Seating Zones and Pricing Strategies for Meany Hall

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Abstract

The Meany Center for Performance Arts is exploring the optimal seating zone arrangement and pricing strategy for its main theater with approximately 1200 seats divided into four zones. Given the historical patron demand, the goal is to maximize net profits by adjusting ticket prices based on zone quality, customers' sensitivity to pricing, the popularity and the type of performance. We first estimate the demand function through linear regression in double log form following the paper *"Revenue and attendance simultaneous optimization in performing arts organization"* (Baldin, Bille, Ellero, and Favaretto 2018). Using this demand function, we then solve a nonlinear maximization problem through PYTHON's Gurobi packages to get the optimal value of ticket price for maximal profits. Overall, this project will enhance profit for Meany Hall, provide pricing insights for future performances, and demonstrate the application of economic theories and discrete math modeling to benefit the community.

1 Problem Description

1.1 Background Information

Meany Hall is a prominent performing arts venue located on the University of Washington campus in Seattle. It was originally constructed in 1909 and named in honor of **Edmond S. Meany**, a professor of botany and history who played a crucial role in the development of the university. The original hall was unfortunately destroyed by fire in 1965. The current Meany Hall was inaugurated in 1974 and was designed as a venue for various art performances (Wikipedia n.d.(a)).

The facility includes the Katharyn Alvord Gerlich Theater, a main stage with a seating capacity of 1,200, known for its excellent acoustics and intimate viewing experience. It hosts an extensive array of performances ranging from university productions to international music and dance events, enhancing its reputation as a cultural hub in the Pacific Northwest (Meany Hall Theater n.d.(a)).

Legally, Meany Hall operates under the university's governance, receiving both public and private funding. It must provide a diverse repertoire of high-quality performances in music, dance, and theater. The hall is also dedicated to education and community engagement, offering programs that include school matinees and artist talks, aimed at audiences of all ages.

The hall's budget and operational strategies are closely tied to its educational mission, ensuring accessibility and involvement from the community. Meany Hall's operational decisions are influenced by a commitment to both preserving classical arts and promoting new and innovative works, reflecting a broad cultural policy that emphasizes inclusivity and artistic excellence (Meany Hall Theater n.d.(a)).

Meany Center for the Performing Arts at the University of Washington fosters innovative performances that advance public engagement, cultural exchange, creative research, and learning through the arts. It provides opportunities for diverse artists, community members, students, and faculty to connect during the discovery and exploration of the boundless power of the arts to create positive impacts in the world.

As a part of a public university, Meany Hall is also aligned with the broader objectives of public engagement and cultural enrichment, maintaining a pricing policy that makes performances accessible to a variety of audiences. This approach not only supports the arts development within the university but also contributes positively to the cultural vitality of the greater Seattle area (Meany Hall Theater n.d.(a)).

1.2 Project Objective

In this project, we delve into the nuanced strategies of seat arrangement and pricing within Meany Hall's main theater which has a capacity of 1200. The primary goal of this project is to help Meany Hall Theater to find the optimal layout of the seating zones and prices for each zone to maximize the net profits based on the historical demand, patrons' sensitivity to pricing, and the fame of the performers for the next year with about a total of 25 performances theoretically. The theater's performances include five types: piano performances, dance, chamber music, crossroads, and special events, so there will be 5 various performances in each type, so-called a series, for a total of 5 series (or 25 performances in total). "Crossroads" is the name given by Meany Hall Theater to the sequence of five events showcasing cross-cultural art elements in a western setting, such as music elements and storytelling from Native American culture. Then the theoretical optimal solution can be tested by the Meany Hall theater in the real world. If the optimal solution works well in the real world, the pricing strategy found in this paper could be implemented in the real world.

All seats in the main theater of Meany Hall are divided into four zones. The Meany Hall theater only sells tickets for zones 1, 2, and 3, each offering a different visual and auditory experience at three different prices. The proposed rough arrangement of seating zones by the Meany Hall theater is shown in Figure 1. In Figure 1, there are two types of seat arrangements, each of which determines zone 1, zone

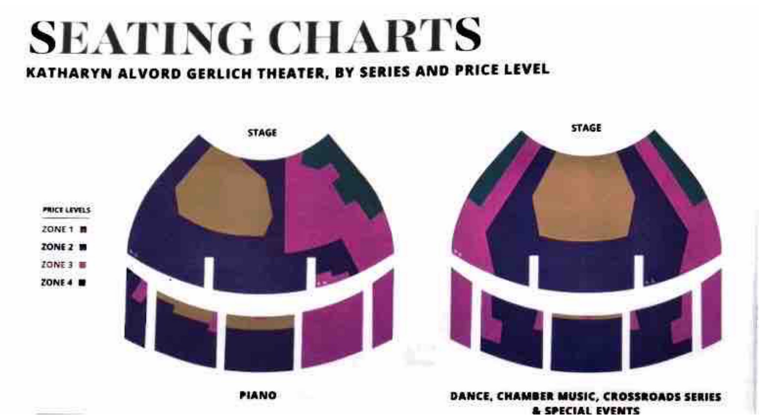


Figure 1: Seating Arrangement in the main theater of Meany Hall (Meany Hall Theater n.d.(b))

2, and zone 3 differently. One arrangement is for piano performances only, and the other is for all other kinds of performances, such as dance performances and chamber music. In zone 1, audiences will have the best overall viewing experience, so the tickets in this zone are the most expensive among the three zones. The viewing experience in Zone 2 will be better than the viewing experience in Zone 3, so the ticket price for Zone 2 is more expensive than the ticket price for Zone 3.

