Outline

1. Specifying lexical structure using regular expressions

2. Finite automata
   - Deterministic Finite Automata (DFAs)
   - Non-deterministic Finite Automata (NFAs)

3. Implementation of regular expressions

   RegExp => NFA => DFA => Tables

1.

Notation

- There is variation in regular expression notation

- Union: $A \mid B \equiv A + B$
- Option: $A + \varepsilon \equiv A?$
- Range: 'a' +' b' +…+' z' $\equiv$ [a-z]
- Excluded range:

  complement of [a-z] $\equiv$ [^a-z]

Regular Expressions => Lexical Spec. (1)

1. Write a rexp for the lexemes of each token
   - Number = digit +
   - Keyword = 'if' + 'else' + …
   - Identifier = letter (letter + digit)*
   - OpenPar = '('
   - …

Regular Expressions => Lexical Spec. (2)

2. Construct R, matching all lexemes for all tokens

$$R = \text{Keyword} + \text{Identifier} + \text{Number} + \ldots$$
$$= R_1 + R_2 + \ldots$$

Regular Expressions => Lexical Spec. (3)

3. Let input be $x_1 \ldots x_n$

   For $1 \leq i \leq n$ check

   $$x_1 \ldots x_i \in L(R)$$

4. If success, then we know that

   $x_1 \ldots x_i \in L(R_j)$ for some j

5. Remove $x_1 \ldots x_i$ from input and go to (3)

Ambiguities (1)

- There are *ambiguities* in the algorithm

- How much input is used? What if
    - $x_1 \ldots x_i \in L(R)$ and also
    - $x_1 \ldots x_K \in L(R)$

- Rule: Pick longest possible string in $L(R)$
  - The "maximal munch"

Ambiguities (2)

- Which token is used? What if
  - $x_1 \ldots x_i \in L(R_j)$ and also
  - $x_1 \ldots x_i \in L(R_k)$

- Rule: use rule listed first (j if $j < k$)
  - Treats "if" as a keyword, not an identifier

Error Handling

- What if

  No rule matches a prefix of input ?
- Problem: Can't just get stuck …

- Solution:
  - Write a rule matching all "bad" strings
  - Put it last (lowest priority)

1.

Summary

- Regular expressions provide a concise notation for string patterns

- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors

- Good algorithms known
  - Require only single pass over the input
  - Few operations per character (table lookup)

2.

Finite Automata

- Regular expressions = specification
- Finite automata = implementation

- A finite automaton consists of
  - An input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions $\text{state} \to^{\text{input}} \text{state}$

Finite Automata

- Transition

$$s_1 \to^a s_2$$

- Is read

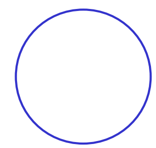
In state s_1 on input "a" go to state s_2

