

# Asteroseismology of GD358 with complex C/O core profiles

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## ABSTRACT

We report on the analysis of 34 years of photometric observations of the pulsating helium atmosphere white dwarf GD358. The complete data set includes archival data from 1982-2006, and 1195.2 hours of new observations from 2007-2016. From this data set, we extract 15 frequencies representing g-mode pulsation modes, adding 4 modes to the 11 modes known previously. We present evidence that these 15 modes are  $\ell = 1$  modes, 13 of which belong to a consecutive sequence in radial overtone  $k$ . We perform a detailed asteroseismic analysis using models that include parameterized, complex carbon and oxygen core composition profiles to fit the periods. Recent spectroscopic analyses place GD358 near the red edge of the DBV instability strip, at  $24,000 \pm 500$  K and a  $\log g$  of  $7.8 \pm 0.08$  dex. The surface gravity translates to a mass range of 0.455 to 0.540  $M_{\odot}$ . Our best fit model has a temperature of 23,650 K and a mass of 0.5706  $M_{\odot}$ . That is slightly more massive than what the most recent spectroscopy suggests. We find a pure helium layer mass of  $10^{-5.50}$ , consistent with the outward diffusion of helium over time.

*Subject headings:* Stars: oscillations — Stars: variables: general — white dwarfs

## 1. Astrophysical Context

White dwarfs are the end product of evolution for around 98% of the stars in our Galaxy. Buried in their interiors are the records of physical processes that take place during earlier stages in the life of the star. Nuclear reaction rates during the core helium burning phase set the core composition of white dwarfs, while the relative time spent burning hydrogen and helium during the asymptotic-giant-branch (AGB) phase and mass-loss episodes determine the thickness of the helium layer (Lawlor & MacDonald 2006; Althaus et al. 2005). Helium atmosphere white dwarfs (DBs) comprise roughly 20% of the population of field white dwarfs, with most of the remaining 80% consisting of their hydrogen atmosphere (DA) cousins. One theory behind the bifurcation into two spectral classes is that during post-AGB evolution, a very late thermal pulse can burn off the residual hydrogen in the envelope, producing a nearly pure helium atmosphere (Iben et al. 1983). Such objects then return to the white dwarf cooling track as PG 1159 stars, which are widely believed to be the precursors of most DB white dwarfs. DBs are found to pulsate at effective temperatures ranging between 21,000 K and 28,000 K (Beauchamp et al. 1999; Castanheira et al. 2005).

The subject of this paper, GD358 (V777 Her) is the brightest ( $m_v = 13.7$ ) and best studied helium atmosphere white dwarf pulsator. It is located near the red edge of the instability strip, with a spectroscopic temperature of  $T_{\text{eff}} = 24000 \pm 500$  K and  $\log g = 7.8$  (Nitta et al. 2012; Koester 2013). GD358’s pulsation spectrum contains a series of independent radial overtones, many with complex frequency structure. For one epoch of data taken during a WET run, models involving magnetic fields and oblique rotation have been proposed to explain such structure (Montgomery et al. 2010).

Since the 2006 Whole Earth Telescope (WET) run reported in Provencal et al. (2009), we have maintained an active observing program of GD358 in an effort to identify additional frequencies in this complex star. These new observations have identified additional periods in GD358’s pulsation spectrum, bringing the total known independent radial overtones to 15. Thirteen of these modes belong to a consecutive  $\ell = 1$  sequence, the longest sequence observed in a DBV.

Paradoxically, among the DBVs with enough detected periods to be fitted asteroseismically, GD358 is the only one that has not been analyzed using the complex C/O profiles adapted and parameterized from stellar evolution calculations (e.g. Salaris et al. (1997); Althaus et al. (2005)). The most recent fits of GD358 (Metcalf et al. 2003) were performed using 11 observed modes and simple models where the oxygen abundance drops linearly from its central value to zero. We present here a new detailed asteroseismic analysis of GD358, taking advantage of its long sequence of  $\ell = 1$  modes to better constrain the asteroseismic fits and define the limits of high precision white dwarf asteroseismology.

The present analysis also allows us to place GD358 in the context of stellar evolution. According to the models, the precursors of DBs emerge from the born-again phase with envelopes containing a nearly uniform mixture of helium (He), carbon (C), and oxygen (O) out to the photosphere (Dreizler & Heber 1998; Herwig et al. 1999). As the PG 1159 stars cool, the helium diffuses upward and gradually accumulates to form a chemically pure surface layer. Through the DBV instability strip, this process is still ongoing so that instead of a pure helium layer surrounding the carbon and oxygen core, one has a region where the carbon and helium are still mixed. This leads to a double layered structure, with the pure He surface layer overlying the remainder of the uniform He/C/O envelope, all above the degenerate C/O core. A key prediction of the diffusion models is that, for a given stellar mass, the pure He surface layer will steadily grow thicker as the DB star cools. GD358 is the fourth DBV we can use to check this theory. The other three DBVs (Bischoff-Kim et al. (2014); Sullivan et al. (2008); Metcalfe et al. (2003)) have painted a picture qualitatively consistent with the diffusion calculations, with the hotter best fit models having thinner pure helium layers. With GD358, we seek to confirm this trend.

In Section 2, we present our new observations and outline the data reduction process. In Section 3, we establish the framework for frequency identification, and present the list of frequencies used for the asteroseismic investigation. We perform further analysis of the observed frequencies in Section 4, and motivate the  $\ell$  and  $m$  identification of the modes. We present the asteroseismic fitting of GD358’s pulsation spectrum in section 5, present the results in section 6 and discuss our results in section 7.

## 2. New Observations and Data Reduction

GD358 was discovered in 1982 (Winget et al. 1982) and has been the target of the Whole Earth Telescope (WET) in 1990, 1994, 2000, and 2006 (Provencal et al. 2009; Kepler et al. 2003; Winget et al. 1994). New observations presented here include 278 individual observing runs (1105.1 hrs) spanning 2007-2016 (Table 1). Each season of observations was obtained as part of multi-site WET campaigns (Nather et al. 1990).

Data reduction follows the steps described in Provencal et al. (2012). In brief, the new observations were obtained with different CCD photometers, each with distinct effective bandpasses. We minimize the bandpass issues by using CCDs with similar detectors whenever possible and employing a red cutoff filter (BG40 or S8612) to normalize wavelength response and reduce extinction effects. We corrected each image for bias and thermal noise, and normalized by the flat field. Aperture photometry using the Maestro photometry pipeline described by Dalessio et al. (2010) was performed on each image, utilizing a range of aperture sizes for the target and comparison stars. For each individual nightly run, we chose the combination of aperture size and comparison star(s) producing the highest signal/noise light curve.

We used the WQED pipeline (Thompson & Mullally 2009) to examine each nightly light curve for photometric quality, remove outlying points, divide by suitable comparison stars, and correct for differential extinction. Our observational technique is therefore not sensitive to oscillations longer than a few hours. The result is a series of light curves with amplitude variations represented as fractional intensity (mmi). The unit is a linear representation of the fractional intensity of modulation ( $1 \text{ mmi} \approx 1 \text{ mmag}$ ). We present our Fourier transforms (FTs) in units of modulation amplitude ( $1 \text{ mma} = 1 \times 10^{-3} \text{ ma} = 0.1\% = 1 \text{ ppt}$ ).

The final reduction step is to combine the individual light curves (an example is shown in Fig. 1) to produce complete light curves for GD358 for each observing season. We assume GD358 oscillates about a mean light level. This assumption allows us to carefully assess overlapping segments from different telescopes and identify and correct any vertical offsets.

As discussed in detail in Provencal et al. (2009), we find no significant differences between the noise levels of amplitude spectra using: 1) the combination of all light curves including overlapping segments from different telescopes, 2) the combination of light curves where we retain only higher signal to noise observations in overlapping segments and 3) combining all light curves using data weighted by telescope aperture.

The complexities associated with GD358’s pulsations (see Section 3) led us to also re-reduce all available archival data (Provencal et al. 2009; Bradley 2004; Kepler et al. 2003; Winget et al. 1994, 1982) to insure continuity in methodology. The final result is a series of light curves for each observing season between 1982 and 2016. For the new observations, 2007 contains 8.1 hrs of observation, 2008 26.1 hrs, 2009 45 hrs, 2010 201.3 hrs, 2011 401.1 hrs, 2012 150.5 hrs, 2013 87 hrs, 2014 184.6 hrs, 2015 55 hrs and 2016 90.6 hrs. Our coverage is not complete, and this incompleteness produces spectral leakage in the amplitude spectra. To quantify this, we sampled single sinusoids using exactly the same times as the original data for each season. The resulting amplitude spectrum, or “spectral window”, is the pattern produced in the FT by a single frequency. The FTs for 2010, 2011, and 2014 are given in Fig. 2.

### 3. Frequency Identification

Our goal is to compile a complete list of GD358’s observed independent and combination frequencies to be used in a comprehensive asteroseismic analysis. GD358 is known for large scale changes in amplitudes and small but not insignificant frequency variations on a variety of timescales (Provencal et al. 2009; Kepler et al. 2003). The amplitude and frequency variations evident in Fig. 2 demonstrate that it is not feasible to analyze the entire data set as one unit. To minimize the effects of the long term amplitude and frequency variations, we analyze the light curves from each observing season individually.

We use *Period04* (Lenz & Breger 2005) for Fourier analysis and nonlinear least squares fitting to select statistically significant peaks in each FT. We adopt the criterion that a peak must have an amplitude at least four times greater than the average noise level in the given frequency range (Provencal et al. 2012). We define “noise” as the frequency-dependent average amplitude after removal of the dominant frequencies. This is a conservative estimate, as it is impossible to ensure that all of the “real” frequencies are removed when determining the noise level. This is certainly true for GD358, where the peaks above  $\approx 2500 \mu\text{Hz}$  are combination frequencies (see Section 4.2, Provencal et al. (2009)). Fig. 3 displays the average noise as a function of frequency for the 2007-2016 observing seasons. A similar plot for the archival data is presented in Fig. 3 of Provencal et al. (2009). To confirm our uncertainty

estimates, we calculated Monte Carlo simulations using the routine provided in *Period04*. This routine generates a set of light curves using the original times, the fitted frequencies and amplitudes, and added Gaussian noise. A least squares fit is performed on each simulated light curve, with the distribution of fit parameters giving the uncertainties. Our Monte Carlo results are consistent with our average noise estimates.

Having established our baseline noise levels, we proceed to frequency selection and identification. Our frequency selection procedure involves identifying the largest amplitude resolved peak in the FT, fitting a sinusoid at that frequency to the data set, subtracting the fit from the light curve, recomputing the FT, examining the residuals, and repeating the process until no significant power remains. This technique, known as prewhitening, must be employed with an over-abundance of caution, especially since we are aware of amplitude and/or frequency modulation in our data set. A detailed discussion of the prewhitening procedure and steps taken to minimize the effects of amplitude modulation is given in Provencal et al. (2009). Our final identifications of independent frequencies detected in each observing season are given in Table 2 and Table 3.

## 4. Frequency Analysis

### 4.1. Frequency Distribution

Perusal of Tables 2 and 3 shows that GD358’s observed frequencies vary in two important ways. First, frequencies detected in a given observing season are not found in all observing seasons. Asteroseismology is based on the assumption that the available pulsation frequencies are linked to stellar structure. Since we are fairly certain that GD358’s internal structure does not change on the timescales of the observations, we can assume that GD358 excites different subsets of its available pulsations at different times. While this is a common phenomenon seen in white dwarf pulsators, the selection mechanism remains unknown. The best way to determine GD358’s complete set of pulsation frequencies is to combine frequency identifications from multiple observing seasons as outlined in Kleinman et al. (1998).

Fig. 4 presents a schematic representation of GD358’s independent frequencies detected between 1982 and 2016. The features of asteroseismic importance are the localized bands between 900 and 2400  $\mu\text{Hz}$  (1100 and 400 s). We interpret the bands to represent a series of modes of radial overtone  $k$ . The bands themselves illustrate the second way in which GD358’s frequencies vary. Each band consists of frequency detections from multiple observing seasons, but each band is significantly wider in frequency space than the frequency error of any single measurement, arguing that some process is acting on the frequencies. The simplest

explanation is rotational splitting. However, with the exception of the two highest frequency (shortest period) bands ( $k = 8$  and  $k = 9$  from Provencal et al. (2009)), we find no clear evidence of stable multiplet structure in the frequency distributions of the lower frequency (longer period) bands (Fig. 5). This does not imply that the structures found by Winget et al. (1994), Kepler et al. (2003), Montgomery et al. (2010) and others are not real. In particular, it is clear that the  $k = 12$  mode can be explained by the oblique rotation model during 2006. However, previous work represents short snapshots of GD358. When examined over timescales of decades, these structures do not regularly recur, and so cannot be interpreted as simple rotational splitting. The lower frequency band widths are most likely the result of various processes producing amplitude and/or frequency modulation that obscures any underlying signatures of rotational splitting.

Fig. 4 also shows that the widths of the bands are not constant, but change from band to band. Bell et al. (2015) presents an interesting analysis of the hydrogen atmosphere pulsator (DAV) KIC4552982, in which they identify 17 bands of pulsation frequencies. KIC4552982 is one of the coolest ZZ Ceti known (Tremblay et al. 2013), and it follows the general pattern exhibited by GD358: the highest frequency (shortest period) mode shows evidence of rotational splitting, while the lower frequency (longer period) modes are complex bands. The authors note that this DAV’s bands have different widths in frequency space, and that there may be astronomical significance to this. Although the observational timebases and coverage are quite different (20 continuous months for KIC4552982 vs 34 incomplete years for GD358), our data presents the opportunity to investigate this for a cooler DB pulsator. We measured the widths of each of GD358’s band where we have more than 10 detections. We define “width” as the difference between the lowest and highest frequency detected in each band. Fig. 6 presents the results. We find no correlation of width with number of detected peaks in each band. Interestingly, the widths of GD358’s bands are commensurate with those found in KIC4552982. We find a general increase in width with decreasing frequency (increasing period), until we reach the band at  $1238 \mu\text{Hz}$  (807 s). This band has a width at least twice as wide as any other.

Montgomery et al. (2016) recently outlined a possible explanation for the behavior seen in the low frequency modes of these cool DAVs that should also apply to GD358. White dwarfs pulsate in nonradial g-modes. As a DBV such as GD358 pulsates, it experiences local surface temperature variations as large as 3000 K. The temperature variation affects different modes in different ways. An appropriate analogy is to consider each mode’s propagation region as a box with a lid. The box is defined as the region where the mode frequency is less than both the buoyancy (Brunt-Väisälä) and acoustic (Lamb) frequencies. The box’s lid represents the mode’s outer turning point. For short period modes (such as  $k = 8$  and  $k = 9$  in GD358), the lid is defined by the acoustic frequency, which is relatively insensitive

to surface temperature variations. As the star pulsates, the box will not change, so these modes should be stable. For the longer period modes, the lid of the box is defined by the buoyancy frequency, which goes to zero at the base of the surface convection zone. This is the important point: for longer period modes, the box lid is actually defined by the base of the convection zone, which is very sensitive to local temperature variations. As GD358 pulsates, its convection zone deepens and thins in response to the local temperature variations. In our analogy, the lid of the box moves, effectively changing the characteristics of the box. Long period modes with outer turning points defined by the base of the convection zone should be perturbed and this is indeed what we see in GD358.

An additional interesting behavior is given by the distribution of average amplitudes for the bands (Fig 4). In particular, the two bands at 1857.7 and 2007.6  $\mu\text{Hz}$  have never been observed at amplitudes above 4 mma. The band at 1369  $\mu\text{Hz}$  is also only observed at lower amplitudes. Interestingly, the band at 1741.5  $\mu\text{Hz}$  was not observed at large amplitude prior to 2006, and it has remained at high amplitude since that time.