There are three types of tickets including seasonal tickets, standard full-price tickets, and student tickets (Meany Center for the Performing Arts 2024). Seasonal tickets, excluding special events, are available for sale, which means that there are only seasonal tickets for chamber music, dance performances, piano performances, and crossroads. Each seasonal ticket is for the same full series of events in one of the seating zones at a discounted total price, and the seasonal ticket for each series can have different ticket price. For example, buying a seasonal ticket for the dance series in Zone 1 includes admission to 5 dance events in the next year with the best viewing experience possible. Based on the proposed price by the Meany Hall Theater for the next year, the seasonal ticket for the dance series in zone 1 is $332 which saves $84 compared with buying 5 standard full-price tickets for the dance series.(Meany Center for the Performing Arts 2024) With that being said, each full-price ticket is for admission into one of 25 performances in one of the three seating zones. Besides seasonal tickets and the standard full-price tickets, student tickets are sold at \$10 for each performance in Zone 2 or Zone 3.

In the following paper, we discuss the simplifications to the problem of finding the best strategy for optimal layout and pricing for the Meany Hall Center's main theater in section 2. In section 3, we translate this real-world problem into a nonlinear optimization problem, define the variables, and explain the constraints. We will explain our data and perform statistical analysis on the data to estimate the demand function in section 4. Then, we will discuss our mathematical solutions and ways to interpret the solutions in the real-world setting in section 5. In section 6, we talk about how we can possibly improve our models and methodologies. Finally, section 7 is our conclusion.

2 Simplification

We will make a list of simplifying assumptions when building our mathematical model. Using the same assumption as the paper "*Revenue and attendance simultaneous optimization in performing arts organizations*", we will assume that the repertoire decisions are already determined by the Meany Hall theater, including the variety of the productions and the number of performances. Then determining the optimal layout and the pricing will be easier. Following the same paper, we will use the price across the seating zones as our decision variables (Baldin, Bille, Ellero, and Favaretto 2018).

The second assumption, as we mentioned previously, is that the student ticket price is considered to be uniform at \$10, and the number of student tickets sold is independent of the number of standard full-price tickets sold. Therefore, the number of student tickets sold are not considered in our demand function.

The third assumption is that we will only take into account the sales of the standard full-price tickets, and consider the attendance for the seasonal ticket holder as constant. This consideration is reasonable because the number of full-price tickets sold is independent of the number of seasonal tickets sold, to some extent. The total revenue earned from seasonal tickets and student tickets will be denoted as α, and we treat it as a constant.

Furthermore, our demand function only focuses on general trends rather than specific fluctuations that could arise from unique events or one-off performances. These simplifications were crucial for maintaining computational feasibility, yet they admittedly might blur some of the finer details relevant to very specific audience groups or exceptional scenarios.

Finally, the most important simplification is that we will frame this project as a teaching tool that illustrates how our methods work through our simulated data that reasonably satisfies real-world intuition. We have to make this simplification due to our challenge in collecting the data from the Meany Hall theater. With real-world data, we believe our methods still work, following the methods similar to the paper "Revenue and attendance simultaneous optimization in performing arts organizations"(Baldin, Bille, Ellero, and Favaretto 2018).

3 Mathematical Model

3.1 Objective function, Demand Function and Variables

The optimization model displayed below focuses on maximizing the revenue (or profit) for the Meany Hall theater, and is expressed through the objective function:

$$\text{Maximize Profit: } \left(\sum_{j=1}^{3}\sum_{i=1}^{5} D_{ij} \cdot p_{ij} + \alpha\right) \tag{1}$$

The revenues are calculated by the sum of revenues over different seating zones (j) and performance type (i) for each performance, with the incorporation of demand function D_{ij} estimated by regression. The following is a list of variables we use throughout this paper.

- $i \in \{1, 2, 3, 4, 5\} = \{$Dance, Chamber Music, Crossroads, Piano, Special Event$\}$
- $j \in \{1, 2, 3\} = \{$Zone 1, Zone 2, Zone 3$\}$
- $m \in \{1, 2, 3\} = \{$Popularity 1, Popularity 2, Popularity 3$\}$
- α: the total revenue earned from seasonal tickets and student tickets
- $\beta_0, \beta_1, \beta_k (k \in A)$: the coefficients for the model that relates the number of tickets sales and the variable denoted in the subscript, where, for example, β_0 is the intercept, β_1 relates the ticket sales and the price in seating zone j and performance type i
- ϵ: the irreducible error term
- p_{ij}: the ticket price for performance type i in seating zone j
- D_{ij}: The estimated demand function (the function to estimate the number of tickets sold) for performance type i in seating zone j. Its output is the expected number of ticket sales that takes on positive integer value between 0 and 1200.
- z_k: the indicator variables representing performance characteristics and seating zones.
- $A \in \{i \cup j \cup m \setminus \{i_2, j_1, m_1\}\}$

In the paper "*Revenue and attendance simultaneous optimization in performing arts organizations*" (Baldin, Bille, Ellero, and Favaretto 2018), the authors use the least square estimate in double log form to construct the demand function, and their objective function is nonlinear. As a teaching tool, we will use least square estimate without the double log form, so the our objective function becomes linear. This change from least square estimates in double log form to the least square estimates without the double log form is possible for our simulated data, because the least square fit R^2, which measures how well the model fits the data, does not change significantly.

We choose to construct our demand function $D_{ij}(p_{ij}, z_i)$ as a function of price, the type of performance, the seating zones and the popularity, because our simulated data only records these features. The variable p_{ij} represents the price, and z_i is the indicator random variables that will be explained later along with the equation of the demand function. It is possible to add other performance characteristics as variables to our demand function, such as whether the performance is on weekends or whether the piano piece to perform is classic, in manners described in the paper by Baldin et al.(2018), if the data include the relevant information. In our demand function, the popularity of a performance, which also depends on the fame of the artists, takes on value 1, 2, 3 where 1 means the performance is less popular, 2 means that the performance is neither popular nor unpopular, and 3 means that the performance is popular. The output of the demand function is the expected tickets sales $E[D_{ij}]$ for price p_{ij}.

The mathematical formula for the demand function for each type of performance i and each seating zone j is given by:

$$D_{ij} = \beta_0 + \beta_1 p_{ij} + \sum_{k \in A} \beta_k z_k + \epsilon \tag{2}$$

where p_{ij} is the price charged for performance type i in seating zone j, A=$\{$crossroad, dance, piano,special_events, zone2, zone3, popularity=2, popularity = 3$\}$ is the set for all categorical variables that takes on value 0 and 1, $\{\beta_0, \beta_1, \beta_k\}$ for $k \in A$ are coefficients to be estimated, and an error term ϵ.

