Figure 1: **E10.2**

Exercise 10.2 Calculate the net torque about point O for the two forces applied as in Fig. 1. The rod and both forces are in the plane of the page.

Solution. This problem is a simple plug and chug to help us practice calculating torques. The net torque on the rod is the sum of the torques due to each of the forces:

$$\boldsymbol{\tau}_{\text{net}} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2. \tag{1}$$

Here, $\boldsymbol{\tau}_1$ is the (vector) torque due to $\mathbf{F}_1$ relative to point O. Let $\mathbf{r}_1$ be the vector from O to where $\mathbf{F}_1$ acts on the rod. Using the coordinate axes drawn in Fig. 1, we have

$$\begin{aligned} \mathbf{r}_1 &= -r_1\,\hat{\mathbf{i}}, \\ \mathbf{F}_1 &= -F_1\,\hat{\mathbf{j}}, \end{aligned}$$

where $r_1 = 5.00\,\text{m}$ and $F_2 = 8.00\,\text{N}$. Then

$$\boldsymbol{\tau}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = r_1 F_1\,\hat{\mathbf{k}}.$$

Now for $\boldsymbol{\tau}_1$, define $\mathbf{r}_2$ as the vector from O to where $\mathbf{F}_2$ acts on the rod. Then

$$\begin{aligned} \mathbf{r}_2 &= -r_2\,\hat{\mathbf{i}}, \\ \mathbf{F}_2 &= -(F_2\cos 30^\circ)\,\hat{\mathbf{i}} + (F_2\sin 30^\circ)\,\hat{\mathbf{j}} = -\frac{\sqrt{3}}{2}F_2\,\hat{\mathbf{i}} + \frac{1}{2}F_2\,\hat{\mathbf{j}}, \end{aligned}$$

where $r_2 = 2.00\,\text{m}$ and $F_2 = 12.0\,\text{N}$, and

$$\boldsymbol{\tau}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = -\frac{1}{2}r_2F_2\,\hat{\mathbf{k}}.$$

Feeding our results into (1),

$$\boldsymbol{\tau}_{\text{net}} = \left(r_1F_1 - \frac{1}{2}r_2F_2\right)\hat{\mathbf{k}} = \tau_{\text{net}}\,\hat{\mathbf{k}},$$

where τ_{net} is the magnitude of the net torque. Plugging everything in,

$$\tau_{\text{net}} = r_1F_1 - \frac{1}{2}r_2F_2 = (5.00\,\text{m})(8.00\,\text{N}) - \frac{1}{2}(2.00\,\text{m})(12.0\,\text{N}) = 28.0\,\text{N}\,\text{m}.$$

Figure 2: **E10.16**

Figure 3: Free-body diagrams for 10.16(a).

Exercise 10.16 A 12.0 kg box resting on a horizontal, frictionless surface is attached to a 2.00 kg weight by a thin, light wire that passes over a frictionless pulley (Fig. 2). The pulley has the shape of a uniform solid disk of mass 2.00 kg and diameter 0.500 m. After the system is released, find

(a) the tension in the wire on both sides of the pulley,

(b) the acceleration of the box, and

(c) the horizontal and vertical components of the force that the axle exerts on the pulley.

Solution. The box and the weight must have the same acceleration a since they are connected by the wire. It is safe to assume the pulley rolls without slipping against the wire (otherwise it would not be a very effective pulley!), so its tangential acceleration is also a. This is the key we need to solve the problem.

We can draw a free-body diagram for each object, as shown in Fig. 3. Using these diagrams, we can write down three equations using Newton's second law: one for the box of mass m_b,

$$m_b a = T_b, \tag{2}$$

one for the weight of mass m_w,

$$m_w a = m_w g - T_w. \tag{3}$$

and one for the pulley (which we will write generally for now),

$$\tau_{\text{net}} = I\alpha = I\frac{a}{r}. \tag{4}$$

Here, I is the moment of inertia about the pulley's center and α its angular acceleration. Since the pulley rolls without slipping, $\alpha = a/r$.

T_w and T_b each exert a torque on the outer edge of the pulley, but in opposite directions. We know the weight must be moving downward, meaning T_w has a greater magnitude, and so the pulley is rotating in the direction due to τ_w. The lever arm for each torque is the radius of the pulley r. Putting this all together,

$$\tau_{\text{net}} = \tau_w + \tau_b = T_w r - T_b r.$$

Figure 4: Free-body diagram for 10.16(c).