To summarize this section in broad brush strokes, the observed global pulsations involve the whole star, but each pulsation samples the star in slightly different ways. Modes with lower frequencies (higher radial  $k$  values) preferentially sample the outer layers, while modes with higher frequencies (lower radial  $k$  values) have outer turning points that are farther from the surface, and so sample the deeper interior. It makes intuitive sense that lower frequency modes would be affected by processes confined to the outer stellar atmosphere, such as the convection zone. We speculate that the observed band widths and pulsation amplitudes contain information about the convection zone and/or any surface magnetic field. Further investigation requires guidance from theory.

## 4.2. Mode Identification

Our current work has produced a well defined sequence of modes, adding to previous studies (Montgomery et al. 2010; Provencal et al. 2009; Metcalfe et al. 2000; Winget et al. 1994). The previous identification of these modes as a series of  $l=1$  radial overtones is based mostly on the pulsation frequency distribution and limited spectroscopic analysis (Kotak et al. 2002; Castanheira et al. 2005). It is important to further investigate these identifications as we initiate an in depth asteroseismic investigation.

GD358’s combination frequencies provide a tool by which we can bolster  $\ell$  identifications. Combination frequencies are typically observed in the FTs of moderate to large amplitude pulsators. They are identified by their exact numerical relationships with parent



frequencies. The combinations themselves are not independent, but result from nonlinear effects associated with the surface convection zone (Brickhill 1992; Brassard et al. 1995; Wu 2001; Ising & Koester 2001). Wu (2001) lays the groundwork, showing that observed amplitudes of the combination frequencies depend on geometric factors such as the  $(\ell, m)$  indices of the parents and the inclination of the pulsation axis to the line of sight.

The methods outlined in Provencal et al. (2012) and Montgomery et al. (2010) work best when applied to larger amplitude frequencies detected in high signal to noise data sets such as provided by extensive WET runs. The primary reason for this is that combination frequencies are lower amplitude than their parents, and so are more difficult to detect in sparse data. We chose the 1990, 1994, 2006, 2010, 2011, and 2014 observing seasons, and looked at pulsation frequencies with amplitudes above 10 mma.

As an example, Fig. 7 shows the probability distribution of  $\ell$  and  $m$  values for the 1735.96  $\mu\text{Hz}$  frequency as detected in 2014. To produce the distribution, we ran the amplitude code (Montgomery et al. 2010) 1000 times, and selected the results having  $Res_{rms} < 9.5 \times 10^{-06}$ , where  $Res_{rms}$  are the root-mean-squared residuals between predicted and observed amplitudes. For our example, this mode is clearly preferred as an  $\ell = 1, m = 1$  mode. We find similar results for all modes above 10 mma in the 1990, 1994, 2006, 2010, and 2014 observing seasons. Combining the results from the combination frequencies with previous evidence, we are confident that the bands in Fig. 4 represent a series of  $\ell = 1$  modes.

Finally, given the  $l=1$  identification and the lack of definitive multiplet structure for the lower frequency modes, the best way to determine the central frequency for each band is simply to average the detected frequencies in the bands. We experimented with numerous weighting techniques, and determined there is no significant difference in our solutions. Our final frequencies for each band are given in Table 4. We use these periods in our asteroseismic fitting.

## 5. Asteroseismic fitting

The basic method in our asteroseismic fitting consists of calculating grids of white dwarf models and running a fitting subroutine to match the periods of the models ( $P^{\text{calc}}$ ) with the observed periods  $P^{\text{obs}}$ . Following standard statistical methods, each fit is assigned a fitness parameter calculated the following way:

$$\sigma_{\text{RMS}} = \sqrt{\frac{1}{W} \sum_1^{n_{\text{obs}}} w_i (P_i^{\text{calc}} - P_i^{\text{obs}})^2}, \quad (1)$$

$$W = \frac{n_{\text{obs}} - 1}{n_{\text{obs}}} \sum_1^{n_{\text{obs}}} w_i \quad (2)$$

where  $n_{\text{obs}}$  is the number of periods present in the pulsation spectrum and the weights  $w_i$  are the inverse square of the uncertainties listed for each period in table 4. We note that the two shortest period modes have 6000 times the weight of the longest period mode. Another way to think about this is to assume that we have a calculated period that matches the highest period mode very poorly, being 20 seconds away. 20 seconds is roughly half the average period spacing for  $\ell = 1$  modes in the relevant area of parameter space and so it is the worse period fit one can get. In order to have the same impact on  $\sigma_{\text{RMS}}$ , the lowest period mode would have to match to within 0.26 seconds. In essence this is almost ignoring the modes that have a period measurement uncertainty of 0.5 s or more. It is, however completely consistent with the relative uncertainty on the periods and it provides us with a true measure of the goodness of fit, while accounting for all the data we have.

### 5.1. The Models

To compute our models, we used the White Dwarf Evolution Code (WDEC). The WDEC uses hot and self-consistently allows them to relax to be solutions of the equations of stellar structure with the temperature of our choice. polytrope models with temperatures above 100,000 K as starting model and numerically evolves them until they are thermally relaxed solutions to the stellar structure equations and have the temperature of our choice. The mass is also fixed as an input, and so is the internal chemical composition profiles (no mass loss, no time dependent diffusion of elements). Each model we compute for our grids is the result of such an evolutionary sequence. The WDEC is described in detail in Lamb & van Horn (1975) and Wood (1990). We used smoother core composition profiles and implemented more complex profiles that result from stellar evolution calculations (Salaris et al. 1997). We updated the envelope equation of state tables from those calculated by Fontaine et al. (1977) to those given by Saumon et al. (1995). We use OPAL opacities (Iglesias & Rogers 1996) and plasmon neutrino rates published by Itoh et al. (1996).

DBVs are younger than their cooler cousins the DAVs. Time-dependent diffusion calculations (e.g. Dehner & Kawaler 1995; Althaus et al. 2005) show that at 24,000 K, a typical

temperature for a DBV, the carbon has not yet fully settled into the core of the star. We expect the helium layer to be separated into a mixed He/C layer with a pure He layer on top, as shown in Fig. 8. Following Metcalfe et al. (2005), we adopted and parameterized this structure in our models.

## 5.2. Initial Grid Search

In our asteroseismic fits, we initially varied six parameters: the effective temperature, the mass and four structure parameters. There are two parameters associated with the shapes of the oxygen (and carbon) core composition profiles: the central oxygen abundance ( $X_o$ ) and the edge of the homogeneous carbon and oxygen core ( $q_{\text{fm}}$ , as a fraction of stellar mass). For envelope structure,  $M_{\text{env}}$  marks the location of the base of the helium layer and  $M_{\text{He}}$  the location where the helium abundance rises to 1 (see Fig. 8).  $M_{\text{env}}$  and  $M_{\text{He}}$  are mass coordinates, defined as e.g.  $M_{\text{env}} = -\log(1 - M(r)/M_*)$ , where  $M(r)$  is the mass enclosed in radius  $r$  and  $M_*$  is the stellar mass.

We started with a master grid (Table 5) chosen so that it covered all relevant area of parameter space and had sufficient resolution to find any region of local minimum in the fitness parameter (Bischoff-Kim & Metcalfe 2011; Bischoff-Kim et al. 2014). We used the maximum resolution that was computationally manageable. The master grid involved the computation of 10,483,200 models. We fit simultaneously all 15 periods, requiring all of them to be  $\ell = 1$  modes. A fitness map of this initial fit are shown in the left panel of Fig. 9, and the parameters of the best fit model are listed in Table 5.

## 5.3. Asymptotic Period Spacing

Before we refine the period-by-period fitting optimization, it is worthwhile to step back and consider what we can learn from the average period spacing of GD358. The average period spacing provides an asteroseismic measure of the mass and temperature of the star, independent of the details of internal chemical composition profiles. Higher  $k$  modes are not strongly trapped in the core and according to asymptotic theory, they should be nearly evenly spaced in period. This spacing is given by Unno et al. (1989).

$$\Delta P = \frac{\pi}{\sqrt{\ell(\ell+1)}} \left[ \int_{r_1}^{r_2} \frac{N}{r} dr \right]^{-1}, \quad (3)$$

where  $r_1$  and  $r_2$  are turning points of the mode and  $N$  is the Brunt-Väisälä frequency. The

asymptotic period spacing is  $\ell$  dependent, with higher  $\ell$  modes having smaller spacing. In the case of GD 358, we have a single  $\ell = 1$  sequence so we only need to worry about the dependence of the asymptotic period spacing on the Brunt-Väisälä frequency. Much if not all of GD 358’s pulsation spectrum is close to the asymptotic limit, because the shortest period observed is a  $k = 8$  mode.

The dependence of  $\Delta P$  on the Brunt-Väisälä frequency leads to higher mass and lower temperature models having a smaller period spacing (their interior is less compressible). This effect appears in asteroseismic fitting of white dwarfs and also sdB stars as a ubiquitous diagonal trend in contour maps of the quality of the fits in the mass-effective temperature plane (e.g Bischoff-Kim et al. 2014; Castanheira & Kepler 2009; Charpinet et al. 2008). One requirement for the periods of the model to match the observed periods is that the average period spacing in the models matches the average period spacing in the observed pulsation spectrum. If a good match occurs for a given mass and effective temperature, then models with lower mass but higher effective temperature will also match well.

We use the sequence of 13 consecutive  $\ell = 1$  modes found in GD358’s pulsation spectrum to calibrate our models (Table 4). Using the  $\ell = 1$  sequence of modes, we compute an average period spacing of 39.08 seconds. We call this  $\Delta P_{\text{obs}}$ . For each model in the master grid, we compute an asymptotic period spacing ( $\Delta P_{\text{calc}}$ ). This asymptotic period spacing is calculated by first discarding the 10 lowest  $k$  modes. The exact value of 10 is somewhat arbitrary, but it is chosen so that the modes we use in our computation are indeed in the asymptotic limit. The higher  $k$  modes show weaker trapping than the lower  $k$  modes. We then fit a line through the set  $(k_i, P_i)$ . The slope gives us the asymptotic period spacing in the model. We also calculate the residuals of the fits and discard the models that have residuals above a certain limit. The limit is chosen by checking the procedure by eye on a few models.

We show a contour map of the location of the models that best match the average period spacing of 39.08 seconds in Fig. 9, right panel. We place it side by side with a contour map showing the location of the best fit model in the same region of parameter space, based on the master grid fitting described in section 5.2. Note how the best fit model falls right within the valley where the period spacings between GD358 and the models match. This should come as no surprise, as in order for 15 periods to fit reasonably well, the model period list should have a spacing similar to that of the observed period spectrum. The most recent spectroscopic determination of Koester (2013) is far off the valley, at 24,000 K and  $0.506^{+0.034}_{-0.051} M_{\odot}$ . Previous spectroscopic determinations, such as that of Bergeron et al. (2011), do fall within the swath where the period spacings match.

One can fit simultaneously the average period spacing and the individual periods formally while performing the fits by using some prescription to calculate the goodness of fit.

This leads to a more complex relation than defined in equation 1. Note that the period spacing is a much weaker constraint than the individual period fit. If one takes 5.0 seconds as an upper limit for goodness of fit, that includes 4% of the models in the period-by-period fit plot (left panel in Fig 9), but the entire parameter plane for the average period spacing fit plot (right panel). In our refined fits, we limit ourselves to a region very near the best fit model found on the initial grid. This essentially limits our search to models that already match the average period spacing. We avoid sophisticated schemes to calculate the fitness parameter and simply use equation 1.

#### 5.4. Optimal Grid Resolution

Having determined a more restricted region of parameter space to search for the best fit models, we now turn to the question of how fine we need to make our refined grid. We want to have a high enough resolution grid that we can be sure we captured a true minimum, but on the other hand, there are computational limitations to how many models we can afford to calculate, save, and process.

One way to gain a sense of how fine the grid needs to be is to make single parameter cuts through parameter space. Fig. 10 shows such cuts for master grid models. The plot was made by fixing 5 of the 6 parameters to the best fit values of the best initial fit (see table 5). For some parameters, the fits seem to settle to a minimum in a smooth way, while for others, they exhibit jumps. For instance, the spike in the effective temperature plot at 27,800 K is due to a period (around 530 s) that goes away and then comes back. The model with the missing period fits poorly compared to the models on either side of it. The jump from  $\sigma_{\text{RMS}} \sim 5$  s to  $\sigma_{\text{RMS}} \sim 30$  s that happens from 28,400 K to 28,600 K is due to a discontinuous change in the period spectrum of the models. Namely, the appearance of a new mode between the 710 and 784 s modes of the 28,400 K model.

Discontinuities like these are unsettling, as it is easy to see that one might miss a best fit model if the grid is not fine enough. In order to quantify "fine enough", we ran systematic scans, computing models with 5 parameter fixed and allowing the 6th parameter to vary in very fine steps. We went down in step sizes to  $\Delta T_{\text{eff}} = 1$  K,  $\Delta M_* = 0.0001 M_{\odot}$ ,  $\Delta M_{\text{env}} = \Delta M_{\text{He}} = 0.01$  dex,  $\Delta X_o = 0.01$ , and  $\Delta q_{\text{fm}} = 0.001$ . Regardless of the behavior of individual modes in the models as the parameters vary, we can assert what step sizes will allow us to minimize our risk of missing a minimum, while minimizing the number of models to compute. We settled on step sizes of  $\Delta T_{\text{eff}} = 50$  K,  $\Delta M_* = 0.0001 M_{\odot}$ ,  $\Delta M_{\text{env}} = \Delta M_{\text{He}} = 0.1$  dex,  $\Delta X_o = 0.1$ , and  $\Delta q_{\text{fm}} = 0.005$  for our refined grid.

## 6. Results of the Period Fitting

The parameters for our best fit model based on the refined grid are listed in Table 5 and the periods of that model in Table 4. The goodness of fit of the model is  $\sigma_{\text{RMS}} = 0.964$  s. We also list the Bayes Information Criterion (BIC) number, a statistic that normalizes the quality of fits by number of free parameters and number of constraints for comparison with other studies. For a discussion applied to this parameter study, see Bischoff-Kim & Metcalfe (2011). A negative BIC indicates a good quality of fit, given the number of constraints. One should keep in mind that the quality of fit is aided by the fact that some of the observed periods have large uncertainties.

In Fig. 11 we focus on the mass dependence of the fitness parameter to illustrate the effect a finer grid has on the fitting. We could have used less of a fine mesh in mass, but with our very fine mesh, we do get a better view of how the discontinuities first shown in Fig. 10 behave and that they are not true discontinuities, just large changes in goodness of fit for small changes in mass.

Finally, we show the interior structure of the best fit model in Fig. 8, with the corresponding Brunt-Väisälä frequency profile.

### 6.1. Validation of the Fitting Method and Error Estimation

With what we have discovered in section 5.4, it is only natural to be concerned about whether we have truly found a best fit. We performed a simple test to validate our fitting method, which consisted of using the exact same procedure to find a best fit to the periods of a model that was not on any of the grids we calculated, but that did have parameters that were very close to the best fit model. We used the period list for a model with parameters  $T_{\text{eff}} = 25630$  K,  $M_*/M_{\odot} = 0.57065$ ,  $M_{\text{env}} = -2.05$ ,  $M_{\text{He}} = -5.55$ ,  $X_o = 0.52$ , and  $q_{\text{fm}} = 0.192$ , including only the subset of 15  $\ell = 1$  periods that match GD358’s pulsation spectrum.

