A Longitudinal Study of the Effect of Self Control on Mathematical Ability of Hispanic Students in Early Childhood Education

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Abstract

Development of mathematical ability in early childhood education is crucial for producing students with high technical competencies for science, technology, engineering, and mathematics (STEM) fields. Hispanic children in particular have been found to have lower mathematical proficiency in the early grades; additionally, a student's self control has been previously linked with their academic achievement. This study took previous research a step further to determine how self control affected the development of mathematical ability over time. Mathematical ability and self control were assessed for 3,761 Hispanic students at five different times between fall of Kindergarten and spring of 5^{th} grade. A generalized linear model with an unstructured covariance matrix was used to create a parsimonious model linking math performance to time, age at entry to Kindergarten, gender, and self control.

It was ultimately found that higher self control levels did predict higher mathematical performance as well as a higher rate of increase in mathematical ability over time for Hispanic students. Additionally, Kindergarten entry age, while not the primary predictor of interest, was found to have a large impact on mathematical ability which persisted over the course of this study. Finally, perhaps most importantly, it was found that a small subset of students had uncharacteristically high mathematical ability at the fall of Kindergarten. Better understanding the reasons behind this initial success might prove to be the most beneficial area for future research. The results from this study may help educators develop more inclusive and effective educational strategies; strategies specifically geared towards those students with lower self-control may help lessen the gap in mathematical ability.

Introduction

A solid mathematics foundation in elementary school is crucial for more complicated concepts in higher education. A better understanding of the characteristics of students who display high mathematical ability in elementary school could help lead to improvements for students who struggle with math. This research examined the possible link between a student's self control and their mathematical ability over time, specifically within the Hispanic student body. The goal was to determine if students with higher self control not only had higher mathematical ability, but also had a higher rate of increase in mathematical ability over time.

Hispanic students in particular have been shown to have lower mathematical ability in early childhood education in previous studies. As immigration from Mexico, Central America, and South America continues, the Hispanic population in the United States continues to grow, ensuring these students have the proper mathematical abilities is crucial for their success as well as the success of the technical sectors of the United States.

Background

Previous research has shown a link between educational achievement and gender, ethnicity, and self-control in early childhood education. Girls tend to have higher levels of self-control, and self-control increases as children age (Swanson, 2014 p. 1935). Hispanic children tend to have lower mathematical proficiency in the early grades (Valiente, 2007 p. 6). Perhaps most importantly, self-control has been tied to academic achievement, with higher self-control corresponding to higher achievement (Swanson, 2014 p.1934).

The goals of prior research were often to tie self-control to various parenting methods and other social behaviors as these are more actionable control mechanisms. Research by Valiente et al. (2007) was a concurrent study that linked self-control, whether students liked school, and academic achievement, but the study failed to explore the effect of self-control on changes in educational achievement over time. Similarly, a study by Swanson et al. (2014), though longitudinal in nature, sought to develop a mediation model to show that self-control in first grade mediated the relationship between parental responses to emotion in Kindergarten and mathematical achievement in second grade. Though informative, this model still did not address the question of how self-control may affect achievement over time.

Thus, this study expanded upon the previous research by developing a single parsimonious model for mathematical achievement over time specifically for Hispanic children. From previous research, it was expected that self-control would have a significant impact on mathematical achievement, but it was not clear how self-control related to changes in achievement over time. Do students with high self-control not only perform better on achievement tests but also increase their proficiency at a higher rate over time? Or do the effects of self-control diminish as children age? Additionally, the effect of self-control may differ over time for males and females. This research attempted to address these questions and more.

Sample Information

Results from the Early Childhood Longitudinal Study – Kindergarten Class of 1998-1999 from the National Center for Education Statistics were used for this analysis. There were 21,409 total students followed in the study at many different public and private schools across the United States, but only 3,761 Hispanic students were included and were the focus of this research. Data were collected in seven waves: fall of Kindergarten, spring of Kindergarten, fall of 1st grade, spring of 1st grade, spring of 3rd grade, spring of 5th grade, and spring of 8th grade. Two variables of interest in this study, mathematical achievement and self-control, were both assessed at only five of the seven collection waves, and thus only these time periods were examined. The two waves which were missing one or both of these metrics were fall of 3rd grade and spring of 8th grade. The other variables of interest in this study (gender and age at entry to Kindergarten) were not time varying and thus largely available.

Since the waves of data collection were not equally spaced, time was re-coded in such a manner as to provide some indication of actual passage of time between collection periods. The grade was used (Kindergarten = 0, 1st grade = 1, and so forth) with fall collection waves corresponding to whole values and spring collection waves corresponding to half values (for example, spring of 3rd grade would have a time value of 3.5).

Mathematical achievement was assessed using Item Response Theory (IRT). IRT takes into account the difficulty, guess-ability (i.e. the probability someone will answer a question correctly without knowing the answer), and discrimination (the ability of a question to correctly separate those whose ability is above or below the difficulty level) to determine the probability that a student with a given level of ability answered the question correctly. These probabilities were used to calculate students' IRT scores at each time. The probabilities were modeled by pooling all time points together to ensure scores would be comparable across all time points – a key consideration for a longitudinal study (Rock, 2002 p. 55-60). The mean math IRT scores, by gender, for Hispanic students are shown in Figure 1 below; as expected, the math scores increased over time, though perhaps not linearly. Though males and females appear to score similarly early-on, males appear to do slightly better by the end of this study.

