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Homework 2

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Problem 1. Error of overrelaxation method.

Let x^* be the value of the function to be found. Thus,

$$x^* = (1+w)f(x^*) - wx^*$$
$$x^* = x_i + \epsilon_i$$

By differentiating the first equation above,

$$\epsilon_{i+1} = \epsilon_i((1+w)f'(x^*) - w)$$

then multiply by ϵ for x close to x^*.

$$\frac{\epsilon_{i+1}}{((1+w)f'(x^*) - w)} = \epsilon_i$$

And we know,

$$x^* = x_i + \epsilon_i$$
$$x^* = x_{i+1} + \epsilon_{i+1}$$
$$x_i + \epsilon_i = x_{i+1} + \epsilon_{i+1}$$
$$x_i + \frac{\epsilon_{i+1}}{((1+w)f'(x^*) - w)} = x_{i+1} + \epsilon_{i+1}$$
$$\frac{\epsilon_{i+1}}{((1+w)f'(x^*) - w)} - \epsilon_{i+1} = x_{i+1} - x_i$$
$$\epsilon_{i+1}\left(\frac{1}{((1+w)f'(x^*) - w)} - 1\right) = x_{i+1} - x_i$$
$$\epsilon_{i+1} = \frac{x_{i+1} - x_i}{(1/((1+w)f'(x^*) - w) - 1)}$$

Using the relaxation method on the function $f(x) = 1 - exp[-2x]$, the algorithm converges to $x = 0.796812$ after 15 iterations with error $\epsilon = 10^{-6}$, but using the overrelaxation method with weight $w = 0.5$, the algorithm converges to the same value after only 5 iterations.

Using a negative weight, w, would be helpful if the initial value was close to a value that almost solved the equation. Thus the negative weight would help it get further away from this false value.

Problem 2. Beginning with the intensity of radiation as a function of wavelength.

$$
\begin{aligned}
I(\lambda) &= \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda k_B T} - 1} \\
\frac{\mathrm{d}}{\mathrm{d}\lambda} I(\lambda) &= \frac{\mathrm{d}}{\mathrm{d}\lambda} \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda k_B T} - 1} \\
&= \frac{\mathrm{d}}{\mathrm{d}\lambda} 2\pi hc^2\lambda^{-5}(e^{hc/\lambda k_B T} - 1)^{-1} \\
&= (e^{hc/\lambda k_B T} - 1)^{-1} \frac{\mathrm{d}}{\mathrm{d}\lambda}[2\pi hc^2\lambda^{-5}] + 2\pi hc^2\lambda^{-5}\frac{\mathrm{d}}{\mathrm{d}\lambda}[(e^{hc/\lambda k_B T} - 1)^{-1}] \\
&= (e^{hc/\lambda k_B T} - 1)^{-1}[2\pi hc^2(-5)\lambda^{-6}] + 2\pi hc^2\lambda^{-5}[(-1)(e^{hc/\lambda k_B T} - 1)^{-2}((-hc/k_B\lambda^2 T)e^{hc/\lambda k_B T})] \\
&= \frac{-10\pi hc^2\lambda^{-6}}{e^{hc/\lambda k_B T} - 1} + \frac{hc}{\lambda k_B T}\frac{2\pi hc^2\lambda^{-6}e^{hc/\lambda k_B T}}{(e^{hc/\lambda k_B T} - 1)^2}
\end{aligned}
$$

Now set this equal to zero and solve for λ.

$$
\begin{aligned}
0 &= \frac{-10\pi hc^2\lambda_{max}^{-6}}{e^{hc/\lambda_{max} k_B T} - 1} + \frac{hc}{\lambda_{max} k_B T}\frac{2\pi hc^2\lambda_{max}^{-6}e^{hc/\lambda_{max} k_B T}}{(e^{hc/\lambda_{max} k_B T} - 1)^2} \\
&= \frac{5}{e^{hc/\lambda_{max} k_B T} - 1} - \frac{hc}{\lambda_{max} k_B T}\frac{e^{hc/\lambda_{max} k_B T}}{(e^{hc/\lambda_{max} k_B T} - 1)^2} \\
&= 5(e^{hc/\lambda_{max} k_B T} - 1) - \frac{hc}{\lambda_{max} k_B T}e^{hc/\lambda_{max} k_B T} \\
&= 5 - 5e^{-hc/\lambda_{max} k_B T} - \frac{hc}{\lambda_{max} k_B T} \\
&= 5e^{-hc/\lambda_{max} k_B T} + \frac{hc}{\lambda_{max} k_B T} - 5
\end{aligned}
$$

Now by substituting $x = \frac{hc}{\lambda k_B T}$, we get the following which we can zero to find the peak wavelength for a given temperature of a black body. Using the binary search method, I found the zero of

$$f(x) = 5e^{-x} + x - 5$$

to be $x = 4.96511$ after 30 iterations with error of 10^{-6}. The zero value for x in this equation relates to Wien's displacement law in the following way,

$$x = \frac{hc}{\lambda_{peak} k_B T}$$

Since the Sun has $\lambda_{peak} = 502$ nm, the approximate temperature of the surface of the Sun is $T = 5772.46$ K.

Problem 3. The gradient descent method is used to minimize the chi squared function of parameters of the Schetcher function and input data points. However this method will on converge for initial values within $10^{\pm 2}$ about the actual value for both M^* and α.

These plots were generated using initial values $\alpha = -1$, $M^* = 10^{11}$, and $\phi = 10^{-3}$, and learning rate, $\gamma = .0001$ with convergence error $\epsilon = 10^{-6}$.

is result robust? (diff starting conditions, parameters)?

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-result from test function?

-best fit values? χ^2?