Problem 1. A young cousin of yours is having a birthday party, and you have been tasked with inflating balloons. Say that you are able to inflate a balloon (which is to be treated as a perfect sphere) to a diameter of 30.0 cm. Inside the balloon, the temperature is 30.0 °C and the absolute pressure is 1.20 atm. Assume that you exhale pure N_2, which has molar mass $28.0\,\mathrm{g\,mol^{-1}}$.

1.1 What is the mass of a single N_2 molecule?

Solution. We can find the mass m of a single molecule by diving the molar mass M by Avogadro's number:

$$m = \frac{M}{N_A} = \frac{28.0\,\mathrm{g\,mol^{-1}}}{6.022\times10^{23}\,\mathrm{molecules\,mol^{-1}}} = 4.65\times10^{-23}\,\mathrm{g}.$$

1.2 What is the (average) kinetic energy per N_2 molecule inside the balloon?

Solution. The average kinetic energy of a single molecule is given by

$$\varepsilon_{\mathrm{av}} = \frac{1}{2}m(v^2)_{\mathrm{av}},$$

where $(v^2)_{\mathrm{av}}$ can be found using the root-mean-square speed of a molecule, which is

$$v_{\mathrm{rms}} = \sqrt{(v^2)_{\mathrm{rms}}} = \sqrt{\frac{3kT}{m}}.$$

Making this substitution and plugging in numbers, we have

$$\varepsilon_{\mathrm{av}} = \frac{m}{2}\frac{3kT}{m} = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23}\,\mathrm{J\,K^{-1}})(303.15\,\mathrm{K}) = 6.28\times10^{-21}\,\mathrm{J},$$

where we have transformed to Kelvin using $0\,\mathrm{K} = -273.15\,°\mathrm{C}$.

1.3 How many N_2 molecules are in the balloon?

Solution. We can find the number of molecules N using the ideal gas law, $PV = NkT$. The volume V of the balloon is just the volume of a sphere of radius $r = 15.0\,\mathrm{cm}$, $V = 4\pi r^3/3$. We also need to write the pressure P in Pa:

$$P = (1.20\,\mathrm{atm})\frac{1.013\times10^5\,\mathrm{Pa}}{1\,\mathrm{atm}} = 1.22\times10^5\,\mathrm{Pa}.$$

Finally, solving the ideal gas law for N and substituting, we find

$$N = \frac{PV}{kT} = \frac{4\pi r^3 P}{3kT} = \frac{4\pi(0.150\,\mathrm{m})^3(1.22\times10^5\,\mathrm{Pa})}{3(1.38\times10^{-23}\,\mathrm{J\,K^{-1}})(303.15\,\mathrm{K})} = \frac{4\pi(3.38)(1.22)\times10^2}{3(1.38)(3.0315)\times10^{-21}} = \frac{51.6}{16.8}\times10^{23} = 3.08\times10^{23},$$

where we have used $1\,\mathrm{Pa} = 1\,\mathrm{J\,m^{-3}}$.

1.4 What is the total kinetic energy of the gas inside the balloon?

Solution. We can simply multiply the kinetic energy per molecule by the total number of molecules to find the total:

$$\mathcal{E} = N\varepsilon = (3.08\times10^{23})(6.28\times10^{-21}\,\mathrm{J}) = 1930\,\mathrm{J}.$$

Problem 2. You have a summer engineering internship at a chemical plant. One of your assignments is to determine the specific heat capacity of an unknown chemical at the plant using only a small metal vat, hot water, and a thermometer. The vat weighs 200.0 g. Both the chemical and the vat are initially at 20.0 °C, and the water is at 80.0 °C. You pour 500.0 g of the chemical and 500.0 g of the water into the vat, and wait for the system to reach thermal equilibrium. You then measure the temperature of both liquids (and the vat) as 58.1 °C. You safely dispose of the mixture, wait for the vat to return to its initial temperature, and repeat the experiment. This time, you use 1000.0 g of the chemical and 500.0 g of the water, and measure the equilibrium temperature as 49.3 °C. Determine the specific heat of the chemical and of the metal vat. (Assume that the specific heat capacities of both are constant over this temperature range, and that no heat is lost to your surroundings.)

