Problem 1. Beta functions in Yukawa theory (P&S 12.1) In the pseudoscalar Yukawa theory studied in Problem 10.2, with masses set to zero,

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^2 - \frac{\lambda}{4!} \phi^4 + \bar{\psi} (i \partial) \psi - i g \bar{\psi} \gamma^5 \psi \phi,$$

compute the Callan-Symanzik  $\beta$  functions for  $\lambda$  and g:

$$\beta_{\lambda}(\lambda, g),$$
  $\beta_{\alpha}(\lambda, g),$ 

to leading order in coupling constants, assuming that  $\lambda$  and  $g^2$  are of the same order. Sketch the coupling constant flows in the  $\lambda$ -g plane.

Problem 2. Beta function of the Gross-Neveu model (P&S 12.2) Compute  $\beta(g)$  in the twodimensional Gross-Neveu model studied in Problem 11.3,

$$\mathcal{L} = \bar{\psi}_i i \partial \psi_i + \frac{1}{2} g^2 (\bar{\psi}_i \psi_i)^2,$$

with i = 1, ..., N. You should find that this model is asymptotically free. How was that fact reflected in the solution to Problem 11.3?

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