- If end of input and in accepting state => accept

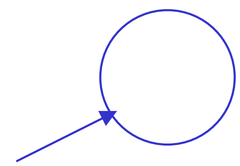
- Otherwise => reject

Finite Automata State Graphs

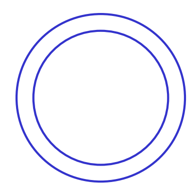
- A state



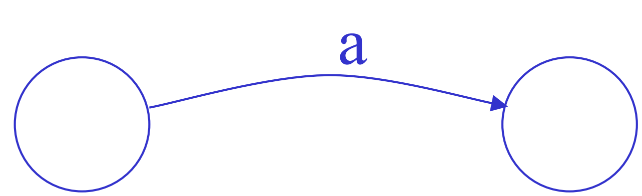
- The start state



- An accepting state

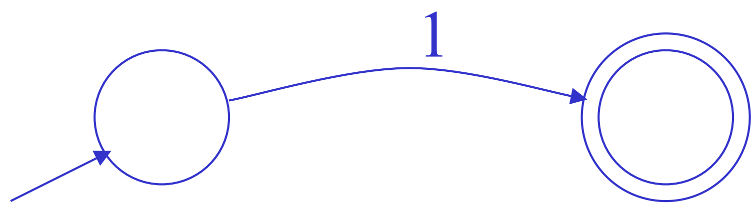


- A transition



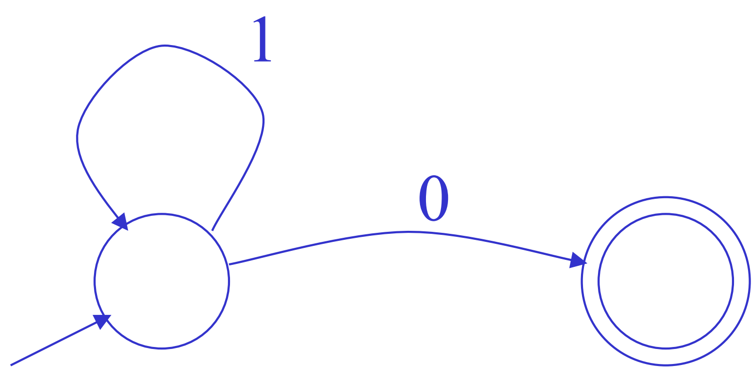
A Simple Example

- A finite automaton that accepts only “1”



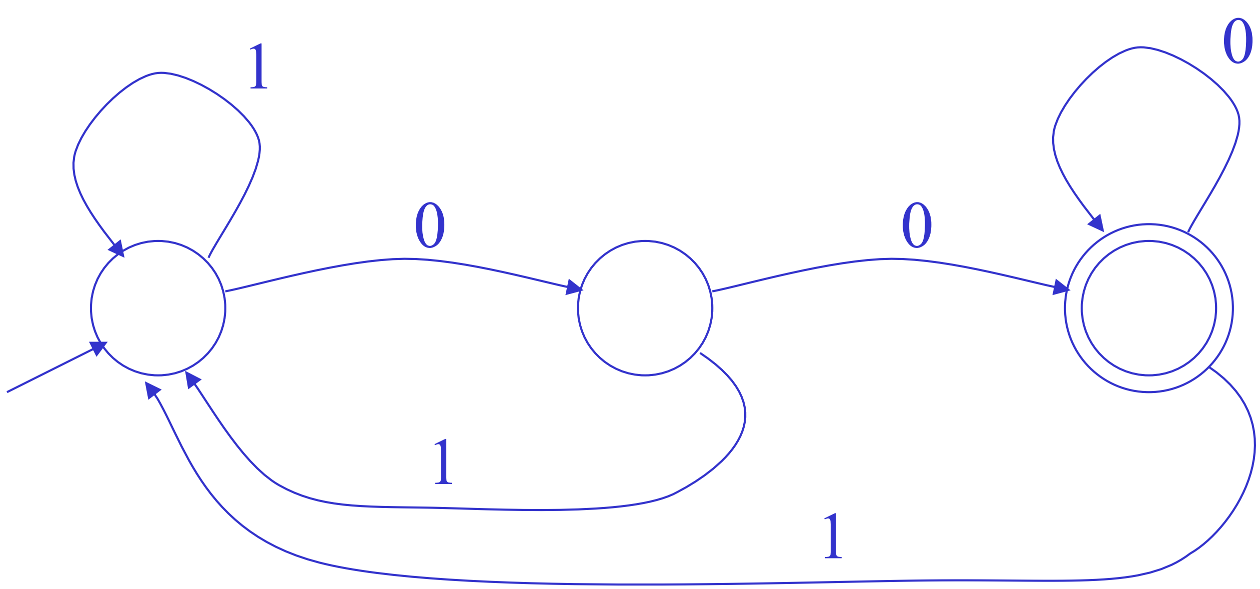
Another Simple Example

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}



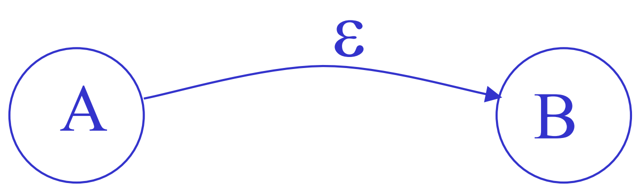
And Another Example

- Alphabet {0,1}
- What language does this recognize?