The pulley is a solid cylinder rotating about its z axis with mass m_p. Then

$$I = \frac{1}{2}m_p r^2,$$

and (4) can be rewritten as follows:

$$(T_w - T_b)r = \frac{1}{2}m_p r^2 \frac{a}{r} \implies T_w - T_b = \frac{1}{2}m_p a. \tag{5}$$

The system of three equations (2), (3), and (5) has three unknowns. The solutions are

$$T_b = 2g\frac{m_b m_w}{2m_b + 2m_w + m_p}, \tag{6}$$

$$T_w = g m_w \frac{2m_b + m_p}{2m_b + 2m_w + m_p}, \tag{7}$$

$$a = 2g\frac{m_w}{2m_b + 2m_w + m_p}. \tag{8}$$

(a) Plugging numbers into (6) and (7) gives us

$$T_b = 2(9.81\,\mathrm{m\,s^{-2}})\frac{(12.0\,\mathrm{kg})(5.00\,\mathrm{kg})}{2(12.0\,\mathrm{kg}) + 2(5.00\,\mathrm{kg}) + (2.00\,\mathrm{kg})} = 32.7\,\mathrm{N},$$

$$T_w = (9.81\,\mathrm{m\,s^{-2}})(5.00\,\mathrm{kg})\frac{2(12.0\,\mathrm{kg}) + (2.00\,\mathrm{kg})}{2(12.0\,\mathrm{kg}) + 2(5.00\,\mathrm{kg}) + (2.00\,\mathrm{kg})} = 35.4\,\mathrm{N}.$$

(b) Plugging numbers into (8) gives us

$$a = 2(9.81\,\mathrm{m\,s^{-2}})\frac{(5.00\,\mathrm{kg})}{2(12.0\,\mathrm{kg}) + 2(5.00\,\mathrm{kg}) + (2.00\,\mathrm{kg})} = 2.73\,\mathrm{m\,s^{-2}}.$$

(c) The pulley is not moving up or down. This means the axle must be exerting a normal force $\mathbf{N}$ upon its center of mass which exactly cancels all other forces acting upon it. The relevant free-body diagram is shown in Fig. 4. Balancing forces in the vertical direction, we have

$$N_y = T_w + m_p g = 35.4\,\mathrm{N} + (2.00\,\mathrm{kg})(9.81\,\mathrm{m\,s^{-2}}) = 55.0\,\mathrm{N},$$

and in the horizontal direction,

$$N_x = T_b = 32.7\,\mathrm{N}.$$

Figure 5: **E10.22**

Exercise 10.22 A string is wrapped several times around the rim of a small hoop with radius 8.00 cm and mass 0.180 kg. The free end of the string is held in place and the hoop is released from rest (Fig. 5). After the hoop has descended 75.0 cm, calculate

(a) the angular speed of the rotating hoop, and

(b) the speed of its center.

Solution. We can solve this problem using conservation of energy. The ring starts from rest and then experiences both translational and rotational motion. The change in its kinetic energy is given by

$$\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2,$$

where v is the (translational) speed of its center, I the moment of inertia about its center of mass, and ω is its angular speed. For a hoop,

$$I = mr^2.$$

Once the ring has descended $\Delta y = 75.0\,\mathrm{cm}$, its gravitational potential energy U_g has decreased:

$$\Delta U_g = -mg\,\Delta y.$$

Invoking conservation of energy,

$$\Delta K + \Delta U = 0 \implies mg\,\Delta y = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2. \tag{9}$$

We need to invoke another condition in order to solve for the *two* unknowns v and ω. Once again, this is rolling without slipping: the edge of the hoop must be moving at the same rate that the string is unwinding. This means

$$v = r\omega, \tag{10}$$

which we can substitute into (9) to obtain

$$mg\,\Delta y = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}mr^2\omega^2 \implies g\,\Delta y = r^2\omega^2. \tag{11}$$

(a) Rearranging (11) and plugging in numbers,

$$\omega = \frac{\sqrt{g\,\Delta y}}{r} = \frac{\sqrt{(9.81\,\mathrm{m\,s^{-2}})(0.750\,\mathrm{m})}}{8.00\times 10^{-2}\,\mathrm{m}} = 33.9\,\mathrm{rad\,s^{-1}}.$$

(b) Now plugging into (10),

$$v = (8.00\times 10^{-2}\,\mathrm{m})(33.9\,\mathrm{rad\,s^{-1}}) = 2.71\,\mathrm{m\,s^{-1}}.$$

Figure 6: **10.19**

Exercise 10.30 (A Ball Rolling Uphill) A bowling ball rolls without slipping up a ramp that slopes upward at and angle β to the horizontal (see Example 10.7 and Fig. 6). Treat the ball as a uniform solid sphere, ignoring the finger holes.

(a) Draw the free-body diagram for the ball. Explain why the friction force must be directed *uphill*.

(b) What is the acceleration of the center of mass of the ball?

(c) What minimum coefficient of static friction is needed to prevent slipping?

Solution. Example 10.7 describes the same situation, except the ball is rolling downhill. We can use that worked example to solve this problem.

(a) The free-body diagram is the same as for Example 10.7, and is shown in Fig. 6(b). The friction force must be directed uphill because it is preventing the ball from sliding down the ramp. This is true even when the ball is rolling uphill instead of downhill, because the force of gravity is *always* trying to make the ball slide downhill.

(b) The free-body diagram is the same as in Example 10.7, so the acceleration of the center of mass is also the same:

$$a_{\text{cm-}x} = \frac{5}{7} g \sin\beta.$$

Taking note of the x axis as drawn in Fig. 6(b), we see that the acceleration points *down* the hill. This means the ball is slowing down as it rolls uphill, which is easily confirmed by physical intuition.