We first performed a fitting of the periods using the master (coarse) grid described in section 5.2. This placed the best fit model in the appropriate region of parameter space. Then we refined our fits, using the grids described at the end of section 5.3. We are able to recover the best fit parameters adequately, with the all top 5 best fit models having parameters  $T_{\text{eff}} = 25000$  K,  $M_{\text{He}} = -2.1$ ,  $M_{\text{env}} = -5.5$ ,  $X_o = 0.50$ , and  $q_{\text{fm}} = 0.190$  and a mass ranging between 0.5730 and 0.5734  $M_{\odot}$ . The best fit model has  $\sigma_{\text{RMS}} = 0.31$  s. We remind the reader that the step sizes in the second phase of the fitting are 50 K for  $T_{\text{eff}}$ , 0.0001  $M_{\odot}$  for stellar mass, 0.1 dex for helium layer masses, 0.1 for  $X_o$ , and 0.005 for  $q_{\text{fm}}$ .

This test also allows us to place minimum error bars on at least some of the parameters found:  $\pm 600$  K in effective temperature, 0.05 and 0.02 dex on  $M_{\text{env}}$  and  $M_{\text{He}}$  respectively, 0.02 on  $X_o$ , and 0.002 on  $q_{\text{fm}}$ .

## 7. Discussion and Conclusions

We analyzed archival data and over a thousand hours of new observations on GD358, together covering a span of 33 years. With data spanning such a long period of time, we learn about the stability of the different modes. We find that the shorter period modes tend to be more stable, while the longer period modes tend to vary more in frequency over time. Bell et al. (2015) have observed and modeled such stochastic behavior in KIC 4552982, a red edge DAV observed nearly continuously for 1.5 years by *Kepler*. In that star, the 361.58 s triplet has sharply defined peaks, while the rest of the modes, with much higher overtone numbers, are less stable. The stability of the low  $k$  modes is likely due to the fact that they are strongly trapped in the core. Unlike their higher  $k$  counterparts, they are affected very little by the convection zone near the surface.

In analyzing the data, we found 4 new modes, adding to the 11 modes known previously. With these 15 modes, we performed a new asteroseismic fit of GD358 with models that include carbon and oxygen core composition profiles based on the stellar evolution models of Salaris et al. (1997). We find a best fit effective temperature of 23,650 K and a mass of  $0.5706 M_{\odot}$  for GD358. While the temperature is close to the recent spectroscopic determination of 24000 K (Koester 2013), the mass is over one sigma above the spectroscopic mass. On the other hand, the mass matches almost exactly that found by Bergeron et al. (2011) (but the effective temperature is low). The spectroscopic data point of Koester (2013) is somewhat off the asymptotic period spacing trend (Fig. 9, right panel).

In this study, we varied the parameters that have traditionally been varied in this type of asteroseismic fitting, so that we can place our results side by side with other studies of DBVs, including KIC 8626021 (Bischoff-Kim et al. 2014), EC20058 (Bischoff-Kim & Metcalfe 2011), and CBS114 (Metcalfe et al. 2005). We now have 4 DBVs that were the object of asteroseismic fitting that used a consistent set of models. We find that cooler best fit models have thicker pure helium envelopes (Fig. 12), in accordance with the outward diffusion of helium over time. To be consistent, we used the effective temperature found through the asteroseismic fitting of each star. Stellar mass also comes into play in diffusion. Ideally, we would like all 4 models to have the same mass. The models range in mass from  $0.525 M_{\odot}$  (EC20058) to  $0.640 M_{\odot}$  (CBS114). More detailed stellar evolution calculations would have to be performed in order to assess the significance of the trend observed with these 4 DBVs.

Numerical experiments of the type presented in section 5.4 have shown that, consistent with the theory of non-radial oscillations, the shapes of the transition zones matter as much as where they occur (Bischoff-Kim 2015). Modern asteroseismic fitting vary parameters attached to the shape of transition zones (Giammichele et al. 2015). We refrained from doing this in the present study because we wanted to compare our results with previous studies of DBVs. It is unclear how much of an effect on structure parameters a change in parameterization would have.

In fitting GD358, we discovered that two very close sets of parameters were able to yield very different fitness parameters. This is due to the fact that some periods can change values drastically for a small change in a given parameter. The periods tend to oscillate between two values and there remains a well defined minimum in parameter space, even if it is a double valued minimum. Any search algorithm that uses a small enough mesh should be able to find the minimum. This phenomenon likely became apparent while fitting GD358 because it is the first white dwarf for which we fit a long consecutive sequence of modes of the same  $\ell$  with high resolution grids.

*Facilities:* MCAO:0.6m (), Struve(), KPNO:2.1m (), , BOAO:1.8m (), Lulin:1.8m (), Beijing:2.16m (), Maidanek:1.5m (), Peak Terskol

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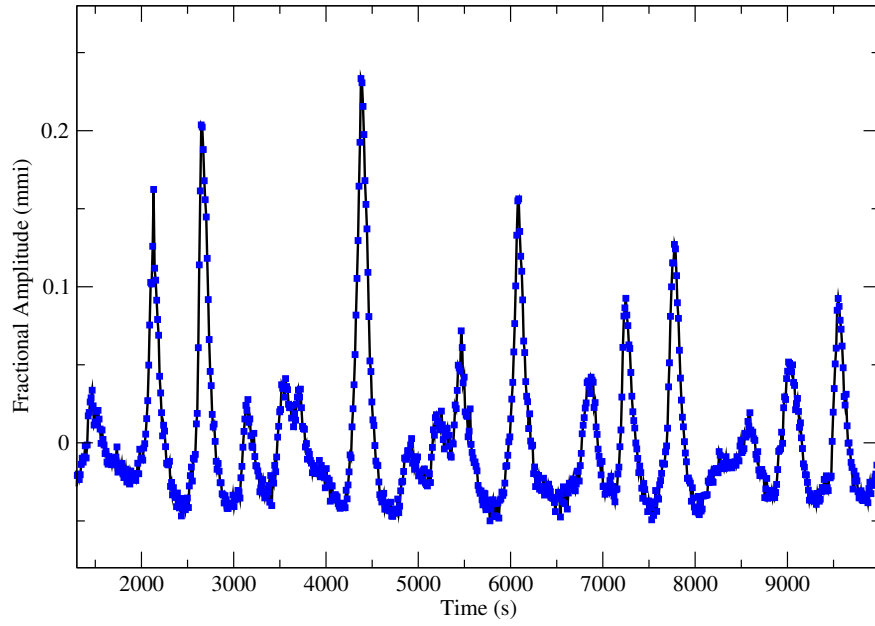


Fig. 1.— Light curve of GD358 obtained with the Peak Terksol 2.0 m telescope. Each point corresponds to a 10 s exposure. The nonlinear, multiperiodic nature of this star is clearly evident. (A color version of this figure is available in the online journal.)

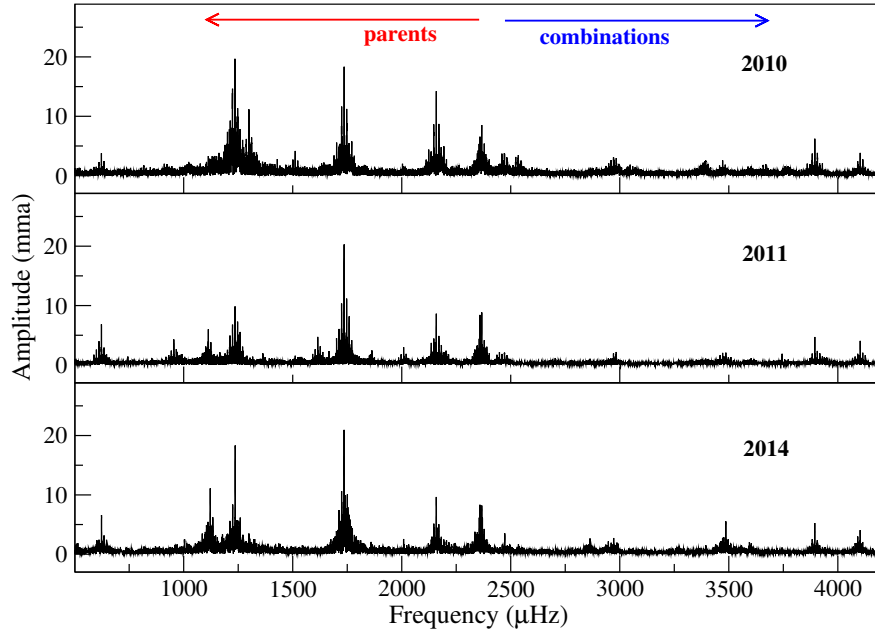


Fig. 2.— Fourier Transforms of GD358 for the 2010, 2011, and 2014 observing seasons. The frequency range of the observed series of  $\ell = 1$  modes discussed in Section 4 is indicated by the left arrow (red). Peaks below  $\approx 2400 \mu\text{Hz}$  are combination frequencies discussed in Section 4.2 (right arrow, blue). (A color version of this figure is available in the online journal.)

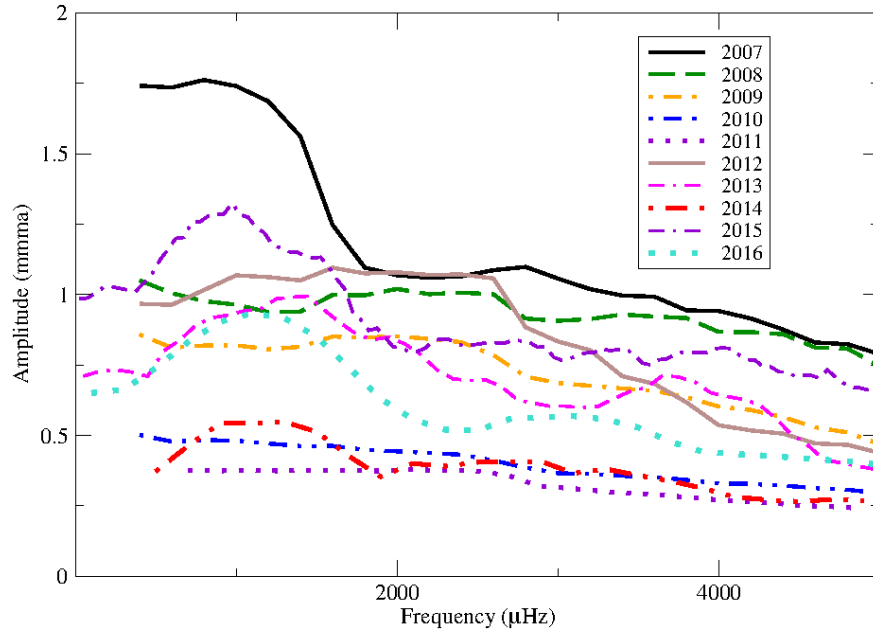


Fig. 3.— A comparison of the average noise as a function of frequency for the 2007-2016 observing seasons. Each data set was prewhitened by its dominant frequencies. The noise levels for each season are somewhat different. This must be taken into account during frequency analysis.

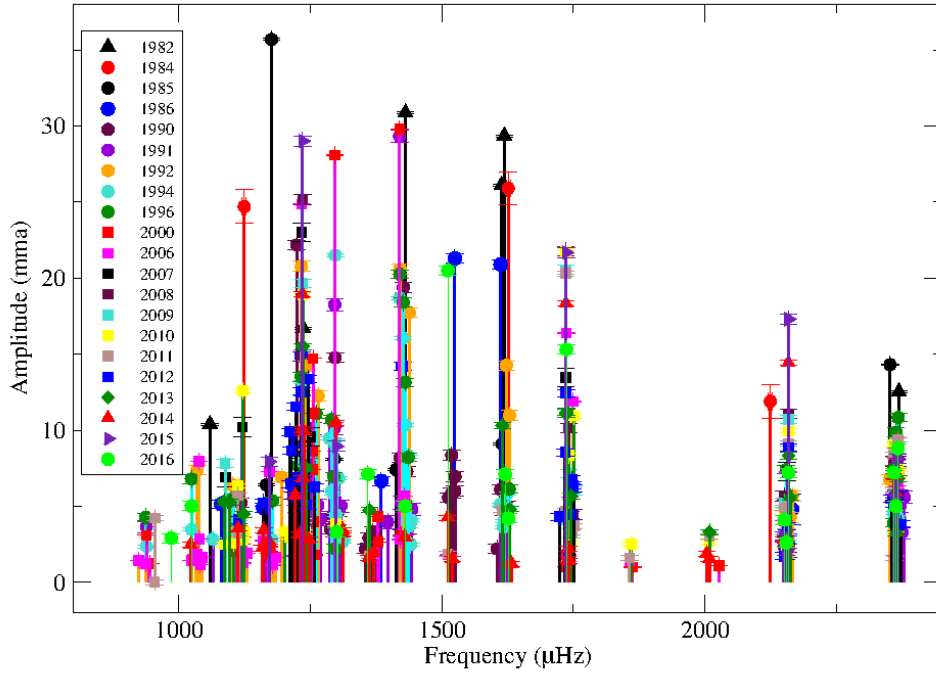


Fig. 4.— A schematic representation of GD358’s pulsation modes for all available data between 1982 and 2015. Systematic patterns of distribution are evident. The bands between 2400 and 1000  $\mu\text{Hz}$  (400 and 1000 s) are of particular importance for this work. (A color version of this figure is available in the online journal.)

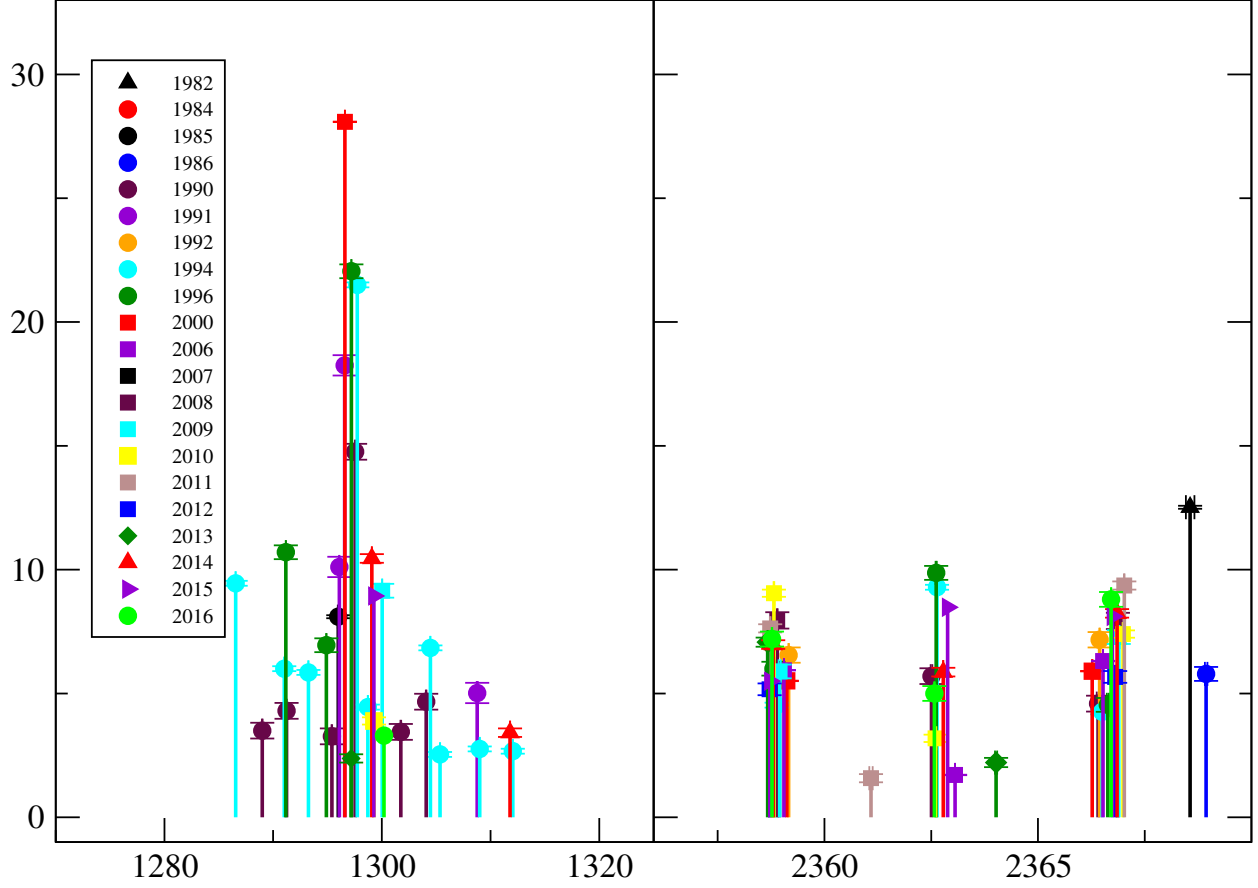


Fig. 5.— The detailed schematic distribution of frequencies for the bands near 1300(*right* and 2360  $\mu\text{Hz}$  (*left*). As an example of the higher frequency bands, the 1300  $\mu\text{Hz}$  band shows no multiplet structure, unlike the 2360  $\mu\text{Hz}$  band. Here we see clear evidence of multiplet structure. Assuming the triplet represents  $\ell = 1$ , this implies a rotation period of  $\approx 1.5$  days. Note changes in the x scale for both axes. (A color version of this figure is available in the online journal.)



