$$\forall x \left(2 \sum_{i=0}^{x} i = x(x+1) \right)$$

Base $x=0$ $\quad 2 \cdot 0 = 0 \quad$ e $\quad 0 \cdot (0+1) = 0 \quad ✓$

Hp.: $2 \sum_{i=0}^{x} i = x(x+1)$ $\qquad$ Tese: $2 \sum_{i=0}^{x+1} i = (x+1)(x+2)$

$$2 \sum_{i=0}^{x+1} = \underbrace{2 \sum_{i=0}^{x}} + 2(x+1) \stackrel{HP}{=} \underbrace{x(x+1)} + 2(x+1) = (x+2)(x+1) \quad ✓$$

$$\downarrow$$

$$= 2 \cdot \left(0+1+\dots+x+(x+1)\right) = 2\left(0+1+\dots+x\right) + 2(x+1)$$

$$\forall n \left(\sum_{i=1}^{n} (2i-1) = n^2 \right)$$

Base $n=1$ $\quad \sum_{i=1}^{1} (2i-1) = 2 \cdot 1 - 1 = 1 \qquad 1^2 = 1 \quad \checkmark$

Hp: $\sum_{i=1}^{n} (2i-1) = n^2 \Rightarrow$ **Tese:** $\sum_{i=1}^{n+1} (2i-1) = (n+1)^2 = n^2 + 2n + 1$

$$\sum_{i=1}^{n+1} (2i-1) = \underbrace{\sum_{i=1}^{n} (2i-1)} + \underbrace{2(n+1) - 1} \overset{HP}{=} \underbrace{n^2} + \underbrace{2n + 1} = (n+1)^2 \checkmark$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$(a+b)^2 = a^2 + 2ab + b^2$$

$n \qquad 1 \qquad\qquad n^2 + 2 \cdot n \cdot 1 + 1^2$

$$\forall n \geq 1 \left(9 \mid 10^n - 1 \right) \Leftrightarrow \forall n \geq 1 \, \exists a \left(10^n - 1 = 9a \right)$$

<u>Base $n = 1$</u> $\quad 10^1 - 1 = 9 = 9 \cdot 1$ vale com $a = 1$

<u>Hp.:</u> $9 \mid 10^n - 1$ $\quad \Rightarrow \quad$ <u>Tese</u> $9 \mid 10^{n+1} - 1$

$$\Updownarrow$$

$$\exists a' \left(10^n - 1 = 9a' \right) \qquad\qquad \exists a \left(10^{n+1} - 1 = 9a \right)$$

$$10^{n+1} - 1 = 10 \cdot 10^n - 1 = 10 \cdot \left(10^n - 1 \right) + 9 = 10 \cdot 9a' + 9 =$$

$$= 9 \left(10a' + 1 \right) \Rightarrow$$ <u>vale com $a = 10a' + 1$</u>

$$\forall n \geq 1 \; \forall m \left(m \mid (m+1)^n - 1 \right) \iff \forall n \geq 1 \; \forall m \; \exists a \left((m+1)^n - 1 = m a \right)$$

Ind. su n

Base $n=1$ $\quad (m+1)^1 - 1 = m+1-1 = m = m \cdot 1$ vale $(a=1)$

Hp. $\exists a' \left((m+1)^n - 1 = m a' \right) \Rightarrow$ **Tesi** $\exists a \left((m+1)^{n+1} - 1 = m a \right)$

$$(m+1)^{n+1} - 1 = (m+1)(m+1)^n - 1 = (m+1)\left((m+1)^n - 1\right) + m \overset{HP}{=}$$

$$= (m+1)\, m a' + m = m \underbrace{\left((m+1)a' + 1\right)}_{a} \quad \checkmark \quad \left(a = (m+1)a' + 1\right)$$

$$\forall n \geq 3 \left(n^2 \geq 2n + 3 \right)$$

$$\boxed{\forall k \left((k+3)^2 \geq 2(k+3) + 3 \right)}$$

<u>Base $n=3$</u> $\quad 3^2 = 9 \quad 2 \cdot 3 + 3 = 6 + 3 = 9$ e $9 \geq 9$ ✓

<u>Hp.:</u> $n^2 \geq 2n + 3 \Rightarrow$ <u>Tese</u> $(n+1)^2 \geq 2(n+1) + 3 = \boxed{2n+5}$

$$(n+1)^2 = \overbrace{\boxed{n^2} + \boxed{2n+1}}^{} \overset{HP}{\geq} \boxed{2n+3} + \boxed{2n+1} = 2n + 4 + 2n$$

$n^2 + 2n + 1$

$a \leq b$
$\Downarrow$
$a + c \leq b + c$

$$\forall n \geq 1 \left(2n \geq 1 \right) \Rightarrow 2n + 4 + 2n \geq 2n + 4 + 1 = 2n + 5.$$

Logo, $(n+1)^2 \geq 2n + 4 + 2n \geq 2n + 5$ ✓

Quero provar que $2n + 3 + 2n + 1 \geq 2n + 5$

$$\underset{\boxed{\cancel{2n+4}} + 2n}{\shortparallel} \qquad \underset{\boxed{\cancel{2n+4}} + 1}{\shortparallel}$$

