Kernel Smoothing of Blip "Survival"

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Introduction

Kernel density estimation is a well-known methodology for recovering an unknown density at a point. The method can be seen as an extension from calculus I.

$$
\begin{aligned}
p_0(x_0) &= \frac{d}{dx}F_0(x_0) \\
&\approx \frac{F_0(x_0-h)-F_0(x_0+h)}{2h} \\
&\approx \frac{F_n(x_0-h)-F_n(x_0+h)}{2h} \\
&= \frac{1}{nh}\sum_{i=1}^{n} k\left(\frac{O_i-x_0}{h}\right)
\end{aligned}
$$

where k is the uniform density from -1 to 1. This is merely a discrete derivative to recover a parameter, $p(x_0)$, which is defined on a set of measure 0. Thus we need to borrow information around x_0, making some modest smoothness assumptions on p, in order to estimate the parameter.

Target Parameter and Smoothed Parameter

Our observed data, $O = (W, A, Y) \sim P$, where W is a set of confounders, A is a binary treatment and Y is a binary outcome. B is the blip function or average treatment effect conditional on W.

Our parameter of interest is the proportion of the target population with conditional average treatment effect bigger than t. This is notated below as a parameter mapping from the model $\mathcal{M}$ to the real numbers:

$$\Psi(P) = \mathbb{E}_w\mathbb{I}(B(W) > t) \text{ for P} \in \mathcal{M}$$

where

$$B(W) = \mathbb{E}\left[Y|A=1,W\right] - \mathbb{E}\left[Y|A=0,W\right]$$

Ψ is not pathwise differentiable so instead we consider the smoothed version of the parameter mapping, using kernel, k, with bandwidth, δ:

$$\Psi_{t,\delta}(P) = \mathbb{E}_w \int_x \frac{1}{\delta}k\left(\frac{x-t}{\delta}\right)\mathbb{I}(B(W) > x)dx = \int_x \frac{1}{\delta}k\left(\frac{x-t}{\delta}\right)S(x)dx$$

The Efficient Influence Curve and Remainder Term

$$
\begin{aligned}
D_{\Psi^\star_{t,\delta}}(P)(O) &= \frac{-\mathbb{I}(t-\delta a < B(W) < t+\delta a)}{\delta}\left(k\left(\frac{B(W)-t}{\delta}\right) - 2k(a)\right) * \frac{2A-1}{g(A|W)}(Y - \bar{Q}(A,W)) + \int \frac{1}{\delta}k\left(\frac{x-t}{\delta}\right)\mathbb{I}(B(W) > x)dx - \Psi_{t,\delta} \\
&= D^\star_2 + D^\star_1
\end{aligned}
$$

We note that we are assuming k is a symmetric kernel with support, $[-a, a]$. Compactness was used in proving the following about our remainder term:

$$R_2(P_0, P) = P_0D^*(P) + \Psi(P) - \Psi(P_0)\frac{1}{\delta}O\left(\|g-g_0\|_{L^2_{P_0}}\|\bar{Q}-\bar{Q}_0\|_{L^2_{P_0}}\right) + \frac{1}{\delta}O\left(\|B-B_0\|^2_\infty\right)$$

We need to estimate both $\bar{Q}_0$ and g_0 at $n^-.25$ to be assured our TMLE will be asymptotically efficient.

