

Problem 25.58 A resistor with resistance R is connected to a battery that has emf 12.0 V and internal resistance $r = 0.40\ \Omega$. For what two values of R will the power in the resistor be 80.0 W ?

Solution. The power P delivered to a resistor is

$$P = I^2 R, \quad (25.18)$$

where I is the current through the resistor and R its resistance. We can find the current from

$$V_{ab} = \mathcal{E} - Ir, \quad (25.17)$$

where V_{ab} is the voltage difference across the resistor, \mathcal{E} is the emf of the battery, and r its internal resistance. We also know that

$$V_{ab} = IR. \quad (25.11)$$

Substituting (25.11) into (25.17), we get

$$IR = \mathcal{E} - Ir \implies \mathcal{E} = I(R + r) \implies I = \frac{\mathcal{E}}{R + r}.$$

Now we can substitute this result into (25.18) and solve for R :

$$\begin{aligned} P &= \frac{\mathcal{E}^2}{(R + r)^2} R \implies \mathcal{E}^2 R = P(R^2 + 2Rr + r^2) \implies 0 = PR^2 + (2Pr - \mathcal{E}^2)R + Pr^2 \\ \implies R &= \frac{\mathcal{E}^2 - 2Pr \pm \sqrt{(2Pr - \mathcal{E}^2)^2 - 4P^2 r^2}}{2P} \end{aligned}$$

Plugging in our numerical values for r , P , and \mathcal{E} , and recalling that $1\text{ W} = 1\text{ V}^2\ \Omega^{-1}$, we get

$$\begin{aligned} R &= \frac{(12.0\text{ V})^2 - 2(80.0\text{ W})(0.40\ \Omega) \pm \sqrt{[2(80.0\text{ W})(0.40\ \Omega) - (12.0\text{ V})^2]^2 - 4(80.0\text{ W})^2(0.40\ \Omega)^2}}{2(80.0\text{ W})} \\ &= \frac{80.0\text{ V}^2 - \pm \sqrt{(80\text{ V}^2)^2 - (64\text{ V}^2)}}{160\text{ V}^2\ \Omega^{-1}} = \frac{80.0\text{ V}^2 \pm \sqrt{2306\text{ V}^4}}{160\text{ V}^2\ \Omega^{-1}} = \frac{80.0 \pm 48.0}{160}\ \Omega = (0.50 \pm 0.30)\ \Omega \\ &= \begin{cases} 0.80\ \Omega, \\ 0.20\ \Omega. \end{cases} \end{aligned}$$

Exercise 26.26 In the circuit shown in **Fig. E26.26**, find

- the current in each branch, and
- the potential difference V_{ab} of point a relative to point b .

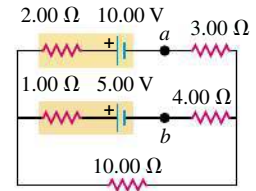


Figure E26.26

Solution. For this problem we need to use Kirchhoff's rules:

$$\sum I = 0 \quad \text{(junction rule),} \quad (26.5)$$

$$\sum V = 0 \quad \text{(loop rule).} \quad (26.6)$$

- Let I_1 be the current through the top branch, I_2 the current through the middle branch, and I_3 the current through the bottom branch. Let's choose I_1 to be flowing from + to – across the 10 V battery, since it has the largest emf. Then we'll choose I_2 to be flowing from + to – across the 5 V battery. Since the 10 V battery has a larger emf, let's choose I_3 to be flowing to the right across the 10 Ω resistor.

First of all, applying the junction rule to point P gives us

$$0 = I_1 - I_2 - I_3 \implies I_1 = I_2 + I_3$$

We will need to apply the loop rule to three loops: the top loop, the bottom loop, and the outer loop.

For the top loop, we have

$$0 = 10 \text{ V} - (2 \Omega)I_1 - (1 \Omega)I_2 - 5 \text{ V} - (4 \Omega)I_2 - (3 \Omega)I_1 = 5 \text{ V} - (5 \Omega)I_1 - (5 \Omega)I_2,$$

where we have subtracted the voltage of the 5 V battery, since we chose the current to flow against it.

For the bottom loop, we have

$$0 = 5 \text{ V} + (1 \Omega)I_2 - (10 \Omega)I_3 + (4 \Omega)I_2 = 5 \text{ V} + (5 \Omega)I_2 - (10 \Omega)I_3,$$

where we note that the current is flowing with the 5 V for this loop.