Solution. We will use the relation

$$\Delta Q = C\,\Delta T = mc\,\Delta T,$$

where ΔQ is the total change in heat, ΔT is the change in temperature, C is the heat capacity, and $C = mc$ where m is the mass and c is the specific heat capacity.

Since no heat is lost to our surroundings in this problem, the total change in heat for all three components of the system must be zero. That is,

$$\Delta Q_c + \Delta Q_w + \Delta Q_v = 0,$$

where Q_c, Q_w, and and Q_v are the heat of the chemical, water, and vat, respectively. By writing this equation for each of the two experiments performed, we will obtain two equations in the two unknowns c_c and c_v, which are the specific heat of the chemical and of the vat, respectively. The specific heat of water is $c_w = 4190\,\mathrm{J\,kg^{-1}\,K^{-1}}$.

For the first experiment, we have

$$\begin{aligned}
\Delta Q_c &= m_c c_c\,\Delta T_c = (0.500\,\mathrm{kg})c_c(58.1\,^\circ\mathrm{C} - 20.0\,^\circ\mathrm{C}) = (19.05\,\mathrm{kg\,K})\,c_c,\\
\Delta Q_w &= m_w c_w\,\Delta T_w = (0.500\,\mathrm{kg})(4190\,\mathrm{J\,kg^{-1}\,K^{-1}})(58.1\,^\circ\mathrm{C} - 80.0\,^\circ\mathrm{C}) = -4.588\times 10^4\,\mathrm{J},\\
\Delta Q_v &= m_v c_v\,\Delta T_v = (0.200\,\mathrm{kg})c_v(58.1\,^\circ\mathrm{C} - 20.0\,^\circ\mathrm{C}) = (7.62\,\mathrm{kg\,K})\,c_v,
\end{aligned}$$

where we have used $1\,\mathrm{K} = 1\,^\circ\mathrm{C}$ on a relative scale. This gives us the equation

$$4.588\times 10^4\,\mathrm{J\,kg^{-1}\,K^{-1}} = 19.05\,c_c + 7.62\,c_v. \tag{1}$$

For the second experiment,

$$\begin{aligned}
\Delta Q_c &= m_c c_c\,\Delta T_c = (1.00\,\mathrm{kg})c_c(49.3\,^\circ\mathrm{C} - 20.0\,^\circ\mathrm{C}) = (29.3\,\mathrm{kg\,K})\,c_c,\\
\Delta Q_w &= m_w c_w\,\Delta T_w = (0.500\,\mathrm{kg})(4190\,\mathrm{J\,kg^{-1}\,K^{-1}})(49.3\,^\circ\mathrm{C} - 80.0\,^\circ\mathrm{C}) = -6.432\times 10^4\,\mathrm{J},\\
\Delta Q_v &= m_v c_v\,\Delta T_v = (0.200\,\mathrm{kg})c_v(49.3\,^\circ\mathrm{C} - 20.0\,^\circ\mathrm{C}) = (5.86\,\mathrm{kg\,K})\,c_v,
\end{aligned}$$

giving us the equation

$$6.432\times 10^4\,\mathrm{J\,kg^{-1}\,K^{-1}} = 29.3\,c_c + 5.86\,c_v. \tag{2}$$

Dividing (1) by 19.05 and (2) by 29.3, we find

$$2408\,\mathrm{J\,kg^{-1}\,K^{-1}} = c_c + 0.400\,c_v, \tag{1}$$

$$2195\,\mathrm{J\,kg^{-1}\,K^{-1}} = c_c + 0.200\,c_v. \tag{2}$$

Subtracting (2) from (1), we find

$$213\,\mathrm{J\,kg^{-1}\,K^{-1}} = 0.200\,c_v \qquad\Longrightarrow\qquad c_v = 1070\,\mathrm{J\,kg^{-1}\,K^{-1}}.$$

Substituting this into (2), we find

$$2195\,\mathrm{J\,kg^{-1}\,K^{-1}} = c_c + 213\,\mathrm{J\,kg^{-1}\,K^{-1}} \qquad\Longrightarrow\qquad c_c = 1980\,\mathrm{J\,kg^{-1}\,K^{-1}}.$$