Epsilon Moves

- Another kind of transition: ε-moves



- Machine can move from state *A* to state *B* without reading input

Deterministic and Nondeterministic Automata

- Deterministic Finite Automata (DFA)
  - One transition per input per state **AFD**
  - No ε-moves

- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves **AFN**

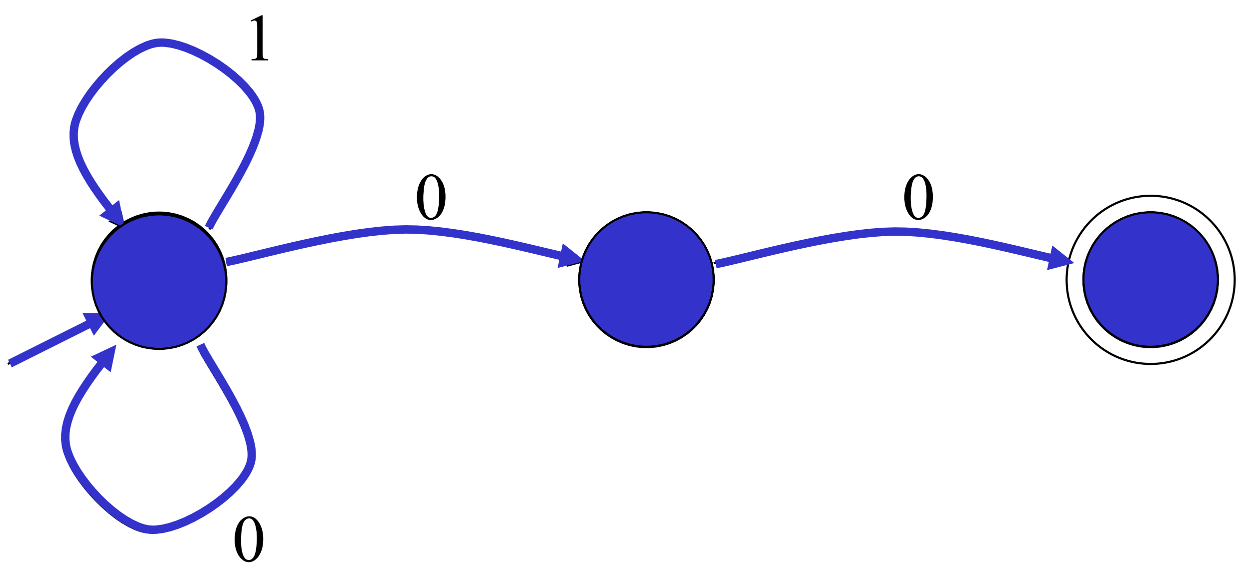
Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input

- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 0

Rule: NFA accepts if it <u>can</u> get to a final state

NFA vs. DFA (1)

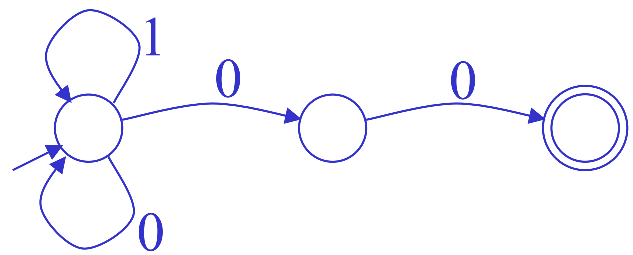
- NFAs and DFAs recognize the same set of languages (regular languages)

- DFAs are faster to execute
  - There are no choices to consider

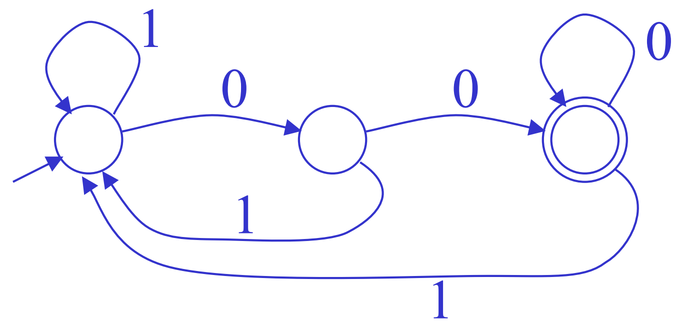
NFA vs. DFA (2)

