Computational Physics Homework #1

Due 9/23, by 9am

Submit via email (jlt12@nyu.edu), a PDF copy of your writeup, with a link to the github repo for this homework set. In your git repo, include your code, your output datafiles, your plots, your latex documents, and the final PDF.

1) Differentiate the functions cos(x) and exp(x) at x = 0.1, 10 using single precision forward-, central- and extrapolated-difference algorithms.
   a) Write a code that implements these three methods.
   b) Make a log-log plot of the relative error ε vs step size *h* and check whether the scaling and the number of significant digits obtained agrees with simple estimates.
   c) Truncation and roundoff error manifest themselves in different regimes in these plots. Clearly identify these regimes.

2) Consider the integral,

$$I = \int_0^1 \exp(-t)dt$$

   And compare the relative error, ε, for the midpoint rule, trapezoid rule, and Simpson's rule for single precision.
   a) Write code that implements each method.
   b) Make a log-log plot of e as a function of number of bins N. (Since this is a log-log plot, choose values that scale in a reasonable fashion). Make N large enough such that you see the effects of roundoff error.
   c) Explain what you see in the plot.

3) In cosmology, density fluctuations in the matter distribution are characterized by a power spectrum, P(k), the rms amplitude fluctuations of the density waves, as a function of wavenumber k (with units of h/Mpc). In configuration space, these density fluctuations are described by the correlation function, ξ(r), at a given scale r, usually in Mpc/h. These two are quantities are related by

$$\xi(r) = \frac{1}{2\pi^2} \int dk\, k^2 P(k) \frac{\sin(kr)}{kr}$$

With this homework, I have attached a tabulated power spectrum. The first column is k and the second column is P(k) (pay no attention to the third column). Using whatever integration method you prefer, use the above equation to calculate ξ(r) in the range r=[50,120] Mpc/h. The power spectrum is tabulated in logarithmic intervals in k, due to it's power-law like nature. You may choose to use an interpolation technique, such as cubic spline, to help evaluate the integral.

Around k~0.1, you can see oscillatory behavior in P(k). We call these the "baryon wiggles," and they manifest as a single "bump" in the correlation function at large scales. Using your calculation for ξ(r), determine the scale, r, of the peak of this bump. Make a plot of $r^2\xi(r)$ over the required range in r (multiplying by r^2 visually enhances the bump). Indicate on this plot the scale of the peak, also known as the "baryon acoustic oscillation" (BAO) peak.

Notes:

a) If you use spline interpolation, you are not required to code that up yourself. You may use a pre-packaged routine (or numerical recipes code).
b) Formally, the limits of the integral are from k=0 to k=infinity. Note that P(0)=0. You may choose a finite upper limit, provided you can determine if your limit is robust.
c) The 'h' in the distance units refers to the Hubble constant $h=H_0/100$, which sets the distance scale and is thus incorporated into the distance units, since its value is unknown.