For the outer loop,

$$0 = 10 \text{ V} - (2 \Omega)I_1 - (10 \Omega)I_3 - (3 \Omega)I_1 = 10 \text{ V} - (5 \Omega)I_1 - (10 \Omega)I_3.$$

Gathering these together, we have a system of three equations in three unknowns:

$$5 \text{ V} = (5 \Omega)I_1 + (5 \Omega)I_2, \quad 5 \text{ V} = (5 \Omega)I_2 + (10 \Omega)I_3, \quad 10 \text{ V} = (5 \Omega)I_1 + (10 \Omega)I_3.$$

We can divide them all by 5 Ω to simplify things a bit:

$$1 \text{ A} = I_1 + I_2, \quad (A)$$

$$1 \text{ A} = I_2 + 2I_3, \quad (B)$$

$$2 \text{ A} = I_1 + 2I_3. \quad (C)$$

Subtracting (??) from (??), we get

$$1 \text{ A} = 2I_3 - I_2.$$

Adding (??) to this gives us

$$2 \text{ A} = 4I_3 \implies I_3 = 2 \text{ A}.$$

Now we can plug I_3 into (??) to get

$$1 \text{ A} = I_2 + 4 \text{ A} \implies I_2 = -3 \text{ A},$$

and we can plug it into (??) to get

$$2 \text{ A} = I_1 + 4 \text{ A} \implies I_1 = -2 \text{ A}$$

Exercise 26.29 In the circuit shown in **Fig. E26.29** the batteries have negligible internal resistance and the meters are both idealized. With the switch S open, the voltmeter reads 15.0 V .

- Find the emf \mathcal{E} of the battery.
- What will the ammeter read when the switch is closed?

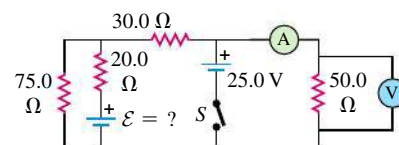


Figure E26.29

Exercise 26.41 In the circuit shown in **Fig. E26.41** both capacitors are initially charged to 45.0 V .

- How long after closing the switch S will the potential across each capacitor be reduced to 10.0 V , and
- what will be the current at that time?

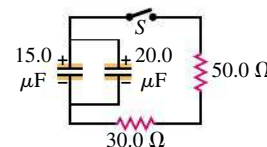


Figure E26.41

Exercise 26.47 In the circuit shown in **Fig. E26.47** the capacitors are initially uncharged, the battery has no internal resistance, and the ammeter is idealized. Find the ammeter reading

- just after the switch S is closed, and
- after S has been closed for a very long time.

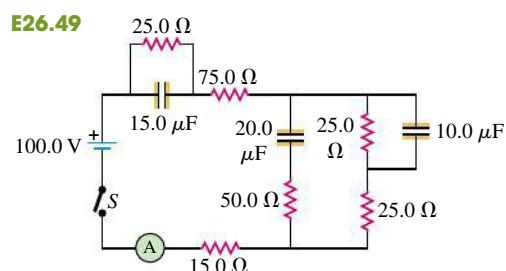


Figure E26.47

Problem 26.53 A capacitor with capacitance C is connected in series to a resistor of resistance R and a battery with emf \mathcal{E} . The circuit is completed at time $t = 0$.

- In terms of \mathcal{E} , R , and C , how much energy is stored in the capacitor when it is fully charged?
- The power output of the battery is $P_{\mathcal{E}} = \mathcal{E}i$, with i given by Eq. (26.13). The electrical energy supplied in an infinitesimal time dt is $P_{\mathcal{E}} dt$. Integrate from $t = 0$ to $t \rightarrow \infty$ to find the total energy supplied by the battery.
- The rate of consumption of electrical energy in the resistor is $P_R = i^2 R$. In an infinitesimal time interval dt , the amount of electrical energy consumed by the resistor is $P_R dt$. Integrate from $t = 0$ to $t \rightarrow \infty$ to find the total energy consumed by the resistor.
- What fraction of the total energy supplied by the battery is stored in the capacitor? What fraction is consumed in the resistor?

Problem 26.59 Calculate the currents I_1 , I_2 , and I_3 indicated in the circuit diagram shown in **Fig. P26.59**.

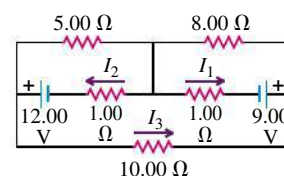


Figure P26.59