- For a given language NFA can be simpler than DFA

NFA



DFA

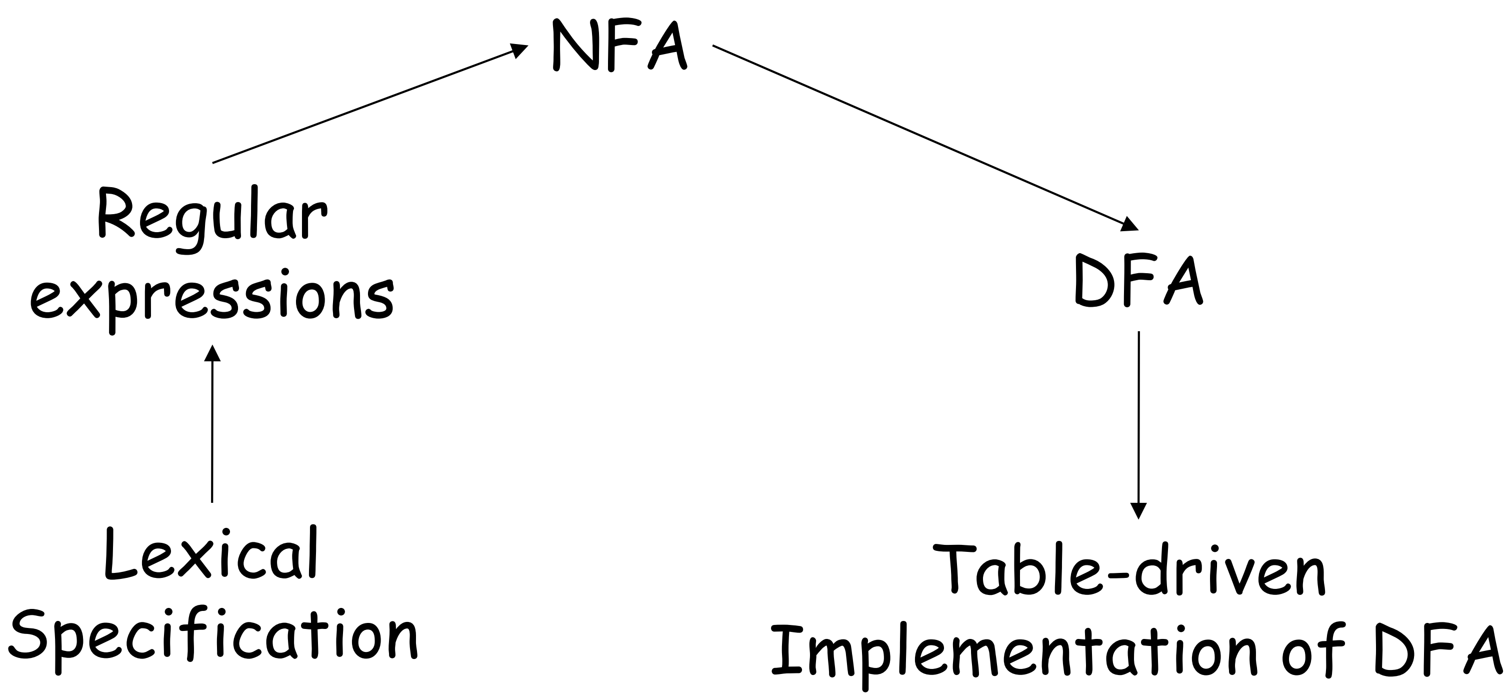


- DFA can be exponentially larger than NFA

3.

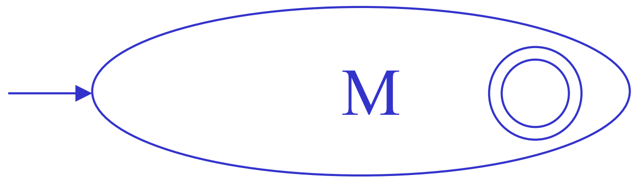
Regular Expressions to Finite Automata

- High-level sketch

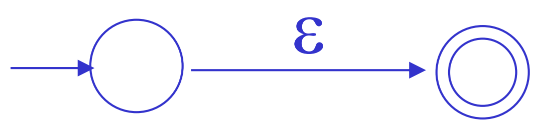


Regular Expressions to NFA (1)

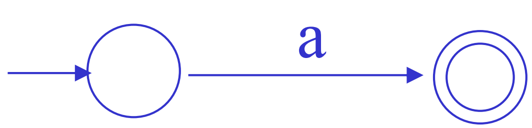
- For each kind of rexp, define an NFA
  - Notation: NFA for rexp M



