Problem 1. A fellow student with a mathematical bent tells you that the wave function of a traveling wave on a thin rope is

$$y(x,t) = (2.30\,\mathrm{mm})\cos[(6.98\,\mathrm{rad\,m^{-1}})x + (742\,\mathrm{rad\,s^{-1}})t]. \tag{1}$$

Being more practical, you measure the rope to have a length of 1.35 m and a mass of 0.003 38 kg. You are then asked to determine the following:

(a) amplitude;

(b) frequency;

(c) wavelength;

(d) wave speed;

(e) direction the wave is traveling;

(f) tension in the rope;

(g) average power transmitted by the wave.

Solution.

(a) A standing wave has the general form

$$y(x,t) = y_0 \sin(kx - \omega t),$$

where y_0 is the amplitude of the wave, k its wavenumber, and ω its angular frequency. However, since sine and cosine differ only by a phase, we might as well write

$$y(x,t) = y_0 \cos(kx - \omega t), \tag{2}$$

which is the form given in the problem, Eq. (1). Then we can easily read off the amplitude:

$$y_0 = 2.30\,\mathrm{mm}.$$

(b) Once again referring to Eq. (2), we can read off the *angular* frequency $\omega = 742\,\mathrm{rad\,s^{-1}}$ from Eq. (1). Then we can easily solve for the frequency f:

$$f = \frac{\omega}{2\pi} = \frac{742\,\mathrm{rad\,s^{-1}}}{2\pi\,\mathrm{rad}} = 118\,\mathrm{Hz}.$$

(c) Reading off the wave number from Eq. (1), we find $k = 6.98\,\mathrm{rad\,m^{-1}}$. Solving for the wavelength λ, we find

$$\lambda = \frac{2\pi}{k} = \frac{2\pi\,\mathrm{rad}}{6.98\,\mathrm{rad\,m^{-1}}} = 0.90\,\mathrm{m} = 90\,\mathrm{cm}.$$

(d) The wave speed is defined as $v = \omega/k$. Plugging in the values of ω and k that we found in (b) and (c),

$$v = \frac{\omega}{k} = \frac{742\,\mathrm{rad\,s^{-1}}}{6.98\,\mathrm{rad\,m^{-1}}} = 106\,\mathrm{m\,s^{-1}}.$$

(e) Equation (2) gives the general expression for a wave traveling in the $+x$ direction. Here, the argument of the cosine function is $kx + \omega t$. However, in the given expression of Eq. (1), the argument has the form $kx - \omega t$. This means that the wave is traveling in the $-x$ direction.

(f) Another expression for the wave speed is

$$v = \sqrt{\frac{T}{\mu}},$$

where μ is the mass density of the rope, and T is the tension in the rope. Solving this definition for T and substituting in $\mu = m/L$ gives us

$$T = \mu v^2 = \frac{mv^2}{L}.$$

Plugging in the given values of m and L, and our result for v from (d), we find

$$T = \frac{(0.003\,38\,\mathrm{kg})(106\,\mathrm{m\,s^{-1}})^2}{1.35\,\mathrm{m}} = 28.3\,\mathrm{N}.$$

(g) The average power $\langle P \rangle$ transmitted by the wave is given by

$$\langle P \rangle = \frac{1}{2}\mu\omega^2 y_0^2 v = \frac{m\omega^2 y_0^2 v}{2L},$$

so plugging in known quantities and previous results gives us

$$\langle P \rangle = \frac{(0.003\,38\,\mathrm{kg})(742\,\mathrm{rad\,s^{-1}})^2(0.0023\,\mathrm{m})^2(106\,\mathrm{m\,s^{-1}})}{2(1.35\,\mathrm{m})} = 0.39\,\mathrm{W}.$$