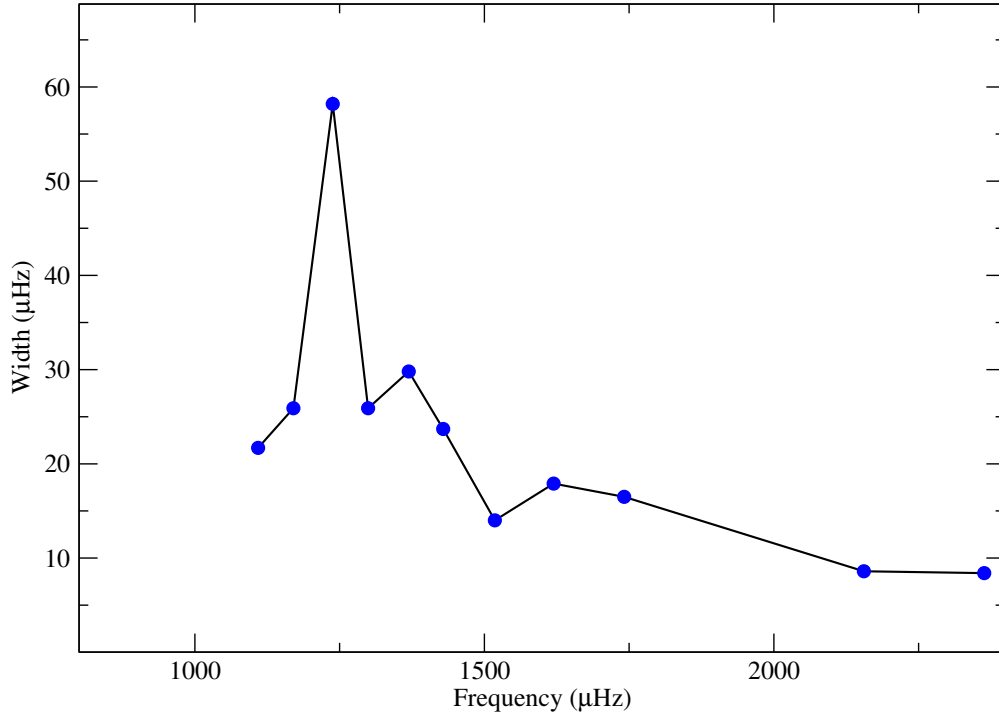


Fig. 6.— Width of each bank of power in Fig. 4. We find an overall increase in band thickness with decreasing frequency (increasing period). The band at 807 s has a width at least twice as large as other bands.

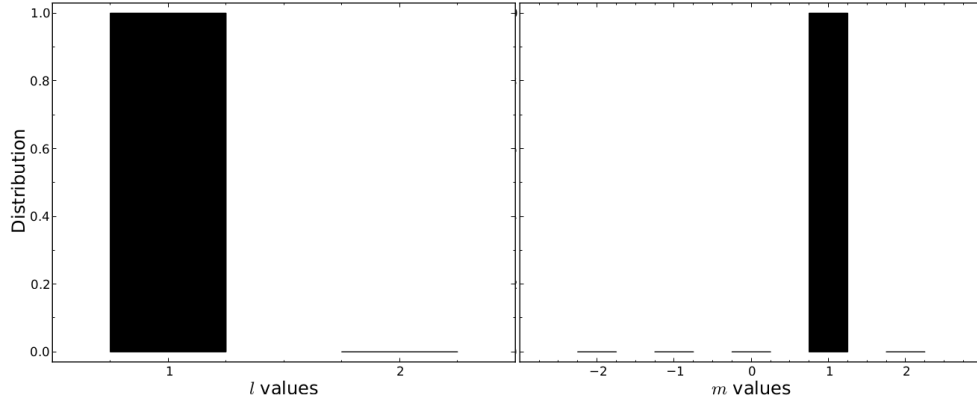


Fig. 7.— Probability distribution (from 0 to 1) of  $\ell$  and  $m$  values for the 1735.96 variation detected in 2014. The distribution is produced from 1000 simulations, selecting with  $Res_{rms} < 9.5^{-06}$ . The amplitudes of the observed combination frequencies argue that this is  $\ell = 1, m=1$ .

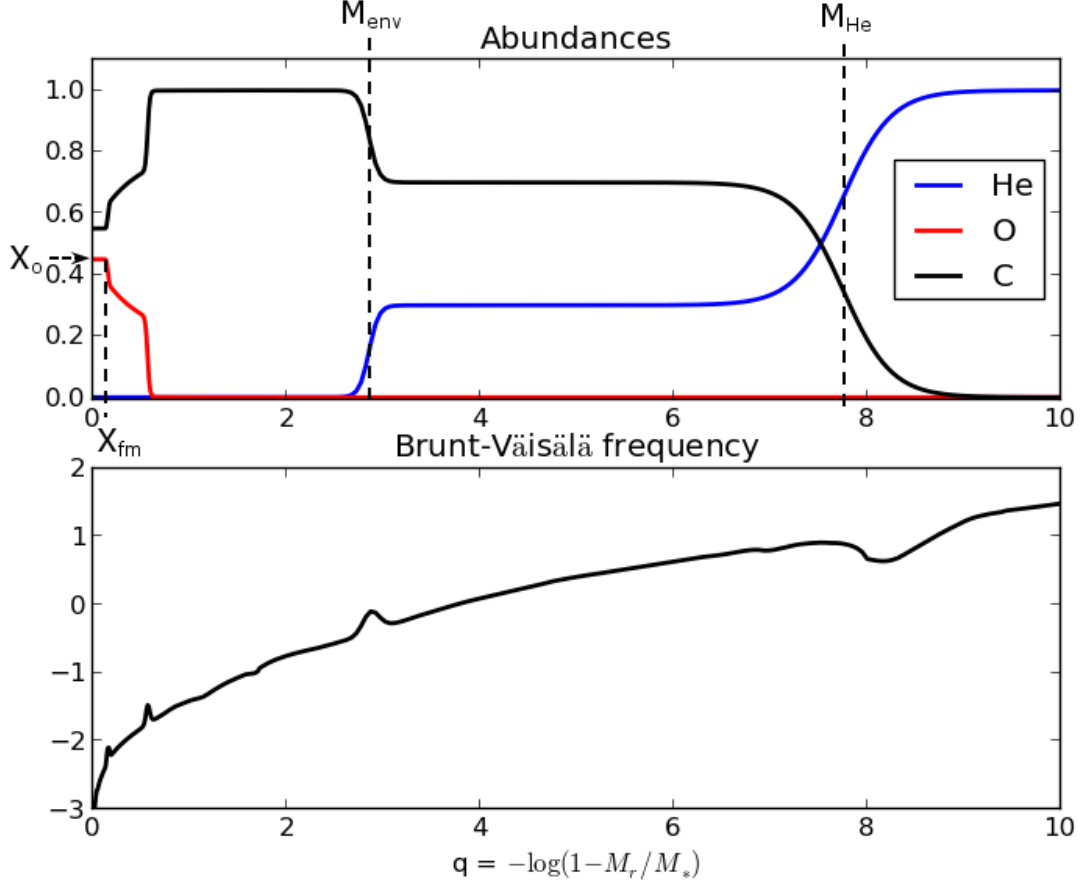


Fig. 8.— *Upper panel:* Chemical composition profiles of the best fit model. The center of the model is to the left and the surface to the right.  $q = 2$  corresponds to  $M_r = 0.99 M_*$ . The vertical axis shows fractional abundances. *Lower panel:* The corresponding Brunt-Väisälä frequency (log  $N^2$  on the vertical axis).

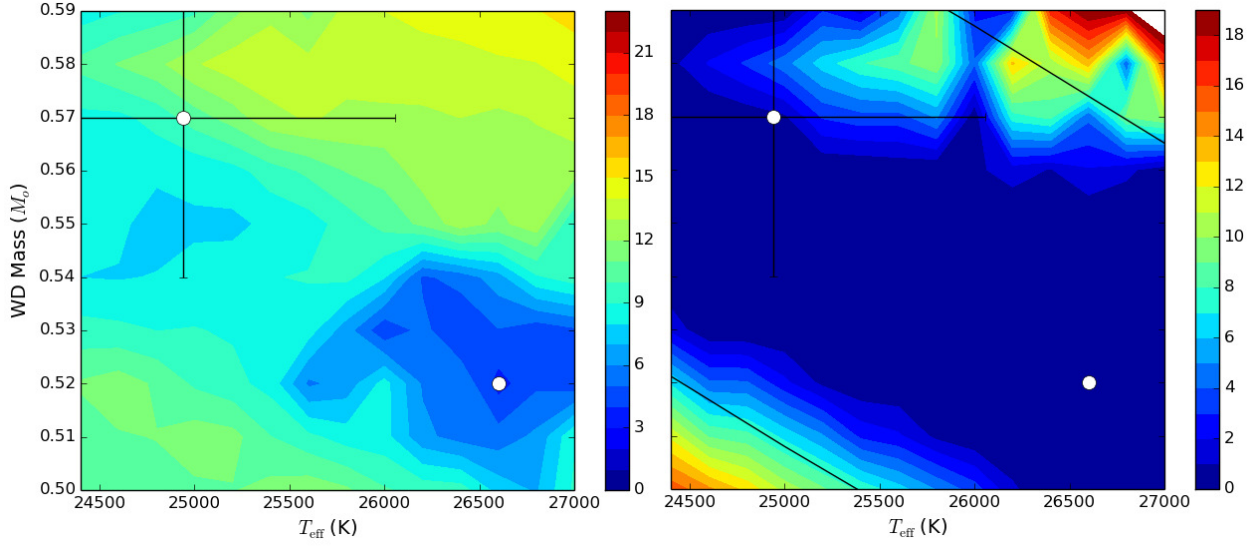


Fig. 9.— *Left panel:* Contour map showing the location of the best fit models. The quantity plotted in the mass-effective temperature plane is the fitness parameter defined in equation 1. *Right panel:* Contour map of the difference between the average periods spacing determined from GD 358’s periods and the asymptotic period spacing of the models, in the mass-effective temperature plane. The scale is in 10ths of seconds, so the worse matches pictured on the plot have  $|\Delta P_{\text{obs}} - \Delta P_{\text{calc}}| \sim 5$  s. We plotted the spectroscopic mass and temperature determinations of Bergeron et al. (2011) (higher mass), and that of Koester (2013) (lower mass). The point without error bars corresponds to the best initial fit model.

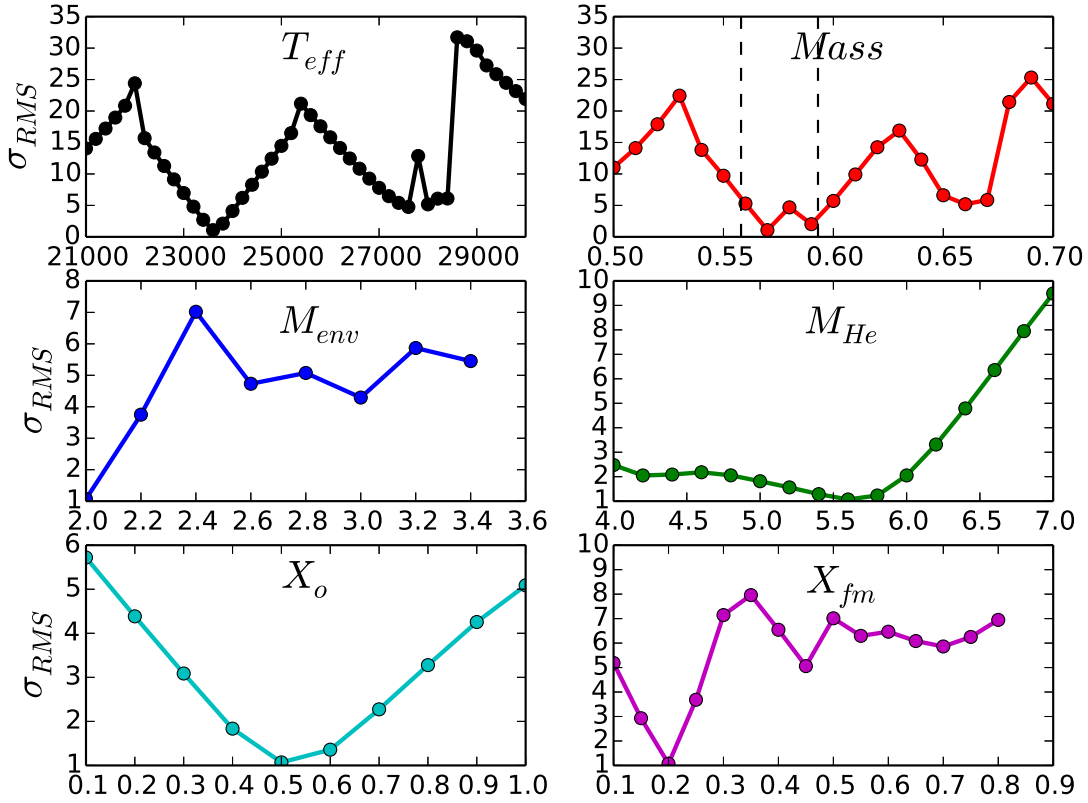


Fig. 10.— Dependence of the fitness parameter on different parameters. The 5 parameters other than the one shown on the horizontal axis in each plot are held to fixed values corresponding to the initial best fit model (table 8). The vertical dashed lines mark the ranges considered in the refined fitting. For the envelope mass, we went from  $M_{env} = -2.0$  to  $-2.3$ .

