Problem 21.96 Two charges are placed as shown in Fig. P21.96. The magnitude of q_1 is $3.00\,\mu\text{C}$, but its sign and the value of the charge q_2 are not known. The direction of the net electric field $\mathbf{E}$ at point P is entirely in the negative y direction.

(a) Considering the different possible signs of q_1 and q_2, four possible diagrams could represent the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ produced by q_1 and q_2. Sketch the four possible electric field configurations.

(b) Using the sketches from part (a) and the direction of $\mathbf{E}$, deduce the signs of q_1 and q_2.

(c) Determine the magnitude of $\mathbf{E}$.

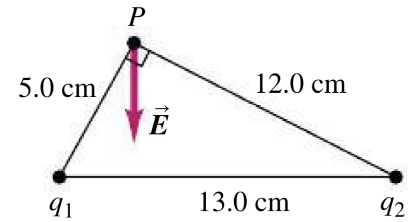
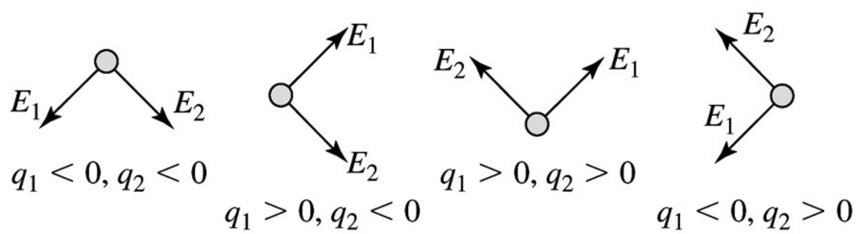


Figure P21.96

Solution

(a) The four possible diagrams are:



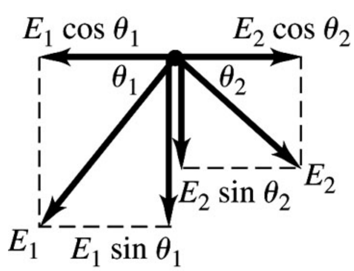
Here E_1 and E_2 indicate the electric fields of q_1 and q_2, respectively.

(b) The first diagram is the only one that has the net $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$ pointing in the $-y$ direction like we want. This means both q_1 and q_2 are negative. (We also could have deduced this by placing a positive test charge at P, and realizing we need q_1 and q_2 to be negative in order to produce an electric field pointing down.)

(c) Using the expression for the electric field due to a point charge, we can find the magnitudes of E_1 and E_2:

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{(5.0\,\text{cm})^2} = -\frac{1}{4\pi\epsilon_0}\frac{3.00\,\mu\text{C}}{(5.0\,\text{cm})^2}, \qquad E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{(12.0\,\text{cm})^2}.$$

Notice that we need to solve for q_2, which we can do by force balance. We need to find and add the x and y components of $\mathbf{E}_1$ and $\mathbf{E}_2$, which we can do by finding their angles from the horizontal. Call these angles θ_1 and θ_2. Building off of our diagram for (a), we can make something similar to a force diagram, but for the electric field:



From Fig. P21.96, we can write down

$$\cos\theta_1 = \frac{5}{13}, \qquad \sin\theta_1 = \frac{12}{13}, \qquad \cos\theta_2 = \frac{12}{13}, \qquad \sin\theta_2 = \frac{5}{12}.$$

First, let's balance forces in the x direction, in which we know there's 0 net electric field. This will allow us to find the magnitude of q_2:

$$0 = E_x = E_{1x} + E_{2x} = E_1 \cos\theta_1 + E_2 \cos\theta_2 = -\frac{5}{13}\frac{1}{4\pi\epsilon_0}\frac{3.00\,\mu\text{C}}{(5.0\,\text{cm})^2} + \frac{12}{13}\frac{1}{4\pi\epsilon_0}\frac{|q_2|}{(12.0\,\text{cm})^2}$$

$$\implies \frac{3.00\,\mu\text{C}}{5.0\,\text{cm}^2} = \frac{|q_2|}{12.0\,\text{cm}^2} \implies |q_2| = \frac{36}{5}\mu\text{C} = 7.2\,\mu\text{C}.$$

