1. (+20) **Length of a graph** of $f(x)$ on $[a, b]$ is $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$. Given $f(x) = ln(x) - \frac{1}{8}x^2$ on $[1, 6]$. (a) Find the length of $f$ on the interval analytically. You will need this to calculate the error in your codes. (b) Write a C++ code parallelized using the **C++ threads** library that numerically evaluates the length of this function on the interval using Riemann sum. Your code should accept the number of points used to partition the interval and the number of threads to spawn. (c) Plot the strong scaling efficiency on 1,2,4,8,16 threads of your code for $n = 1.e8$ partition points, i.e. time versus thread count. (d) Plot the *log* of the numerical error for $10, 100, ...1.e6$ partition points (increasing by factors of $10x$ each time). Submit your code and plots.

$$f'(x) = \frac{1}{x} - \frac{1}{4}x = \frac{4 - x^2}{4x},$$

$$1 + (f'(x))^2 = 1 + \frac{x^4 - 8x^2 + 16}{16x^2} = \frac{x^4 + 8x^2 + 16}{16x^2} = \frac{(x^2+4)^2}{16x^2}$$

$$\sqrt{\frac{(x^2+4)^2}{16x^2}} = \frac{x^2+4}{4x}$$

$$L = \int_1^6 \frac{x^2+4}{4x} dx = \frac{35}{8} + \ln(6) \approx 6.1668$$