This model incorporates performance characteristics denoted by indicator random variable (or categorical variable) z_i which is the compact notation for $z_{crossroad}$, z_{dance}, z_{piano}, $z_{special\_events}$, z_{zone2}, z_{zone3}, $z_{popularity=2}$, $z_{popularity=3}$ that only take on the value of 0 or 1. The indicator random variables for performance type (chamber music), zone (zone 1), and popularity (popularity 3) are excluded, because the expected ticket sales for these categories are predicted through the base demand model when the existing indicator random variables of certain categories, such as performance type, zone, and popularity, are set to zero. For example, when we estimate the expected ticket sales for one of chamber music event with popularity score 3 in zone 1, we have that $z_{crossroad} = z_{dance} = z_{piano} = z_{special\_events} = z_{zone2} = z_{zone3} = z_{popularity=2} = 0$ and $z_{z_{popularity=3}} = 1$, so our demand function is

$$D_{ij} = \beta_0 + \beta_1 p_{ij} + \beta_{popularity=3} * z_{popularity=3};$$

if we estimate the demand for a piano performance in zone 2 with normal popularity, we have that $z_{piano} = z_{zone2} = z_{normal\_popularity} = 1$, and all other indicator random variables except price takes on value 0; when we estimate the expected ticket sales for one of the less popular crossroad series in zone 1, our demand function is

$$D_{ij} = \beta_0 + \beta_1 p_{ij} + \beta_{crossroad} * z_{crossroad}$$

where $z_{dance} = z_{piano} = z_{special\_events} = z_{zone2} = z_{zone3} = 0$, $z_{crossroad} = 1$, and $z_{popularity=2} = 0$ and $z_{z_{popularity=3}} = 1$.

3.2 Constraints

1. $$\sum_{j=1}^{3} D_{ij} + \alpha \leq 1200$$

   This constraint ensures that the total number of tickets sold across three zones in each performance at Meany Hall does not exceed its seating capacity. The variable α represents the total number of seasonal tickets and student tickets sold for a event series and is treated as a constant. This constraint is crucial for managing both the physical limitations of the venue and customer segmentation, and operates as a safeguard against overselling.

2. $$\text{For all } i \text{ and } j, \quad p_{ij}^{\min} \leq p_{ij} \leq p_{ij}^{\max}$$

   where p_{ij} represents the ticket price for the performance type i in the seating zone j. The ticket price for each performance is set at a lower bound $p_{ij}^{\min}$ that covers the cost, and an upper bound $p_{ij}^{\max}$ to keep the performances accessible and attractive to the desired audiences. We use the lowest and the highest price in our simulated data for $p_{ij}^{\min}$ and $p_{ij}^{\max}$, respectively.

3. $$\text{For all } i, \quad p_{i1} > p_{i2} > p_{i3}$$

   The price for each zone is set in a manner that price in zone 1 is higher than the price in zone 2, and the price in zone 2 is higher than price in zone 3. We do this to reflect the tiered quality of seating, visibility, comfort, and acoustic effects.

4. $$D_{ij} \geq \text{Minimum Sales Requirement}$$

   The number of tickets sold for each performance must be a positive integer and meet a lower bound, and in this case, we arbitrarily set the lower bound to be 100. For each performance, it is important to have a minimum attendance so that the art performances will benefit more people, and have a greater positive impacts on the communities.

4 Data and Implementation

4.1 Data Simulation

One of the challenging part of this project is to collect data. We aren't able to receive the real data from our community partner, so we instead frame our project as a teaching tool with simulated data and hope to gain some useful insights. We simulate our data in R, which is the most commonly used statistical programming software. The codes for simulating the data are included in the Appendix. Our data has 600 samples, which is equivalent to 8 years of performance data with 25 performances each year, and 8 columns. A fraction of data is shown below in table 1:

Performance	Price	Tickets_Sold	Zone	Type	Year	Revenue	Popularity
1	80	135	1	Piano	2015	12480	3
1	73	610	2	Piano	2015	25550	3
1	58	324	3	Piano	2015	20300	3
2	77	97	1	Dance	2015	9933	2
2	62	390	2	Dance	2015	9796	2
2	44	235	3	Dance	2015	6952	2

Table 1: A fraction of performance data

Our simulated data satisfies the real-world intuition reasonably. Firstly the data for ticket price is simulated through sampling from normal distribution with some noise. In the process, we ensure that, for any performance, the ticket price for zone 1 is always greater than zone 2 which is greater than zone 3. In particular, the price used in the data simulation process is based on the proposed ticket price for the 2024 -2025 performance season (Meany Center for the Performing Arts 2024). Secondly, the number of tickets sold for all performances are also sampled from normal distribution, and we ensure that there will be more seats for zone 2 than zone 3 which has more seats than zone 1, based on the proposed seating zones in Figure 1 from the Meany Hall.

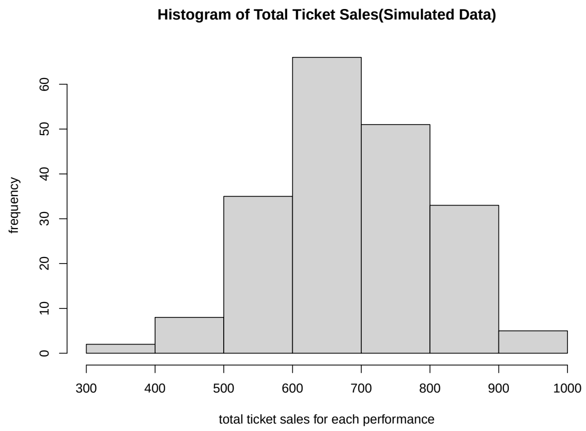
The reason to sample from the normal distribution is the Central Limit Theorem, which states that "under appropriate conditions, the distribution of a normalized version of the sample mean converges to a standard normal distribution" (Wikipedia Contributors n.d., "Central Limit Theorem"), and the random noise are added to increase the randomness in the data. From Figure 2, we can see that the total number of tickets sold and the total revenue for each performance follows a normal distribution, both of which satisfies and is justified by the Central Limit Theorem.