Figure 1. Mean Math Score by Gender

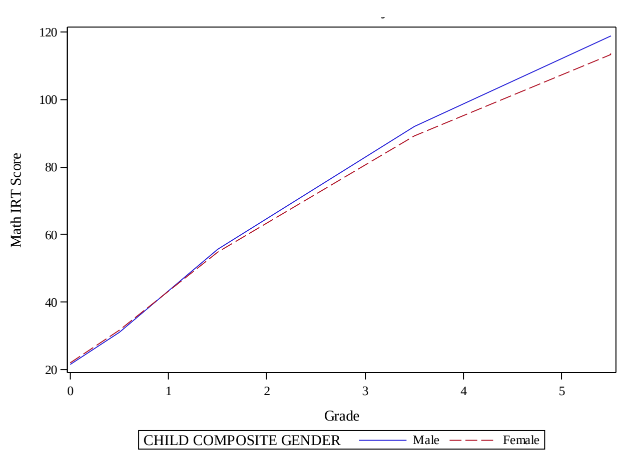
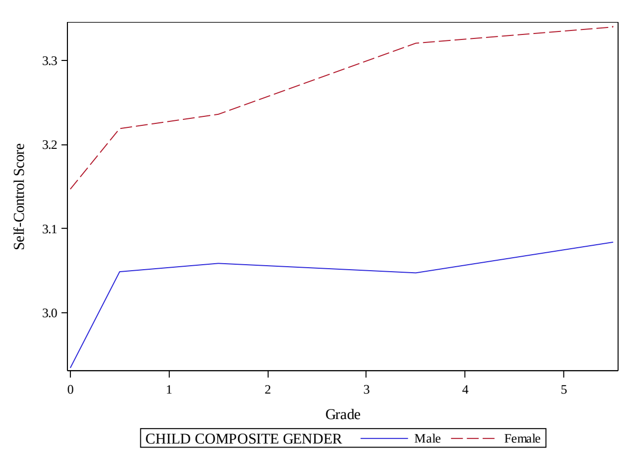


Figure 2. Mean Self Control by Gender



A student's self-control was assessed using an adaptation of the Social Skills Rating System. Both parents and teachers completed a survey which asked them to select how often a given behavior was demonstrated, ranging from never (score of 1) to very often (score of 4) (Rock, 2002 p. 110-111). Mean self-control over time by gender, in Figure 2 above, matches previous patterns seen in previous studies. Females tended to have a higher level of self-control than males at all time points in this study. Both genders appear to have undergone a rapid increase in self-control between the fall and spring of Kindergarten, but the rate of change decreases after this point, perhaps more so for males.

Methodology

One of the main goals of this research was to produce a single parsimonious model relating self-control to mathematical achievement. To do this, a generalized linear model was used with several differences compared to ordinary least squares (OLS) regression. First, OLS regression assumes that each observation is independent and uncorrelated with other observations. In longitudinal studies, however, repeated measures are performed on the same individuals over time. Thus, observations are not independent, and observations from the same individuals are typically positively correlated. For example, a student who has a high math score in Kindergarten often has high math scores in all grades. Second, simple linear regression assumes constant variance for the response; this assumption is often not tenable for longitudinal studies. In many cases, variability increases over time, and thus it may be necessary to use a different variance at different time points.

For example, Figure 3 shows individual level math scores over time for a few select students. Though math scores increase over time for nearly all students, there is considerable variability in the initial math score and the rate of increase. Figure 4 shows individual self-control scores over time for the same select students. The patterns for self-control are even more variable with some increasing and some decreasing over time, with lots of variability even within the same individual. Additionally, it should be clear from both graphs that many individuals have missing data. Some individuals have far fewer than 5 observations. Some individuals, such as the student with ID 1189007C, are missing math or self control scores but not at the same occasions. These missing data can cause potential complications with the analysis.

There was substantial drop out in this study as well as intermittent missing data for students who did not drop out. Table I below gives a breakdown of the missing data by collection wave for the math IRT score. Students who were considered to have dropped out had no data available at a given data collection wave and all subsequent waves. The other students with missing data had scores recorded in a future data wave. As can be seen, only 56.0% of students had math scores recorded in the 5th grade data collection wave. However, an additional 5.4% of these students were missing the self-control assessment score, leaving only 50.7% of the initial study population for this time point.

For the model to be valid, the missing data mechanism must be missing at random (MAR) or missing completely at random (MCAR). For data to be MCAR, missing responses may be related to observed values of the explanatory variables, but not to the observed or unobserved responses. For MAR, the missingness may be related to both the explanatory and observed responses, but not the unobserved responses.