$2n \geq 1$

$$\forall n \geq 3 \left(2n \geq 1 \right).$$

<u>Base $n=3$</u> $\quad 2 \cdot 3 = 6 \geq 1$

Hp.: $2n \geq 1 \Rightarrow$ Tese $2(n+1) \geq 1$

$$2n \geq 1 \Rightarrow \exists c \left(2n = 1 + c \right)$$

$$2(n+1) = 2n + 2 \overset{HP}{=} 1 + (c + 2) \geq 1$$

Seja X o conjunto dos meses de um ano (não bissexto) e seja, $\forall x \in X$, $\boxed{m(x)}$ o número dos dias do mês x. Seja $R \subseteq X^2$:

$$x R y \text{ sse } m(x)+m(y) \geq 59.$$

Verificar as props.: refl., irrefl., sim., antis., trans.

R é reflexiva se $\forall x (xRx)$ sse $\forall x (m(x)+m(x) \geq 59)$

$2 \cdot m(FEV) = 2 \cdot 28 = 56 < 59$. R não é reflexiva

R é irreflexiva sse $\forall x (x \not R x)$ sse $\forall x (2 \cdot m(x) < 59)$

$2 \cdot m(JAN) = 2 \cdot 31 = 62 \geq 59 \Rightarrow Jan\, R\, Jan \Rightarrow R$ não é irreflexiva

R é simétrica sse $\forall x \forall y (xRy \to yRx)$ sse $((m(x)+m(y) \geq 59) \to (m(y)+m(x) \geq 59))$

$m(x)+m(y) = m(y)+m(x)$, então, xRy implica yRx.

R é antissimétrica sse $\forall x \forall y ((xRy \text{ e } yRx) \to x=y)$ sse

$\forall x \forall y ((m(x)+m(y) \geq 59 \text{ e } m(y)+m(x) \geq 59) \to x=y)$

Não vale, pois, por exemplo $JAN\, R\, FEV$ e $FEV\, R\, JAN$, mas $JAN \neq FEV$.

R é transitiva sse $\forall x \forall y \forall z ((xRy \text{ e } yRz) \to xRz)$ sse

$\forall x \forall y \forall z ((m(x)+m(y) \geq 59 \text{ e } m(y)+m(z) \geq 59) \to m(x)+m(z) \geq 59)$

$FEV\, R\, JAN$ e $JAN\, R\, ABR$, pois $m(FEV)+m(JAN) = 59$ e $m(JAN)+m(ABR) = 61$, porém

$m(FEV)+m(ABR) = 58 < 59$, então $FEV \not R\, ABR$ e R não é transitiva.

Prove, por indução, que $\forall n \left(11 \mid 9^{n+1} + 2^{6n+1} \right)$

(Dica: pode usar o fato que $9 = 2^6 - 55$ ou escrever a hp. de indução como: $\exists k \left(9^{n+1} = 11k - 2^{6n+1} \right)$)

Base $n=0$ $\quad 11 \mid 9^{0+1} + 2^{6\cdot 0+1} \qquad 9^{0+1} + 2^{6\cdot 0+1} = 9+2 = 11$ e $11 \mid 11$ ✓

Hp.: $\quad 11 \mid 9^{n+1} + 2^{6n+1} \Rightarrow$ Tese: $11 \mid 9^{n+2} + 2^{6n+7}$

$\exists h \left(9^{n+1} + 2^{6n+1} = 11h \right) \Rightarrow \exists k \left(9^{n+2} + 2^{6n+7} = 11k \right)$

Como $9 = 2^6 - 55$, $9^{n+2} + 2^{6n+7} = 9 \cdot 9^{n+1} + 2^6 \cdot 2^{6n+1} = \left(2^6 - 55\right) 9^{n+1} + 2^6 \cdot 2^{6n+1}$

$= 2^6 \cdot 9^{n+1} + 2^6 \cdot 2^{6n+1} - 55 \cdot 9^{n+1} = 2^6 \left(9^{n+1} + 2^{6n+1} \right) - 55 \cdot 9^{n+1} \overset{HP}{=}$

$= 2^6 \cdot 11h - 5 \cdot 11 \cdot 9^{n+1} = 11 \underbrace{\left(2^6 h - 5 \cdot 9^{n+1} \right)}_{k} = 11k \Rightarrow 11 \mid 9^{n+2} + 2^{6n+7}$

$9^{n+2} + 2^{6n+7} = 9 \cdot 9^{n+1} + 2^{6n+7} \overset{HP}{=} 9 \cdot \left(11h - 2^{6n+1} \right) + 2^{6n+7} = 9 \cdot 11h - 9 \cdot 2^{6n+1} + 2^{6n+7} =$

$= 9 \cdot 11h - 9 \cdot 2^{6n+1} + 2^6 \cdot 2^{6n+1} = 9 \cdot 11h + 2^{6n+1} \left(2^6 - 9 \right) = 9 \cdot 11h + 2^{6n+1} \cdot 55 =$

$= 11 \left(9h + 5 \cdot 2^{6n+1} \right)$