Thirdly, in our simulated data, we ensure that there will be 25 performances for each year for 5 series of events, similar to the theater's repertoire for the next performance season. In addition, we assigned the popularity score of 1, 2, and 3, based on the total number of tickets sold for each performance. If more tickets are sold in total for a given performance, we assign a popularity score of 3; we assign other scores similarly, where 1 means less popular and 2 means neither popular nor unpopular. When performing the statistical analysis on the simulated data, the multiple linear regression fits the data reasonably well with $R^2 \approx 0.75$ which is similar to the fit generated by Baldin et al in their paper (2018).

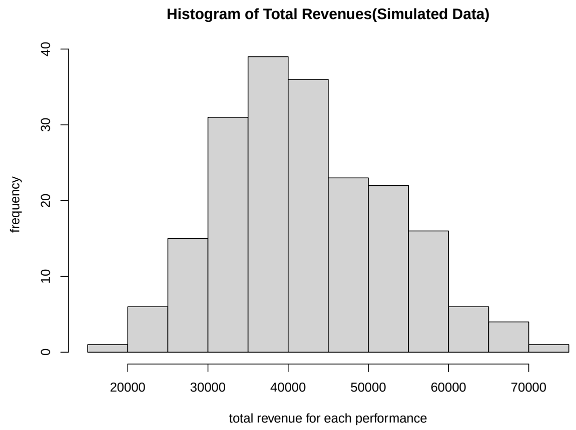
4.2 Implementation in R and Statistical Analysis

In this project as a teaching tool, we will perform least square estimate on our simulated data. Our demand function is estimated through the multiple linear regression in R, and the output of the results are shown in Table 2, Table 3, and Figure 3.

In table 2, the coefficient of the predicted price is -0.05190, meaning that 1 unit increase in price will decrease the ticket sales by 0.05190. The p-value of $0.8652 > 0.05$ for price means that price does not significantly impact demand, but we choose to keep price as an independent input for the demand



(a) histogram of the total number of tickets sold for each performance for the simulated data



(b) histogram of total revenue of each performance for the simulated data

Figure 2: Two histograms that shows the distribution of data

Coefficients	**Estimate**	**Std. Error**	**t value**	$Pr(>\|t\|)$
(Intercept)	61.81306	28.26923	2.187	0.0292
as.numeric(data$Price)	-0.05190	0.34862	-0.170	0.8652
as.factor(data$Type)Crossroads	-0.27542	8.16954	-0.034	0.9731
as.factor(data$Type)Dance	-6.67675	8.20290	-0.814	0.4160
as.factor(data$Type)Piano	-2.26656	8.15882	-0.278	0.7813
as.factor(data$Type)Special Events	-3.00700	8.30599	-0.362	0.7175
as.factor(data$Zone)2	243.63804	8.92298	27.305	$< 2e-16$
as.factor(data$Zone)3	123.30416	13.00119	9.483	$< 2e-16$
as.factor(data$Popularity)2	51.91845	6.54074	7.938	1.04e-14
as.factor(data$Popularity)3	126.80846	8.02591	15.800	$< 2e-16$

Table 2: multiple linear regression of the number of tickets sold against (price, performance type, zones, and popularity)

function. Except price, all other predictors, such as Type, Zone, and Popularity, are qualitative predictors (or categorical variables). The coefficient of approximately 243.98 for zone 2 means that zone 2 tends to sell 243.97924 more tickets than zone 1; we can make similar interpretation for coefficient of zone3.

Coefficients	**Estimate**	**Std. Error**	**t value**	$Pr(>|t|)$
(Intercept)	57.38933	27.08108	2.119	0.0345
as.numeric(data$Price)	-0.04035	0.34521	-0.117	0.9070
as.factor(data$Zone)2	243.97942	8.86750	27.514	$< 2e-16$
as.factor(data$Zone)3	123.91874	12.89771	9.608	$< 2e-16$
as.factor(data$Popularity)2	52.74021	6.33576	8.324	$5.85e-16$
as.factor(data$Popularity)3	127.24326	7.97378	15.958	$< 2e-16$

Table 3: Linear Regression of number of tickets sold against (price, zones, and popularity)

```
Analysis of Variance Table

Model 1: as.numeric(data$Tickets_Sold) ~ as.numeric(data$Price) + as.factor(data$Type) +
    as.factor(data$Zone) + as.factor(data$popularity)
Model 2: as.numeric(data$Tickets_Sold) ~ as.numeric(data$Price) + as.factor(data$Zone) +
    as.factor(data$popularity)
  Res.Df     RSS Df Sum of Sq      F Pr(>F)
1    590 2345807
2    594 2349201 -4   -3394.8 0.2135  0.931
```

Figure 3: Analysis of Variance Results from anova() function that compares the regression model with event type considered and the regression model without considering the performance type

The p-value of $2e-16$, which is smaller than 0.05, means that there is enough statistical evidence that seating zones will impact the number of tickets sold for a given performance at a significance level of 5%.

The coefficient of popularity 2 is 52.74021, which means that, compared with event with popularity score of 1, the performances with popularity score 2 will tend to sell 52.74021 more tickets for each zone; the coefficient of popularity3 is 127.24326, which means that, compared with event with popularity score of 1, performance with popularity score 3 will tend to sell 127.24326 more tickets for each zone. The p-value of $5.58e-16 << 0.05$ for the predictor popularity2 and the p-value of $2e-16 << 0.05$ for the predictor popularity3 means that we can confidently say popularity2 influences the number of tickets sold at a 5% significance level.