Figure 3. Individual Level Graphs of Math Scores for Select Students

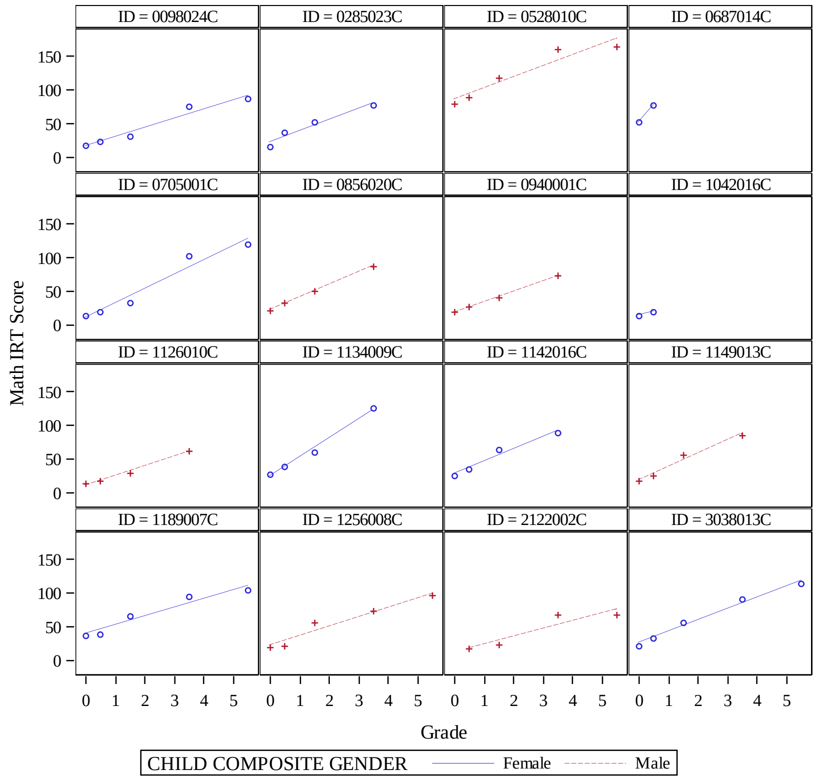
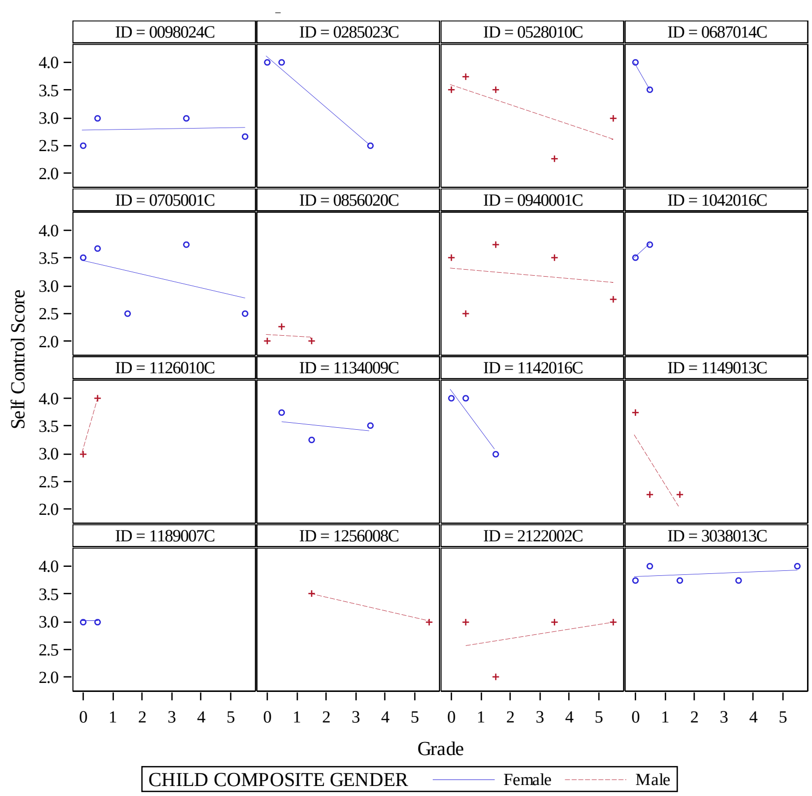


Figure 4. Individual Level Graphs of Self Control Scores for Select Students



Status	K – Fall	K – Spring	1^{st} – Spring	3^{rd} – Spring	5^{th} – Spring
Not Missing	3399 (90.4%)	3470 (92.3%)	2880 (76.6%)	2597 (69.1%)	2106 (56.0%)
Dropout	0 (0%)	163 (4.3%)	723 (19.2%)	1051 (27.9%)	1556 (41.4%)
Other Missing	362 (9.6%)	128 (3.4%)	158 (4.2%)	113 (3.0%)	99 (2.6%)

Table I. Frequency of Missing Data and Drop Out by Data Collection Wave

Logistic regression was used to determine if the probability of a student dropping out was related to previous math score responses. Prior math scores were found to have a significant effect on the probability of drop out. For example, for drop out in the spring of Kindergarten, the previous math score in the fall of Kindergarten was a significant factor ($p = 0.0286$) for predicting the log odds of drop out. For each 10 point increase in the fall math IRT score, the odds of a student remaining in the study were 1.37 times higher. Thus, students who had lower math scores were more likely to drop out of the study. Similar results were achieved at the other time points.

Thus, the missing data pattern was clearly not MCAR. Since it was not possible to follow up on the individuals who dropped out, some assumptions about the missing data mechanism had to be made. There are many reasons why a student may have dropped out of a study. The student's family may have relocated to a different school district. The student's parents may have decided to stop participating because their student was struggling in school and did not want the study to be a distraction. In either of these cases, the missing data mechanism would not necessarily be related to the unobserved responses. It was assumed for this analysis that the data were missing at random.

