**Problem 1. Non-equilibrium entropies of Fermi, Bose, and Boltzmann distributions** Consider a gas out of equilibrium with a slightly non-uniform density in $n(x)$ and mean density $\bar{n} = V^{-1} \int n(x)\, d^3x$. We know that if the gas obeys Boltzmann statistics, its entropy is $S = -\int n \log n \, dV$.

**1.1** Argue that this formula is valid only if the gradients are small: $|\boldsymbol{\nabla}_x n| \ll \bar{n}^{4/3}$ ("coarse-graining condition") and that $|n(x) - \bar{n}| \ll \bar{n}$.

**1.2** Remove the second condition in 1.1 and obtain the general formula for the entropy for both Fermi and Bose gases.

**Problem 2. Quantum correction to the Boltzmann thermodynamics** Find the the quantum correction to the free energy of the Boltzmann gas (the leading $\hbar$-dependent term in the expansion of the free energy at small $\hbar$) for Bose and Fermi gases. From there, find the correction to the pressure. Does the quantum correction increase or decrease the pressure (and why is the answer predictable)?

**Problem 3. Degenerate Fermi gas** Consider a Fermi gas in 1, 2, and 3 spatial dimensions with density $\bar{n} = N/V$.

**3.1** First, set the temperature to zero ($T = 0$) and find the Fermi momentum, Fermi energy, and the total energy in all three cases as a function of density.

**3.2** Then compute the leading terms of the small temperature corrections to the basic thermodynamic quantities: thermodynamic potential, free energy, energy, pressure, entropy, and specific heat.

**Problem 4. Degenerate Bose gas**

**4.1** The chemical potential of the degenerate Bose gas vanishes below $T^*$ (the critical temperature of the BEC). Find its temperature dependence at temperatures slightly above $T^*$.

**4.2** Find the discontinuities in the derivatives of thermodynamic quantities at the BEC transition. Which order is this phase transition?

**4.3 (*)** Can the ideal Bose gas condense in spatial dimensions 1 and 2? Discuss what happens in these cases.

**Problem 5. Thermodynamics of radiation** Compute the following thermodynamic quantities of a radiation field in a 1D and a 2D cavity and compare it with the textbook example of a 3D cavity.

**5.1** Planck formula and the Rayleigh-Jeans and Wien limits of the distribution over frequencies.

**5.2** Free energy and the Stefan-Boltzmann constant.

**5.3** The relation between the free energy and energy (Boltzmann law).

**5.4** Specific heat.

**5.5** Pressure.

**5.6** The total number of photons in the cavity.

**Problem 6. Thermodynamics of solids** Compute the following thermodynamic quantities for the harmonic photonic modes in a 1D and a 2D crystal at low temperatures (a.k.a. phonons) and compare with the textbook example of a 3D crystal.

**6.1** Free energy.

**6.2** Entropy.

**6.3** Energy.

**6.4** Specific heat.