From the analysis of variance table, we can see that the p-value from F-test is $0.931 > 0.05$, which means that we can confidently say that event type does not affect the number of tickets sold significantly. The R codes used to estimate the demand function and perform statistical tests are as follows:

```
```{r statistical analysis}
model2 <- lm(as.numeric(data$Tickets_Sold) ~ as.numeric(data$Price) +
as.factor(data$Type) + as.factor(data$Zone) +
as.factor(data$popularity), data = data)
summary(model2)

model3 <- lm(as.numeric(data$Tickets_Sold) ~ as.numeric(data$Price) +
as.factor(data$Zone) + as.factor(data$popularity), data = data)
summary(model3)

anova(model2, model3)
```
```

4.3 Solving Optimization Problem through PYTHON

Incorporating our estimated demand function, we will solve our linear objective function described in section 3.1 using the Gurobi optimization package in PYTHON, which is a commercial tool for solving large-scale linear, integer, and quadratic optimization problems. The code for solving this optimization problem is included in the Appendix.

5 Discussion of Results

5.1 Discussion of Model Outcomes

In our exploration of optimal seating arrangements and ticket pricing at Meany Hall, our model was developed with the goal of maximizing revenue. We use the number of tickets sold when the art performances are very popular, or performance with popularity score 3, to estimate the proportion of seats in each of the three seating zones, because the expected number of ticket sales for each type of art performance can serve as an upper bound. The results are shown in table 4. Compared with the average ticket sales across the performances in simulated data, our model suggests a number of possible change in seating arrangement. A proposed 44% increase in the number of seats for Zone 1 reflects the high demand for premium seats; a 15% decrease in seats for Zone 2 and a 17% increase in seats in Zone 3 promotes the accessibility of art performances to the general public. The increase in accessibility is solely due to the sales of more tickets in zone 3. The model also unexpectedly recommends a decrease in number of ticket sold in Zone 2.

Zone	Actual Number of Seats	Proposed Ticket Sell	Percentage Change
1	180	260	Increase by 44%
2	610	460	Decrease by 15%
3	410	480	Increase by 17%

Table 4: Tickets Sales for Art Performance with Popularity Score 3 (i.e. popular art performances)

When the popularity increases, the optimal solution suggests that the number of tickets sold in each zone increases but the ticket prices remain constant. The minimal impact of price on sales is due to the weak correlation between price and the number of tickets sold for each performance series; the increase in ticket sales due to popularity means that more customers are willing to buy tickets for popular shows. In table 5, we showed the suggested change in ticket price for each performance type i and seating zone j.

Event Type	Zone 1 Increase ($)	Zone 2 Increase ($)	Zone 3 Increase ($)
Dance	10~15	1~5	1~5
Piano	15~20	10~15	10~15
Chamber Music	15~20	10~15	10~15
Special Events	2~7	6~11	2~7
Crossroads	15~50	5~28	7~30

Table 5: Proposed Price Increases for Different Events and Zones for Popularity 1

5.2 Validation and Comparative Analysis

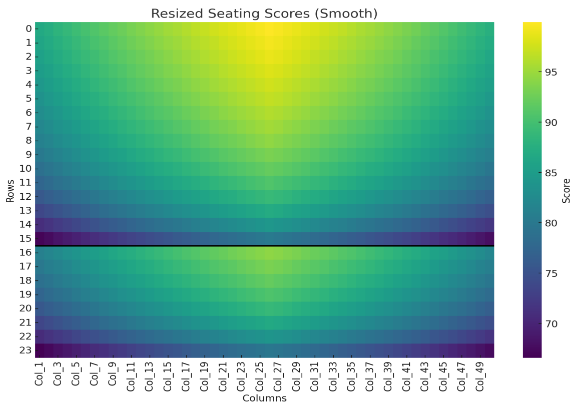
To assess the efficacy of our model, we compare the predictions against the average ticket sales and revenues for each performance type in our simulated data, which had not applied pricing optimizations. By comparing the revenue computed from the raw data and our proposed results (in Appendix A), we conclude that, for art performances with popularity score 2 and popularity score 3, our strategy increase the revenue and attendance .

5.3 Model Evaluation

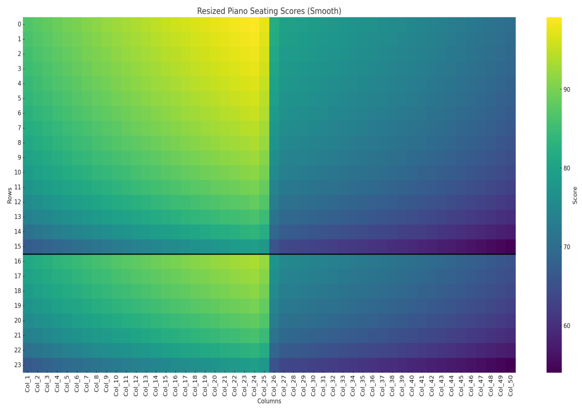
Despite the promising outcomes, the model's assumptions warrant scrutiny. It presumes a level of price elasticity that might not uniformly apply across all types of performances, particularly during events with unusually high demand. Here the price elasticity is a measure of how sensitive the quantity demanded is to its price (Wikipedia n.d.(b)). Moreover, the model's heavy reliance on simulated data might not adequately capture future shifts in market conditions, such as economic changes or evolving consumer preferences.

6 Detailed Seating Analysis and Application on Meany Hall

In our analysis of Meany Hall's seating layout, you may observe a discontinuity in the graphical representations beyond row 16. Row 16 marks the transition from the lower floor level to the upper floor level. This division impacts the viewing experiences. In fact, the first row of the second floor's customers' seating satisfaction is higher than the back row of the first floor. Thus, such experience is reflected in Figure a. Consequently, this affects ticket pricing and demand forecasting, as the desirability of seats varies. We conducted on-site assessments and simulations to identify the most sought-after seating locations for each performance in Meany Hall's main theater. It became evident that the seating demand for piano performances varies significantly from other types of shows. Unlike other performances that use the entire stage and have symmetrical seating demands, piano performances create unique seating preferences due to the placement of the pianist on the left side of the stage. The figures below illustrate these seating demand variations:



(a) Heatmap of the Seating demand during regular performances



(b) Heatmap of the Seating demand during piano events

Figure 4: Seating demand variations

Our seating strategy at Meany Hall is based on an analytical approach that uses patron rankings derived from demand forecasts. This ensures that our seating zones reflect the real-world preferences and demands of our audience.