For the actual model building process, a maximal model including age at entry to Kindergarten, gender, self-control , and time, as well as all three-way interactions, was constructed using restricted maximum likelihood (REML). Because of the correlated nature of the observations within individuals, a covariance structure allowing correlation between the repeated measures was used. Determining the proper covariance structure, using the maximal model for the mean, was the first step of building the model. The least prohibitive covariance structure, unstructured covariance, allowed each time to have a unique variance and each pair of points to have a unique correlation. However, this flexibility came at a cost of 15 unique parameters that must be estimated. Less complex structures for the covariance were compared to the unstructured model using a likelihood ratio test to determine if a simpler structure was adequate. The compound symmetry heterogeneous structure was trialed, which allows unique variances at each time but assumes a constant correlation between all time points. However, this structure was too simple. The likelihood ratio test for compound symmetry heterogeneous versus the unstructured covariance yielded a test statistic of 798.7 with 9 degrees of freedom ($p < 0.0001$). Thus, the more complex, but necessary, unstructured covariance was used in the model.

Reduction to the final model was made using a backwards selection type approach, only retaining those terms where the Type 3 Analysis of Effects indicated a significant effect (using $\alpha = 0.05$). REML and the unstructured covariance were used throughout. When a potential final model was reached, it was compared to the maximal model using generalized linear hypotheses. The test for the simpler model yielded a test statistic $F(16, 3108) = 1.48$, with $p = 0.0981$; thus, the reduced model was found to be adequate. Time was treated as a factor rather than a continuous variable in the model. Various attempts were undertaken to treat time as a linear effect, a quadratic effect, and a linear spline effect; however, none of these produced an adequate fit versus treating time as a factor.

Validity of the final model was assessed by residuals analysis. Because of the non-constant variance and correlation among observations, analysis of the raw residuals can be difficult and misleading. Therefore, Cholesky residuals were used, which standardize and adjust for these factors.

Results

The estimated covariance and correlation matrices are shown in Tables II and III respectively. The variances at each time point are on the diagonal of the covariance matrix, and they show how much the variance grows over time. For the fall of kindergarten, the estimated variance for the conditional math IRT scores is only 50.1; in the spring of 5th grade, the variance is over ten times larger at 550.5. The correlation matrix reveals that the observations were strongly correlated between grades, but this correlation lessens over time. For example, kindergarten fall math IRT scores have a correlation of 0.79 with spring kindergarten IRT scores; this correlation decreases to 0.57 for fall of Kindergarten and spring of 5th grade scores, though it is still quite substantial.

Table II. Estimated Covariance Matrix

Time	*0*	*0.5*	*1.5*	*3.5*	*5.5*
0	50.1				
0.5	55.7	98.4			
1.5	69.4	108.9	235.7		
3.5	98.2	152.3	260.3	511.6	
5.5	95.1	149.2	252.1	448.6	550.5

Table III. Estimated Correlation Matrix

Time	*0*	*0.5*	*1.5*	*3.5*	*5.5*
0	1				
0.5	0.79	1			
1.5	0.64	0.72	1		
3.5	0.61	0.68	0.75	1	
5.5	0.57	0.64	0.70	0.85	1

The final model for math IRT scores included main effects for time, gender, self-control, and age at entry into kindergarten as well as interactions between gender and time, self-control and time, and kindergarten entry age and time. The parameter estimates are in Table IV on the next page, and the general model form is as follows:

$$\widehat{Math\,IRT} = \widehat{\beta_0} + \widehat{\beta_{1\iota}} * time + \widehat{\beta_{2J}} * gender + \widehat{\beta_3} * control + \widehat{\beta_4} * age + \widehat{\beta_{5\iota J}} * gender * time + \widehat{\beta_{6\iota}} * control * time + \widehat{\beta_{7\iota}} * age * time$$

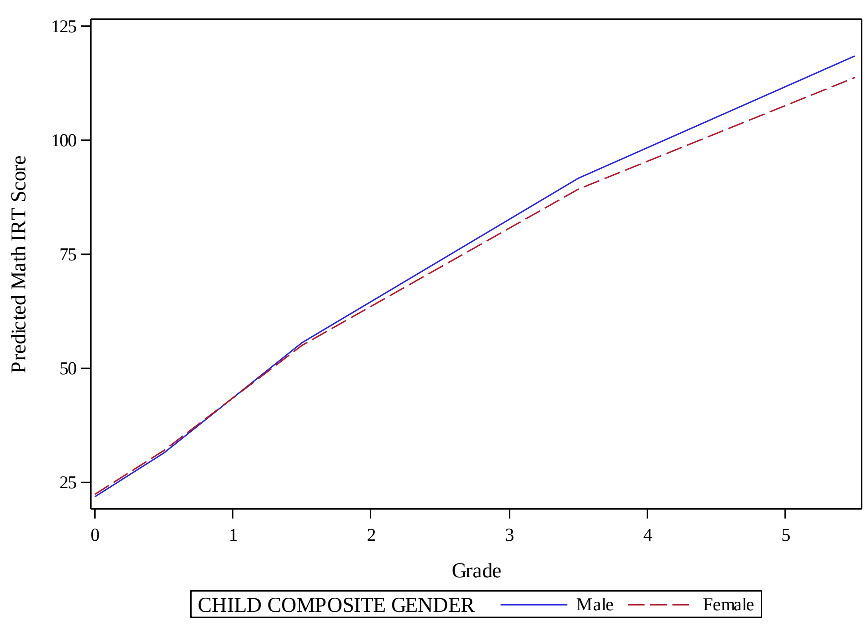
where *MathIRT* is the estimated mean math IRT score, *time* is the assessment time or grade (and a categorical factor), *gender* is a binary factor that is 1 for females and 0 for males, *control* is the self control score, and *age* is the age at entry to Kindergarten in months.

