

Comp Physics HW 2

Marcus DuPont

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1. Newman 6.11:

Solution:

In general, when evaluating the relaxation method we have equations of the form $x = f(x)$. Let us define a parameter Δx that is described by

$$\Delta x = x' - x = f(x) - x. \quad (1)$$

According to Newman, the overrelaxation method involves the iteration of the following equation

$$x' = x + (1 + \omega)\Delta x = (1 + \omega)f(x) - \omega x \quad (2)$$

Let us define ϵ to be the error on our current solution estimate. This implies that the true solution x^* differs by said error by $x^* = x + \epsilon$. On the next iteration, let's define ϵ' to be the error on next estimate, so that $x^* = x' + \epsilon'$. Performing a Taylor expansion, the value x' after an iteration is given in terms of the previous values by

$$\begin{aligned} x' &= f(x^*) = f(x^*) + f'(x^*)(x - x^*) = x^* + f'(x^*)(x - x^*) \\ \Rightarrow x' - x^* &= f'(x^*)(x - x^*) = ((1 + \omega)f'(x) - \omega)(x - x^*) \end{aligned} \quad (3)$$

Now, finally applying values near the solution x^* , we get

$$\epsilon' = \epsilon [(1 + \omega)f'(x^*) - \omega]$$

Now, we revisit the solutions that deviate slightly by error ϵ

$$x^* = x + \epsilon = x + \frac{\epsilon'}{(1 + \omega) - \omega} = x' + \epsilon'$$

Finally, we get

$$\epsilon' \approx \frac{x - x'}{1 - 1/[(1 + \omega)f'(x) - \omega]} \quad (4)$$

Results:

Relaxation results...

Answer: 0.7968126311118457, Iterations: 14

Overrelaxation Results...

Answer: 0.7968123729832619, Iterations: 4

One may need to use a negative value of ω to avoid cancellation error if the function $f(x)$ is complex polynomial.

① ✓
② → what's ω for this?

2. **Newman 6.13:** Planck's radiation law says that the intensity of radiation per unit area per unit wavelength λ a black body emits at a temperature T is

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/k_B T \lambda} - 1}$$

- (a) At maximum intensity, the derivative of the radiation law must go to zero implying

$$\begin{aligned} \frac{dI}{d\lambda} &= \frac{-10\pi hc^2 \lambda^{-6} (e^{hc/k_B T \lambda} - 1) - 2\pi hc^2 \lambda^{-5} (-hc/k_B T \lambda^{-2} e^{hc/k_B T \lambda})}{(e^{hc/k_B T \lambda} - 1)^2} = 0 \\ &= -5(e^{hc/k_B T \lambda} - 1) + \frac{hc}{k_B T \lambda} e^{hc/k_B T \lambda} = 0 \\ &= -5 + 5e^{-hc/k_B T \lambda} + \frac{hc}{k_B T \lambda} = 0 \end{aligned}$$

If we define a parameter $x = hc/k_B T \lambda$, we can rewrite the above equation as

$$-5 + 5e^{-x} + x = 0,$$

which can be solved numerically. From the numeric value of x , we arrive at

$$\lambda = \frac{b}{T}; \quad b := \frac{hc}{k_B x} \tag{5}$$

with b being a constant of proportionality.

Results:

The value of x is: 4.965113639831543

The value of the displacement constant is: 0.0028977732599822937 K m

At a wavelength of 502nm, the Sun's temperature is roughly: 5772.45669319182 K

3. Could not get the Schecter function to fit at all.

✓
are all these
digits
significant.

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