Bassel Heiba Elfeky **Homework 2** October 11, 2019

10/10

6.11

a) Derivation of ϵ', equivalent to 6.81, for overrelaxation method:

$$x' = (1+w)f(x) - wx$$

Using $f(x) = f(x^*) + (x - x^*)f'(x^*)$ we can write this as:

$$x' = (1+w)(f(x^*) + (x - x^*)f'(x^*)) - wx$$

Since $f(x^*) = x^*$ and $x^* = x' + \epsilon' = x + \epsilon$ we can rewrite it as:

$$x' = (1+w)(x^* + (x - x^*)f'(x^*)) - wx$$

$$= (1+w)(x^* - \epsilon f'(x^*)) - wx$$

$$x^* - \epsilon' = x^* - \epsilon f'(x^*) + wx^* - w\epsilon f'(x^*) - wx$$

$$\implies \epsilon' = \epsilon(f'(x^*) - w + wf'(x^*))$$

$$= \epsilon((1+w)f'(x^*) - w))$$

Substituting ϵ as $x^* - x$:

$$\epsilon' = (x^* - x)((1+w)f'(x^*) - w))$$

$$= (x' + \epsilon' - x)((1+w)f'(x^*) - w))$$

$$\implies \epsilon'(((1+w)f'(x^*) - w) - 1) = (x - x')((1+w)f'(x^*) - w))$$

$$\epsilon' = \frac{(x - x')((1+w)f'(x^*) - w))}{(((1+w)f'(x^*) - w)) - 1)}$$

$$= \frac{(x - x')}{(1 - 1/[(1+w)f'(x^*) - w)])}$$

Using the assumption that $f'(x^*) \simeq f'(x)$, we can write our equation in terms of $f'(x)$ which gives us the error on x' for the overrelaxation method:

$$\epsilon' = \frac{(x - x')}{(1 - 1/[(1+w)f'(x) - w)])}$$

b) For $x = 1 - e^{-cx}$, for the case where $c = 2$ and starting value is $x = 1$, it took 14 iterations to converge to reach a result accurate to 10^{-6}. With a starting value as $x = 2$, it took 15 iterations to converge. With starting values $x = 9$, the code breaks down as the value is too far from the solution.

c) Using the overrelaxtion method, it took 5 iterations to reach a solution accurate to 10^{-6} for a value of $w = 0.5$. It took 12 iterations for $w = 0.1$ and 8 iterations for $w = 0.4$. This means that as we moved further south of $w = 0.5$, the number iterations it took to converge increased which means that the optimum is greater than $w = 0.5$. With some trial and error for values of w and using the same logic, I found that $w = 0.75$ is the optimal value, with the code taking only 4 iterations to reach a solution. This agrees with the assumption that the overrelaxation method reaches a solution more than twice as fast as the relaxation method.

d) If we find that the value overshoots the solution then a negative value of w would help reach a solution faster than a positive value of w. For example, if our solution is close to 1 and the starting point is 0.5 and after the iteration it is 1.5, it means that the code overshoot beyond the solution and a negative value of w would give us a more efficient result.

Show example?

6.13

a) Starting with Planck's radiation law where I is the intensity per unit area per unit wavelength, λ is the wavelength, T is the black body temperature, h is Planck's constant, c is the speed of light, and k_b is Boltzmann's constant:

$$I(\lambda) = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda k_B T} - 1}$$

Taking the derivative of Plank's I with respect to λ:

$$\frac{dI}{d\lambda} = \frac{(-10\pi hc^2\lambda^{-6})}{(e^{hc/\lambda k_b T - 1})} + \frac{(2\pi hc^2\lambda^{-5})(hc/kbT)e^{hc/\lambda kbT}}{\lambda^2(e^{hc/\lambda k_b T} - 1)^2} = 0$$

$$(-10\pi hc^2\lambda^{-6})(e^{hc/\lambda k_b T} - 1) + \frac{(2\pi hc^2)\lambda^{-7}e^{hc/\lambda k_b T}}{k_b T} = 0$$

$$5e^{hc/\lambda k_b T} - 5 + \frac{hc}{\lambda k_b T} = 0$$

Which can be written in terms of x by making the substitution $x = hc/\lambda k_b T$:

$$5e^{-x} + x - 5 = 0$$

Since x is the solution to our equation, λ_{max} can be written in terms of x as:

$$\lambda_{max} = \frac{hc}{k_b x T}$$

and by writing $b = hc/k_b x$ we get:

$$\lambda_{max} = \frac{b}{T}$$

b) Using the binary search method, we calculate the value of x to be $x = 4.96511448174715$ and 0. Plugging that in our equation $b = \dfrac{hc}{k_b x}$ we find that the displacement constant is $b = 0.002897664723899897$ which agrees with the actual displacement constant up to 10^{-6}.

c) To estimate the value of the surface temperature of the sun, we plug our value of the displacement constant into $\lambda = \dfrac{b}{T}$. With a wavelength peak of 502nm, our estimated surface temperature of the sun is $T = 5772.24048585637K$ which is very close to the actual value of about $5778K$.

✓

maybe not all
these digits are
significant

3

more informative, I think, to show

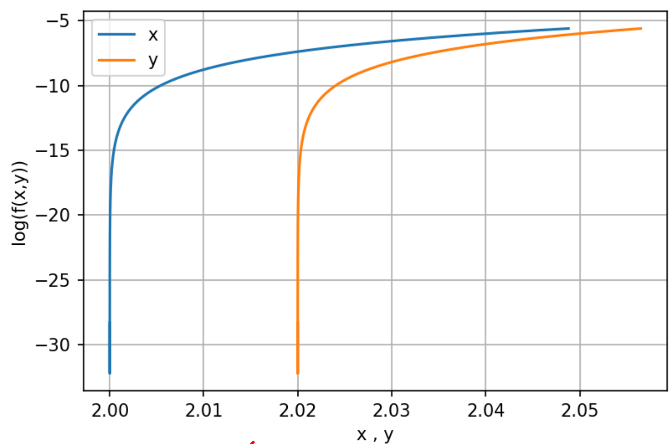

Figure 1: Minimum value of test function with initial values $x = 10$ and $y = 8$

$$\text{Test function: } f(x, y) = (x - 2)^2 + (y - 2.02)^2$$

For our test function, I calculated the values to be x= 1.999999500085 and y= 2.01999950006. Figure 1 shows us that at these values we get a value very close to zero for $f(x, y)$ which means that we have successfully found the minimum of our test function. I choose the minimum values to be $x = 2$ and $y = 2.02$, so that we can see that $f(x, y)$ going to zero at the calculated values of x and y and so that they would show on the same graph.

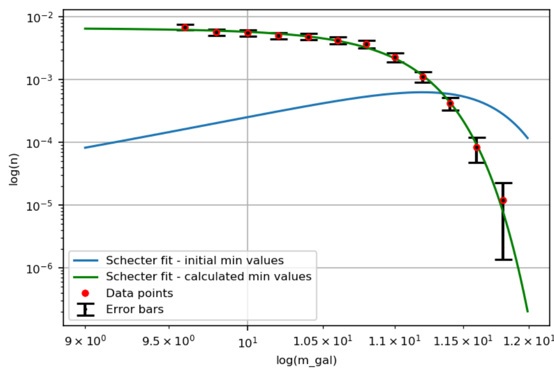

Figure 2: Schecter function with initial conditions: $\phi^* = -3.5$, $M^* = 12$ and $\alpha = -0.75$

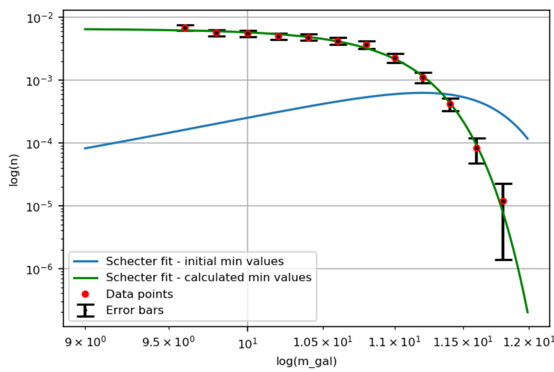

Figure 3: Schecter function with initial conditions: $\phi^* = -3.2$, $M^* = 11.5$ and $\alpha = -0.5$