Thus, the change in math performance over time differed between the sexes. Figure 5 below shows the predicted average response profiles for males and females both at the mean Kindergarten entry age and mean self-control level for each time point. Even when Kindergarten entry age and self control are adjusted for, differences between the sexes persevere and appear most noticeable at later grades. In the spring of 5th grade, the estimated math score was 4.7 points higher for males than for females, controlling for Kindergarten entry age and self-control. In the fall of Kindergarten, the difference between males and females is negligible, with females estimated to score 0.5 higher than males on math assessments on average.

Table IV. Solution for Fixed Effects						
Effect	**Gender**	**Time**	**Estimate**	**Std Error**	**t Value**	**Pr > |t|**
Intercept			67.22	7.847	8.57	<.0001
Gender	Female		-4.73	1.004	-4.71	<.0001
Time		0	-72.32	7.171	-10.08	<.0001
Time		0.5	-72.47	6.844	-10.59	<.0001
Time		1.5	-60.43	6.511	-9.28	<.0001
Time		3.5	-35.43	5.782	-6.13	<.0001
Self Control			3.45	0.580	5.95	<.0001
Age			0.62	0.117	5.29	<.0001
Gender*Time	Female	0	5.26	0.913	5.77	<.0001
Gender*Time	Female	0.5	5.26	0.868	6.06	<.0001
Gender*Time	Female	1.5	4.14	0.820	5.05	<.0001
Gender*Time	Female	3.5	2.33	0.730	3.19	0.0014
Control*Time		0	-3.17	0.596	-5.31	<.0001
Self Control*Time		0.5	-2.85	0.603	-4.73	<.0001
Self Control*Time		1.5	-2.25	0.656	-3.44	0.0006
Self Control*Time		3.5	-1.29	0.696	-1.85	0.0648
Age*Time		0	-0.22	0.106	-2.05	0.0407
Age*Time		0.5	-0.083	0.101	-0.82	0.4140
Age*Time		1.5	0.076	0.0953	0.79	0.4274
Age*Time		3.5	0.20	0.0827	2.39	0.0167

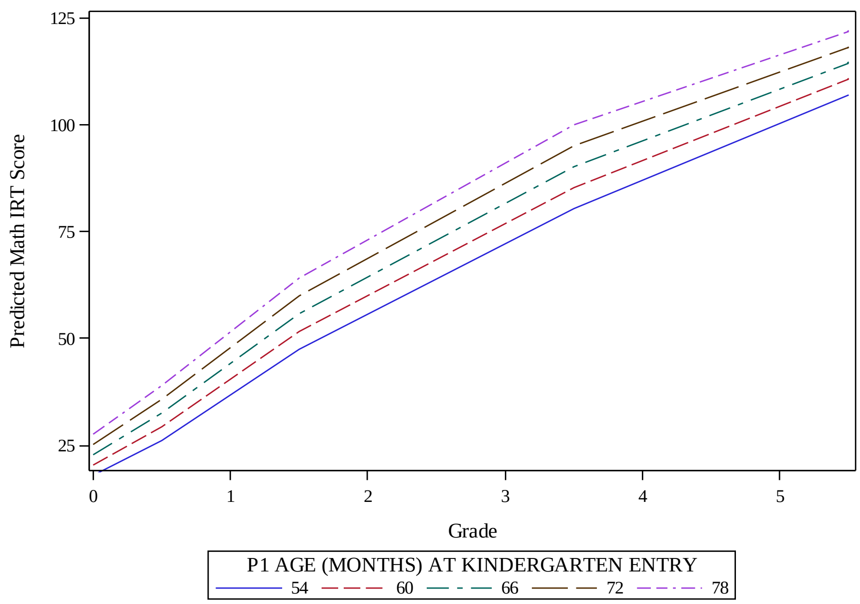
Note: Age = the age at entry to Kindergarten in months.