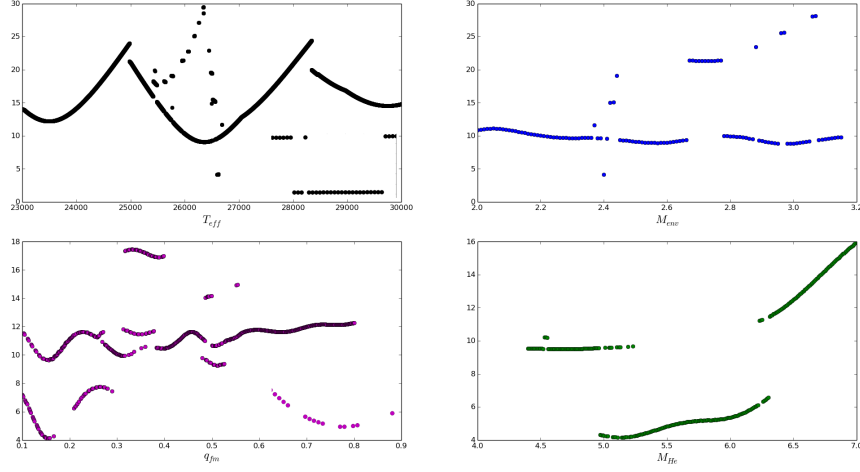


Fig. 11.— Dependence of the fitness parameter on mass alone based on the master grid of models (dashed line) and the refined mesh (solid line).

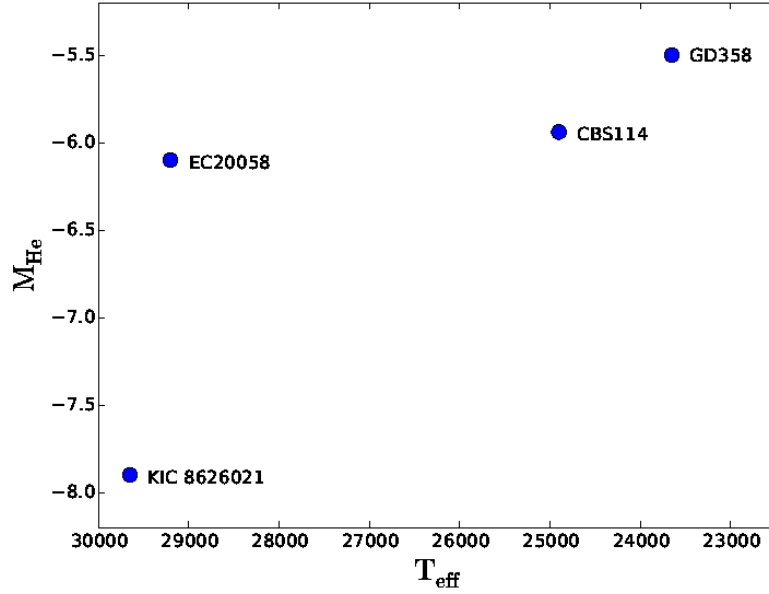


Fig. 12.— Relationship between effective temperature and pure helium layer mass in 4 DBVs fitted using similar models as GD358 in the present study.

Table 1.

Run Name	Telescope	Detector	Date	Length (hrs)
mcao070524-01	MCAO 0.6	CCD	2007-05-24	4.0
mcao070531-02	MCAO 0.6	CCD	2007-05-31	4.1
mcao080613-01	MCAO 0.6	CCD	2008-06-13	2.0
mcao080617-02	MCAO 0.6	CCD	2008-07-25	1.2
mcao080726-01	MCAO 0.6	CCD	2008-07-26	1.7
mcao080730-01	MCAO 0.6	CCD	2008-07-30	3.5
mcao080802-01	MCAO 0.6	CCD	2008-08-02	3.7
pjmo080706-03	PJMO 0.6	CCD	2008-07-06	5.1
suho080810-19	Suhora 0.6	CCD	2008-08-10	4.7
suho080811-19	Suhora 0.6	CCD	2008-08-11	2.5
boao090528-17	BOAO 1.8	CCD	2009-05-28	2.0
kore090529-16	BOAO 1.8	CCD	2009-05-29	2.4
mcao090513-01	MCAO 0.6	CCD	2009-05-13	2.4
mcao090519-03	MCAO 0.6	CCD	2009-05-19	3.2
mcao090520-01	MCAO 0.6	CCD	2009-05-20	4.8
mcao090521-01	MCAO 0.6	CCD	2009-05-21	4.9
mcao090522-01	MCAO 0.6	CCD	2009-05-22	3.1
mcao090531-02	MCAO 0.6	CCD	2009-05-31	2.1
mcao090601-01	MCAO 0.6	CCD	2009-06-01	3.3
mole090526-20	Moletai 1.65	CCD	2009-05-26	3.7
suho090520-19	Suhora 0.6	CCD	2009-05-20	2.2
suho090521-19	Suhora 0.6	CCD	2009-05-21	5.1
vien090525-19	Vienna 0.6	CCD	2009-05-25	6.2
krak100523-20	Krakow 0.4	CCD	2010-05-23	3.4
krak100526-20	Krakow 0.4	CCD	2010-05-26	2.3
krak100616-20	Krakow 0.4	CCD	2010-06-16	4.3
krak100617-20	Krakow 0.4	CCD	2010-06-17	4.5
mcao100516-01	MCAO 0.6	CCD	2010-05-16	2.0
mcao100521-01	MCAO 0.6	CCD	2010-05-21	0.9
mcao100521-05	MCAO 0.6	CCD	2010-05-21	0.8
mcao100526-01	MCAO 0.6	CCD	2010-05-26	3.3
mcdo100517-06	McDonald 2.1	CCD	2010-05-17	4.8

Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
mole100516-22	Moletai 1.65	CCD	2010-05-26	1.8
mole100517-22	Moletai 1.65	CCD	2010-05-17	2.0
mole100520-21	Moletai 1.65	CCD	2010-05-20	3.3
mole100521-21	Moletai 1.65	CCD	2010-05-21	3.5
pjmo100519-04	PJMO 0.6	CCD	2010-05-19	4.0
pjmo100520-02	PJMO 0.6	CCD	2010-05-20	3.8
pjmo100521-05	PJMO 0.6	CCD	2010-05-21	3.3
pjmo100522-02	PJMO 0.6	CCD	2010-05-22	7.0
pjmo100523-04	PJMO 0.6	CCD	2010-05-23	6.0
pjmo100524-02	PJMO 0.6	CCD	2010-05-24	7.5
pjmo100527-02	PJMO 0.6	CCD	2010-05-27	8.3
pjmo100528-02	PJMO 0.6	CCD	2010-05-28	8.3
pjmo100619-03	PJMO 0.6	CCD	2010-06-19	4.0
pjmo100622-02	PJMO 0.6	CCD	2010-06-22	0.7
suho100617-21	Suhora 0.6	CCD	2010-06-17	4.3
terb100516-17	Terskol 2.0	CCD	2010-05-16	2.4
terb100520-17	Terskol 2.0	CCD	2010-05-20	8.9
terb100521-20	Terskol 2.0	CCD	2010-05-21	4.0
terb100524-19	Terskol 2.0	CCD	2010-05-24	5.6
terb100525-17	Terskol 2.0	CCD	2010-05-25	7.3
tueb100522-20	Tuebingen 0.8	SBig	2010-05-22	5.9
tueb100523-20	Tuebingen 0.8	SBig	2010-05-23	5.9
tueb100524-20	Tuebingen 0.8	SBig	2010-05-24	6.0
tueb100604-20	Tuebingen 0.8	SBig	2010-06-04	5.8
tueb100605-20	Tuebingen 0.8	SBig	2010-06-05	5.3
turk100512-18	Canakkale 1.2	CCD	2010-05-12	6.6
turk100513-22	Canakkale 1.2	CCD	2010-05-13	2.8
turk100516-19	Canakkale 1.2	CCD	2010-05-16	6.4
turk100520-19	Canakkale 1.2	CCD	2010-05-20	1.1
turk100522-21	Canakkale 1.2	CCD	2010-05-22	2.6
turk100523-19	Canakkale 1.2	CCD	2010-05-23	6.0
turk100611-19	Canakkale 1.2	CCD	2010-06-11	5.2

Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
turk100613-19	Canakkale 1.2	CCD	2010-06-13	5.3
turk100615-20	Canakkale 1.2	CCD	2010-06-15	4.8
turk100618-19	Canakkale 1.2	CCD	2010-06-18	4.5
turk100620-20	Canakkale 1.2	CCD	2010-06-20	4.8
hvar110521-22	Hvar 1.0	CCD	2011-05-21	2.6
hvar110526-19	Hvar 1.0	CCD	2011-05-26	6.6
hvar110527-19	Hvar 1.0	CCD	2011-05-27	3.5
hvar110529-19	Hvar 1.0	CCD	2011-05-29	6.7
hvar110531-19	Hvar 1.0	CCD	2011-05-31	4.3
hvar110602-19	Hvar 1.0	CCD	2011-06-02	3.8
hvar110604-19	Hvar 1.0	CCD	2011-06-04	6.6
hvar110606-20	Hvar 1.0	CCD	2011-06-06	4.4
krak110506-20	Krakow 0.4	CCD	2011-05-06	5.4
krak110509-19	Krakow 0.4	CCD	2011-05-09	6.7
krak110510-19	Krakow 0.4	CCD	2011-05-10	6.2
krak110511-19	Krakow 0.4	CCD	2011-05-11	6.7
krak110512-19	Krakow 0.4	CCD	2011-05-12	3.1
krak110516-19	Krakow 0.4	CCD	2011-05-16	6.1
krak110517-19	Krakow 0.4	CCD	2011-05-17	5.6
krak110518-19	Krakow 0.4	CCD	2011-05-18	5.7
krak110519-20	Krakow 0.4	CCD	2011-05-19	5.4
krak110520-20	Krakow 0.4	CCD	2011-05-20	5.2
krak110522-21	Krakow 0.4	CCD	2011-05-22	4.3
krak110523-23	Krakow 0.4	CCD	2011-05-23	1.4
krak110524-19	Krakow 0.4	CCD	2011-05-24	4.0
krak110525-20	Krakow 0.4	CCD	2011-05-25	5.4
krak110526-20	Krakow 0.4	CCD	2011-05-26	5.3
krak110529-20	Krakow 0.4	CCD	2011-05-29	5.0
krak110530-20	Krakow 0.4	CCD	2011-05-30	4.8
krak110531-20	Krakow 0.4	CCD	2011-05-31	0.8
krak110604-20	Krakow 0.4	CCD	2011-06-04	5.3
mole110426-22	Moletai 1.65	CCD	2011-04-26	1.7



Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
mole110427-21	Moletai 1.65	CCD	2011-04-27	3.7
mole110430-20	Moletai 1.65	CCD	2011-04-30	0.8
mole110502-20	Moletai 1.65	CCD	2011-05-02	0.4
mole110502-21	Moletai 1.65	CCD	2011-05-02	2.6
mole110505-20	Moletai 1.65	CCD	2011-05-05	1.1
mole110510-21	Moletai 1.65	CCD	2011-05-10	3.5
mtlm110426-05	Mt. Lemmon 1.0	CCD	2011-04-26	6.3
mtlm110427-06	Mt. Lemmon 1.0	CCD	2011-04-27	5.5
mtlm110428-04	Mt. Lemmon 1.0	CCD	2011-04-28	7.1
mtlm110429-05	Mt. Lemmon 1.0	CCD	2011-04-29	6.8
mtlm110430-05	Mt. Lemmon 1.0	CCD	2011-04-30	6.4
mtlm110501-05	Mt. Lemmon 1.0	CCD	2011-05-01	6.6
mtlm110502-05	Mt. Lemmon 1.0	CCD	2011-05-02	6.5
naoc110426-13	NAOC 0.5	CCD	2011-04-26	6.9
naoc110427-11	NAOC 0.5	CCD	2011-04-27	3.3
naos110426-13	NAOC 0.85	CCD	2011-04-26	6.9
naos110427-11	NAOC 0.85	CCD	2011-04-27	3.3
naos110428-12	NAOC 0.85	CCD	2011-04-28	2.6
naos110501-13	NAOC 0.85	CCD	2011-05-01	7.1
naos110502-12	NAOC 0.85	CCD	2011-05-02	8.3
naos110505-13	NAOC 0.85	CCD	2011-05-05	7.0
pjmo110426-07	PJMO 0.6	CCD	2011-04-26	3.0
pjmo110428-04	PJMO 0.6	CCD	2011-04-28	2.0
pjmo110429-03	PJMO 0.6	CCD	2011-04-29	2.5
pjmo110430-04	PJMO 0.6	CCD	2011-04-30	3.0
pjmo110503-04	PJMO 0.6	CCD	2011-05-03	2.6
pjmo110518-03	PJMO 0.6	CCD	2011-05-18	6.7
pjmo110519-03	PJMO 0.6	CCD	2011-05-19	2.7
pjmo110521-03	PJMO 0.6	CCD	2011-05-21	2.7
pjmo110524-02	PJMO 0.6	CCD	2011-05-24	5.2
pjmo110525-07	PJMO 0.6	CCD	2011-05-25	3.2
pjmo110526-03	PJMO 0.6	CCD	2011-05-26	7.0

Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
pjmo110527-02	PJMO 0.6	CCD	2011-05-27	0.8
pjmo110527-03	PJMO 0.6	CCD	2011-05-27	6.7
suho110521-20	Suhora 0.6	CCD	2011-05-21	5.2
suho110522-19	Suhora 0.6	CCD	2011-05-22	5.9
suho110523-20	Suhora 0.6	CCD	2011-05-23	5.6
suho110527-20	Suhora 0.6	CCD	2011-05-27	2.0
suho110529-20	Suhora 0.6	CCD	2011-05-29	4.6
suho110530-20	Suhora 0.6	CCD	2011-05-30	4.9
terb110527-20	Terskol 2.0	CCD	2011-05-27	2.4
terb110529-17	Terskol 2.0	CCD	2011-05-27	4.4
terb110530-18	Terskol 2.0	CCD	2011-05-30	2.6
terb110531-17	Terskol 2.0	CCD	2011-05-31	1.8
terb110601-19	Terskol 2.0	CCD	2011-06-01	3.3
tueb110504-19	Tuebingen 0.8	SBig	2011-05-04	6.8
tueb110505-19	Tuebingen 0.8	SBig	2011-05-05	6.8
tueb110506-19	Tuebingen 0.8	SBig	2011-05-06	6.7
tueb110508-19	Tuebingen 0.8	SBig	2011-05-08	6.6
tueb110509-19	Tuebingen 0.8	SBig	2011-05-09	6.7
tueb110513-19	Tuebingen 0.8	SBig	2011-05-13	2.2
tueb110518-22	Tuebingen 0.8	SBig	2011-05-18	4.1
tueb110523-20	Tuebingen 0.8	SBig	2011-05-23	5.9
tueb110524-20	Tuebingen 0.8	SBig	2011-05-24	5.9
tueb110525-20	Tuebingen 0.8	SBig	2011-05-24	6.0
tueb110529-20	Tuebingen 0.8	SBig	2011-05-29	5.9
tueb110530-20	Tuebingen 0.8	SBig	2011-05-30	3.4
tubi110602-00	Tubitak 1.0	CCD	2011-06-02	1.1
tubi110602-23	Tubitak 1.0	CCD	2011-06-02	2.0
tubi110603-20	Tubitak 1.0	CCD	2011-06-03	5.2
boao120418-18	BOAO 1.8	CCD	2012-04-18	1.7
krak120418-00	Krakow 0.4	CCD	2012-04-18	1.8
krak120421-00	Krakow 0.4	CCD	2012-04-21	4.6
krak120423-00	Krakow 0.4	CCD	2012-04-23	1.7

Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
krak120428-00	Krakow 0.4	CCD	2012-04-28	6.6
krak120429-00	Krakow 0.4	CCD	2012-04-29	6.1
krak120430-00	Krakow 0.4	CCD	2012-04-30	6.7
mtlm120419-10	Mt. Lemmon 1.0	CCD	2012-04-19	1.7
mtlm120420-10	Mt. Lemmon 1.0	CCD	2012-04-20	1.4
mtlm120421-05	Mt. Lemmon 1.0	CCD	2012-04-21	6.2
mtlm120422-07	Mt. Lemmon 1.0	CCD	2012-04-22	4.8
na50120524-12	NAOC 0.5	CCD	2012-05-24	4.5
na50120525-12	NAOC 0.5	CCD	2012-05-25	4.4
na50120526-14	NAOC 0.5	CCD	2012-05-26	3.3
na50120527-12	NAOC 0.5	CCD	2012-05-27	4.9
na50120530-13	NAOC 0.5	CCD	2012-05-30	6.0
pjmo120425-03	PJMO 0.6	CCD	2012-04-25	7.2
pjmo120426-03	PJMO 0.6	CCD	2012-04-26	4.3
prom120430-04	PROMPT 0.4	CCD	2012-04-30	2.2
prom120430-07	PROMPT 0.4	CCD	2012-04-30	1.1
prom120501-04	PROMPT 0.4	CCD	2012-05-01	4.5
prom120502-04	PROMPT 0.4	CCD	2012-05-02	4.4
prom120503-04	PROMPT 0.4	CCD	2012-05-03	0.9
prom120504-04	PROMPT 0.4	CCD	2012-05-04	4.5
prom120509-04	PROMPT 0.4	CCD	2012-05-09	4.3
prom120510-04	PROMPT 0.4	CCD	2012-05-10	4.1
prom120511-03	PROMPT 0.4	CCD	2012-05-11	4.1
prom120512-03	PROMPT 0.4	CCD	2012-05-12	4.0
prom120513-03	PROMPT 0.4	CCD	2012-05-13	2.1
prom120514-03	PROMPT 0.4	CCD	2012-05-14	4.0
prom120515-03	PROMPT 0.4	CCD	2012-05-15	4.0
suho120501-20	Suhora 0.6	CCD	2012-05-01	6.0
suho120502-21	Suhora 0.6	CCD	2012-05-02	5.1
suho120503-23	Suhora 0.6	CCD	2012-05-03	2.4
suho120505-19	Suhora 0.6	CCD	2012-05-05	2.1
tubi120420-01	Tubitak 1.0	CCD	2012-04-20	1.0

Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
tubi120423-19	Tubitak 1.0	CCD	2012-04-23	6.6
tubi120512-20	Tubitak 1.0	CCD	2012-04-23	5.2
caam130412-21	Cannakkale 1.2	CCD	2013-04-12	4.0
caam130417-21	Cannakkale 1.2	CCD	2013-04-17	3.2
caam130418-21	Cannakkale 1.2	CCD	2013-04-18	4.5
krak130418-00	Krakow 0.4	CCD	2013-04-18	6.7
krak130421-00	Krakow 0.4	CCD	2013-04-18	7.4
krak130425-00	Krakow 0.4	CCD	2013-04-25	6.7
krak130426-00	Krakow 0.4	CCD	2013-04-26	2.1
naos130503-13	NAOC 0.85	CCD	2013-05-03	6.0
pjmo130429-06	PJMO 0.6	CCD	2013-04-29	2.7
pjmo130430-07	PJMO 0.6	CCD	2013-04-30	2.1
pjmo130503-07	PJMO 0.6	CCD	2013-05-03	3.6
pjmo130505-04	PJMO 0.6	CCD	2013-05-05	6.8
suho130422-21	Suhora 0.6	CCD	2013-04-22	4.6
suho130425-20	Suhora 0.6	CCD	2013-04-25	5.9
suho130426-20	Suhora 0.6	CCD	2013-04-26	5.4
suho130501-19	Suhora 0.6	CCD	2013-05-01	6.1
suho130504-21	Suhora 0.6	CCD	2013-05-04	4.4
suho130505-19	Suhora 0.6	CCD	2013-05-05	5.5
caam140610-19	Cannakkale 1.2	CCD	2014-04-10	5.5
krak140530-21	Krakow 0.4	CCD	2014-05-30	2.4
krak140604-20	Krakow 0.4	CCD	2014-06-04	5.5
krak140606-20	Krakow 0.4	CCD	2014-06-06	4.7
krak140607-20	Krakow 0.4	CCD	2014-06-07	5.1
krak140608-20	Krakow 0.4	CCD	2014-06-08	5.2
krak140609-19	Krakow 0.4	CCD	2014-06-09	4.7
krak140610-20	Krakow 0.4	CCD	2014-06-10	2.4
mcao140602-05	MCAO 0.6	CCD	2014-06-02	3.0
mcao140603-01	MCAO 0.6	CCD	2014-06-03	4.1
mcao140607-01	MCAO 0.6	CCD	2014-06-07	4.0
mcao140608-01	MCAO 0.6	CCD	2014-06-08	3.9

Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
mole140526-20	Moletai 1.65	CCD	2014-05-26	2.5
mole140527-20	Moletai 1.65	CCD	2014-05-27	3.7
mole140605-20	Moletai 1.65	CCD	2014-06-05	3.3
mole140607-20	Moletai 1.65	CCD	2014-06-07	2.9
naos140605-15	NAOC 0.85	CCD	2014-06-05	5.7
naos140606-17	NAOC 0.85	CCD	2014-06-06	3.9
pjmo140519-04	PJMO 0.6	CCD	2014-05-19	4.0
pjmo140529-03	PJMO 0.6	CCD	2014-05-29	5.4
pjmo140530-03	PJMO 0.6	CCD	2014-05-30	4.3
pjmo140531-02	PJMO 0.6	CCD	2014-05-31	5.0
pjmo140601-03	PJMO 0.6	CCD	2014-06-01	3.3
pjmo140602-02	PJMO 0.6	CCD	2014-06-02	4.5
pjmo140603-02	PJMO 0.6	CCD	2014-06-03	7.1
pjmo140604-04	PJMO 0.6	CCD	2014-06-04	5.8
pjmo140611-03	PJMO 0.6	CCD	2014-06-11	5.9
suho140604-19	Suhora 0.6	CCD	2014-06-04	4.4
suho140606-20	Suhora 0.6	CCD	2014-06-06	4.7
ters140604-20	Terskol 0.6	CCD	2014-06-04	2.0
ters140619-20	Terskol 0.6	CCD	2014-06-19	6.0
tsao140524-17	Tien Shan 1.0	CCD	2014-05-24	4.7
tsao140525-16	Tien Shan 1.0	CCD	2014-05-25	6.0
tsao140610-15	Tien Shan 1.0	CCD	2014-06-10	1.0
tsao140611-16	Tien Shan 1.0	CCD	2014-06-11	6.0
tsao140613-17	Tien Shan 1.0	CCD	2014-06-13	4.6
tsao140616-19	Tien Shan 1.0	CCD	2014-06-16	1.4
tsao140619-17	Tien Shan 1.0	CCD	2014-06-19	4.8
tsao140620-18	Tien Shan 1.0	CCD	2014-06-20	3.1
tsao140627-16	Tien Shan 1.0	CCD	2014-06-27	5.9
tsao140628-16	Tien Shan 1.0	CCD	2014-06-28	5.9
krak150420-20	Krakow 0.4	CCD	2015-05-20	2.1
krak150421-19	Krakow 0.4	CCD	2015-05-21	7.5
pjmo150420-03	PJMO 0.6	CCD	2015-04-20	7.3

Table 1—Continued

Run Name	Telescope	Detector	Date	Length (hrs)
pjmo150421-05	PJMO 0.6	CCD	2015-04-21	5.4
pjmo150422-03	PJMO 0.6	CCD	2015-04-22	4.7
pjmo150426-03	PJMO 0.6	CCD	2015-04-26	7.4
prom150428-08	PROMPT 0.4	CCD	2015-04-28	6.5
prom150429-03	PROMPT 0.4	CCD	2015-04-29	6.6
prom150501-03	PROMPT 0.4	CCD	2015-05-01	6.2
mcao160710-01	MCAO 0.6	CCD	2016-07-10	2.0
mcao160711-01	MCAO 0.6	CCD	2016-07-11	1.5
mcao160711-03	MCAO 0.6	CCD	2016-07-11	2.0
mcao160712-01	MCAO 0.6	CCD	2016-07-12	1.2
mcao160718-01	MCAO 0.6	CCD	2016-07-18	1.2
mcao160721-03	MCAO 0.6	CCD	2016-07-21	2.0
mcao160805-02	MCAO 0.6	CCD	2016-08-05	1.2
pjmo160802-03	PJMO 0.6	CCD	2016-08-02	4.6
pjmo160804-02	PJMO 0.6	CCD	2016-08-04	5.7
pjmo160805-02	PJMO 0.6	CCD	2016-08-05	5.6
pjmo160806-02	PJMO 0.6	CCD	2016-08-06	6.0
pjmo160808-03	PJMO 0.6	CCD	2016-08-08	4.8
suho160723-19	Suhora 0.6	CCD	2016-07-23	5.5
tueb160716-21	Tuebingen 0.8	CCD	2016-07-16	2.0
tueb160718-20	Tuebingen 0.8	CCD	2016-07-18	4.5
tueb160719-20	Tuebingen 0.8	CCD	2016-07-19	6.0
tueb160729-22	Tuebingen 0.8	CCD	2016-07-29	4.2
tueb160730-20	Tuebingen 0.8	CCD	2016-07-30	4.0
tueb160801-20	Tuebingen 0.8	CCD	2016-08-01	2.6
tueb160803-20	Tuebingen 0.8	CCD	2016-08-03	2.5
tueb160807-20	Tuebingen 0.8	CCD	2016-08-07	4.9
warw160802-20	Warwick 1.0m	CCD	2016-08-02	5.0
warw160803-20	Warwick 1.0m	CCD	2016-08-03	4.6
warw160804-20	Warwick 1.0m	CCD	2016-08-04	5.0

Note. — Data from the Warwick 1.0m telescope was obtained during an engineering run.

Table 2. 1982-2006 Detected Independent Frequencies

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
1982			
$1236.483 \pm 0.07$	808.75	$16.70 \pm 0.06$	9.4
$1431.112 \pm 0.04$	698.76	$30.86 \pm 0.06$	11.4
$1613.842 \pm 0.05$	619.65	$26.12 \pm 0.06$	9.7
$1618.845 \pm 0.06$	617.72	$29.34 \pm 0.06$	10.9
$2368.563 \pm 0.10$	422.20	$12.52 \pm 0.06$	5.8
1984			
$1124.701 \pm 3.5$	889.13	$24.7 \pm 1.1$	7.2
$1626.474 \pm 3.5$	614.83	$25.9 \pm 1.1$	6.8
1985			
$1166.944 \pm 0.05$	856.94	$6.43 \pm 0.06$	11
$1176.489 \pm 0.03$	849.99	$35.70 \pm 0.06$	16.0
$1614.554 \pm 0.04$	619.37	$9.15 \pm 0.06$	4.6
$2351.762 \pm 0.04$	425.22	$14.34 \pm 0.06$	12
1986			
$1081.025 \pm 0.03$	925.05	$5.1 \pm 0.4$	4.6
$1160.678 \pm 0.03$	861.57	$4.9 \pm 0.4$	4.5
$1223.803 \pm 0.02$	817.12	$9.3 \pm 0.4$	9.1
$1233.585 \pm 0.01$	810.65	$14.9 \pm 0.4$	14.5
$1385.240 \pm 0.03$	721.90	$6.7 \pm 0.4$	6.4
$1426.181 \pm 0.01$	701.17	$14.2 \pm 0.4$	14.3
$1525.030 \pm 0.01$	655.72	$21.3 \pm 0.4$	21.4
$1611.691 \pm 0.01$	620.47	$20.9 \pm 0.4$	20.9



Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$2157.783 \pm 0.04$	463.44	$2.4 \pm 0.4$	4
$2165.564 \pm 0.03$	461.77	$4.8 \pm 0.4$	4
$2368.938 \pm 0.03$	422.13	$5.8 \pm 0.4$	5.6
1990			
$1112.933 \pm 0.03$	898.53	$2.31 \pm 0.32$	4.0
$1114.180 \pm 0.03$	897.52	$2.44 \pm 0.32$	4.1
$1118.324 \pm 0.02$	894.20	$5.24 \pm 0.32$	8.9
$1119.089 \pm 0.03$	893.58	$2.83 \pm 0.32$	4.8
$1224.216 \pm 0.03$	816.83	$22.17 \pm 0.32$	4.0
$1233.413 \pm 0.03$	810.76	$4.80 \pm 0.32$	8.4
$1245.395 \pm 0.03$	802.96	$2.17 \pm 0.32$	4.0
$1288.995 \pm 0.03$	775.80	$3.50 \pm 0.32$	6.3
$1291.229 \pm 0.03$	774.46	$4.31 \pm 0.32$	7.6
$1295.400 \pm 0.03$	771.96	$3.27 \pm 0.32$	5.8
$1297.540 \pm 0.01$	770.69	$14.76 \pm 0.32$	26.1
$1304.075 \pm 0.03$	766.83	$4.67 \pm 0.32$	8.3
$1355.447 \pm 0.03$	737.76	$2.20 \pm 0.32$	4.0
$1361.728 \pm 0.03$	734.36	$2.90 \pm 0.32$	5.2
$1368.588 \pm 0.03$	730.68	$3.32 \pm 0.32$	5.9
$1375.434 \pm 0.03$	727.04	$3.18 \pm 0.32$	5.7
$1421.041 \pm 0.02$	703.71	$8.22 \pm 0.32$	15.1
$1423.704 \pm 0.03$	702.39	$3.11 \pm 0.32$	5.8
$1427.365 \pm 0.01$	700.59	$19.39 \pm 0.32$	35.8
$1428.663 \pm 0.03$	699.96	$2.61 \pm 0.32$	4.8
$1433.729 \pm 0.02$	697.48	$7.29 \pm 0.32$	13.5
$1435.209 \pm 0.03$	696.76	$3.50 \pm 0.32$	6.5
$1512.798 \pm 0.02$	661.03	$5.57 \pm 0.32$	10.7
$1518.661 \pm 0.02$	658.47	$8.34 \pm 0.32$	15.9

Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1519.372 \pm 0.02$	658.17	$5.88 \pm 0.32$	11.2
$1524.924 \pm 0.02$	655.77	$5.98 \pm 0.32$	11.2
$1525.498 \pm 0.02$	655.52	$6.95 \pm 0.32$	13.0
$1611.741 \pm 0.02$	620.45	$6.09 \pm 0.32$	12.3
$1617.380 \pm 0.03$	618.28	$4.69 \pm 0.32$	9.7
$1623.709 \pm 0.02$	615.87	$5.04 \pm 0.32$	10.2
$2154.009 \pm 0.03$	464.25	$4.40 \pm 0.32$	11.2
$2157.765 \pm 0.03$	463.44	$2.27 \pm 0.32$	5.8
$2358.946 \pm 0.02$	423.92	$5.63 \pm 0.32$	14.1
$2362.507 \pm 0.02$	423.28	$5.71 \pm 0.32$	14.3
$2366.408 \pm 0.03$	422.58	$4.59 \pm 0.32$	11.5
1991			
$1296.087 \pm 0.13$	771.55	$10.11 \pm 0.41$	17.7
$1296.577 \pm 0.13$	771.26	$18.25 \pm 0.41$	29.8
$1308.777 \pm 0.05$	764.07	$5.02 \pm 0.41$	9.9
$1396.945 \pm 0.05$	715.85	$3.95 \pm 0.41$	8.4
$1419.934 \pm 0.01$	704.26	$29.3 \pm 0.41$	45.6
$1423.333 \pm 0.04$	702.58	$5.23 \pm 0.41$	6.3
$1427.062 \pm 0.04$	700.74	$5.02 \pm 0.41$	7.4
$1443.381 \pm 0.05$	692.82	$4.81 \pm 0.41$	9.7
$2150.307 \pm 0.05$	465.05	$2.36 \pm 0.41$	4.1
$2154.389 \pm 0.05$	464.17	$2.79 \pm 0.41$	4.6
$2157.939 \pm 0.05$	463.41	$2.62 \pm 0.41$	4.4
$2370.121 \pm 0.04$	421.92	$5.01 \pm 0.41$	9.2
$2374.211 \pm 0.05$	421.19	$3.26 \pm 0.41$	5.7
$2378.195 \pm 0.04$	420.49	$5.58 \pm 0.41$	10.7
1992			

Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1035.357 \pm 0.05$	965.85	$7.35 \pm 0.31$	4.1
$1101.952 \pm 0.05$	907.48	$6.40 \pm 0.31$	3.7
$1195.349 \pm 0.003$	836.58	$6.90 \pm 0.31$	4.1
$1233.263 \pm 0.001$	810.86	$20.79 \pm 0.31$	13.5
$1242.854 \pm 0.003$	804.60	$14.20 \pm 0.31$	8.8
$1265.279 \pm 0.003$	790.34	$12.25 \pm 0.31$	7.1
$1420.845 \pm 0.001$	703.81	$20.59 \pm 0.31$	13.1
$1438.411 \pm 0.003$	695.21	$17.71 \pm 0.31$	11.9
$1622.795 \pm 0.003$	616.22	$14.26 \pm 0.31$	9.2
$1628.812 \pm 0.004$	613.94	$10.9 \pm 0.317$	6.5
$2162.104 \pm 0.007$	462.51	$4.26 \pm 0.31$	4.2
$2166.094 \pm 0.007$	461.66	$5.66 \pm 0.31$	4.8
$2351.041 \pm 0.007$	425.34	$6.75 \pm 0.31$	5.2
$2359.162 \pm 0.007$	423.88	$6.55 \pm 0.31$	9.2
$2366.443 \pm 0.007$	422.58	$7.17 \pm 0.31$	6.3
1994			
$939.264 \pm 0.007$	1064.66	$2.36 \pm 0.10$	5.7
$1024.773 \pm 0.011$	975.83	$3.46 \pm 0.10$	8.8
$1064.931 \pm 0.012$	939.03	$2.82 \pm 0.10$	7.4
$1106.833 \pm 0.013$	903.48	$2.56 \pm 0.10$	7.0
$1113.548 \pm 0.012$	898.03	$3.95 \pm 0.10$	9.4
$1164.637 \pm 0.012$	858.64	$3.12 \pm 0.10$	8.6
$1176.684 \pm 0.013$	849.85	$2.79 \pm 0.10$	7.2
$1224.306 \pm 0.012$	816.79	$3.28 \pm 0.10$	9.1
$1234.488 \pm 0.013$	810.05	$2.70 \pm 0.10$	7.7
$1235.491 \pm 0.005$	809.39	$13.13 \pm 0.10$	37.3
$1242.357 \pm 0.013$	804.92	$3.33 \pm 0.10$	9.1

Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1246.494 \pm 0.013$	802.25	$2.69 \pm 0.10$	7.8
$1286.550 \pm 0.003$	777.27	$9.45 \pm 0.10$	27.6
$1291.023 \pm 0.009$	774.58	$6.07 \pm 0.10$	17.8
$1293.244 \pm 0.009$	773.25	$5.85 \pm 0.10$	16.1
$1297.737 \pm 0.005$	770.57	$21.5 \pm 0.10$	61.9
$1298.710 \pm 0.010$	769.99	$4.45 \pm 0.10$	12.5
$1304.464 \pm 0.009$	766.64	$6.84 \pm 0.10$	19.9
$1305.352 \pm 0.013$	766.08	$2.54 \pm 0.10$	7.0
$1309.003 \pm 0.013$	763.94	$2.76 \pm 0.10$	8.4
$1312.045 \pm 0.013$	762.17	$2.67 \pm 0.10$	7.9
$1419.641 \pm 0.003$	704.40	$18.70 \pm 0.10$	54.4
$1422.947 \pm 0.013$	702.77	$2.86 \pm 0.10$	8.2
$1426.395 \pm 0.003$	701.07	$16.05 \pm 0.10$	46.6
$1430.851 \pm 0.005$	698.88	$10.35 \pm 0.10$	30.1
$1433.167 \pm 0.010$	697.75	$4.06 \pm 0.10$	11.6
$1437.607 \pm 0.006$	695.60	$8.28 \pm 0.10$	24.1
$1438.523 \pm 0.011$	695.16	$3.68 \pm 0.10$	10.7
$1440.997 \pm 0.012$	693.96	$2.48 \pm 0.10$	7.2
$1441.910 \pm 0.009$	693.52	$4.09 \pm 0.10$	11.9
$1611.351 \pm 0.009$	620.60	$5.12 \pm 0.10$	14.2
$1617.450 \pm 0.010$	618.26	$3.61 \pm 0.10$	9.5
$1618.545 \pm 0.009$	617.84	$4.04 \pm 0.10$	11.2
$1624.624 \pm 0.009$	615.53	$5.83 \pm 0.10$	16.1
$1625.634 \pm 0.009$	615.14	$4.89 \pm 0.10$	13.9
$2150.498 \pm 0.010$	465.01	$3.22 \pm 0.10$	9.4
$2154.130 \pm 0.010$	464.22	$4.75 \pm 0.10$	13.7
$2157.844 \pm 0.012$	463.43	$2.70 \pm 0.10$	7.6
$2358.880 \pm 0.010$	423.93	$4.53 \pm 0.10$	13.5
$2362.636 \pm 0.005$	423.26	$9.29 \pm 0.10$	31.2
$2366.505 \pm 0.011$	422.56	$4.26 \pm 0.10$	13.2

Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
1996			
$937.695 \pm 0.028$	1066.44	$4.27 \pm 0.28$	3.5
$1024.276 \pm 0.028$	976.30	$6.78 \pm 0.28$	6.4
$1104.483 \pm 0.025$	905.40	$5.55 \pm 0.28$	5.1
$1178.461 \pm 0.025$	848.56	$5.34 \pm 0.28$	5.5
$1227.936 \pm 0.022$	814.37	$7.66 \pm 0.28$	7.5
$1233.14 \pm 0.015$	810.94	$13.50 \pm 0.28$	12.1
$1234.506 \pm 0.015$	810.04	$12.53 \pm 0.28$	11.2
$1247.959 \pm 0.022$	801.31	$7.84 \pm 0.28$	4.1
$1261.275 \pm 0.015$	792.85	$11.08 \pm 0.28$	11.6
$1291.169 \pm 0.015$	774.49	$10.71 \pm 0.28$	9.7
$1294.899 \pm 0.025$	772.26	$6.95 \pm 0.28$	7.9
$1297.194 \pm 0.010$	770.89	$22.05 \pm 0.28$	2.4
$1420.057 \pm 0.010$	704.20	$20.26 \pm 0.28$	19.7
$1426.443 \pm 0.010$	701.04	$18.41 \pm 0.28$	8.3
$1430.253 \pm 0.012$	699.18	$13.15 \pm 0.28$	12.8
$1436.261 \pm 0.022$	696.25	$8.20 \pm 0.28$	9.1
$1628.296 \pm 0.028$	614.14	$6.13 \pm 0.28$	6.3
$2154.117 \pm 0.028$	464.23	$7.40 \pm 0.28$	7.1
$2358.806 \pm 0.025$	423.94	$6.01 \pm 0.28$	4.7
$2362.617 \pm 0.014$	423.26	$9.87 \pm 0.28$	9.0
$2367.288 \pm 0.025$	422.42	$8.92 \pm 0.28$	11.6
2000			
$938.991 \pm 0.015$	1064.95	$3.09 \pm 0.01$	9.4
$946.238 \pm 0.015$	1056.82	$1.20 \pm 0.01$	4.0
$1110.999 \pm 0.015$	900.09	$1.98 \pm 0.01$	7.0

Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1171.564 \pm 0.015$	853.56	$1.89 \pm 0.01$	7.0
$1173.021 \pm 0.012$	852.50	$2.32 \pm 0.01$	8.8
$1251.851 \pm 0.004$	798.82	$3.21 \pm 0.01$	12.5
$1254.503 \pm 0.002$	797.13	$8.60 \pm 0.01$	33.1
$1255.583 \pm 0.002$	796.44	$14.72 \pm 0.01$	67.1
$1256.248 \pm 0.005$	796.02	$8.04 \pm 0.01$	42.6
$1257.268 \pm 0.010$	795.38	$3.05 \pm 0.01$	15.2
$1258.288 \pm 0.010$	794.73	$2.06 \pm 0.01$	5.7
$1296.603 \pm 0.001$	771.25	$28.08 \pm 0.01$	109.5
$1378.795 \pm 0.008$	725.27	$4.33 \pm 0.01$	17.3
$1379.737 \pm 0.010$	724.78	$2.64 \pm 0.01$	10.4
$1420.101 \pm 0.001$	704.18	$29.78 \pm 0.01$	118.8
$1423.597 \pm 0.010$	702.45	$3.08 \pm 0.01$	12.3
$1736.660 \pm 0.014$	575.82	$1.02 \pm 0.01$	4.0
$2150.515 \pm 0.013$	465.01	$2.99 \pm 0.01$	11.2
$2154.040 \pm 0.013$	464.24	$5.38 \pm 0.01$	19.9
$2157.736 \pm 0.012$	463.45	$2.51 \pm 0.01$	9.5
$2359.118 \pm 0.008$	423.89	$5.51 \pm 0.01$	23.7
$2366.271 \pm 0.010$	422.61	$5.90 \pm 0.01$	25.1
2006			
$923.976 \pm 0.001$	1082.28	$1.43 \pm 0.01$	5.2
$938.216 \pm 0.002$	1065.85	$1.23 \pm 0.01$	4.4
$1024.496 \pm 0.002$	976.09	$1.44 \pm 0.01$	4.8
$1033.760 \pm 0.002$	967.34	$1.85 \pm 0.01$	6.9
$1039.075 \pm 0.001$	962.39	$7.95 \pm 0.01$	27.2
$1039.474 \pm 0.001$	962.02	$2.84 \pm 0.01$	9.7
$1041.535 \pm 0.001$	960.12	$1.18 \pm 0.01$	4.3
$1044.381 \pm 0.002$	957.50	$1.63 \pm 0.01$	6.2

Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1113.582 \pm 0.001$	898.02	$2.71 \pm 0.01$	9.4
$1120.404 \pm 0.001$	892.53	$2.09 \pm 0.01$	7.3
$1120.902 \pm 0.001$	892.14	$2.98 \pm 0.01$	10.2
$1121.704 \pm 0.002$	891.50	$1.26 \pm 0.01$	4.4
$1130.144 \pm 0.002$	884.84	$1.91 \pm 0.01$	7.3
$1161.552 \pm 0.001$	860.92	$2.74 \pm 0.01$	9.6
$1173.015 \pm 0.001$	852.50	$7.26 \pm 0.01$	25.4
$1178.096 \pm 0.002$	848.83	$1.13 \pm 0.01$	4.3
$1184.470 \pm 0.002$	844.26	$1.64 \pm 0.01$	6.2
$1222.199 \pm 0.002$	818.20	$1.72 \pm 0.01$	6.3
$1222.751 \pm 0.001$	817.83	$5.04 \pm 0.01$	18.3
$1222.945 \pm 0.001$	817.70	$4.59 \pm 0.01$	5.1
$1228.185 \pm 0.002$	814.21	$2.71 \pm 0.01$	9.4
$1228.791 \pm 0.001$	813.81	$5.27 \pm 0.01$	19.0
$1234.124 \pm 0.001$	810.29	$24.87 \pm 0.01$	88.0
$1239.510 \pm 0.001$	806.77	$5.07 \pm 0.01$	18.3
$1240.237 \pm 0.002$	806.30	$2.85 \pm 0.01$	9.9
$1244.790 \pm 0.002$	803.35	$1.90 \pm 0.01$	6.9
$1245.219 \pm 0.001$	803.07	$4.75 \pm 0.01$	17.1
$1246.032 \pm 0.001$	802.55	$4.44 \pm 0.01$	15.2
$1421.012 \pm 0.002$	703.72	$2.81 \pm 0.01$	7.1
$1429.209 \pm 0.001$	699.69	$5.65 \pm 0.01$	22.5
$1512.141 \pm 0.002$	661.31	$1.79 \pm 0.01$	6.4
$1736.301 \pm 0.001$	575.94	$16.38 \pm 0.01$	75.2
$1737.962 \pm 0.002$	575.39	$1.80 \pm 0.01$	7.6
$1741.666 \pm 0.001$	574.16	$11.01 \pm 0.01$	49.7
$1743.733 \pm 0.001$	573.48	$5.59 \pm 0.01$	8.4
$1749.083 \pm 0.001$	571.73	$11.88 \pm 0.01$	50.2
$1856.845 \pm 0.002$	538.55	$1.44 \pm 0.01$	6.4
$2150.393 \pm 0.001$	465.03	$4.06 \pm 0.01$	17.3

Table 2—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$2154.223 \pm 0.001$	464.20	$5.50 \pm 0.01$	21.8
$2158.073 \pm 0.001$	463.38	$7.24 \pm 0.01$	29.0
$2359.052 \pm 0.001$	423.90	$5.94 \pm 0.01$	22.1
$2363.058 \pm 0.002$	423.18	$1.7 \pm 0.011$	6.0
$2366.524 \pm 0.001$	422.56	$6.31 \pm 0.01$	23.0



Table 3. 2007-2016 Detected Independent Frequencies

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
2007			
$1088.951 \pm 0.05$	918.31	$6.9 \pm 0.7$	5
$1121.169 \pm 0.04$	891.93	$10.19 \pm 0.7$	8
$1233.956 \pm 0.02$	810.40	$23 \pm 0.7$	20
$1251.193 \pm 0.05$	799.24	$9.56 \pm 0.7$	8
$1735.444 \pm 0.03$	576.22	$13.47 \pm 0.7$	13
$2156.288 \pm 0.03$	463.76	$7.26 \pm 0.7$	7
2008			
$1235.745 \pm 0.004$	809.23	$25.14 \pm 0.33$	23.9
$1735.716 \pm 0.004$	576.13	$21.77 \pm 0.33$	22.6
$1741.459 \pm 0.009$	574.23	$10.14 \pm 0.33$	9.8
$1750.185 \pm 0.012$	571.37	$4.47 \pm 0.33$	5.6
$2150.245 \pm 0.011$	465.06	$5.66 \pm 0.33$	7.7
$2158.416 \pm 0.01$	463.30	$11.07 \pm 0.33$	11
$2358.895 \pm 0.01$	423.93	$7.95 \pm 0.33$	10.2
$2366.872 \pm 0.01$	422.50	$7.93 \pm 0.33$	10.2
2009			
$1088.445 \pm 0.018$	918.74	$7.82 \pm 0.28$	9
$1235.845 \pm 0.007$	809.16	$9.64 \pm 0.28$	22.4
$1236.849 \pm 0.018$	808.51	$7.91 \pm 0.28$	9.1
$1300.029 \pm 0.018$	576.29	$4.10 \pm 0.28$	4.7
$1735.699 \pm 0.006$	576.14	$20.60 \pm 0.28$	22.8
$1741.415 \pm 0.018$	574.25	$7.15 \pm 0.28$	8.8
$1750.525 \pm 0.012$	571.26	$5.98 \pm 0.28$	6.8
$2150.261 \pm 0.014$	465.06	$5.22 \pm 0.28$	6.5