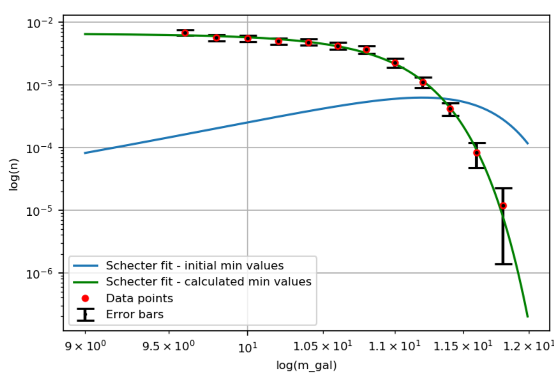

Figure 4: Schecter function with initial conditions: $\phi^* = -3$, $M^* = 11$ and $\alpha = -1$

For the Schecter function, with the starting values of $\phi^* = -3.2$, $M^* = 12$ and $\alpha = -0.75$, the gradient descent gives values of optimal values of $\phi^* = -2.568623993493128$, $M^* =$

10.976483189119367 and $\alpha = -1.0097925044663667$ which gives us a minimum of value of $\chi^2 = 2.908097865742 56$. On Figure 2, the Schecter function fit is plotted and matches the data points very well and lies within the error bars. Using starting values for ± 1 for ϕ^*, α and M^*, calculated optimal values are very similar and result in almost identical plots. Otherwise, the fit starts to become worse and worse until the initital values are to far away to to give us well estimated results.

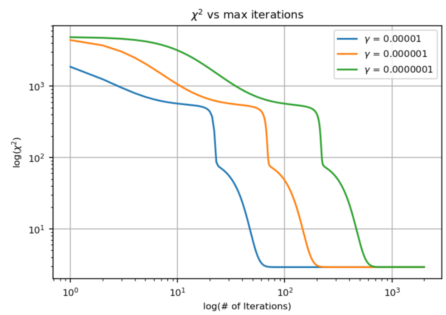

Figure 5: Initial conditions: $\phi^* = -3.5$, $M^* = 12$ and $\alpha = -0.75$

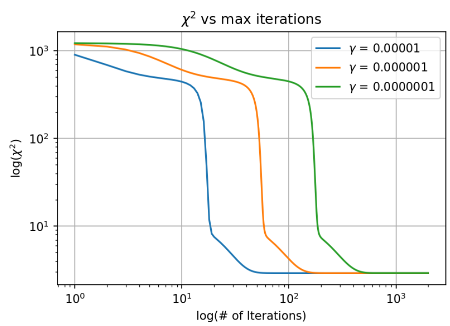

Figure 6: Initial conditions: $\phi^* = -3.2$, $M^* = 11.5$ and $\alpha = -0.5$

nice! good work

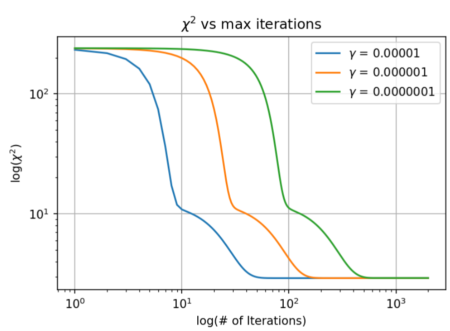

Figure 7: Initial conditions: $\phi^* = -3$, $M^* = 11$ and $\alpha = -1$

Setting the precision to 10^{-7} and step size to 10^{-5}, we have that with a single iteration, the value of χ^2 starts out at a few multiples of 10^3. As we increase the number of iterations, the value of χ^2 starts to sharply decrease until it converges after a sufficient number of iterations. This happens at about 70 iterations for $\gamma = 10^{-5}$, about 200 iterations for $\gamma = 10^{-6}$ and about 800 iterations for $\gamma = 10^{-7}$. The number of iterations of which χ^2 begins to converge seems to depend on γ with smaller values of γ taking longer to converge to our minimum value of $\chi^2 = 2.91$.