Lista 3, 3h

Domínio de $z = \dfrac{x-y}{\operatorname{sen} x - \operatorname{sen} y}$ é $\mathbb{R}^2 - L$

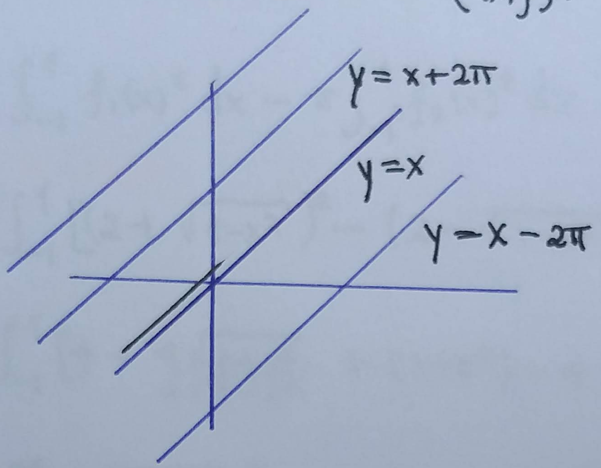
onde L é o conjunto $\{(x,y) : \operatorname{sen} x = \operatorname{sen} y\}$

$$\parallel$$

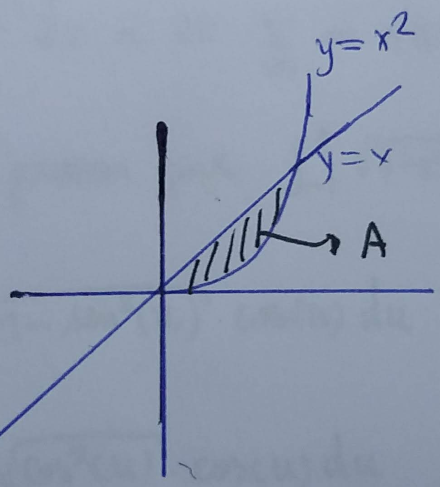
$$\{(x,y) : x = y + k2\pi\}$$

$$\parallel$$

reunião de retas.



Lista 1, g).

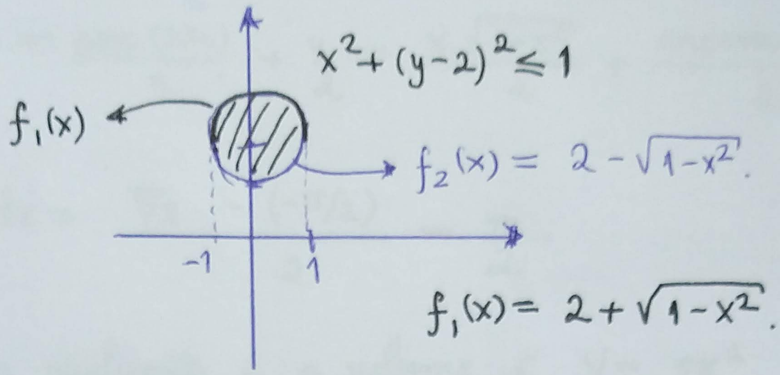


$x^2 \leq y \leq x$
conjunto A

V = volume rotação A, eixo $O_x = \pi \int_0^1 x^2 dx - \pi \int_0^1 x^4 dx$

$$= \pi \frac{x^3}{3}\Big|_0^1 - \pi \frac{x^5}{5}\Big|_0^1 = \frac{2\pi}{15}.$$

Lista 1, 1 m).



$$V = \pi\int_{-1}^{1} f_1(x)^2\,dx - \pi\int_{-1}^{1} f_2(x)^2\,dx$$

$$= \pi\int_{-1}^{1}\left[\left(2+\sqrt{1-x^2}\right)^2 - \left(2-\sqrt{1-x^2}\right)^2\right]dx$$

$$= \pi\int_{-1}^{1}\left[4 + \underset{4\sqrt{1-x^2}}{\cancel{4\sqrt{1+x^2}}} + (1-x^2) - 4 + 4\sqrt{1-x^2} - (1-x^2)\right]dx$$

$$= \pi\int_{-1}^{1} 8\sqrt{1-x^2}\,dx = 8\pi\cdot\frac{\pi}{2} = 4\pi^2.$$

Agora precisamos provar que $\int_{-1}^{1}\sqrt{1-x^2}\,dx = \frac{\pi}{2}$. De fato:

$$\int\sqrt{1-x^2}\,dx = \int\sqrt{1-\operatorname{sen}^2(u)}\,\cos(u)\,du$$

$x = \operatorname{sen}(u)$
$-\frac{\pi}{2} \le u \le \pi/2$
$dx = \cos(u)\,du$.

$$= \int\sqrt{\cos^2(u)}\cdot\cos(u)\,du$$

$\cos(u) \ge 0$

$$= \int\cos^2(u)\,du = \int\left(\frac{\cos(2u)+1}{2}\right)du$$

$$= \frac{\operatorname{sen}(2u)}{4} + \frac{u}{2}$$

$\operatorname{sen}(u) = x$, $\cos(u) = \sqrt{1-x^2}$

$\operatorname{sen}(2u) = 2\operatorname{sen}u\cdot\cos u$
$= 2x\sqrt{1-x^2}$.