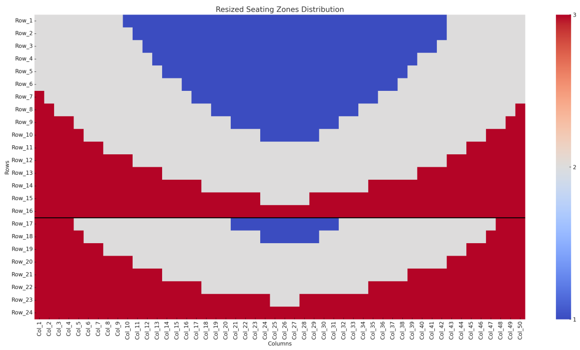
Seats are allocated into zones as follows:

- **Zone 1**: The top-ranked seats based on visibility, acoustic quality, and overall desirability. This zone includes the first few rows that typically offer the best viewing and listening experience. The number of seats in this premier zone is determined by the cumulative demand percentile.
- **Zone 2**: The next tier of seats, which offer favorable but slightly lesser viewing and acoustic advantages. This zone captures a significant segment of patrons seeking a balance between cost and quality.

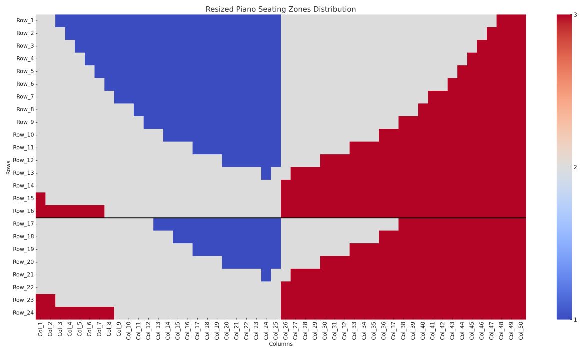
- **Zone 3**: The remaining seats, generally farther from the stage or with obstructed views, representing the most economical options. This zone caters to patrons prioritizing attendance over specific seating preferences or those with limited budgets.

This zone allocation strategy maximizes revenue by aligning ticket prices with market demand and enhancing customer satisfaction. We allocate seats to different zones based on the data obtained from our analysis. To ensure suitability for all levels of popularity, we apply a fraction derived from the most popular performances (popularity level 3).

Seats are allocated as follows: The top-ranked seats, based on visibility, acoustic quality, and overall desirability, are assigned to Zone 1. The next best seats are assigned to Zone 2. The remaining seats are allocated to Zone 3. The number of seats in each zone is determined by the same ratio used for performances with popularity level 3, as these performances attract the highest number of attendees. This approach prevents a shortage of high-demand seats and ensures that all patrons have seating options that meet their preferences and expectations



(a) Zone arrangement for regular performances



(b) Zone arrangement for piano events

Figure 5: Zone Arrangement

7 Improvements

To enhance the effectiveness of the current seating and pricing model at Meany Hall, several improvements to our methods could be considered. Each improvement aims to refine our approach to maximize revenue, improve customer satisfaction, and engage more deeply with the community.

7.1 Dynamic Pricing

Implementing dynamic pricing strategies that adjust ticket prices based on real-time demand and performance popularity can significantly optimize revenue and seat utilization. This approach allows for a flexible response to fluctuating demand, potentially increasing both sales and customer satisfaction by offering prices that buyers perceive as fair under varying circumstances.

7.2 Enhanced Forecasting Models

The adoption of advanced forecasting methods, such as machine learning algorithms, is proposed to improve the accuracy of demand predictions. These technologies can analyze vast amounts of data and identify patterns that traditional methods might miss, leading to more effective pricing and marketing strategies.

7.3 User Experience Feedback

Collecting and analyzing feedback on seating experiences is crucial. This data can inform adjustments in seat pricing and placement, ensuring that prices align more closely with the perceived value offered to patrons. Understanding customer preferences and discomforts can help tailor the venue to better meet audience expectations.

7.4 Community Cultural Enrichment

Enhancing the focus on community cultural enrichment involves prioritizing attendance and participation of local community members in cultural events. This strategy aims to foster greater understanding, appreciation, and engagement with diverse cultural expressions. In turn, this can promote a more inclusive, supportive, and culturally rich community environment, strengthening unity and shared understanding among its members.

Addressing challenges such as data acquisition, analysis capabilities, and market acceptance of dynamic pricing strategies will be essential. Nonetheless, these enhancements could substantially refine the pricing model and improve the overall customer experience.

8 Conclusions

Our experiments on simulated data showed that the ticket price and the performance type does not significantly impact the ticket price, while the seating zones and the popularity can significantly influence the ticket demand. The implications of these optimized pricing strategies are significant. The elevated prices in Zone 1 is based on attendees' desire to have better visual and auditory experiences, while the price reduction in Zone 3 is an attempt to enhance accessibility. Maintaining steady prices in Zone 2 ensures a steady influx of revenue.

9 Acknowledgement

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A Appendix A: Optimization Outcome

Performance Series	Zone 1 Price	Zone 2 Price	Zone 3 Price	Zone 1 Sales	Zone 2 Sales	Zone 3 Sales	Revenue
Piano	99.0	73.0	61.0	53	298	178	$37859.0
Dance	98.0	64.0	55.0	53	298	179	$34111.0
Chamber Music	90.0	71.0	64.0	53	298	178	$37320.0
Crossroads	100.0	72.0	60.0	53	298	178	$37436.0
Special Events	83.0	73.0	55.0	54	298	179	$36081.0

Table 6: Popularity Level 1

Performance Series	Zone 1 Price	Zone 2 Price	Zone 3 Price	Zone 1 Sales	Zone 2 Sales	Zone 3 Sales	Revenue
Piano	99.0	73.0	61.0	106	351	231	$50208.0
Dance	98.0	64.0	55.0	106	351	231	$45557.0
Chamber Music	90.0	71.0	64.0	106	351	231	$49245.0
Crossroads	100.0	72.0	60.0	106	351	231	$49732.0
Special Events	83.0	73.0	55.0	106	351	231	$47126.0

