Problem 1. **(Jackson 12.3)** A particle with mass m and charge e moves in a uniform, static, electric field $\mathbf{E}_0$.

1(a) Solve for the velocity and position of the particle as explicit functions of time, assuming that the initial velocity $\mathbf{v}_0$ was perpendicular to the electric field.

1(b) Eliminate the time to obtain the trajectory of the particle in space. Discuss the shape of the path for short and long times (define "short" and "long" times).

Problem 2. **(Jackson 12.5)** A particle of mass m and charge e moves in the laboratory in crossed, static, uniform, electric and magnetic fields. $\mathbf{E}$ is parallel to the x axis; $\mathbf{B}$ is parallel to the y axis.

2(a) For $|\mathbf{E}| < |\mathbf{B}|$ make the necessary Lorentz transformation described in Section 12.3 to obtain explicitly parametric equations for the particle's trajectory.

2(b) Repeat the calculation of part (a) for $|\mathbf{E}| > |\mathbf{B}|$.

Problem 3. **(Jackson 12.19)** Source-free electromagnetic fields exist in a localized region of space. Consider the various conservation laws that are contained in the integral of $\partial_\alpha M^{\alpha\beta\gamma} = 0$ over all space, where

$$M^{\alpha\beta\gamma} = \Theta^{\alpha\beta} x^\gamma - \Theta^{\alpha\gamma} x^\beta.$$

3(a) Show that when β and γ are both space indices conservation of the total field angular momentum follows.

3(b) Show that when $\beta = 0$ the conservation law is

$$\frac{d\mathbf{X}}{dt} = \frac{c^2 \mathbf{P}_{\text{em}}}{E_{\text{em}}},$$

where $\mathbf{X}$ is the coordinate of the center of mass of the electromagnetic fields, defined by

$$\mathbf{X} \int u \, d^3x = \int \mathbf{x} u \, d^3x \, ,$$

where u is the electromagnetic energy density and E_{em} and $\mathbf{P}_{\text{em}}$ are the total energy and momentum of the fields.

Problem 4. We discussed in class the construction of linearly polarized electromagnetic waves.

4(a) Generalize the discussion to circularly polarized waves (see also Wald Sec. 5.5). Discuss both right-handed and left-handed polarizations.

4(b) Compute the angular momentum of the circularly polarized waves of part (a) using the formula for angular momentum derived in class.

Problem 5. We wrote in class the Lagrangian of a charged particle coupled to the electromagnetic field (see pp. 159–160) in the lecture notes).

5(a) Show that the Euler-Lagrange equations that follow from this Lagrangian give rise to the Lorentz force law

$$\frac{dp_i}{dt} = q\left[E^i + \frac{1}{c}(\mathbf{v} \times \mathbf{B})^i\right].$$

5(b) Show that the Lorentz force law can be written covariantly in the form

$$\frac{dU^\mu}{d\tau} = \frac{q}{mc} F^{\mu\nu} U_\nu.$$