

**Problem 3.** Consider a sinusoidal electromagnetic wave propagating in the  $+x$  direction, whose electric field is parallel to the  $y$  axis. The wave has wavelength 475 nm, and the electric field has amplitude  $3.20 \times 10^{-3} \text{ V m}^{-1}$ . What is the frequency of the wave? What is the amplitude of the magnetic field? What are the vector equations for  $\mathbf{E}(x, t)$  and  $\mathbf{B}(x, t)$ ?

**Solution.** Frequency is related to wavelength by  $v = \lambda f$ , where the wave speed  $v = c$  for an electromagnetic wave in vacuum. So

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m s}^{-1}}{475 \times 10^{-9} \text{ m}} = \frac{3.00 \times 10^8}{4.75 \times 10^{-7}} \text{ Hz} = 6.32 \times 10^{14} \text{ Hz}.$$

The amplitudes of the fields are related by  $E_0 = cB_0$ , so

$$B_0 = \frac{E_0}{c} = \frac{3.20 \times 10^{-3} \text{ V m}^{-1}}{3.00 \times 10^8 \text{ m s}^{-1}} = \frac{3.20}{3.00} \times 10^{-11} \text{ T} = 1.07 \times 10^{-11} \text{ T}.$$

where we have used  $1 \text{ T} = 1 \text{ V s m}^{-2}$ .

The direction of propagation of the wave is  $\mathbf{E} \times \mathbf{B}$ . We know from the problem statement that the wave is propagating in the  $\hat{\mathbf{x}}$  direction, and that the electric field points in the  $\hat{\mathbf{y}}$  direction. Since  $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ , the magnetic field must point in the  $\hat{\mathbf{z}}$  direction. Then the vector equations are

$$\mathbf{E}(x, t) = E_0 \cos(kx - \omega t), \quad \mathbf{B}(x, t) = B_0 \cos(kx - \omega t).$$

We can find the wave number  $k$  and angular frequency  $\omega$  as follows:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{4.75 \times 10^{-7} \text{ m}} = 1.32 \times 10^7 \text{ rad m}^{-1},$$

$$\omega = 2\pi f = (2\pi \text{ rad})(6.32 \times 10^{14} \text{ Hz}) = 3.97 \times 10^{15} \text{ rad s}^{-1}.$$

Then we have

$$\mathbf{E}(x, t) = (3.20 \times 10^{-3} \text{ V m}^{-1}) \cos[(1.32 \times 10^7 \text{ rad m}^{-1})x - (3.97 \times 10^{15} \text{ rad s}^{-1})t],$$

$$\mathbf{B}(x, t) = (1.07 \times 10^{-11} \text{ T}) \cos[(1.32 \times 10^7 \text{ rad m}^{-1})x - (3.97 \times 10^{15} \text{ rad s}^{-1})t].$$