Problem 1. Alternative regulators in QED (Peskin & Schroeder 7.2) In Section. 7.5, we saw that the Ward identity can be violated by an improperly chosen regulator. Let us check the validity of the identity $Z_1 = Z_2$, to order α, for several choices of the regulator. We have already verified that the relation holds for Paul-Villars regularization.

1(a) Recompute δZ_1 and δZ_2, defining the integrals (6.49) and (6.50) by simply placing an upper limit Λ on the integration over ℓ_E. Show that, with this definition, $\delta Z_1 \neq \delta Z_2$.

1(b) Recompute δZ_1 and δZ_2, defining the integrals (6.49) and (6.50) by dimensional regularization. You may take the Dirac matrices to be 4×4 as usual, but note that, in d dimensions,

$$g_{\mu\nu}\gamma^{\mu}\gamma^{\nu} = d.$$

Show that, with this definition, $\delta Z_1 = \delta Z_2$.

Problem 2. (Peskin & Schroeder 7.3) Consider a theory of elementary fermions that couple both to QED and to a Yukawa field ϕ:

$$H_{\text{int}} = \int d^3x \, \frac{\lambda}{\sqrt{2}} \phi\bar{\psi}\psi + \int d^3x \, eA_\mu \bar{\psi}\gamma^\mu\psi.$$

2(a) Verify that the contribution to Z_1 from the vertex diagram with a virtual ϕ equals the contribution to Z_1 from the diagram with a virtual ϕ. Use dimensional regularization. Is the Ward identity generally true in this theory?

2(b) Now consider the renormalization of the $\phi\bar{\psi}\psi$ vertex. Show that the rescaling of this vertex at $q^2 = 0$ is *not* canceled by the correction to Z_2. (It suffices to compute the ultraviolet-divergent parts of the diagrams.) In this theory, the vertex and field-strength rescaling give additional shifts of the observable coupling constant relative to its bare value.