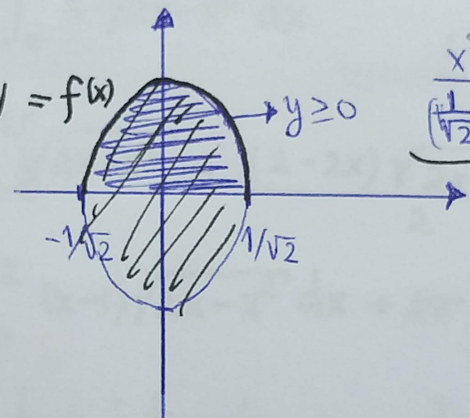
Logo,

$$\int \sqrt{1-x^2}\,dx = \frac{\text{sen}(2u)}{4} + \frac{u}{2} = \frac{x\sqrt{1-x^2}}{2} + \frac{\text{arcsen}(x)}{2} + C.$$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{\pi/2 - (-\pi/2)}{2} = \frac{\pi}{2}.$$

Isso conclui a resolução e o volume é $V = 4\pi^2$.

Lista 1, d)



$2x^2 + y^2 \leq 1, \; y \geq 0$

$$\underbrace{\frac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{y^2}{1^2} \leq 1}_{\text{interior de elipse}}, \; y \geq 0$$

$f(x) = \sqrt{1-2x^2}.$

$$V = \pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} f(x)^2\,dx = \pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1-2x^2)\,dx$$

$$= \pi \cdot \frac{2}{\sqrt{2}} - 2\pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} x^2\,dx = \sqrt{2}\cdot\pi - 2\pi \cdot 2 \cdot \frac{(1/\sqrt{2})^3}{3}$$

$$= \pi\left(\sqrt{2} - \frac{2}{\sqrt{2}}\cdot\frac{1}{3}\right) = \frac{2\pi\sqrt{2}}{3}.$$

Lista 2, 1 g). Rotação de

$$A = \{(x,y) : y^2 \leq 2x - x^2, \ y \geq 0\}$$

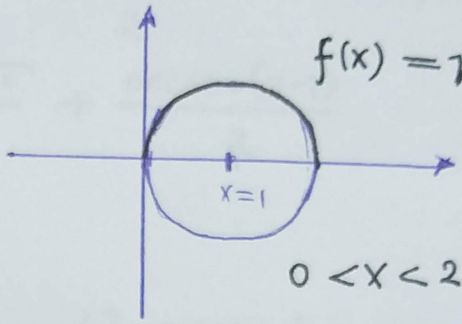
com relação ao eixo y.

$y^2 \leq 2x - x^2$

$x^2 - 2x + y^2 \leq 0$

$x^2 - 2x + 1 + y^2 \leq 1$

$(x-1)^2 + y^2 \leq 1$



$f(x) = \sqrt{2x - x^2}$

$0 < x < 2.$

$$V = 2\pi \int_0^2 x \cdot \sqrt{2x - x^2}\, dx$$

usando $x = 1 - \frac{1}{2}(2 - 2x)$

$$= 2\pi \int_0^2 \left[\sqrt{2x - x^2} - \frac{(2-2x)\sqrt{2x-x^2}}{2}\right] dx$$

$$= 2\pi \int_0^2 (x-1)\sqrt{2x - x^2}\, dx + 2\pi \int_0^2 \sqrt{2x - x^2}\, dx$$

Vamos calcular as duas integrais separadamente.

$$\int (x-1)\sqrt{2x-x^2}\, dx = -\frac{1}{2}\int \sqrt{u}\, du$$

$$= -\frac{u^{3/2}}{3} = -\frac{(2x-x^2)^{3/2}}{3}.$$

$u = 2x - x^2$

$dx = \frac{1}{2-2x} dx$

$$\int \sqrt{2x - x^2}\, dx = \int \sqrt{1 - (x-1)^2}\, dx$$

$$= \int \sqrt{1 - u^2}\, du$$

$u = x - 1$

$dx = du$

$$\int \sqrt{1-u^2}\,du = \int \cos(v)\sqrt{1-\text{sen}^2(v)}\,dv$$

$u = \text{sen}(v)$
$du = \cos(v)\,dv$
$-\frac{\pi}{2} \leq v \leq \pi/2$.

$$= \int \cos^2(v)\,dv = \frac{\cos v \cdot \text{sen}\, v}{2} + \frac{v}{2}$$

$$= \frac{u\sqrt{1-u^2}}{2} + \frac{\text{arcsen}(u)}{2}$$

$\cos(v) = \sqrt{1-u^2}$

$$= \frac{(x-1)\sqrt{1-(x-1)^2}}{2} + \frac{\text{arcsen}(x-1)}{2}$$

Portanto,

$$V = 2\pi \int_0^2 (x-1)\sqrt{2x-x^2}\,dx + 2\pi \int_0^2 \sqrt{2x-x^2}\,dx$$

$$= 2\pi \cdot \left[-\frac{(2x-x^2)^{3/2}}{3} + \frac{\sqrt{1-(x-1)^2}\,(x-1)}{2} + \frac{\text{arcsen}(x-1)}{2} \right]_0^2$$

$$= 2\pi \cdot \frac{\pi}{2} = \pi^2.$$

$$\boxed{V = \pi^2}$$