- For ε

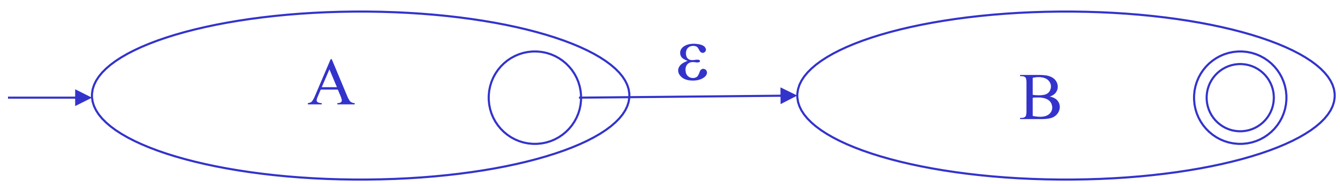


- For input a



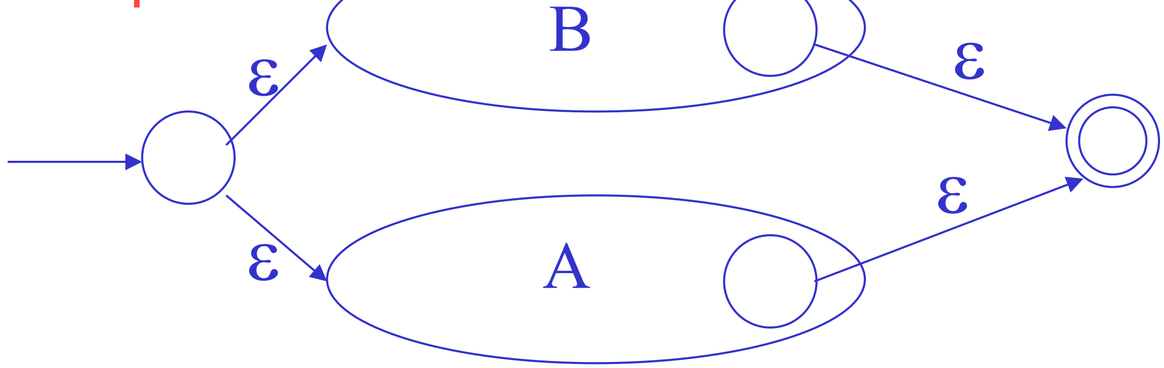
Regular Expressions to NFA (2)

- For *AB*



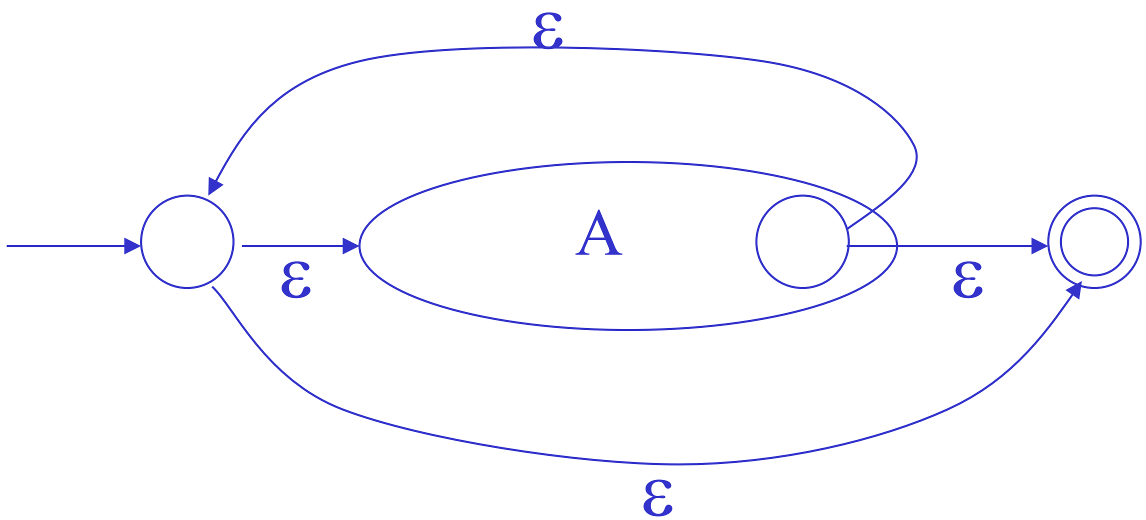
- For *A* + *B*

A | B



Regular Expressions to NFA (3)