The Derivation of the Efficient IC

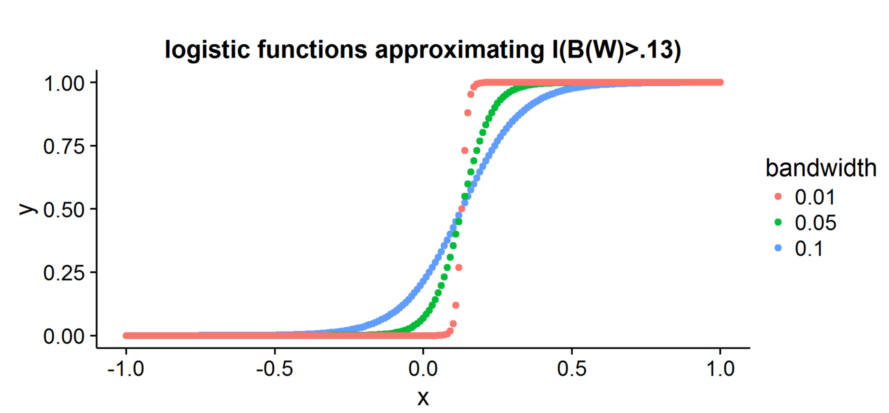
The derivation revolves around the use of fabulous, logistic function

Let us assume we want to estimate one or more points on the blip survival curve so our target parameter is

$$\Psi_{t,\delta}(P) = \lim_{h\to 0}\mathbb{E}_w \int_{-1}^{1} \frac{1}{\delta}k\left(\frac{x-t}{\delta}\right)\Phi(\frac{B(W)-x}{h})dx$$

where we approximate the $\mathbb{I}(B(W) > t)$ by the logistic function

$$\Phi_{\delta,t}(x) = \frac{1}{1 + exp(-fracB(W) - th)}$$



We then compute

$$lim_{\epsilon\to 0}\frac{\Psi_{t,\delta}(P_\epsilon)-\Psi_{t,\delta}(P)}{\epsilon} = lim_{\epsilon\to 0}\frac{1}{\epsilon}lim_{h\to 0}\mathbb{E}_w \int_x \tfrac{1}{\delta}k\left(\tfrac{x-t}{\delta}\right)\left(\Phi(\tfrac{B_\epsilon(W)-x}{h}) - \Phi(\tfrac{B(W)-x}{h})\right)dx$$

$$= \langle D^\star_{\Psi_{t,\delta}}, s\rangle_{L^2_0(P)}$$

where s and $D^\star_{\Psi_{t,\delta}}$ are scores in $L^2_0(P)$

The TMLE: Locally Least Favorable Model

We can estimate many points at once on the blip "survival" curve or merely one, all using a single epsilon locally least favorable model.

We could have used a universal least favorable model but each update requires computing an entire integral, which would be quite expensive since the one-step TMLE requires many recursions.

$$\frac{d}{d\epsilon}P_nL(P_{n,\epsilon})\Big|_{\epsilon=0} = \|P_nD^\star(P_n)\|_2 \tag{1}$$

Submodel of our initial estimate, P^0_n:

$$P^0_{n,Y}(Y|A,W) = \bar{Q}^0_n(A,W)^Y(1-\bar{Q}^0_n(A,W))^{1-Y}$$

$$\bar{Q}_{n,dt}(A,W) = expit\left(logit(\bar{Q}_n(A,W) - dt\langle H(A,W), \frac{P_nD^\star(P_n)}{\|D^\star(P_n)\|_2}\rangle_2\right)$$

$$q_{n,\epsilon}(W) = q_n(W) + \epsilon(q_n(W) - P_nq_n)$$

The clever covariates for each parameter are:

$$H(A,W) = (H_1(A,W), ..., H_m(A,W)$$

with components

$$H_k(A,W) = \frac{-\mathbb{I}(t_k-\delta a < B(W) < t_k+\delta a)}{\delta}\left(k\left(\frac{B(W)-t_k}{\delta}\right) - 2k(a)\right) * \frac{2A-1}{g(A|W)}$$

Methods

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Conclusion

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Additional Information

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- Curabitur pellentesque dignissim
- Eu facilisis est tempus quis
- Duis porta consequat lorem

References

[1] J. M. Smith and A. B. Jones.
Book Title.
Publisher, 7th edition, 2012.

[2] A. B. Jones and J. M. Smith.
Article Title.
Journal title, 13(52):123–456, March 2013.

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