Figure 5. Predicted Average Math Performance by Gender



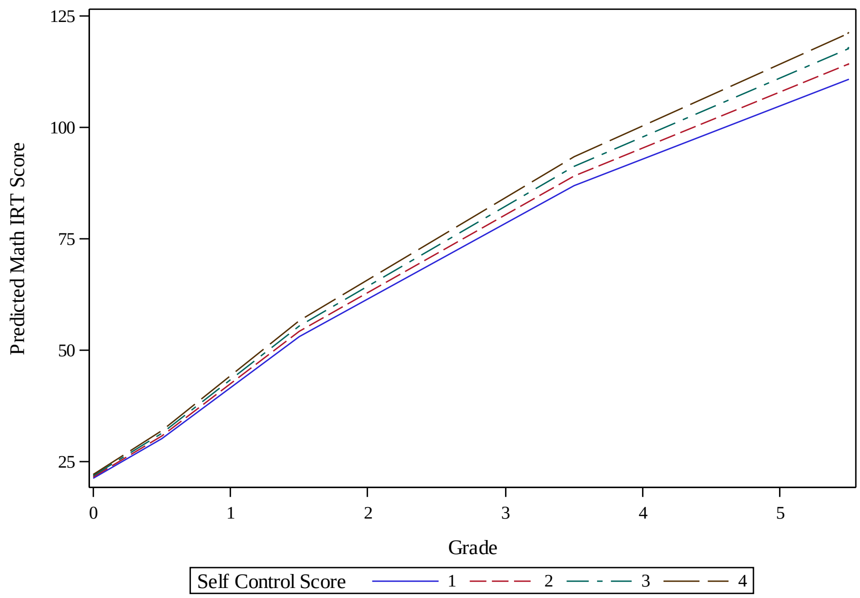
Kindergarten entry age had perhaps the most profound and surprising effect on changes in the mean math scores over time. Figure 6 shows estimated mean math scores for females using average self-control levels at each time and a range of entry ages that are representative of those in the study (note the plot for males is very similar and thus not shown). Older children were expected to perform better in Kindergarten, but it was not anticipated that this effect would carry through to such a large degree in subsequent grades. Indeed, older children showed higher mathematical ability at every time occasion, with this effect being most pronounced at the spring of 3rd grade measurement occasion. For the spring of 3rd grade, the estimated math score increased 9.8 points on average for every year increase in age, everything else being equal; in the fall of Kindergarten, the estimated average math score only increased 4.8 points on average for a one year increase in age, everything else being equal.

Figure 6. Predicted Average Math Performance by Kindergarten Entry Age



Finally, these results indicated that self-control, the variable of primary interest, was also related to different rates of increase in mathematics competency over time. The Type 3 analysis of effects (the full table of which can be found in the Appendix) found the time*self-control interaction to be significant with $p < 0.0001$. A plot of predicted values at different values of self-control (held constant over time) for males assuming average Kindergarten entry age shows that the effect of self-control becomes more pronounced over time (see Figure 7). Students who have a higher degree of self-control appear to have a higher rate of increase in mathematical ability than those with a low degree of self-control. In spring of Kindergarten, the estimated mean math score increases 0.6 on average for every one unit increase in self-control, everything else being equal. Considering the full range of self-control only spans three units, there does not appear to be a practically important difference based on a child's level of self control. However, for the spring of 5th grade, this difference is larger; math scores increased 3.45 points on average for a one unit increase in self-control.

Figure 7. Predicted Average Math Performance by Self Control Score

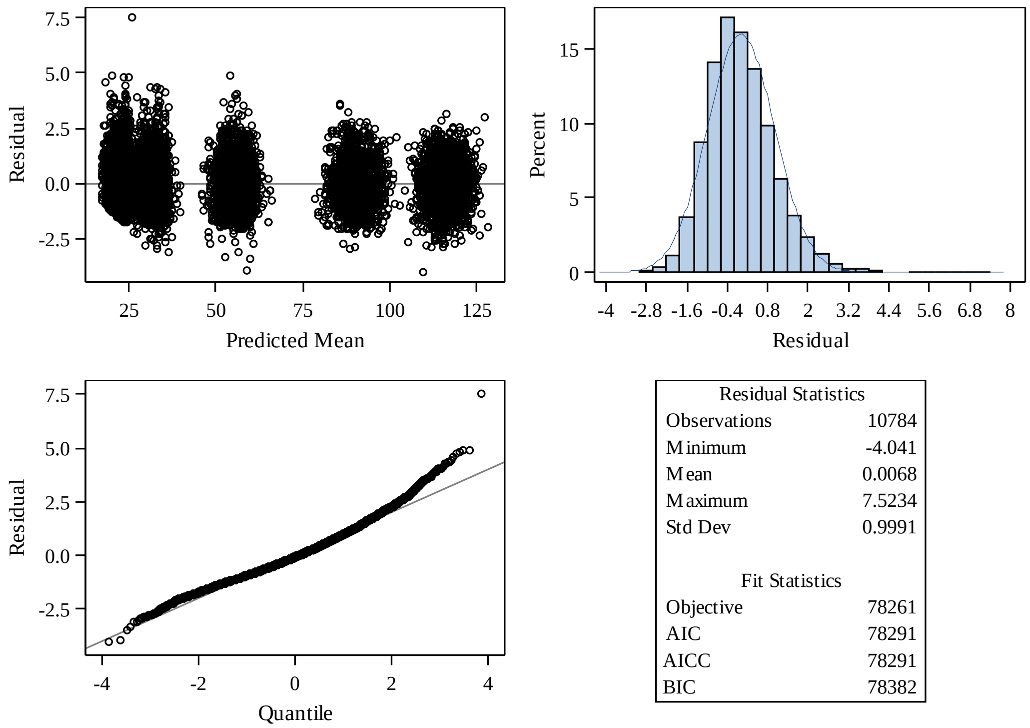


To assess the validity of the model, Cholesky residuals were reviewed. As Figure 8 below shows, there was some positive skew to the residuals, especially at low predicted values. One observation in particular had a Cholesky residual of 7.52. The individual level graphics for this student, a male with ID 0528010C, can be reviewed in Figures 3 and 4 above; this student had a very high math score in the fall of Kindergarten (79.3 versus the predicted mean value of 26.0), which is what produced the large residual. The other residuals for this student were reasonable. The model estimation was repeated deleting this one observation. The new table of parameter estimates can be found in the Appendix, but these were found to be very similar to the results obtained when this observation was included.