Table 7: Popularity Level 2

Performance Series	Zone 1 Price	Zone 2 Price	Zone 3 Price	Zone 1 Sales	Zone 2 Sales	Zone 3 Sales	Revenue
Piano	99.0	73.0	61.0	180	425	306	$67511.0
Dance	98.0	64.0	55.0	180	426	306	$61734.0
Chamber Music	90.0	71.0	64.0	180	425	305	$65895.0
Crossroads	100.0	72.0	60.0	180	425	306	$66960.0
Special Events	83.0	73.0	55.0	181	425	306	$62878.0

Table 8: Popularity Level 3

B Appendix B: Data Simulation Code

```
```{r data simulation}
# 25 performances per year (5 type of performance for each), 8 years
# 3 seating zones for each performance
# 600 values in total

# generate price randomly between min and max
sample_price <- function(mean1, sd1, min, max){
  x <- rnorm(n = 1, mean = mean1, sd = sd1)
  noise <- rnorm(n = 1, mean = 0, sd = 3)
  while ((x + noise) <= min) {
    x <- rnorm(n = 1, mean = mean1, sd = sd1)
    noise <- rnorm(n = 1, mean = 0, sd = 3)
  }
  while ((x + noise) >= max) {
    x <- rnorm(n = 1, mean = mean1, sd = sd1)
```
```

```r
    noise <- rnorm(n = 1, mean = 0, sd = 3)
  }
  return(round(x + noise, digits = 0))
}

# generate attendance randomly between min and max
sample_attendence <- function(mean1, sd1, min, max){
  x <- rnorm(n = 1, mean = mean1, sd = sd1)
  noise <-rnorm(n = 1, mean = 0, sd = 30)
  while ((x + noise) <= min) {
    x <- rnorm(n = 1, mean = mean1, sd = sd1)
    noise <-rnorm(n = 1, mean = 0, sd = 30)
  }
  while ((x + noise) >= max) {
    x <- rnorm(n = 1, mean = mean1, sd = sd1)
    noise <- rnorm(n = 1, mean = 0, sd = 3)
  }
  return(round(x + noise, digits = 0))
}
```

```{r data simulation}
total_sales = matrix(NA, nrow = 1, ncol = 200)
total_revenue = matrix(NA, nrow = 1, ncol = 200)

set.seed(13)
mat1 <- matrix(NA, nrow = 600, ncol = 8)
colnames(mat1) <- c("Performance", "Price", "Tickets_Sold", "Zone", "Type", "year", "revenue",
    "popularity")

mat1[, 6] <- rep(2015:2022, each = 75) # it works as expected

iter = 1
for (i in seq(1, 600, by = 3)){
  mat1[i, 1] <- iter
  mat1[i + 1, 1] <- iter
  mat1[i + 2, 1] <- iter

  mat1[i, 2] <- sample_price(75, 10, 50, 100)
  mat1[i + 1, 2] <- sample_price(55, 10, 40, 80)
  mat1[i + 2, 2] <- sample_price(45, 10, 25, 65)
  while (mat1[i, 2] <= mat1[i + 1, 2] | mat1[i + 1, 2] <= mat1[i + 2, 2]) {
    mat1[i, 2] <- sample_price(75, 5, 50, 100)
    mat1[i + 1, 2] <- sample_price(55, 5, 40, 80)
    mat1[i + 2, 2] <- sample_price(45, 5, 25, 65)
  }

  mat1[i, 3] <- sample_attendence(100, 40, 40, 200)
  mat1[i + 1, 3] <- sample_attendence(350, 90, 100, 600)
  mat1[i + 2, 3] <- sample_attendence(220, 80, 60, 400)
  total_sales[1, iter] <- mat1[i, 3] + mat1[i + 1, 3] + mat1[i + 2, 3]

  mat1[i, 7] <- mat1[i, 2]*mat1[i, 3]
  mat1[i + 1, 7] <- mat1[i + 1, 2]*mat1[i+1, 3]
  mat1[i + 2, 7] <- mat1[i + 2, 2]*mat1[i+1, 3]
  total_revenue[1, iter] = mat1[i, 7] + mat1[i + 1, 7] + mat1[i + 2, 7]

  mat1[i, 4] <- 1
  mat1[i + 1, 4] <- 2
  mat1[i + 2, 4] <- 3
  iter = iter + 1
```

```
}
```

```
```{r data simulation}
set.seed(13)
iter = 1
for (i in seq(1, 600, by = 3)) {
  a1 <- sample_attendence(100, 40, 40, 200)
  a2 <- sample_attendence(350, 90, 100, 600)
  a3 <- sample_attendence(220, 80, 60, 400)
  total_sales[1, iter] <- a1 + a2 + a3
  if (total_sales[1, iter] > 800) {
    p <- runif(n = 1, min = 1.05, max = 1.2)
    mat1[i, 3] <- round(p*a1, digits = 0)
    mat1[i + 1, 3] <- round(p*a2, digits = 0)
    mat1[i + 2, 3] <- round(a3, digits = 0)
    total_sales[1, iter] = a1 + a2 + a3
    mat1[i, 8] <- 3
    mat1[i + 1, 8] <- 3
    mat1[i + 2, 8] <- 3
  } else if (total_sales[1, iter] < 800 & total_sales[1, iter] > 600) {
    p <- runif(n = 1, min = 0.95, max = 1.05)
    mat1[i, 3] <- round(p*a1, digits = 0)
    mat1[i + 1, 3] <- round(p*a2, digits = 0)
    mat1[i + 2, 3] <- round(p*a3, digits = 0)
    total_sales[1, iter] = a1 + a2 + a3
    mat1[i, 8] <- 2
    mat1[i + 1, 8] <- 2
    mat1[i + 2, 8] <- 2
  } else {
    mat1[i, 8] <- 1
    mat1[i + 1, 8] <- 1
    mat1[i + 2, 8] <- 1
    mat1[i, 3] <- a1
    mat1[i + 1, 3] <-a2
    mat1[i + 2, 3] <- a3
  }
  iter = iter + 1
}
```

