Lista 1

Questão d).

$$2x^2 + y^2 \leq 1 \quad e \quad y \geq 0$$

$$0 \leq y \leq \sqrt{1-2x^2}$$

$$f(x) = \sqrt{1-2x^2}$$

$$\text{Volume} = \pi \int_a^b f(x)^2 dx = \pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1 - 2x^2)\, dx$$

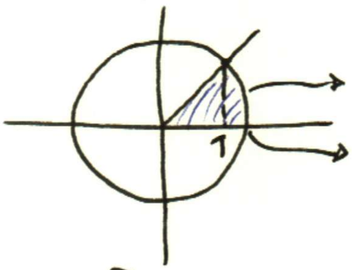
$$= \pi \left[x - \frac{2x^3}{3} \right]_{-1/\sqrt{2}}^{1/\sqrt{2}}$$

$$= \frac{\sqrt{8}}{3} \pi$$

Questão h)

$$0 \leq y \leq x \qquad x^2 + y^2 \leq 2$$

(abaixo da reta e dentro do círculo)



$$V = \pi \int_0^1 x^2 dx + \pi \int_1^{\sqrt{2}} (2 - x^2) dx$$

$$= \frac{\pi}{3} + \pi \left(\frac{\sqrt{32} - 5}{3} \right).$$

Lista 2.

1 e) $$V = 2\pi \int_a^b xy\, dx.$$

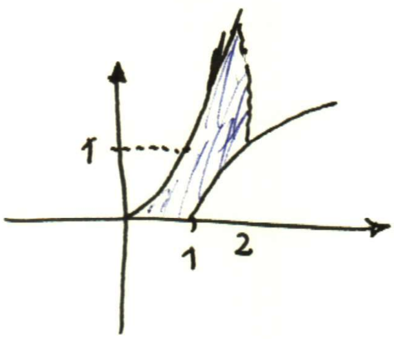
$0 \le x \le 1 \qquad 0 \le y \le \operatorname{arctg} x$

$$V = 2\pi \int_0^1 x \operatorname{arctg} x = 2\pi \left[\frac{x(x \operatorname{arctg} x - 1) + \operatorname{arctg} x}{2} \right]_0^1$$

Integração por Partes

$$= 2\pi \cdot \frac{(\pi - 2)}{4} = \frac{\pi(\pi-2)}{2}.$$

1 h) $0 \le x \le 2 \qquad y \ge \sqrt{x-1} \qquad 0 \le y \le x^2$



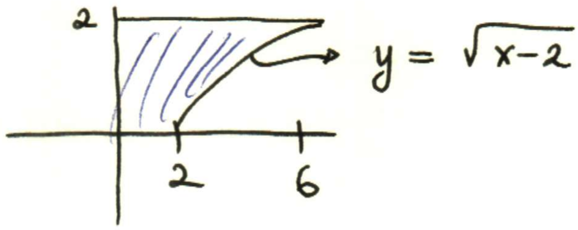
$$V = 2\pi \int_0^1 x^2 \cdot x\, dx + 2\pi \int_1^2 x^2 \cdot x\, dx - 2\pi \int_1^2 x\sqrt{x-1}\, dx$$

$$= 2\pi \int_0^2 x^3 dx - 2\pi \int_1^2 x\sqrt{x-1}\, dx$$ substituição $u = x-1$.

$$= 2\pi \cdot 4 + 2\pi \left(\frac{2(x-1)^{3/2}(3x+2)}{15} \right)_1^2 = 8\pi + \frac{32\pi}{15}$$

$$= \frac{152\pi}{15}.$$

2 a)



$$V = 2\pi \int_0^2 2 \cdot x \, dx + 2\pi \int_2^6 2 \cdot x \, dx - 2\pi \int_2^6 x\sqrt{x-2} \, dx$$

$$= 2\pi \int_0^6 2x \, dx - 2\pi \int_2^6 x\sqrt{x-2} \, dx$$ → subst. $u = x-2$