Further investigation into the Cholesky residuals revealed no major concerns with regards to patterns with gender or age at Kindergarten entry (see Appendix for corresponding graphs). However, the residuals versus time period did show a potential concern. Most of the positive skew, in the form of large positive residuals, appeared to be focused at the first time point, fall of Kindergarten. Indeed, normal quantile plots (Figures 9 and 10) at the different time points reveal significant non-normality in the residuals at the first time point, with the residuals becoming more and more normally distributed as time increases. By the last measurement occasion, the normality assumption appears very reasonable (Figure 10). Resolving this normality issue is quite difficult since a transformation of the response may fix the issues at the first time point but cause later time points to no longer be normally distributed.

Essentially, the issue for the fall of Kindergarten testing comes down to a subset of students doing much better than expected on the initial mathematics testing. Logically, their performance may be the result of a number of circumstances, and inclusion of this unknown factor would likely resolve the fit issues observed. Some potential causes might include prior mathematics exposure, either at home or through a pre-school program, or simply an innate gift for mathematical reasoning. Both of these possible explanations could be tested in a future study where learning experiences prior to Kindergarten as well as gifted-talented status (or similar testing results) are included as variables in the study.

Figure 8. Summary Plots of Cholesky Residuals



Residual Statistics	
Observations	10784
Minimum	-4.041
Mean	0.0068
Maximum	7.5234
Std Dev	0.9991
Fit Statistics	
Objective	78261
AIC	78291
AICC	78291
BIC	78382

Figure 9. Normal Quantile Plot of Cholesky Residuals for Fall of Kindergarten Responses

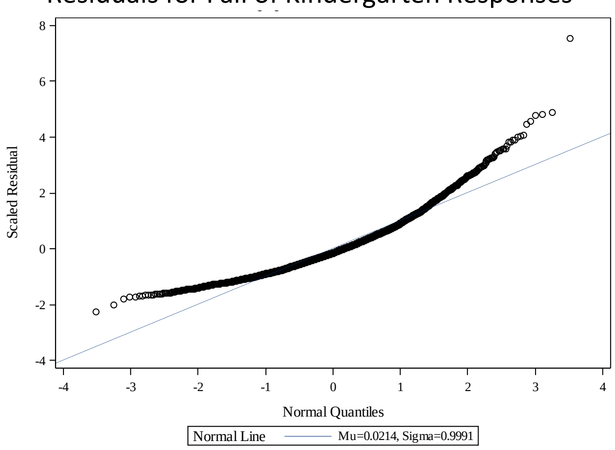
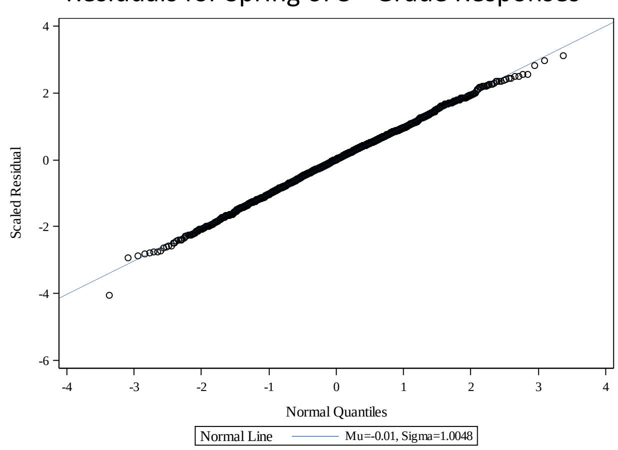


Figure 10. Normal Quantile Plot of Cholesky Residuals for Spring of 5th Grade Responses



Conclusions

In conclusion, self control was found to have a significant impact on mathematics ability over time in the Hispanic student body for early childhood education. Higher self-control was linked to a higher level of mathematical ability, with these differences becoming more pronounced over time. Age at entry to kindergarten was also found to have a significant impact on mathematical ability over time, and this effect persisted well beyond kindergarten, producing the largest difference in average mathematical ability at the spring of 3rd grade. Finally, gender also appeared to relate to differences in average mathematical ability, with males displaying higher mathematical proficiency than females on average at the 3rd and 5th grade measurements.

These results have possible important implications. The natural course of action may be to consider what drives certain students to have higher levels of self-control over others, but another possible avenue would be to investigate alternative education methods that may be better suited to students with lower levels of self control. Finally, perhaps the most interesting finding from this study was the presence of many students with higher than expected mathematical ability at the fall of kindergarten. Future research should focus in on these students to try and better understand what drives such high performance with little exposure to public education.

References

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Swanson, J., Valiente, C., Lemery-Chalfant, K., Bradley, R. H., & Eggum-Wilkens, N. D. (2014). Longitudinal relations among parents' reactions to chlidren's negative emotions, effortful control, and math achievement in early elementary school. *Child Development, 85* (5), 1932-1947.

