Computational Physics : Assignment 1

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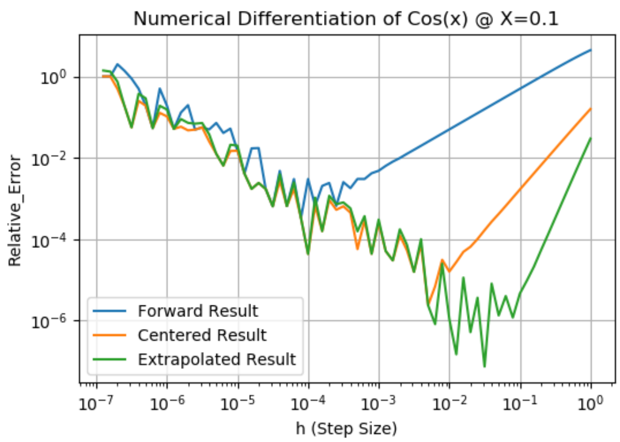
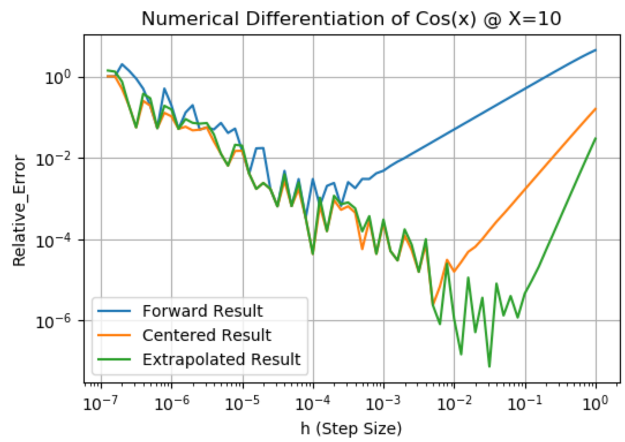
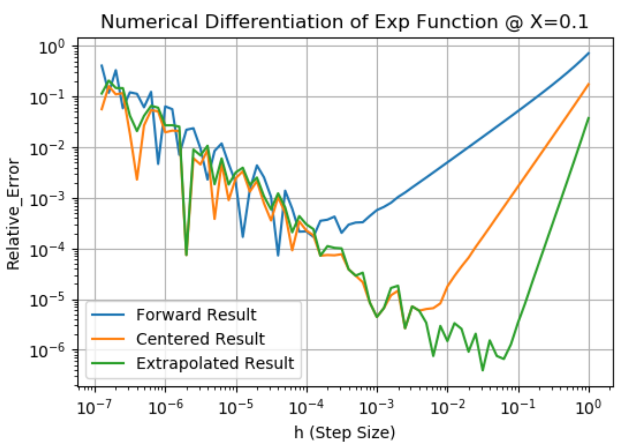
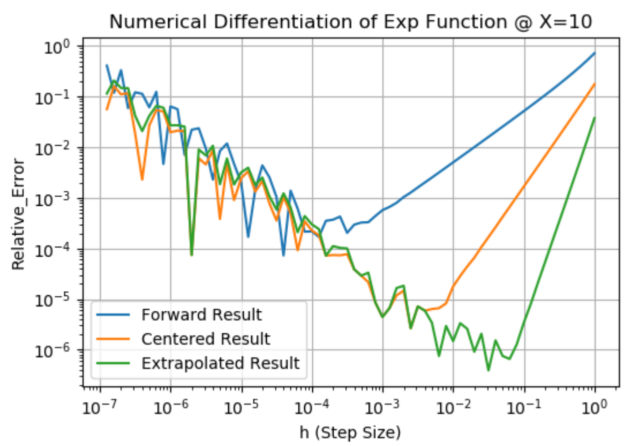
Question 1: Numerical Differentiation :









Commentary on Graphs & Precision :

The results of the three method confirmed our theoretical knowledge of how each method works. It wasn't surprising to yield more accurate results using the Extrapolated Difference Method and the least accurate results using the Forward Difference Method. There's a limit to our accuracy imposed by the usage of single precision number representation, though. The single precision representation have only 24 bits of precision which is equivalent to 7 or 8 decimal digits. We can see how the number of significant figures obtained in this problem is in agreement with our knowledge of the limits of single precision representation as our relative errors don't go beyond 10^{-7}.

Commentary on Errors:

Truncation error is essentially an error that is caused by using simple approximations to represent exact mathematical formulas. Round-off error in a numerical method is error that is caused by using a discrete number of significant digits to represent real numbers on a computer. Round-off errors are dominant when we tend to subtract two numbers that are nearly identical. This is what happens when we apply an approximation to intervals that are too small (Small step size[h]).