- For *A**

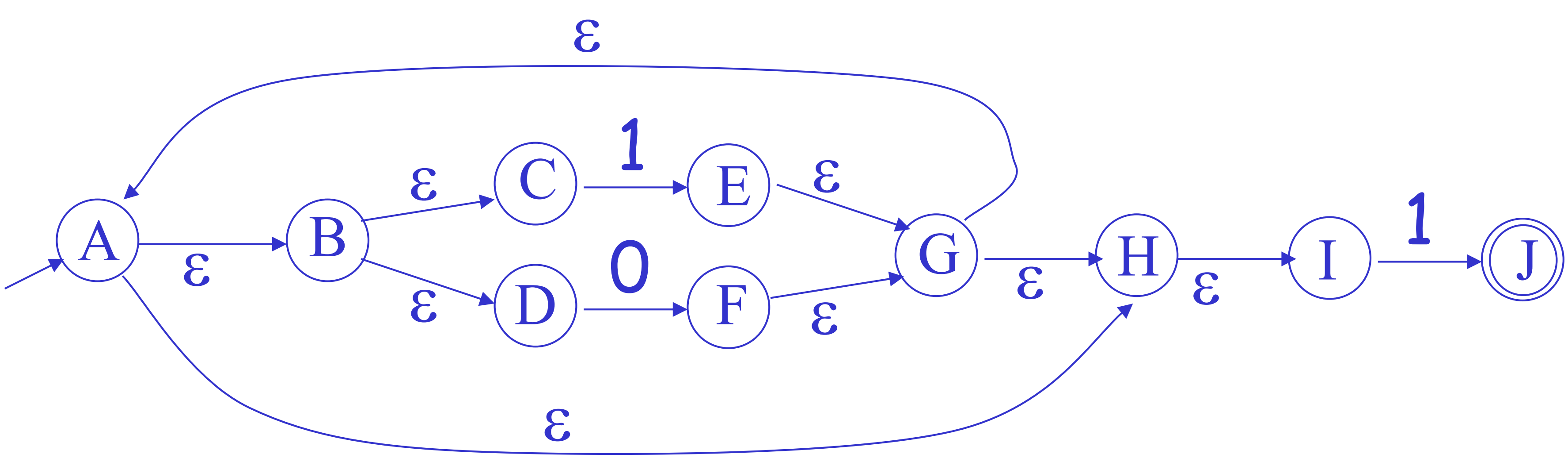


Example of RegExp -> NFA conversion

- Consider the regular expression

$$(1+0)^*1$$

- The NFA is



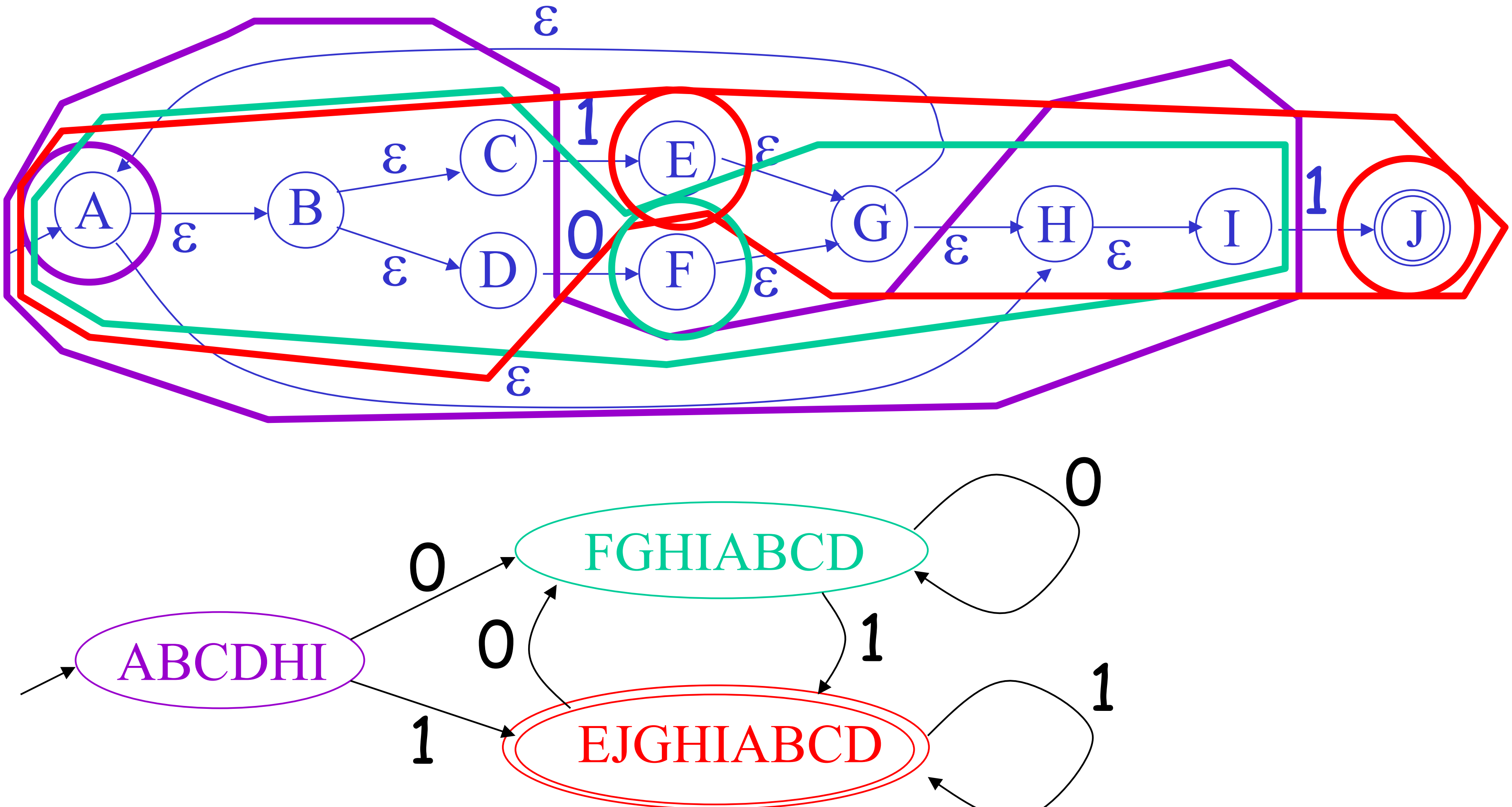
NFA to DFA: *The Trick*

- Simulate the NFA
- Each state of DFA
  = a non-empty subset of states of the NFA
- Start state
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \rightarrow^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from any state in S after seeing the input a, considering ε-moves as well

