

Problem 25.58 A resistor with resistance R is connected to a battery that has emf 12.0 V and internal resistance $r = 0.400\ \Omega$. For what two values of R will the power in the resistor be 80.0 W ?

Solution. The power P delivered to a resistor is

$$P = I^2 R, \quad (25.18)$$

where I is the current through the resistor and R its resistance. We can find the current from

$$V_{ab} = \mathcal{E} - Ir, \quad (25.17)$$

where V_{ab} is the voltage difference across the resistor, \mathcal{E} is the emf of the battery, and r its internal resistance. We also know that

$$V_{ab} = IR. \quad (25.11)$$

Substituting (25.11) into (25.17), we get

$$IR = \mathcal{E} - Ir \implies \mathcal{E} = I(R + r) \implies I = \frac{\mathcal{E}}{R + r}.$$

Now we can substitute this result into (25.18) and solve for R :

$$\begin{aligned} P &= \frac{\mathcal{E}^2}{(R + r)^2} R \implies \mathcal{E}^2 R = P(R^2 + 2Rr + r^2) \implies 0 = PR^2 + (2Pr - \mathcal{E}^2)R + Pr^2 \\ &\implies R = \frac{\mathcal{E}^2 - 2Pr \pm \sqrt{(2Pr - \mathcal{E}^2)^2 - 4P^2 r^2}}{2P} \end{aligned}$$

Plugging in our numerical values for r , P , and \mathcal{E} , and recalling that $1\text{ W} = 1\text{ V}^2\ \Omega^{-1}$, we get

$$\begin{aligned} R &= \frac{(12.0\text{ V})^2 - 2(80.0\text{ W})(0.400\ \Omega) \pm \sqrt{[2(80.0\text{ W})(0.400\ \Omega) - (12.0\text{ V})^2]^2 - 4(80.0\text{ W})^2(0.400\ \Omega)^2}}{2(80.0\text{ W})} \\ &= \frac{80.0\text{ V}^2 - \pm \sqrt{(80\text{ V}^2)^2 - (64\text{ V}^2)}}{160\text{ V}^2\ \Omega^{-1}} = \frac{80.0\text{ V}^2 \pm \sqrt{2306\text{ V}^4}}{160\text{ V}^2\ \Omega^{-1}} = \frac{80.0 \pm 48.0}{160}\ \Omega = (0.50 \pm 0.30)\ \Omega \\ &= \begin{cases} 0.80\ \Omega, \\ 0.20\ \Omega. \end{cases} \end{aligned}$$

Exercise 26.26 In the circuit shown in Fig. E26.28, find

- the current in each branch, and
- the potential difference V_{ab} of point a relative to point b .

Figure E26.28

Solution.

- We need to use Kirchhoff's rules. Since this circuit has more than one loop, we need to use both the junction rule,

$$\sum I = 0, \quad (26.5)$$

and the loop rule,

$$\sum V = 0. \quad (26.6)$$

Let's choose the current to be flowing to the right across the 10.00 V battery, and start with the loop rule. For the top loop, we have

$$0 = 10\text{ V} - I_1(2\ \Omega) - I_1(1\ \Omega) - 5\text{ V} - I_1(4\ \Omega) - I_1(3\ \Omega) \quad (\text{A})$$