Problem 1. (Jackson 14.1) Verify by explicit calculation that the Liénard-Wiechert expressions for *all* components of $\mathbf{E}$ and $\mathbf{B}$ for a particle moving with constant velocity agree with the ones obtained in the text by means of a Lorentz transformation. Follow the general method at the end of Section 14.1.

Solution. The Liénard-Wiechert expressions for the fields are given by Jackson (14.13–14):

$$\mathbf{B} = [\mathbf{n} \times \mathbf{E}]_{\text{ret}}, \qquad \mathbf{E}(\mathbf{x}, t) = e\left[\frac{\mathbf{n} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta}\cdot\mathbf{n})^3 R^2}\right]_{\text{ret}} + \frac{e}{c}\left[\frac{\mathbf{n} \times \{(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \boldsymbol{\beta}\cdot\mathbf{n})^3 R}\right]_{\text{ret}}.$$

For a particle moving with constant velocity, $\dot{\boldsymbol{\beta}} = 0$ and so $\mathbf{E}(\mathbf{x}, t)$ becomes

$$\mathbf{E}(\mathbf{x}, t) = e\left[\frac{\mathbf{n} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta}\cdot\mathbf{n})^3 R^2}\right]_{\text{ret}}.$$

The expressions for the components of $\mathbf{E}$ and $\mathbf{B}$ obtained by a Lorentz transformation are given by Jackson (11.152):

$$E_1 = -\frac{e\gamma vt}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}, \qquad E_2 = \frac{e\gamma b}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}, \qquad E_3 = B_1 = B_2 = 0, \qquad B_3 = \beta E_2,$$

where the particle is moving in the x_1 direction at impact parameter b, as shown in Fig. (1). From Jackson (14.16), note that

$$[(1 - \boldsymbol{\beta}\cdot\mathbf{n})R]^2 = b^2 + v^2 t^2 - \beta^2 b^2 = \frac{b^2 + \gamma^2 v^2 t^2}{\gamma}$$

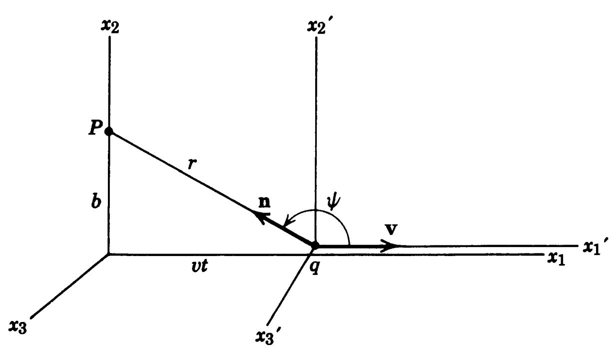

Figure 1: (Jackson Fig. 11.8) Particle of charge q moving at constant velocity $\mathbf{v}$ passes an observation point P at impact parameter b.

Problem 2. **(Jackson 14.3)** The Heaviside-Feynman expression for the electric field of a particle of charge e in arbitrary motion, an alternative to the Liénard-Wiechert expression (14.14) is

$$\mathbf{E} = e\left[\frac{\mathbf{n}}{R^2}\right]_{\text{ret}} + e\left[\frac{R}{c}\right]_{\text{ret}} \frac{d}{dt}\left[\frac{\mathbf{n}}{R^2}\right]_{\text{ret}} + \frac{e^2}{c^2}\frac{d^2}{dt^2}[\mathbf{n}]_{\text{ret}},$$

where the time derivatives are with respect to the time at the observation point. The magnetic fields are given by (14.3).

Using the fact that the retarded time is $t' = t - R(t')/c$ and that, as a result,

$$\frac{dt}{dt'} = 1 - \boldsymbol{\beta}(t') \cdot \mathbf{n}(t'),$$

show that the form above yields (14.14) when the time differentiations are performed.

Problem 3. **(Jackson 14.4)** Using the Liénard-Wiechart fields, discuss the time-averaged power radiated per unit solid angle in nonrelativistic motion of a particle with charge e. Sketch the angular distribution of the radiation and determine the total power radiated in each case.

3(a) The particle is moving along the z axis with instantaneous position $z(t) = \alpha \cos \omega_0 t$.

3(b) The particle is moving in a circle of radius R in the xy plane with constant angular frequency ω_0.