**Nonlinear Regression Overview:** The overall goal of nonlinear regression is the same as linear regression, in that you are seeking to find a relationship between a response variable *y* to a predictor variable (or set of predictor variables) *x*. Nonlinear regression is defined by the fact that the response variable depends nonlinearly on one or more unknown parameters, not the relationship between the covariates and the response. Nonlinear models are in given in the form: , where are the responses, is a known function of the predictor variables, , and the parameter vector, , and are random errors. This type of modeling is often used when a more complex or curved relationship is apparent in observed data. It is frequently used in biology, chemistry, finance, and psychological and social sciences.

There are a plethora of nonlinear models to choose from including logarithmic, exponential, spline models, and combinations of those mentioned. For our demonstration, we will focus on the exponential and spline model.

**Data Requirements:** Exponential and exponential spline nonlinear regression have several data requirements, including a dependent variable that is quantitative and continuous; the values must be numerical and take in a value within the range. The relationship between the independent and dependent variables must be nonlinear and from the same data distribution. To best demonstrate the utility of exponential nonlinear regression, at least 30 observations are favorable. Assumptions include choosing the correct model to represent the relationship between the dependent and independent variables. The errors or residuals must be independent and normally distributed and remain constant across all levels of the independent variables. To assess the assumptions, plotting residuals against independent variables can help detect violations in assessing independence. Performing a Shapiro-Wilks test may determine the normality across the independent variables.

**Exponential and Spline Function Details:** The general form of the equation used for the exponential regression function is: , where is the response variable, is the predictor variable, is a scale factor, and is the rate of growth or decay. The output from the nls() function used in R for the exponential regression model is largely interpreted the same as with the lm() function for linear regression. The two items specific to the nls function are number of iterations to convergence (how many times the algorithm adjusted parameter values to minimize RSS) and achieved convergence tolerance.

A spline is a piecewise function conducted by splitting the data into multiple ranges at points called knots and fitting a model to the data within each range, then combining; these will account for differences in the relationships between variables in a way that a single function fitted to the entire plot may not. Spline functions are versatile in that you have the option to specify the number of knots you wish to use for your model.

When determining goodness of fit, the R2 coefficient should not be used in nonlinear regression. Some models, such as the spline, will give the R2 coefficient, however in nonlinear regression SS Regression + SS Error does not equal SS Total. Thus the R2 coefficient is invalid and using a method such as AIC or BIC is a better method to compare goodness of fit between models.

**Nonlinear Regression Handout**

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