted advertising, I show that the consequences of technological innovation such as targeted advertising may be better sorting but at the cost of greater market power.

References

ACEMOGLU, D. (2009): *Introduction to Modern Economic Growth*, no. v. 1 in Introduction to Modern Economic Growth, Princeton University Press.

ARGENTE, D., M. LEE, AND S. MOREIRA (2018): "How do firms grow? The life cycle of products matters," .

ARKOLAKIS, C. (2010): "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of Political Economy*, 118, pp. 1151–1199.

——— (2016): "A unified theory of firm selection and growth," *The Quarterly Journal of Economics*, 131, 89–155.

ATKESON, A. AND A. BURSTEIN (2007): "Pricing-to-Market in a Ricardian Model of International Trade," *American Economic Review*, 97, 362–367.

——— (2008): "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, 98, 1998–2031.

BERGEMANN, D. AND J. VÄLIMÄKI (2006): "Dynamic Pricing of New Experience Goods," *Journal of Political Economy*, 114, 713–743.

BERNARD, A. B. AND T. OKUBO (2016): "Product Switching and the Business Cycle," Working Paper 22649, National Bureau of Economic Research.

BERNARD, A. B., S. J. REDDING, AND P. K. SCHOTT (2010): "Multiple-product firms and product switching," *American Economic Review*, 100, 70–97.

BRODA, C. AND D. E. WEINSTEIN (2010): "Product Creation and Destruction: Evidence and Price Implications," *American Economic Review*, 100, 691–723.

BURDETT, K. AND M. G. COLES (1997): "Steady State Price Distributions in a Noisy Search Equilibrium," *Journal of Economic Theory*, 72, 1 – 32.

BURDETT, K. AND K. L. JUDD (1983): "Equilibrium Price Dispersion," *Econometrica*, 51.

DE LOECKER, J. AND J. EECKHOUT (2017): "The rise of market power and the macroeconomic implications," Tech. rep., National Bureau of Economic Research.

DE LOS SANTOS, B., A. HORTAÇSU, AND M. R. WILDENBEEST (2012): "Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior," *American Economic Review*, 102, 2955–80.

DINLERSOZ, E. M. AND M. YORUKOGLU (2012): "Information and Industry Dynamics," *American Economic Review*, 102, 884–913.

DROZD, L. A. AND J. B. NOSAL (2012): "Understanding International Prices: Customers as Capital," *American Economic Review*, 102, 364–95.

EATON, J., M. ESLAVA, C. J. KRIZAN, M. KUGLER, AND J. TYBOUT (2014): "A Search and Learning Model of Export Dynamics," .

EDMOND, C., V. MIDRIGAN, AND D. Y. XU (2015): "Competition, Markups, and the Gains from International Trade," *American Economic Review*.

FABER, B. AND T. FALLY (2017): "Firm Heterogeneity in Consumption Baskets: Evidence from Home and Store Scanner Data," Working Paper 23101, National Bureau of Economic Research.

FAJGELBAUM, P., G. M. GROSSMAN, AND E. HELPMAN (2011): "Income Distribution, Product Quality, and International Trade," *Journal of Political Economy*, 119, 721 – 765.

GABAIX, X., D. LAIBSON, D. LI, H. LI, S. RESNICK, AND C. G. DE VRIES (2016): "The impact of competition on prices with numerous firms," *Journal of Economic Theory*, 165, 1 – 24.

GILBUKH, S. AND P. ROLDAN (2017): "Firm Dynamics and Pricing under Customer Capital Accumulation," .

Goeree, M. S. (2008): "Limited Information and Advertising in the U.S. Personal Computer Industry," *Econometrica*, 76, 1017–1074.

Goolsbee, A. D. and P. J. Klenow (2018): "Internet rising, prices falling: Measuring inflation in a world of e-commerce," in *AEA Papers and Proceedings*, vol. 108, 488–92.

Gourio, F. and L. Rudanko (2014a): "Can Intangible Capital Explain Cyclical Movements in the Labor Wedge?" *American Economic Review*, 104, 183–88.

——— (2014b): "Customer Capital," *The Review of Economic Studies*, 81, 1102–1136.

Kaplan, G., G. Menzio, L. Rudanko, and N. Trachter (2019): "Relative price dispersion: Evidence and theory," *American Economic Journal: Microeconomics*, Forthcoming.

Kaplan, G. and S. Schulhofer-Wohl (2017): "Inflation at the household level," *Journal of Monetary Economics*, 91, 19–38.

Kleinrock, L. (1975): *Theory, Volume 1, Queueing Systems*, Wiley-Interscience.

Klemperer, P. (1995): "Competition When Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade," *Review of Economic Studies*, 62, 515–39.

Luttmer, E. G. (2006): *Consumer search and firm growth*, Minneapolis Fed Working Paper 645.

Mahajan, V., E. Muller, and F. M. Bass (1990): "New Product Diffusion Models in Marketing: A Review and Directions for Research," *Journal of Marketing*, 54, pp. 1–26.

Michelacci, C., L. Paciello, and A. Pozzi (2019): "The extensive margin of aggregate consumption demand," .

Mongey, S. (2018): "Market structure and monetary non-neutrality," .

Paciello, L., A. Pozzi, and N. Trachter (2019): "Price dynamics with customer markets," *International Economic Review*, 60, 413–446.

Peters, M. (2018): "Heterogeneous Markups, Growth and Endogenous Misallocation," .

Rob, R. and A. Fishman (2005): "Is Bigger Better? Customer Base Expansion through Word-of-Mouth Reputation," *Journal of Political Economy*, 113, pp. 1146–1162.

Rudanko, L. (2017): "Pricing in a frictional product market," .