Valiente, C., Lemery-Chalfant, K., & Castro, K. S. (2007). Children's effortful control and academic competence: mediation through school liking. *Merrill-Palmer Quarterly, 53* (1), 1-25.

Appendix: Supplementary Tables and Figures

Table A1. Type 3 Tests of Fixed Effects for Final Model

Effect	Num DF	Den DF	F Value	Pr > F
Gender	1	3109	6.08	0.0137
Time	4	3109	26.25	<.0001
Self Control	1	3109	56.41	<.0001
Age	1	3109	94.66	<.0001
Gender*Time	4	3109	9.24	<.0001
Self Control*Time	4	3109	8.81	<.0001
Age*Time	4	3109	9.82	<.0001

Table A2. Parameter Estimates for Fixed Effects with Outlier Deleted

Effect	Gender	Time	Estimate	Standard Error	t Value	Pr > |t|
Intercept			67.18	7.847	8.56	<.0001
Gender	Female		-4.74	1.004	-4.72	<.0001
Time		0	-71.94	7.166	-10.04	<.0001
Time		0.5	-72.46	6.843	-10.59	<.0001
Time		1.5	-60.43	6.51	-9.28	<.0001
Time		3.5	-35.46	5.7820	-6.13	<.0001
Self Control			3.44	0.5800	5.94	<.0001
Age			0.62	0.117	5.29	<.0001
Gender*Time	Female	0	5.29	0.912	5.80	<.0001
Gender*Time	Female	0.5	5.27	0.868	6.07	<.0001
Gender*Time	Female	1.5	4.14	0.820	5.05	<.0001
Gender*Time	Female	3.5	2.33	0.730	3.20	0.0014
Self Control*Time		0	-3.17	0.596	-5.31	<.0001
Self Control*Time		0.5	-2.84	0.603	-4.72	<.0001
Self Control*Time		1.5	-2.25	0.656	-3.43	0.0006
Self Control*Time		3.5	-1.29	0.696	-1.86	0.0635
Age*Time		0	-0.22	0.106	-2.11	0.0352
Age*Time		0.5	-0.083	0.101	-0.82	0.4107
Age*Time		1.5	0.075	0.0953	0.79	0.4288
Age*Time		3.5	0.20	0.0827	2.40	0.0163

Figure A1. Cholesky Residuals versus Time

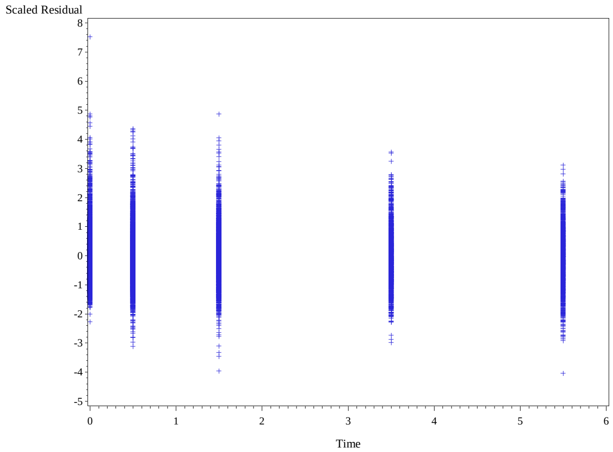

Figure A2. Cholesky Residuals versus Gender

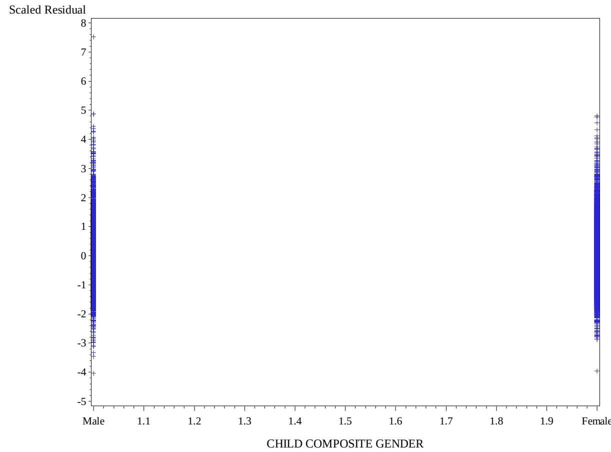

Figure A3. Cholesky Residuals versus Age at Kindergarten Entry

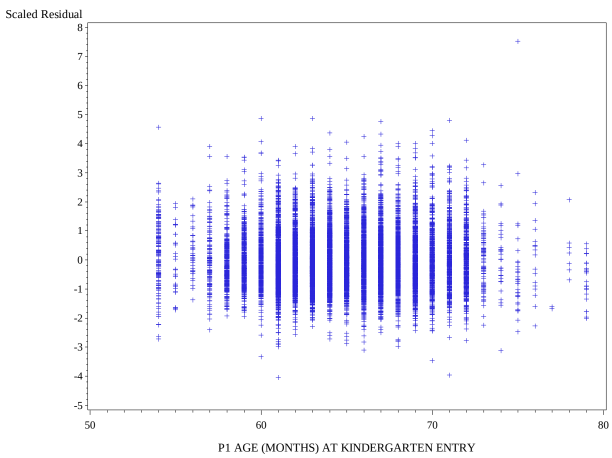

Figure A4. Cholesky Residuals versus Self Control