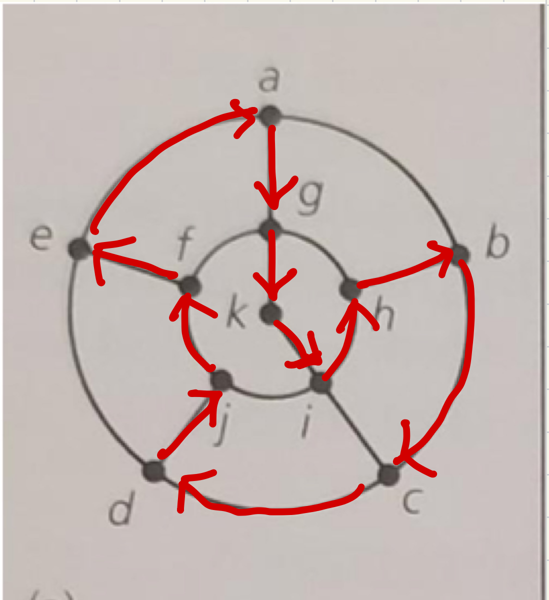
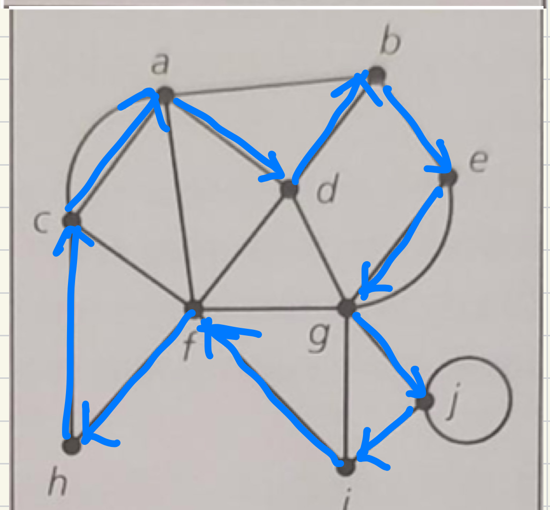


(a)

a) Has Hamilton cycle

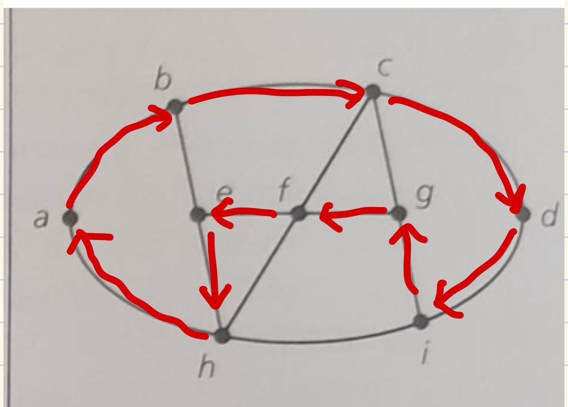
$a \to g \to k \to i \to h \to b \to c \to d \to j \to f \to e \to a$



(b)

b) Has Hamilton cycle

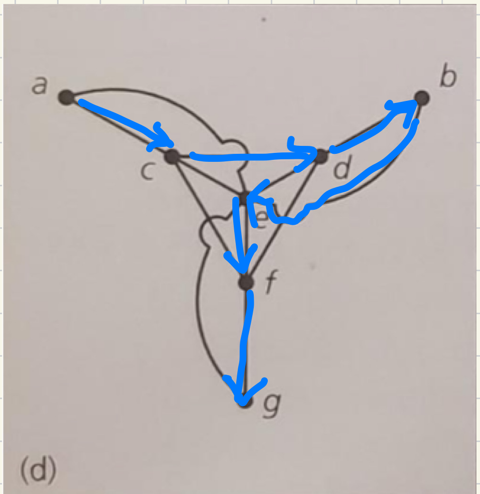
$a \to d \to b \to e \to g \to j \to i \to f \to h \to c \to a$



(c)

c) Has Hamilton cycle

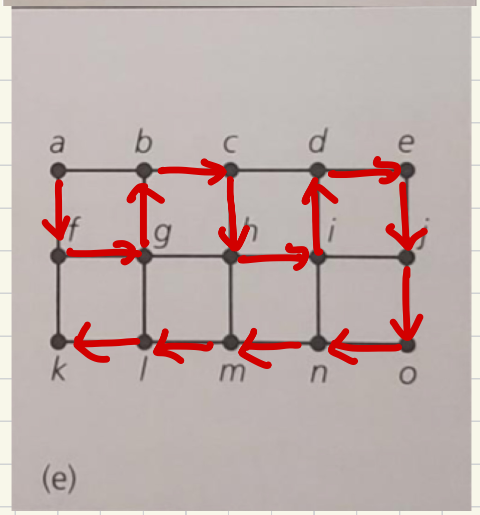
$a \to b \to c \to d \to i \to g \to f \to e \to h \to a$



(d)

d) No hamilton cycle, but hamilton path

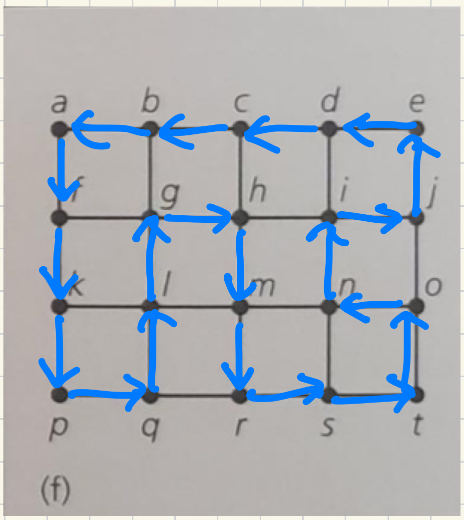
$a \to c \to d \to b \to e \to f \to g$



(e)

e) No Hamilton cycle, but hamilton path

$a \to f \to g \to b \to c \to h \to i \to d \to e \to j \to o \to n \to m \to l \to k$



(f)

f) Has hamilton cycle

$a \to f \to k \to p \to q \to l \to g \to h \to m \to r \to s \to t \to o \to n \to i \to j \to e \to d \to c \to b \to a$

2. (+10) **function fun**. $\forall x$ on the real line, find all real valued continuously differentiable functions f such that:

$$(f(x))^2 = 2025 + \int_0^x \left((f(t))^2 + (f'(t))^2\right)\, dt$$

$$\frac{d}{dx} f(x)^2 = \frac{d}{dx}\left(2025 + \int_0^x (f(t)^2 + f'(t)^2)\, dt\right)$$

$$2f(x)f'(x) = f(x)^2 + f'(x)^2$$

$$f(x)^2 - 2f(x)f'(x) + f'(x)^2$$

$$(f(x) - f'(x))^2 = 0 \qquad f(x) = f'(x) = Ce^x$$

$u = 2t$

$du = 2\, dt$

$f(x) = Ce^x$

$f^2(x) = C^2e^{2x}$

$f(t)^2 + f'(t)^2 = 2C^2e^{2x}$

substitute $\Rightarrow$

$$C^2e^{2x} = 2025 + \int_0^x 2C^2e^{2t}\, dt$$

$$C^2e^{2x} = 2025 + \left[C^2e^u\right]_0^{2x}$$

$$C^2e^{2x} = 2025 + C^2e^{2x} - C^2$$

$$C^2 = 2025, \qquad C = \pm 45$$

$f(x) = 45e^x$ or $f(x) = -45e^x$ //