Problem 25.58 A resistor with resistance R is connected to a battery that has emf 12.0 V and internal resistance $r = 0.40\,\Omega$. For what two values of R will the power in the resistor be 80.0 W?

Solution. The power P delivered to a resistor is

$$P = I^2 R, \tag{25.18}$$

where I is the current through the resistor and R its resistance. We can find the current from

$$V_{ab} = \mathcal{E} - Ir, \tag{25.17}$$

where V_{ab} is the voltage difference across the resistor, $\mathcal{E}$ is the emf of the battery, and r its internal resistance. We also know that

$$V_{ab} = IR. \tag{25.11}$$

Substituting (25.11) into (25.17), we get

$$IR = \mathcal{E} - Ir \implies \mathcal{E} = I(R + r) \implies I = \frac{\mathcal{E}}{R + r}.$$

Now we can substitute this result into (25.18) and solve for R:

$$\begin{aligned} P = \frac{\mathcal{E}^2}{(R+r)^2} R &\implies \mathcal{E}^2 R = P(R^2 + 2Rr + r^2) \implies 0 = PR^2 + (2Pr - \mathcal{E}^2)R + Pr^2 \\ &\implies R = \frac{\mathcal{E}^2 - 2Pr \pm \sqrt{(2Pr - \mathcal{E}^2)^2 - 4P^2r^2}}{2P} \end{aligned}$$

Plugging in our numerical values for r, P, and $\mathcal{E}$, and recalling that $1\,\mathrm{W} = 1\,\mathrm{V}^2\,\Omega^{-1}$, we get

$$\begin{aligned} R &= \frac{(12.0\,\mathrm{V})^2 - 2(80.0\,\mathrm{W})(0.40\,\Omega) \pm \sqrt{[2(80.0\,\mathrm{W})(0.40\,\Omega) - (12.0\,\mathrm{V})^2]^2 - 4(80.0\,\mathrm{W})^2(0.40\,\Omega)^2}}{2(80.0\,\mathrm{W})} \\ &= \frac{80.0\,\mathrm{V}^2 - \pm\sqrt{(80\,\mathrm{V}^2)^2 - (64\,\mathrm{V}^2)}}{160\,\mathrm{V}^2\,\Omega^{-1}} = \frac{80.0\,\mathrm{V}^2 \pm \sqrt{2306\,\mathrm{V}^4}}{160\,\mathrm{V}^2\,\Omega^{-1}} = \frac{80.0 \pm 48.0}{160}\,\Omega = (0.50 \pm 0.30)\,\Omega \\ &= \begin{cases} 0.80\,\Omega, \\ 0.20\,\Omega. \end{cases} \end{aligned}$$

Exercise 26.26 In the circuit shown in **Fig. E26.26**, find

(a) the current in each branch, and

(b) the potential difference V_{ab} of point a relative to point b.

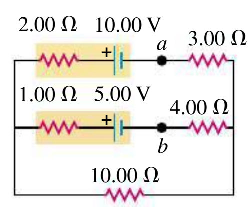

Figure E26.26

Exercise 26.29 In the circuit shown in **Fig. E26.29** the batteries have negligible internal resistance and the meters are both idealized. With the switch S open, the voltmeter reads 15.0 V.

(a) Find the emf $\mathcal{E}$ of the battery.

(b) What will the ammeter read when the switch is closed?

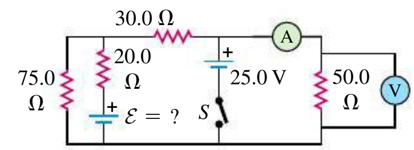

Figure E26.29

Exercise 26.41 In the circuit shown in **Fig. E26.41** both capacitors are initially charged to 45.0 V.

(a) How long after closing the switch S will the potential across each capacitor be reduced to 10.0 V, and

(b) what will be the current at that time?

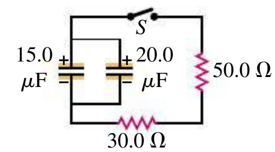

Figure E26.41

Exercise 26.47 In the circuit shown in **Fig. E26.47** the capacitors are initially uncharged, the battery has no internal resistance, and the ammeter is idealized. Find the ammeter reading

(a) just after the switch S is closed, and

(b) after S has been closed for a very long time.

E26.49

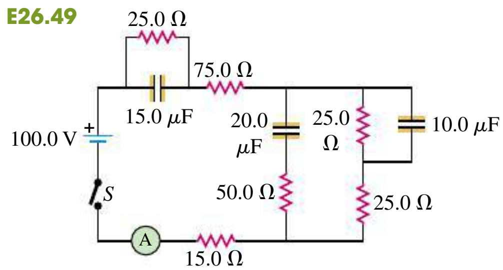

Figure E26.47

Problem 26.53 A capacitor with capacitance C is connected in series to a resistor of resistance R and a battery with emf $\mathcal{E}$. The circuit is completed at time $t = 0$.

(a) In terms of $\mathcal{E}$, R, and C, how much energy is stored in the capacitor when it is fully charged?

(b) The power output of the battery is $P_{\mathcal{E}} = \mathcal{E}i$, with i given by Eq. (26.13). The electrical energy supplied in an infinitesimal time dt is $P_{\mathcal{E}}\,dt$. Integrate from $t = 0$ to $t \to \infty$ to find the total energy supplied by the battery.

(c) The rate of consumption of electrical energy in the resistor is $P_R = i^2R$. In an infinitesimal time interval dt, the amount of electrical energy consumed by the resistor is $P_R\,dt$. Integrate from $t = 0$ to $t \to \infty$ to find the total energy consumed by the resistor.

(d) What fraction of the total energy supplied by the battery is stored in the capacitor? What fraction is consumed in the resistor?

Problem 26.59 Calculate the currents I_1, I_2, and I_3 indicated in the circuit diagram shown in **Fig. P26.59**.

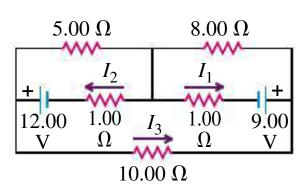

Figure P26.59