AMATH 483 / 583 (Roche) - Homework Set 1

Due Wednesday April 9, 5pm PT

April 2, 2025

Homework 1 (85 points)

1. (+15) Integrate the following integrals. Show work.
   (a) $\int_0^{\frac{\pi}{2}} ln\, sin(x)\, dx$.
   (b) $\int_0^{\pi} \frac{x\, sin(x)}{1+cos^2(x)}\, dx$. Use $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$.
   (c) $\int_0^{\frac{\pi}{2}} \frac{1}{1+(tan(x))^{\sqrt{2}}}\, dx$.
2. (+15) Show work.
   (a) Find $a, b \in \mathbb{R} \ni (1+i\sqrt{3})^{11} = a+ib$.
   (b) Find values of $(1+i\sqrt{3})^{\frac{1}{5}}$.
   (c) Solve for $w \in \mathbb{C}$ given $w^{\frac{4}{3}} + 2i = 0$.
3. (+15) Write the following as the ratio of integers. Show work.
   (a) $1 + 10^{-2} + 10^{-4} + 10^{-6} + \cdots$
   (b) $376.376376\ldots$
   (c) $.999\overline{9}$
4. (+10) Estimate the following as the ratio of integers using the secant approximation. Show work.
   (a) $(1.1)^{\frac{1}{3}}$
   (b) $\sqrt{8.5}$
5. (+10) Given $(x+y+z)^7$, find the expansion coefficients of the following terms. Show work.
   (a) $x^2y^2z^3$
   (b) x^3z^4
6. (+5) Given $(x+2y-3z+2w+5)^{16}$, find the expansion coefficient of the following term. Show work.
   (a) $x^2y^3z^2w^5$
7. (+10) Two numbers $a, b \in \mathbb{Z}$ are relatively prime when $gcd(a,b) = 1$, or $\exists x, y \in \mathbb{Z}$ with $ax + by = 1$. Recall, $c = gcd(a,b)$ when $c|a$ and $c|b$, and for any other divisor d of a, b then $d|c$.
   (a) For any $n \in \mathbb{Z}^+$, prove $8n+3$ and $5n+2$ are relatively prime. Show work. Hint: try Euclid's algorithm
   (b) Find the $gcd(250, 111)$ and show result as linear combination of these integers. Show work.
8. (+5) Write the prime factorization of 980220.