Problem 1. You found a tiny (2.50 mm long) insect in your kitchen, and you want to get a better look at it in order to identify its species. Luckily, you happen to have on hand a collection of magnifying glasses of various focal lengths. You plan to place the insect at the focal point of one of your magnifying lenses, and you want its image to have an angular size of 0.0500 rad. What focal length should you choose for your lens?

Solution. The angular size of the image, θ_i, is given by

$$\theta_i = \frac{\ell_s}{f},$$

where ℓ_s is the height (or length, in this case) of the object and f is the focal length of the lens. Solving this for the focal length and plugging in numbers,

$$f = \frac{\ell_s}{\theta_i} = \frac{2.50\,\mathrm{mm}}{0.0500\,\mathrm{rad}} = 50.0\,\mathrm{mm} = 5.00\,\mathrm{cm}.$$

Problem 2. Let's look at what happens when we combine two lenses. A figurine 1.50 cm tall is located 60.0 cm to the left of a converging lens L_1. A second converging lens, L_2, is located 400.0 cm to the right of L_1. The focal length of L_1 is 50.0 cm, and the focal length of L_2 is 70.0 cm. What is the location and height of I_1, the image formed by L_1? The final image is formed by L_2 with I_1 as the object. What is the location and height of the final image?

Solution. We will make good use of the lens object-image relation

$$\frac{1}{s} + \frac{1}{i} = \frac{1}{f}, \tag{1}$$

where s is the distance to the object, i is the distance to the image, and f is the focal length of the lens. We will also use the definition of magnification M, which is

$$M = \frac{h_i}{h_s}, \tag{2}$$

where h_i is the height of the image and h_s is the height of the object. For a thin lens, we also have

$$M = -\frac{i}{s}. \tag{3}$$

Let's begin with the properties of I_1. To find its distance from L_1, we solve Eq. (1) for i:

$$\frac{1}{s_1} + \frac{1}{i_1} = \frac{1}{f_1} \quad \Longrightarrow \quad \frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{s_1} \quad \Longrightarrow \quad i_1 = \frac{1}{1/f_1 - 1/s_1} = \frac{f_1 s_1}{s_1 - f_1}.$$

Plugging in numbers gives us

$$i_1 = \frac{(50.0\,\text{cm})(60.0\,\text{cm})}{(60.0\,\text{cm}) - (50.0\,\text{cm})} = \frac{3000}{10.0}\,\text{cm} = 300.0\,\text{cm}.$$

Since this number is positive, I_1 is located 300.0 cm to the right of L_1.

For the height, we first use Eq. (2) to find the magnification of I_1:

$$M_1 = -\frac{i_1}{s_1} = -\frac{300.0\,\text{cm}}{60.0\,\text{cm}} = -5.00.$$

We can solve Eq. (3) for i and use our result for M_1 to find the height of I_1:

$$M_1 = \frac{h_{i1}}{h_{s1}} \quad \Longrightarrow \quad h_{i1} = M_1 h_{s1} = (-5.00)(1.50\,\text{cm}) = -7.50\,\text{cm}.$$

The minus sign tells us that I_1 is 7.50 cm tall and inverted.

For the final image, we simply repeat the steps for L_2 with I_1 as the object. Since I_1 is 300.0 cm to the right of L_1, which is 400.0 cm to the left of L_2, I_1 is $s_2 = 100.0$ cm to the right of L_2. Once again applying Eq. (1), we find

$$i_2 = \frac{f_2 s_2}{s_2 - f_2} = \frac{(70.0\,\text{cm})(100.0\,\text{cm})}{(100.0\,\text{cm}) - (70.0\,\text{cm})} = \frac{7000}{30.0}\,\text{cm} \approx 233\,\text{cm}.$$

This tells us that the final image is located 233 cm to the right of L_2.

Now for the height, we use Eqs. (2) and (3), and note that $h_{s2} = h_{i1}$:

$$M_2 = -\frac{i_2}{s_2} = -\frac{233\,\text{cm}}{100.0\,\text{cm}} = -2.33,$$

$$h_{i2} = M_2 h_{s2} = M_2 h_{i1} = (-2.33)(-7.50\,\text{cm}) \approx 17.5\,\text{cm}.$$

So the final image is 17.5 cm tall and upright (with respect to the original object).