Now we can add the forces in the y direction to find the magnitude of $\mathbf{E}$:

$$\begin{aligned}|\mathbf{E}| = E_y = E_{1y} + E_{2y} = E_1 \sin\theta_1 + E_2 \sin\theta_2 &= \frac{12}{13}\frac{1}{4\pi\epsilon_0}\frac{3.00\,\mu\text{C}}{(5.0\,\text{cm})^2} + \frac{5}{12}\frac{1}{4\pi\epsilon_0}\frac{7.2\,\mu\text{C}}{(12.0\,\text{cm})^2} \\ &= \frac{1}{4\pi\epsilon_0}\left(\frac{12}{13}\frac{3.00\times10^{-6}\,\text{C}}{25\times10^{-4}\,\text{m}^2} + \frac{5}{12}\frac{7.2\times10^{-6}\,\text{C}}{144\times10^{-4}\,\text{m}^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{36}{325} + \frac{36}{1728}\right)\times10^{-2}\,\text{C}\,\text{m}^{-2} \\ &= (8.99\times10^9\,\text{N}\,\text{m}^2\,\text{C}^{-2})(0.131\times10^{-2}\,\text{C}\,\text{m}^{-2}) \\ &= 1.18\times10^7\,\text{N}\,\text{C}^{-1}.\end{aligned}$$

Problem 23.82 A hollow, thin-walled insulating cylinder of radius R and length L (like the cardboard tube in a roll of toilet paper) has charge Q uniformly distributed over its surface.

(a) Calculate the electric potential at all points along the axis of the tube. Take the origin to be at the center of the tube, and take the potential to be zero at infinity.

(c) Use the result of part (a) to find the electric field at all points along the axis of the tube.

Solution

(a) We can think of the tube as being made of a stack of charged rings. Example 23.11 in the textbook derives the expression for the electric potential due to a ring, which is

$$V_{\text{ring}} = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{ring}}}{\sqrt{x^2+a^2}},$$

where Q_{ring} is the total charge of the ring, x is the distance from the center of the ring along its axis, and a is the ring's radius (Fig. 23.20).

Let the ring have charge dQ and be at position z along the x axis. We will need to integrate z from $-L/2$ to $L/2$. The infinitesimal charge dQ of the ring is related to the entire charge of the tube through their respective lengths:

$$\frac{dQ}{dz} = \frac{Q}{L} \implies dQ = \frac{Q}{L}\,dz\,.$$

Our integral is then

$$V = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon_0}\frac{dQ}{\sqrt{(x-z)^2+R^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\int_{-L/2}^{L/2}\frac{dz}{\sqrt{(x-z)^2+R^2}}.$$

The denominator of the integrand looks kind of gross, so we probably want to do a u substitution. Let $u = x - z$, so $du = -\,dz$. Then our integral becomes

$$V = -\frac{1}{4\pi\epsilon_0}\frac{Q}{L}\int_{x+L/2}^{x-L/2}\frac{du}{\sqrt{u^2+R^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\int_{x-L/2}^{x+L/2}\frac{du}{\sqrt{u^2+R^2}},$$

which is possible to do, but we would have to look it up in a table of integrals, and the form is really gross (look it up and see). Let's try to find an easier way, by finding the electric field first instead of second.

The electric field due to a ring of charge was found in Ex. 21.9:

$$E_{\text{ring}} = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{ring}} x}{(x^2 + a^2)^{3/2}}.$$

Setting up the integral over the length of the tube as we did before, we get

$$E = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon_0} \frac{dQ}{[(x-z)^2 + R^2]^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \int_{-L/2}^{L/2} \frac{dz}{[(x-z)^2 + R^2]^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \int_{x-L/2}^{x+L/2} \frac{du}{(u^2 + R^2)^{3/2}}.$$

Problem 22.32 A cube has sides of length $L = 0.300\,\mathrm{m}$. One corner is at the origin (Fig. E22.6). The nonuniform electric field is given by $\mathbf{E} = (-5.00\,\mathrm{N\,C^{-1}\,m^{-1}})x\,\hat{\mathbf{i}} + (23.00\,\mathrm{N\,C^{-1}\,m^{-1}})z\,\hat{\mathbf{k}}$.

(a) Find the electric flux through each of the six cube faces S_1, S_2, S_3, S_4, S_5, and S_6.

(b) Find the total electric charge inside the cube.

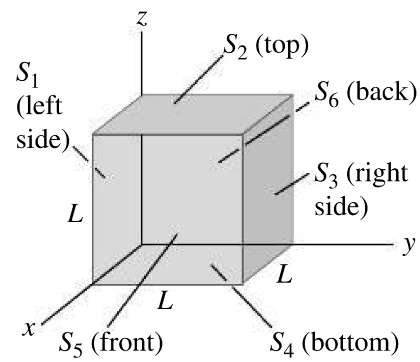


Figure E22.6

Problem 24.11

Problem 4 A spherical capacitor contains a charge of $3.30\,\mathrm{nC}$ when connected to a potential difference of $220\,\mathrm{V}$. If its plates are separated by vacuum and the inner radius of the outer shell is $4.00\,\mathrm{cm}$, calculate

(a) the capacitance,

(b) the radius of the inner sphere, and

(c) the electric field just outside the surface of the inner sphere.