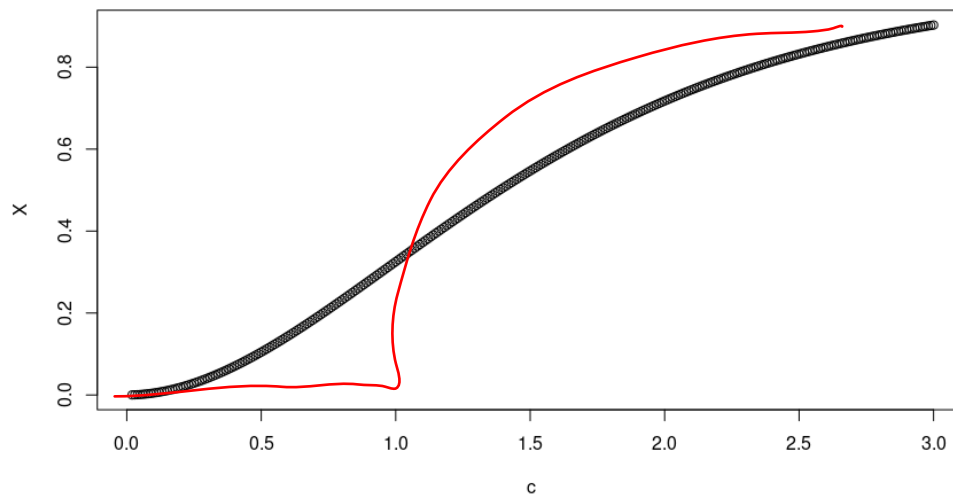


*please include
your name.***Homework 2****Introduction***9.5/10*

After having less success than desired on the previous assignment I managed to successfully achieve the desired results here. My final fitted Schechter function fits visually and analytically.

1

From Newman 6.10, part a) I get a solution converging to order 10^{-6} of $x = 0.7968121$. In part b) plotting the convergence value of x as a function of c , with c ranging from 0 to 3 generated the following plot. The plot has a tail where x is near zero before rising as the regime changes.

*this wasn't
required for
writeup, so ok.*

From Newman 6.11, part a) the derivation of the error is as follows.

$$x' = (1 + \omega)f(x) - \omega x$$

$$x^* + \epsilon' = (1 + \omega)f(x) - \omega(w^* + \epsilon)$$

$$x^* + \epsilon' = (1 + \omega)f(x^* + \epsilon') - \omega(w^* + \epsilon)$$

$$x^* + \epsilon' = (1 + \omega)[f(x^*) + \epsilon' f'(x^*) \dots] - \omega(w^* + \epsilon)$$

$$x^* + \epsilon' = (1 + \omega)[f(x^*) + (x' - x)f'(x^*)] - \omega(w^* + \epsilon)$$

Then our equivalent to 6.81 becomes:

$$\epsilon' = \epsilon[f'(x^*) + w f'(x^*) - w]$$

Homework 2

Which in turn implies our equivalent of 6.83:

$$\epsilon' = \frac{x - x'}{1 - 1/[(1 + \omega)f'(x) - \omega]}$$

From Newman 6.11 part b) the fewest number of iterations needed to converge were 16. However in part c) applying the overrelaxation technique I was able to reduce number of iterations to as few as 5. Which is remarkably less than a third the previous required number. This was achieved with an Omega value $\omega = 0.70$. For part d) a negative value of ω is useful for a function if you're starting point is to the right of the root. Meaning your starting value of x is greater than the root value.

2

From Newman 6.13, part a) the derivative is as follows. Simply taking the derivative of I with respect to λ using the quotient rule then setting that equal to zero yields

$$5e^{-hc/\lambda k_b T} + \frac{hc}{\lambda k_b T}$$

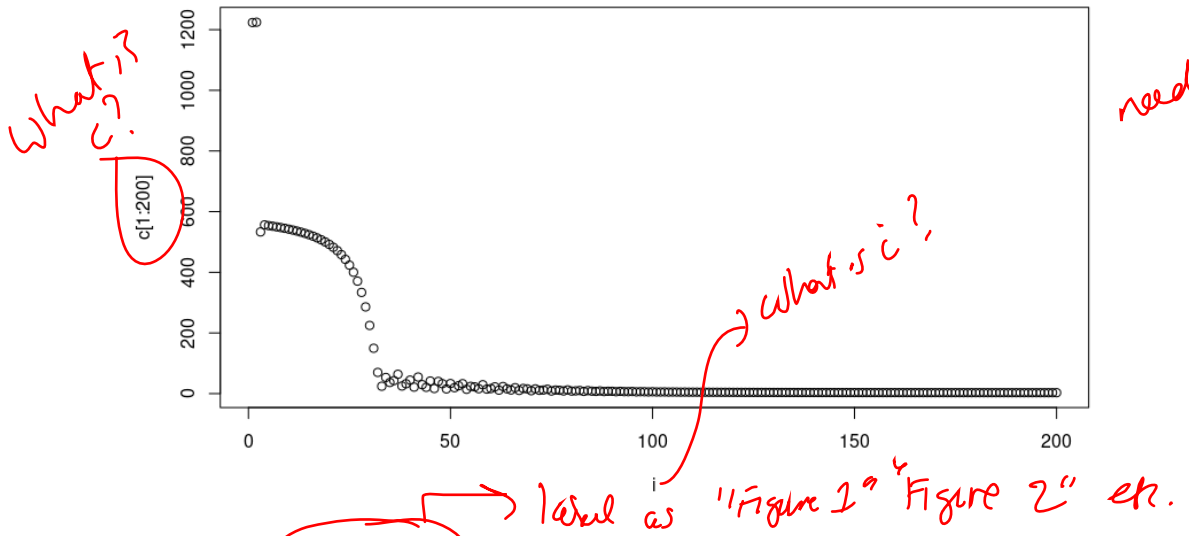
From Newman 6.13 part b) implementing the binary search algorithm to the Wien Displacement constant gives a value for $x = 4.961$.

From Newman 6.13 part c) substituting in values to find the temperature of the Sun. This was simply substituting in an rearranging the simple Wien Displacement law, which yielded $T_{sun} = 5759K$.

3

For problem 3, before importing the COSMOS data I applied my gradient descent method to the test function which converged, unsurprisingly, to the point (2, 2). It was much more of a task to correctly implement this to the χ^2 of the Schechter function to minimize the error and create a best fit curve to provided data points.

My code first imports, and formats the provided data into a Dataframe, which is a special array type in R. From here I define the Schechter function in log space of M^* and ϕ^* . Then I define the general χ^2 function. I used some test values in both to test they were working. I then constructed my gradient descent process in 3 dimensions which converged. The convergence value was $\chi^2 = 2.908056$. This is a plot of χ^2 vs step number.



And, lastly this is a plot of my best fit Schechter function overlaying the provided data with the given error bars associated with each point, in a log-log scale. The overall fit is very much within the error range for all but the first data point.