$$= 2\pi \cdot 36 - 2\pi \left[\frac{2(x-2)^{3/2}(3x+4)}{15} \right]_2^6$$

$$= 72\pi - 2\pi \cdot \frac{352}{15} = \frac{376\pi}{15}.$$

3 c) O comprimento da curva $\gamma(t)$ é

$$\ell(\gamma) = \int_0^\pi \|\gamma'(t)\| \, dt$$

$$= \int_0^\pi \|(\text{sen}\, t, 1 - \cos t)\| \, dt$$

$$= \int_0^\pi \sqrt{\text{sen}^2 t + (1-\cos t)^2} \, dt$$

$$= \int_0^\pi \sqrt{\text{sen}^2 t + 1 - 2\cos t + \cos^2 t} \, dt$$

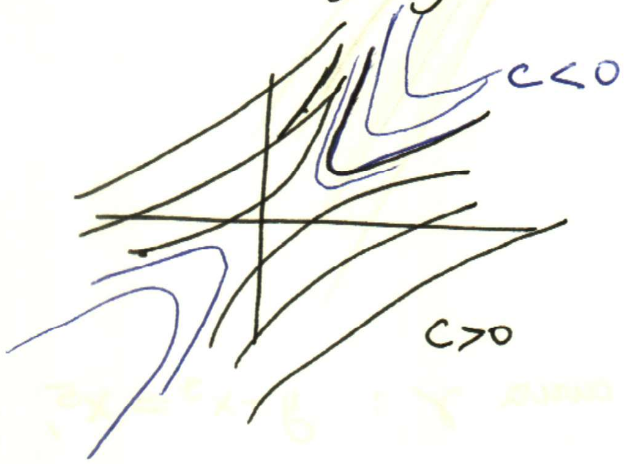
$$= \int_0^\pi \sqrt{2 - 2\cos t} \, dt$$ → use $1 - \cos x = 2\,\text{sen}^2\left(\frac{x}{2}\right)$ obtido de $\cos 2x = \cos^2 x - \text{sen}^2 x$

$$= \left[-4\cos\left(\frac{t}{2}\right)\right]_0^\pi = 4$$

Lista 3

5h) As curvas de nível de $f(x,y) = 3x^2 - 4xy + y^2$ são as hipérboles

$$3x^2 - 4xy + y^2 = c$$



Este é um exercício típico de G.A. e não cai na prova.

3h) O domínio de $f(x,y) = \dfrac{x-y}{\operatorname{sen} x - \operatorname{sen} y}$ é

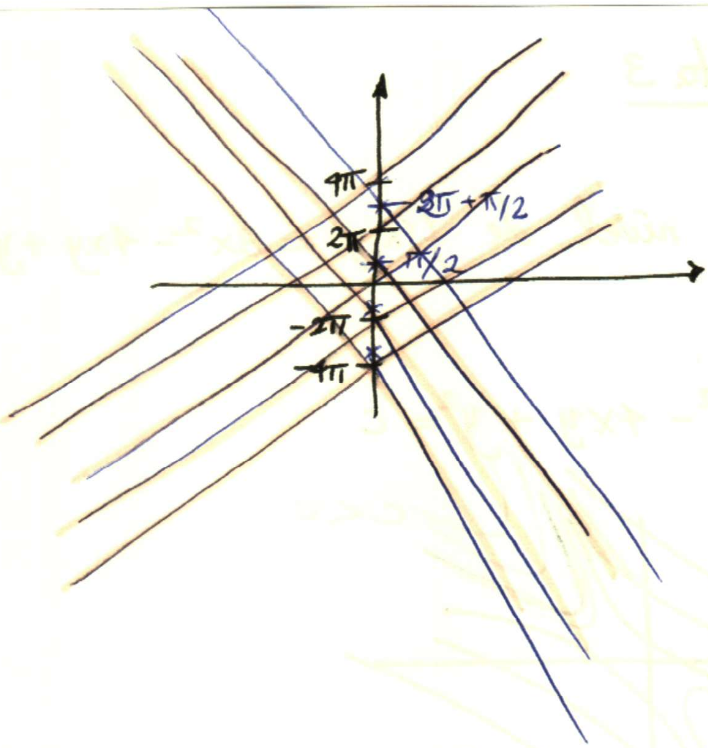
$\mathbb{R}^2 -$ conjunto de retas.

Conjunto de retas $= \{(x,y) : \operatorname{sen} x = \operatorname{sen} y\}$

$= \{(x,y) : \operatorname{sen} x = \operatorname{sen} y\}$

$= \{(x,y) : y = x + 2\pi \cdot k \text{ ou } y = \left(\frac{\pi}{2} - x\right) + 2k\pi\}$

- $y = x + 2k\pi$
- $y = -x + \frac{\pi}{2} + 2k\pi$



10 g) Considere a curva γ: $y - x^3 = x^5$, que passa por $(0,0)$.

f calculada em γ vale:

$$f(x,y) = \frac{x\,(x^5+x^3)}{x^5} = \frac{x^4\,(x^2+1)}{x^5}$$

$$= \frac{x^2+1}{x} \longrightarrow \pm\infty \text{ quando } x \to 0^{\pm}.$$

<u>Conclusão</u>: o limite não existe!

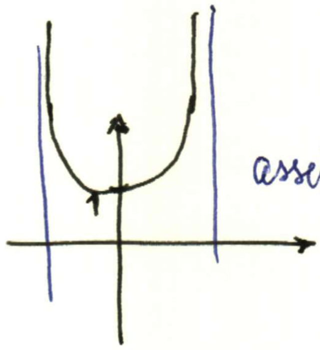
4 m) Curvas de nível: $\frac{1}{\sqrt{1-x^2-y^2}} = c \Rightarrow \frac{1}{c^2} = 1-x^2-y^2$

$$x^2+y^2 = 1 - \frac{1}{c^2} \qquad |c|>1.$$

círculos

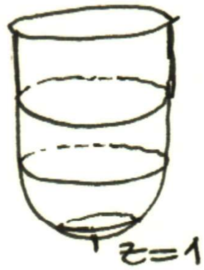
Interseção com o plano $x=0$:

$$z=\frac{1}{\sqrt{1-y^2}}$$



assíntotas.

