10.2 Calculate the net torque about point O for the two forces applied as in Fig. **??**. The rod and both forces are in the plane of the page.

Solution. The net torque is the sum of the torques due to each of the forces. Let $\boldsymbol{\tau}_1$ be the (vector) torque due to $\mathbf{F}_1$ relative to point O, and $\mathbf{r}_1$ the vector from O to where $\mathbf{F}_1$ acts. Using the coordinate axes drawn in Fig. **??**, we have

$$\mathbf{r}_1 = -r_1\,\hat{\mathbf{i}},$$
$$\mathbf{F}_1 = -F_1\,\hat{\mathbf{j}},$$

where $r_1 = 5.00\,\mathrm{m}$ and $F_2 = 8.00\,\mathrm{N}$. Then

$$\boldsymbol{\tau}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = r_1F_1\,\hat{\mathbf{k}}.$$

Now define $\boldsymbol{\tau}_1$, $\boldsymbol{\tau}_2$, and $\mathbf{r}_2$ similarly for $\mathbf{F}_2$. Then

$$\mathbf{r}_2 = -r_2\,\hat{\mathbf{i}},$$
$$\mathbf{F}_2 = -(\cos 30^\circ F_2)\,\hat{\mathbf{i}} + (\sin 30^\circ F_2)\,\hat{\mathbf{j}} = -\frac{\sqrt{3}}{2}F_2\,\hat{\mathbf{i}} + \frac{1}{2}F_2\,\hat{\mathbf{j}},$$

where $r_2 = 2.00\,\mathrm{m}$ and $F_2 = 12.0\,\mathrm{N}$, and

$$\boldsymbol{\tau}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = -\frac{1}{2}r_2F_2\,\hat{\mathbf{k}}.$$

The net torque $\boldsymbol{\tau}_{\text{net}}$ is then

$$\boldsymbol{\tau}_{\text{net}} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 = \left(r_1F_1 - \frac{1}{2}r_2F_2\right)\hat{\mathbf{k}} = \tau_{\text{net}}\,\hat{\mathbf{k}},$$

where τ_{net} is the magnitude of the net torque. Plugging everything in,

$$\tau_{\text{net}} = r_1F_1 - \frac{1}{2}r_2F_2 = (5.00\,\mathrm{m})(8.00\,\mathrm{N}) - \frac{1}{2}(2.00\,\mathrm{m})(12.0\,\mathrm{N}) = 28.0\,\mathrm{N\,m}.$$

10.16 A 12.0 kgbox resting on a horizontal, frictionless surface is attached to a 2.00 kgweight by a thin, light wire that passes over a frictionless pulley (Fig. **??**). The pulley has the shape of a uniform solid disk of mass 2.00 kgand diameter 0.500 m. After the system is released, find

(a) the tension in the wire on both sides of the pulley,

(b) the acceleration of the box, and

(c) the horizontal and vertical components of the force that the axle exerts on the pulley.

Solution. The box and the weight must have the same acceleration a since they are connected by the wire. We also assume the pulley rolls without slipping, so its tangential acceleration is also a. Using the notation of the free-body diagrams in Fig. **??**, let T_b be the tension in the side of the wire connected to the box and T_w the tension in the side connected to the weight.

Let m_b denote the mass of the box and m_w that of the weight. We can write down three equations using Newton's second law: one equation from the free-body diagram for the box in Fig. **??**,

$$m_ba = T_b, \tag{1}$$

one from the diagram for the weight,

$$m_w a = m_w g - T_w. \tag{2}$$

and one from the diagram for the pulley,

$$\tau_{\text{net}} = I\alpha \iff (T_w - T_b)r = I\frac{a}{r} \iff T_w - T_b = I\frac{a}{r^2},$$

where r is the pulley's radius, I the moment of inertia about its center, and α its angular acceleration. For rolling without slipping, $\alpha = a/r$.

The pulley is a solid cylinder rotating about its z axis. Let m_p be its mass. Then

$$I = \frac{1}{2}m_p r^2,$$

and (**??**) can be rewritten as

$$T_w - T_b = \frac{1}{2}m_p a. \tag{3}$$

The system of three equations (**??**), (**??**), and (**??**) has three unknowns. The solutions are

$$T_b = 2g\frac{m_b m_w}{2m_b + 2m_w + m_p}, \tag{4}$$

$$T_w = g m_w \frac{2m_b + m_p}{2m_b + 2m_w + m_p}, \tag{5}$$

$$a = 2g\frac{m_w}{2m_b + 2m_w + m_p}. \tag{6}$$

(a) Plugging numbers into (**??**) and (**??**), we have

$$T_b = \frac{m_b m_w}{I + m_b + m_w} g$$

(b) Torque due to T_b is $T_b R$, also $T_w R$

10.22 A string is wrapped several times around the rim of a small hoop with radius 8.00 cm and mass 0.180 kg. The free end of the string is held in place and the hoop is released from rest (Fig. **??**). After the hoop has descended 75.0 cm, calculate

(a) the angular speed of the rotating hoop, and

(b) the speed of its center.

Solution.