Problem 1. A particle is initially in the the ground state of an infinite one-dimensional potential box with walls at $x = 0$ and $x = L$. During the time interval $0 \leq t \leq \infty$, the particle is subject to a perturbation $V(t) = x^2 e^{-t/\tau}$, where τ is a time constant. Calculate, to first order in perturbation theory, the probability of finding the particle in its first excited state as a result of this perturbation.

Problem 2. Consider a system of two electrons, which is described by the Hamiltonian

$$H = H_a + H_b + V, \qquad H_i = \frac{\mathbf{p}_i^2}{2m} - \frac{Z\alpha\hbar c}{r_i}, \qquad V = \frac{\alpha\hbar c}{r_{ab}}.$$

Here, we label two electrons by $i = a, b$; $r_i = |\mathbf{x}_i|$ and $r_{ab} = |\mathbf{x}_a - \mathbf{x}_b|$ where $\mathbf{x}_i$ is the spatial coordinate for electron i; and Z and α are constants. To find an approximate ground state of H, let us try a variational wave function

$$\Psi(\mathbf{x}_a, \mathbf{x}_b) = \frac{A}{4\pi} e^{-B(r_a + r_b)},$$

where A is a normalization constant and B is your variational parameter.

2.1 Compute the variational energy for the given variational parameter B.

2.2 By minimizing the variational energy, find the optimal value of B.

Problem 3. Consider a two-dimensional harmonic oscillator described by the Hamiltonian

$$H_0 = \frac{p_x^2 + p_y^2}{2m} + m\omega^2 \frac{x^2 + y^2}{2}.$$

3.1 How many single-particle states are there for the first excited level?

3.2 Write down the many-body ground state for two electrons (with spin). What is the eigenvalue of $(\mathbf{S}_1 + \mathbf{S}_2)^2$ for this state? Here $\mathbf{S}_i$ are the spin operators of the electrons.

3.3 Write down all the first excited many-body states of two electrons (with spin). Choose them to be eigenstates of the total spin operator, and compute their eigenvalues of $(\mathbf{S}_1 + \mathbf{S}_2)^2$ and $S_1^z + S_2^z$ (where S_i^z is the z componenet of the spin operator $\mathbf{S}_i$).

I consulted Sakurai's *Modern Quantum Mechanics*, Shankar's *Principles of Quantum Mechanics*, and Wolfram MathWorld while writing up these solutions.