Gráfico:

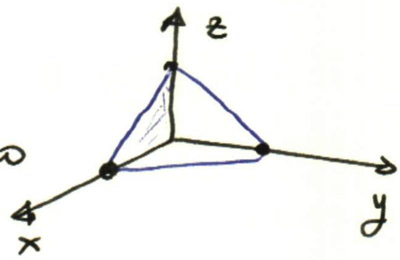


7 d) $w=\sqrt{1-|x|-|y|-|z|}$

Domínio: $1-|x|-|y|-|z|\geq 0$

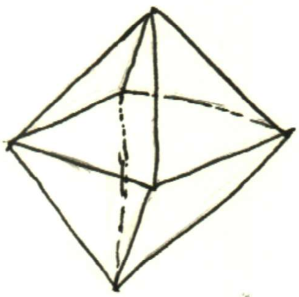
$$|x|+|y|+|z|\leq 1$$

Note que $\begin{cases} x+y+z\leq 1 \\ x\geq 0 \\ y\geq 0 \\ z\geq 0 \end{cases}$ é o tetraedro



Logo

$|x|+|y|+|z|\leq 1$ é o octaedro



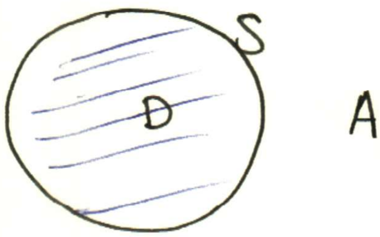
11 a) $f(\gamma(t)) = f(at, bt)$

$$= \frac{2at \cdot b^2t^2}{a^2t^2 + b^4t^4}$$

$$= \frac{2ab^2t^3}{t^2(a^2+b^2t^2)} = \underbrace{t}_{\to 0}\underbrace{\left(\frac{2ab^2}{a^2+b^2t^2}\right)}_{\to \frac{2ab^2}{a^2} = \frac{2b^2}{a}}$$

$$\Longrightarrow \lim_{t\to 0} f(\gamma(t)) = 0$$

12 g)



D $\quad x^2+y^2<1$

A $\quad x^2+y^2>1.$

S $\quad x^2+y^2=1.$

- $f(x,y) = e^{\frac{1}{x^2+y^2-1}}$ é composta de funções contínuas em D, logo é contínua em D.

  Note: $x^2+y^2-1 \neq 0$ em D.

- $f(x,y) = 0$ é contínua em D.

- Seja $(x_0,y_0) \in S$. Se $(x,y) \in D$ e $(x,y) \to (x_0,y_0)$ então $r = \sqrt{x^2+y^2} \to 1-$.

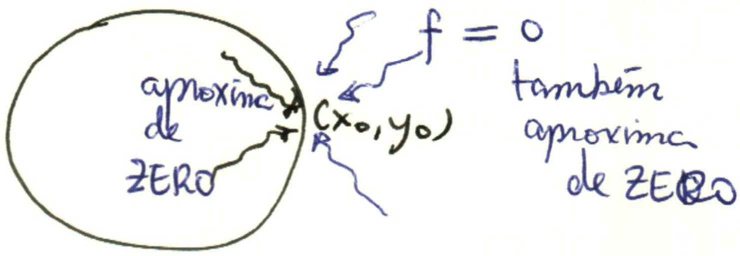
Analogamente, $(x,y) \in A$ e $(x,y) \to (x_0,y_0)$

$$\Longrightarrow r = \sqrt{x^2+y^2} \to 1+.$$

Nos dois casos $r^2-1 \to 0+$ ou $r^2-1 \to 0-$.

$$\Longrightarrow \frac{1}{r^2-1} \to \pm\infty$$

O que nos interessa é $f(x,y) = e^{\frac{1}{r^2-1}} \to e^{-\infty} = 0$ quando $(x,y) \in D$ e $(x,y) \to (x_0,y_0)$.



<u>Conclusão</u>: $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = 0 = f(x_0,y_0)$

$\therefore$ f é contínua em todo $\mathbb{R}^2$.

Lista 4.

7) $z = f(\rho \cos\theta, \rho \,\text{sen}\,\theta) = e^{\rho^2}$

$x = \rho \cos\theta$
$y = \rho \,\text{sen}\,\theta$

$$\frac{\partial z}{\partial \rho} = 2\rho \, e^{\rho^2} = 2\rho \, e^{x^2+y^2}$$

Como $x\cos\theta + y\,\text{sen}\,\theta = \rho\cos^2\theta + \rho\,\text{sen}^2\theta$
$= \rho$

Segue

$$2\rho = 2x\cos\theta + 2y\,\text{sen}\,\theta$$

$$\frac{\partial z}{\partial \rho} = e^{x^2+y^2}\,(2x\cos\theta + 2y\,\text{sen}\,\theta).$$

Sabemos que $\dfrac{\partial z}{\partial x} = e^{x^2+y^2}\cdot 2x$ e $\dfrac{\partial z}{\partial y} = e^{x^2+y^2}\cdot 2y$.

Conclusão:

$$\frac{\partial z}{\partial \rho} = \cos\theta\,\frac{\partial z}{\partial x} + \text{sen}\,\theta\,\frac{\partial z}{\partial y}.$$