```
```{r data simulation}
for (i in seq(1, 600, by = 15)) {
  mat1[i, 5] <- "Piano"
  mat1[i + 1, 5] <- "Piano"
  mat1[i + 2, 5] <- "Piano"

  mat1[i + 3, 5] <- "Dance"
  mat1[i + 4, 5] <- "Dance"
  mat1[i + 5, 5] <- "Dance"

  mat1[i + 6, 5] <- "Chamber Music"
  mat1[i + 7, 5] <- "Chamber Music"
  mat1[i + 8, 5] <- "Chamber Music"

  mat1[i + 9, 5] <- "Crossroads"
  mat1[i + 10, 5] <- "Crossroads"
  mat1[i + 11, 5] <- "Crossroads"

  mat1[i + 12, 5] <- "Special Events"
  mat1[i + 13, 5] <- "Special Events"
```

```
  mat1[i + 14, 5] <- "Special Events"
}

mat1[, 2] <- as.numeric(mat1[, 2])
mat1[, 3] <- as.numeric(mat1[, 3])
mat1[, 7] <- as.numeric(mat1[, 7])
data <- as.data.frame(mat1)
write.csv(data, file="Meany_Hall_Data.csv")
```
```

# C Appendix C: Optimization Code

```
import pandas as pd
import gurobipy as gp
from gurobipy import GRB

# Load the data
data = pd.read_csv('/Users/chris/Library/CloudStorage/OneDrive-UW/3. Junior/MATH 381/Group
    Project/theater_data.csv')

# Extract necessary parameters
performance_categories = data['Type'].unique()
zones = data['Zone'].unique()
pops = data['popularity'].unique()
print(performance_categories)
print(zones)

# Function to define and optimize the model for a given popularity level
def optimize_for_popularity(popularity_level):
    # Define the Gurobi model
    model = gp.Model('theater_pricing')

    # Define decision variables: price and sales for each performance category and zone
    prices = model.addVars(performance_categories, zones, name="prices", vtype=GRB.CONTINUOUS)
    sales = model.addVars(performance_categories, zones, name="sales", vtype=GRB.CONTINUOUS)

    # Extract price bounds from historical data
    price_bounds = data.groupby(['Type', 'Zone'])['Price'].agg(['min', 'max']).reset_index()
    pij_min = price_bounds.pivot(index='Type', columns='Zone', values='min')
    pij_max = price_bounds.pivot(index='Type', columns='Zone', values='max')

    # Define the demand function based on the given coefficients including popularity
    def demand_expr(price, zone2, zone3, popularity_level):
        if popularity_level == 1:
            pop_2 = 0
            pop_3 = 0
        elif popularity_level == 2:
            pop_2 = 1
            pop_3 = 0
        elif popularity_level == 3:
            pop_2 = 0
            pop_3 = 1
        return 57.38933 + (-0.04035 * price) + (243.97924 * zone2) + (123.91874 * zone3) +
             (52.74 * pop_2) + (127.24 * pop_3)

    # Define the objective function: Maximize Profit
    profit = gp.quicksum(sales[category, zone] * prices[category, zone]
                    for category in performance_categories for zone in zones)

    model.setObjective(profit, GRB.MAXIMIZE)
```
```

```python
# Add constraints
# Constraint 1: Each performance type should not exceed 1200 tickets sold
for category in performance_categories:
    category_demand = gp.quicksum(sales[category, zone] for zone in zones)
    model.addConstr(category_demand <= 1200, f"category_capacity_{category}")

# Constraint 2: Ensure each performance series has more than 0 sales
min_sales_threshold = 1 # You can set this to a higher number if needed
for category in performance_categories:
    category_sales = gp.quicksum(sales[category, zone] for zone in zones)
    model.addConstr(category_sales >= min_sales_threshold, f"min_sales_{category}")

# Constraint 4: Sales must match the demand function
for category in performance_categories:
    for zone in zones:
        zone2_flag = 1 if zone == 2 else 0
        zone3_flag = 1 if zone == 3 else 0
        model.addConstr(sales[category, zone] == demand_expr(prices[category, zone],
            zone2_flag, zone3_flag, popularity_level), f"demand_{category}_{zone}")

# Constraint 5: Price bounds
for category in performance_categories:
    for zone in zones:
        model.addConstr(prices[category, zone] >= pij_min.loc[category, zone],
            f"min_price_{category}_{zone}")
        model.addConstr(prices[category, zone] <= pij_max.loc[category, zone],
            f"max_price_{category}_{zone}")

# Constraint 6: Price hierarchy among zones for each performance category
for category in performance_categories:
    if 1 in zones and 2 in zones:
        model.addConstr(prices[category, 1] >= prices[category, 2],
            f"zone_hierarchy_{category}_1_2")
    if 2 in zones and 3 in zones:
        model.addConstr(prices[category, 2] >= prices[category, 3],
            f"zone_hierarchy_{category}_2_3")

# Optimize the model
model.optimize()

# Output the optimal prices and sales
optimal_prices = pd.DataFrame(index=performance_categories, columns=zones)
optimal_sales = pd.DataFrame(index=performance_categories, columns=zones)
for category in performance_categories:
    for zone in zones:
        optimal_prices.loc[category, zone] = prices[category, zone].X
        optimal_sales.loc[category, zone] = sales[category, zone].X

print(f"Optimal Prices for popularity {popularity_level}:")
print(optimal_prices)
print(f"Optimal Sales for popularity {popularity_level}:")
print(optimal_sales)

# Calculate profit for each performance series
profits = {}
for category in performance_categories:
    profit = 0
    for zone in zones:
        profit += optimal_sales.loc[category, zone] * optimal_prices.loc[category, zone]
    profits[category] = profit

print(f"Profits for each performance series for popularity {popularity_level}:")
for category, profit in profits.items():
```

```
        print(f"{category}: ${profit:.2f}")

# Run the optimization for each popularity level
for pop_level in [1, 2, 3]:
    optimize_for_popularity(pop_level)
```