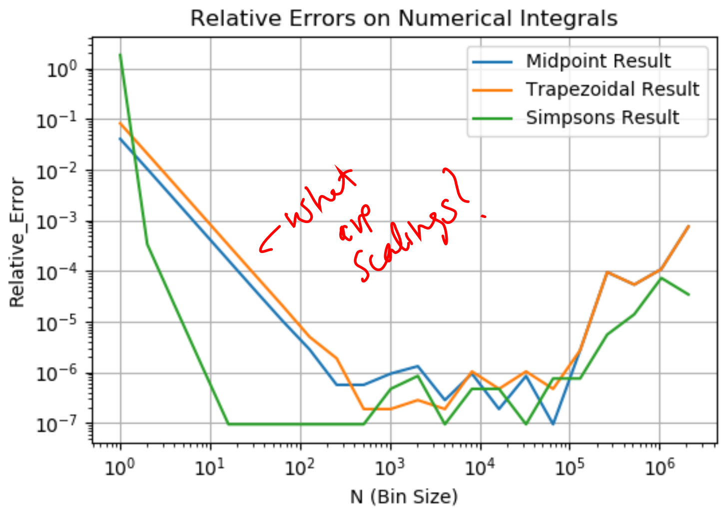
As we can see on the graphs, in the regions where our chosen step size is small (Left part of the graphs), round-off errors are more dominant leading to having comparable results using the three methods. However, as we increase our step size (h), we see how different the accuracy is for each method. As we expected, the forward difference method gave the least accurate results, while the extrapolated difference method gave the best results. The extrapolated difference method gives its least error when h is chosen to be lying between 10^{-2} to 10^{-1} . However, increasing the step size, while it successfully minimizes the round off errors; it will unintendedly lead to having major truncation errors as we can clearly notice on the right side of the graph. So, while doing numerical differentiation, one must always be careful in finding the optimum h value that would yield the most accurate results.

want more details on scalings and locations of minimum error.

Question 2: Numerical Integration :

Graphical Results for the Integral :

$$I = \int_0^1 e^{-t}\, dt$$



← what are scalings?

✓ good.

Commentary on the Graph:

As we were expecting,Simpsons Method yielded the most accurate results in almost all the bin sizes, but significantly so when our bin size was small.As we increased our bin size, the three methods started giving comparable results, then round off errors started to become dominant. The surprising result, however, was that the Midpoint Method giving more accuracy leaving the trapezoid Method to give us the worst or least accurate results in almost all the bin sizes.

Commentary on Errors :

As with any numerical calculations, we essentially have two sources of error as discussed before;a rounding error resulting from calculating the integral in addition to the approximation error. However, the main source of error in calculating our integrals is the approximation error because the integration rules themselves (any of them) are approximation to the integrals themselves.

Our estimation of the integral gives more accurate results when we take large number of Bins (N) which means that the approximation errors get reduced. The backlash from this could result in increasing the computing time without improving the accuracy in a significant manner because by reducing the approximation error, we have unintentionally made the round-off errors more dominant. Therefore, increasing the bin size will be efficient up to the point where the two errors are equal or comparable in magnitude.

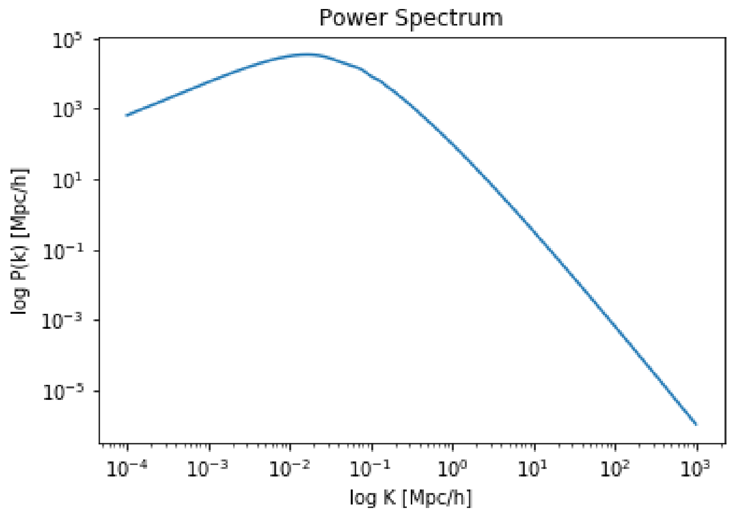
As we can see in the graphs, I think this could be the reason why the three graphs of the three different methods did curve up and started to oscillate at large bin sizes leading to an increased error value in calculating the integral.

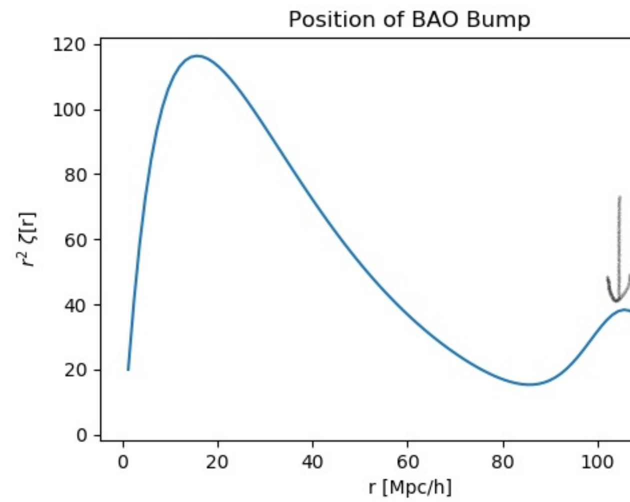
fit plot on page. the most important part is being cut off.

Question 3: Cosmology & Correlation Function

Graphical Results :

$$\zeta(r) = \frac{1}{2\pi}\int k^2 P(k)\frac{sin(kr)}{kr}\,dk$$





Commentary on the Graphs :

Around k 0.1, you can see oscillatory behavior in P(k). We call these the baryon wiggles, and they manifest as a single bump in the correlation function at large scales. Using your calculation for (r), determine the scale, r, of the peak of this bump. Make a plot of r 2 (r) over the required range in r (multiplying by r$^2 visuallyenhancesthebump).Indicateonthisplotthescaleofthepeak, alsoknownasthebaryonacousticoscillation(BAO$

proof read.