Problem 25.58 A resistor with resistance R is connected to a battery that has emf 12.0 Vand internal resistance $r = 0.400\,\Omega$. For what two values of R will the power in the resistor be 80.0 W?

Solution. The power P delivered to a resistor is

$$P = I^2 R, \tag{25.18}$$

where I is the current through the resistor and R its resistance. We can find the current from

$$V_{ab} = \mathcal{E} - Ir, \tag{25.17}$$

where V_{ab} is the voltage difference across the resistor, $\mathcal{E}$ is the emf of the battery, and r its internal resistance. We also know that

$$V_{ab} = IR. \tag{25.11}$$

Substituting (**??**) into (**??**), we get

$$IR = \mathcal{E} - Ir \implies \mathcal{E} = I(R+r) \implies I = \frac{\mathcal{E}}{R+r}.$$

Now we can substitute this result into (**??**) and solve for R:

$$P = \frac{\mathcal{E}^2}{(R+r)^2}R \implies \mathcal{E}^2 R = P(R^2 + 2Rr + r^2) \implies 0 = PR^2 + (2Pr - \mathcal{E}^2)R + Pr^2$$
$$\implies R = \frac{\mathcal{E}^2 - 2Pr \pm \sqrt{(2Pr - \mathcal{E}^2)^2 - 4P^2r^2}}{2P}$$

Plugging in our numerical values for r, P, and $\mathcal{E}$, and recalling that $1\,\mathrm{W} = 1\,\mathrm{V}^2\,\Omega^{-1}$, we get

$$R = \frac{(12.0\,\mathrm{V})^2 - 2(80.0\,\mathrm{W})(0.400\,\Omega) \pm \sqrt{[2(80.0\,\mathrm{W})(0.400\,\Omega) - (12.0\,\mathrm{V})^2]^2 - 4(80.0\,\mathrm{W})^2(0.400\,\Omega)^2}}{2(80.0\,\mathrm{W})}$$
$$= \frac{80.0\,\mathrm{V}^2 - \pm\sqrt{(80\,\mathrm{V}^2)^2 - (64\,\mathrm{V}^2)}}{160\,\mathrm{V}^2\,\Omega^{-1}} = \frac{80.0\,\mathrm{V}^2 \pm \sqrt{2306\,\mathrm{V}^4}}{160\,\mathrm{V}^2\,\Omega^{-1}} = \frac{80.0 \pm 48.0}{160}\,\Omega = (0.50 \pm 0.30)\,\Omega$$
$$= \begin{cases} 0.80\,\Omega, \\ 0.20\,\Omega. \end{cases}$$

Exercise 26.26 In the circuit shown in Fig. E26.28, find

(a) the current in each branch, and
(b) the potential difference V_{ab} of point a relative to point b.

Figure E26.28

Solution.

(a) We need to use Kirchhoff's rules. Since this circuit has more than one loop, we need to use both the junction rule,

$$\sum I = 0, \tag{26.5}$$

and the loop rule,

$$\sum V = 0. \tag{26.6}$$

Let's choose the current to be flowing to the right across the 10.00 V battery, and start with the loop rule. For the top loop, we have

$$-I_1(2\,\Omega) - I_1(1\,\Omega) - 5\,\mathrm{V} - I_1(4\,\Omega) - I_1(3\,\Omega) = 0 \tag{A}$$