Table 3—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$2158.474 \pm 0.003$	463.29	$10.72 \pm 0.28$	13.4
$2358.952 \pm 0.026$	423.92	$5.89 \pm 0.28$	8.1
$2366.893 \pm 0.018$	422.49	$7.28 \pm 0.28$	10.1
2010			
$1087.790 \pm 0.024$	919.29	$2.47 \pm 0.15$	5.1
$1104.397 \pm 0.023$	891.34	$2.57 \pm 0.15$	5.1
$1112.868 \pm 0.009$	898.58	$6.22 \pm 0.15$	13.2
$1121.901 \pm 0.005$	891.34	$12.6 \pm 0.15$	26.6
$1122.674 \pm 0.021$	889.89	$2.73 \pm 0.15$	5.2
$1123.731 \pm 0.020$	891.33	$3.31 \pm 0.15$	7.1
$1200.999 \pm 0.017$	832.64	$3.33 \pm 0.15$	7.1
$1211.906 \pm 0.021$	825.15	$2.99 \pm 0.15$	6.4
$1236.278 \pm 0.003$	808.88	$18.9 \pm 0.15$	39.7
$1237.011 \pm 0.023$	808.41	$4.10 \pm 0.15$	9.2
$1241.721 \pm 0.014$	805.33	$3.32 \pm 0.15$	7.0
$1299.274 \pm 0.018$	769.66	$3.89 \pm 0.15$	7.6
$1735.786 \pm 0.003$	576.11	$21.72 \pm 0.015$	46.9
$1741.367 \pm 0.004$	574.26	$8.34 \pm 0.15$	17.2
$1745.084 \pm 0.025$	573.04	$2.53 \pm 0.15$	5.2
$1750.642 \pm 0.005$	571.19	$10.93 \pm 0.15$	25.2
$1859.431 \pm 0.023$	537.80	$2.50 \pm 0.15$	5.2
$2008.645 \pm 0.025$	497.85	$2.66 \pm 0.15$	5.2
$2150.138 \pm 0.025$	465.09	$4.52 \pm 0.15$	9.7
$2154.432 \pm 0.025$	464.16	$2.17 \pm 0.15$	5.1
$2158.524 \pm 0.006$	463.28	$9.95 \pm 0.15$	23.4
$2358.818 \pm 0.008$	423.94	$9.05 \pm 0.15$	23.1
$2362.603 \pm 0.017$	423.26	$3.19 \pm 0.15$	6.7
$2366.990 \pm 0.008$	422.48	$7.40 \pm 0.15$	19

Table 3—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
2011			
$954.564 \pm 0.008$	1047.59	$5.2 \pm 0.17$	10.9
$954.922 \pm 0.02$	1047.21	$4.22 \pm 0.17$	8.7
$1113.07 \pm 0.013$	898.45	$5.72 \pm 0.17$	11.6
$1113.641 \pm 0.02$	897.96	$3.2 \pm 0.17$	6.4
$1235.415 \pm 0.004$	809.44	$9.76 \pm 0.17$	20.1
$1362.917 \pm 0.03$	733.72	$1.66 \pm 0.17$	4.0
$1511.537 \pm 0.03$	661.58	$1.85 \pm 0.17$	4.3
$1614.827 \pm 0.016$	619.26	$4.40 \pm 0.17$	10.5
$1735.871 \pm 0.003$	576.08	$20.3 \pm 0.17$	150.5
$1736.14 \pm 0.037$	575.99	$4.70 \pm 0.17$	11.1
$1747.123 \pm 0.037$	572.37	$3.30 \pm 0.17$	7.9
$1750.833 \pm 0.013$	571.16	$3.77 \pm 0.17$	8.9
$1856.872 \pm 0.03$	538.54	$1.62 \pm 0.17$	3.8
$2008.742 \pm 0.02$	497.82	$3.00 \pm 0.17$	7.4
$2150.138 \pm 0.016$	465.09	$4.98 \pm 0.17$	13.4
$2154.294 \pm 0.028$	464.19	$3.73 \pm 0.17$	8.9
$2158.571 \pm 0.007$	463.27	$9.10 \pm 0.17$	24.6
$2358.707 \pm 0.009$	423.96	$7.63 \pm 0.17$	27.4
$2359.911 \pm 0.04$	423.74	$1.58 \pm 0.17$	2.47
$2366.986 \pm$	422.48	$8.9 \pm 0.17$	26.4
2012			
$1113.254 \pm 0.014$	898.27	$4.1 \pm 0.24$	6.2
$1161.469 \pm 0.01$	860.98	$5.21 \pm 0.24$	7.7
$1211.371 \pm 0.006$	825.51	$9.89 \pm 0.24$	14.7
$1212.822 \pm 0.009$	824.52	$6.46 \pm 0.24$	9.6

Table 3—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1213.995 \pm 0.006$	823.73	$8.66 \pm 0.24$	12.8
$1215.915 \pm 0.01$	822.43	$5.72 \pm 0.24$	8.5
$1222.598 \pm 0.004$	817.930	$11.53 \pm 0.24$	17.2
$1226.836 \pm 0.008$	815.11	$6.89 \pm 0.24$	10.3
$1233.355 \pm 0.004$	810.80	$12.53 \pm 0.24$	18.6
$1235.431 \pm 0.009$	809.43	$6.70 \pm 0.24$	9.9
$1246.397 \pm 0.004$	802.31	$13.35 \pm 0.24$	19.9
$1258.482 \pm 0.009$	794.61	$6.26 \pm 0.24$	9.3
$1723.487 \pm 0.015$	580.22	$4.30 \pm 0.24$	6.1
$1734.393 \pm 0.006$	576.57	$8.54 \pm 0.24$	12.1
$1735.975 \pm 0.004$	576.05	$12.4 \pm 0.24$	17.7
$1745.543 \pm 0.012$	572.89	$4.44 \pm 0.24$	6.3
$1747.152 \pm 0.008$	572.36	$6.74 \pm 0.24$	9.5
$1748.895 \pm 0.009$	571.79	$6.36 \pm 0.24$	9.0
$1750.345 \pm 0.008$	571.32	$6.19 \pm 0.24$	8.9
$2150.072 \pm 0.02$	465.10	$2.74 \pm 0.24$	4.0
$2155.981 \pm 0.015$	463.83	$3.68 \pm 0.24$	5.2
$2158.563 \pm 0.006$	463.27	$8.86 \pm 0.24$	12.2
$2181.89 \pm 0.015$	458.32	$3.60 \pm 0.24$	4.9
$2355.788 \pm 0.01$	424.47	$5.48 \pm 0.24$	8.7
$2358.721 \pm 0.01$	423.96	$5.17 \pm 0.24$	8.4
$2366.807 \pm 0.009$	422.51	$5.67 \pm 0.24$	9.4
$2372.715 \pm 0.015$	421.46	$3.81 \pm 0.24$	6.2
2013			
$1086.678 \pm 0.007$	920.24	$5.28 \pm 0.18$	6.5
$1097.639 \pm 0.014$	911.05	$5.28 \pm 0.18$	5.4
$1123.417 \pm 0.023$	890.14	$4.46 \pm 0.18$	5.9
$1235.981 \pm 0.005$	809.07	$15.5 \pm 0.18$	15.0

Table 3—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1241.938 \pm 0.013$	805.19	$7.50 \pm 0.18$	6.4
$1362.571 \pm 0.027$	733.91	$4.74 \pm 0.18$	5.2
$1614.872 \pm 0.007$	619.24	$10.3 \pm 0.18$	10.2
$1629.282 \pm 0.015$	613.77	$4.80 \pm 0.18$	5.3
$1736.468 \pm 0.008$	575.88	$11.1 \pm 0.18$	11.4
$1737.008 \pm 0.008$	575.70	$11.2 \pm 0.18$	11.4
$1746.007 \pm 0.024$	572.73	$5.66 \pm 0.18$	5.7
$2158.532 \pm 0.006$	463.28	$8.32 \pm 0.18$	9.4
$2162.432 \pm 0.014$	462.44	$5.55 \pm 0.18$	6.4
$2358.635 \pm 0.007$	423.97	$7.50 \pm 0.18$	9.2
$2364.019 \pm 0.026$	423.013	$3.20 \pm 0.18$	6.4
$2367.037 \pm 0.014$	422.47	$6.82 \pm 0.18$	8.5
2014			
$1023.122 \pm 0.010$	977.40	$2.38 \pm 0.17$	6.1
$1113.15 \pm 0.008$	898.35	$3.53 \pm 0.17$	8.7
$1158.561 \pm 0.014$	863.14	$2.33 \pm 0.17$	6.0
$1161.856 \pm 0.008$	860.69	$3.33 \pm 0.17$	8.6
$1171.782 \pm 0.010$	853.40	$2.96 \pm 0.17$	8.0
$1172.914 \pm 0.011$	852.58	$2.20 \pm 0.17$	5.7
$1221.633 \pm 0.004$	818.58	$5.19 \pm 0.17$	13.6
$1230.125 \pm 0.008$	812.93	$3.11 \pm 0.17$	8.1
$1234.599 \pm 0.003$	809.98	$10.08 \pm 0.17$	26.4
$1235.145 \pm 0.001$	809.62	$8.63 \pm 0.17$	48.7
$1236.228 \pm 0.004$	808.96	$5.96 \pm 0.17$	15.6
$1237.719 \pm 0.001$	807.94	$9.60 \pm 0.17$	25.0
$1248.182 \pm 0.01$	801.17	$2.79 \pm 0.17$	7.3
$1299.081 \pm 0.003$	769.77	$10.58 \pm 0.17$	27.6
$1311.786 \pm 0.008$	762.32	$3.29 \pm 0.17$	8.5

Table 3—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$1312.442 \pm 0.008$	761.94	$3.10 \pm 0.17$	8.3
$1361.767 \pm 0.018$	734.34	$1.61 \pm 0.17$	4.1
$1371.106 \pm 0.014$	729.34	$1.93 \pm 0.17$	5.0
$1429.540 \pm 0.009$	699.53	$2.97 \pm 0.17$	7.5
$1511.465 \pm 0.006$	661.61	$4.25 \pm 0.17$	10.2
$1520.357 \pm 0.018$	657.74	$1.53 \pm 0.17$	3.7
$1633.117 \pm 0.023$	612.33	$1.19 \pm 0.17$	2.7
$1735.955 \pm 0.001$	576.05	$18.4 \pm 0.17$	43.1
$1739.822 \pm 0.019$	574.77	$1.47 \pm 0.17$	3.5
$1740.357 \pm 0.012$	574.59	$2.22 \pm 0.17$	5.3
$1741.406 \pm 0.019$	574.25	$1.3 \pm 0.17$	3.3
$2003.559 \pm 0.009$	499.11	$1.9 \pm 0.17$	4.4
$2008.717 \pm 0.018$	497.83	$1.53 \pm 0.17$	3.5
$2150.280 \pm 0.010$	465.06	$2.78 \pm 0.17$	6.7
$2154.315 \pm 0.012$	464.18	$2.55 \pm 0.17$	6.1
$2158.634 \pm 0.002$	463.26	$14.41 \pm 0.17$	34.8
$2358.801 \pm 0.010$	423.94	$6.97 \pm 0.17$	18.2
$2362.782 \pm 0.005$	423.23	$5.86 \pm 0.17$	15.3
$2366.854 \pm 0.003$	422.50	$8.24 \pm 0.17$	21.4
2015			
$1172.340 \pm 0.024$	852.99	$7.92 \pm 0.40$	9.1
$1235.760 \pm 0.006$	809.22	$29.01 \pm 0.33$	35.4
$1299.269 \pm 0.021$	769.66	$8.94 \pm 0.63$	10.6
$1736.429 \pm 0.008$	575.89	$21.68 \pm 0.33$	26.7
$2150.059 \pm 0.060$	465.10	$3.004 \pm 0.70$	4
$2154.861 \pm 0.090$	464.07	$2.06 \pm 0.73$	4
$2158.634 \pm 0.011$	463.26	$17.3 \pm 0.3$	22.9
$2358.722 \pm 0.034$	423.96	$5.501 \pm 0.7$	8.1

Table 3—Continued

Frequency $\mu\text{Hz}$	Period s	Amplitude mma	Signal/Noise
$2362.884 \pm 0.023$	423.21	$8.48 \pm 0.70$	12.45
$2366.742 \pm 0.024$	422.52	$8.11 \pm 0.7$	11.9
2016			
$985.856 \pm 0.015$	1014.35	$2.9 \pm 0.3$	4.0
$1024.527 \pm 0.01$	976.06	$5.0 \pm 0.3$	5.3
$1300.192 \pm 0.013$	769.12	$3.3 \pm 0.3$	4.0
$1360.071 \pm 0.01$	735.26	$7.1 \pm 0.3$	8.0
$1430.250 \pm 0.01$	699.18	$5.0 \pm 0.3$	5.7
$1511.674 \pm 0.002$	661.52	$20.5 \pm 0.3$	24.4
$1620.449 \pm 0.006$	617.11	$7.1 \pm 0.3$	9.0
$1626.906 \pm 0.01$	614.66	$4.2 \pm 0.3$	5.3
$1736.647 \pm 0.003$	575.82	$15.3 \pm 0.3$	21.2
$2150.461 \pm 0.01$	465.02	$4.1 \pm 0.3$	5.2
$2154.493 \pm 0.016$	464.15	$2.6 \pm 0.3$	4.9
$2158.579 \pm 0.003$	463.27	$7.2 \pm 0.3$	24.9
$2358.772 \pm 0.006$	423.95	$7.2 \pm 0.3$	13.6
$2362.575 \pm 0.01$	423.27	$5.0 \pm 0.3$	9.4
$2366.713 \pm 0.005$	422.53	$8.8 \pm 0.3$	16.4

Table 4. List of 15 Periods Used in Fitting GD358 and Corresponding Best Fit Periods.

$k$	Frequency ( $\mu\text{Hz}$ )	Period (s)	Uncertainty (s)	Best fit periods (s)
8	2363.318	423.13	0.04	423.12
9	2155.544	463.92	0.04	463.87
10	2007.628	497.83	0.2	493.14
11	1857.716	538.30	0.3	540.80
12	1741.505	574.22	0.1	574.90
13	1619.700	617.40	0.2	615.98
14	1518.160	658.69	0.5	656.61
15	1428.943	699.82	0.2	701.43
16	1369.336	730.28	0.8	741.67
17	1299.147	769.74	0.2	768.84
18	1238.129	807.67	0.2	809.40
19	1170.207	854.55	0.6	854.01
20	1109.271	901.49	0.8	893.00
22	1032.967	968.09	1.4	967.71
24	941.333	1062.32	3.1	1051.16
			$\sigma_{\text{rms}}$	0.964 s
			BIC ( $n_{\text{obs}} = 15$ )	-0.0303 s



Table 5. Master Grid and Best Fit Parameters

	$T_{\text{eff}}[\text{K}]$	Mass [ $M_{\odot}$ ]	$M_{\text{env}}$	$M_{\text{He}}$	$X_o$	$q_{\text{fm}}$	
Minimum	21,000	0.500	−2.00	−4.00	0.10	0.10	
Maximum	30,000	0.700	−3.40	−7.00	1.00	0.80	
Step size	200	0.010	0.20	0.20	0.10	0.05	
Best fit parameters							
Initial fit	23,600	0.57	−2.0	−5.6	0.50	0.20	1.069 s
Refined fit	23,650.0	0.5706	−2.0	−5.5	0.500	0.195	0.964 s