(c) We are looking to saturate the inequality

$$f \leq \mu_s n, \tag{12}$$

where f is the magnitude of the static friction force, μ_s is the coefficient of static friction, and n is the magnitude of the normal force acting on the ball. From Fig. 6(b),

$$n = Mg\cos\beta,$$

and from Example 10.7,

$$f = \frac{2}{7} Mg\sin\beta.$$

Now we just need to solve for μ_s in (12):

$$\mu_s = \frac{f}{n} = \frac{2}{7}\frac{Mg\sin\beta}{Mg\cos\beta} = \frac{2}{7}\tan\beta.$$

Exercise 10.48 (Asteroid Collision!) Suppose that an asteroid traveling straight toward the center of the earth were to collide with our planet at the equator and bury itself just below the surface. What would have to be the mass of this asteroid, in terms of the earth's mass M, for the day to become 25.0% longer than it presently is as a result of the collision? Assume that the asteroid is very small compared to the earth and that the earth is uniform throughout.

Solution. We can solve this problem using conservation of angular momentum:

$$I_i\omega_i = I_f\omega_f. \tag{13}$$

When the asteroid hits the earth, its moment of inertia will change from I_i to I_f, which will cause its angular velocity to change from ω_i to ω_f.

According to the problem statement, we can model the earth before the collision as a solid sphere. This means

$$I_i = \frac{2}{5}MR^2,$$

where R is the radius of the earth.

After the collision, the asteroid is essentially a point mass m stuck to the side of the earth at the equator. This earth-asteroid system has a center of mass that is some distance d away from the earth's center of mass. If we fix the origin at the earth's center of mass, then

$$d = \frac{mR}{m+M}.$$

This means the asteroid is orbiting the center of mass with orbital radius $R-d$, so its moment of inertia is

$$I_a = m(R-d)^2.$$

We can use the parallel axis theorem to find the new moment of inertia of the earth:

$$I_e = I_i + Md^2,$$

because I_i is the moment of inertia of the earth rotating about its center of mass. Finally, the moment of inertia of the entire system is just their sum,

$$I_f = I_e + I_a = \frac{2}{5}MR^2 + Md^2 + m(R-d) = \frac{2}{5}MR^2 + M\left(\frac{mR}{m+M}\right)^2 + m\left(R - \frac{mR}{m+M}\right)^2.$$

An earth day is equivalent to the period of the earth's rotation about its axis:

$$T = \frac{2\pi}{\omega}.$$

For the day to become 25.0% longer, we need

$$\frac{T_f}{T_i} = 1.25 \implies \frac{\omega_i}{\omega_f} = 1.25.$$

This suggests that we rewrite (13) as

$$I_f = \frac{\omega_i}{\omega_f}I_i = \frac{5}{4}I_i.$$

Feeding in our results for the moments of inertia and solving for m, we find

$$m = \frac{M}{9}.$$

Figure 7: **P10.72**

Problem 10.72 A thin-walled, hollow spherical shell of mass m and radius r starts from rest and rolls without slipping down a track (Fig. 7). Points A and B are on a circular part of the track having radius R. The diameter of the shell is very small compared to h_0 and R, and the work done by rolling friction is negligible.

(a) What is the minimum height h_0 for which this shell will make a complete loop-the-loop on the circular part of the track?

(b) How hard does the track push on the shell at point B, which is at the same level as the center of the circle?

(c) Suppose that the track had no friction and the shell was released from the same height h_0 you found in part (a). Would it make a complete loop-the-loop? How do you know?

(d) In part (c), how hard does the track push on the shell at point A, the top of the circle? How hard did it push in part (a)?

Problem 10.76 You are designing a system for moving aluminum cylinders from the ground to a loading dock. You use a sturdy wooden ramp that is 6.00 m long and inclined at 37.0° above the horizontal. Each cylinder is fitted with a light, frictionless yoke through its center, and a light (but strong) rope is attached to the yoke. Each cylinder is uniform and has mass 460 kg and radius 0.300 m. The cylinders are pulled up the ramp by applying a constant force **F** to the free end of the rope. **F** is parallel to the surface of the ramp and exerts no torque on the cylinder. The coefficient of static friction between the ramp surface and the cylinder is 0.120.

(a) What is the largest magnitude **F** can have so that the cylinder still rolls without slipping as it moves up the ramp?

(b) If the cylinder starts from rest at the bottom of the ramp and rolls without slipping as it moves up the ramp, what is the shortest time it can take the cylinder to reach the top of the ramp?

Problem 10.80 A 5.00 kg ball is dropped from a height of 12.0 m above one end of a uniform bar that pivots at its center. The bar has mass 8.00 kg and is 4.00 m in length. At the other end of the bar sits another 5.00 kg ball, unattached to the bar. The dropped ball sticks to the bar after the collision. How high will the other ball go after the collision?