NFA to DFA. Remark

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?
  - $2^N - 1$ = finitely many

NFA -> DFA Example

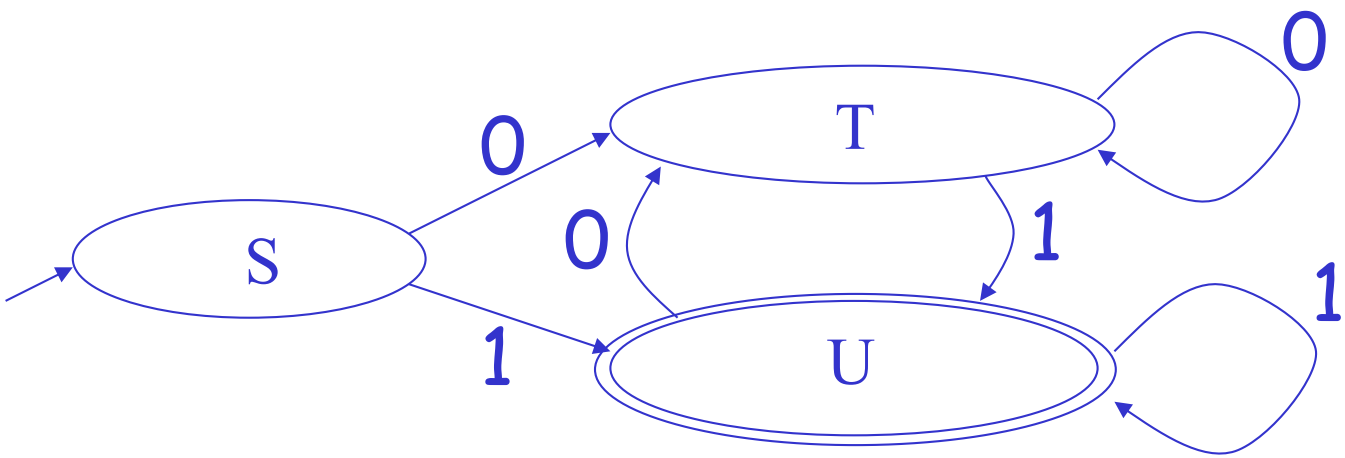


Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is "states"
  - Other dimension is "input symbol"
  - For every transition $S_i \rightarrow^a S_k$ define T[i,a] = k

- DFA "execution"
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA -> DFA conversion is at the heart of tools such as flex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations