

#### Universidade Federal da Bahia - UFBA Instituto de Matemática e Estatística - IME Departamento de Matemática



MAT A07 - Álgebra Linear A Aula 13

Subespaços Vetoriais: Intersecção, União, Soma

Dependência e Independência Linear, Bases

Professora: Isamara C. Alves

Data: 15/04/2021

Subespaço Gerado

Exercícios:

Subespaço Gerado

#### Exercícios:

Sejam  $\mathcal{V}=\mathcal{M}_2(\mathbb{R})$  um espaço vetorial sobre o corpo  $\mathbb{K}=\mathbb{R}$ ,  $\mathcal{W}_1=\{A\in\mathcal{M}_2(\mathbb{R})|A=A^t\}$ 

Subespaço Gerado

#### Exercícios:

Sejam  $\mathcal{V} = \mathcal{M}_2(\mathbb{R})$  um espaço vetorial sobre o corpo  $\mathbb{K} = \mathbb{R}$ ,  $\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$  e  $\mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$  subespaços vetoriais de  $\mathcal{V}$ .

1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .

#### EXERCÍCIOS:

- 1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ .

- 1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ . ( DICA: utilize a propriedade  $[S_1] + [S_2] = [S_1 \cup S_2]$  )

- 1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ . ( DICA: utilize a propriedade  $[S_1] + [S_2] = [S_1 \cup S_2]$ )
- 3. Determine um conjunto de geradores para  $\mathcal{V} = \mathcal{M}_2(\mathbb{R})$ .

- 1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ . ( DICA: utilize a propriedade  $[S_1] + [S_2] = [S_1 \cup S_2]$ )
- 3. Determine um conjunto de geradores para  $\mathcal{V} = \mathcal{M}_2(\mathbb{R})$ .

Subespaço Gerado

Exercícios: (respostas)

$$\text{Exercícios:} \big( \text{Respostas} \big) \ \, \mathcal{W}_1 = \{ A \in \mathcal{M}_2(\mathbb{R}) | A = A^t \} \, \, \text{e} \, \, \mathcal{W}_2 = \{ A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t \}$$

Exercícios: (Respostas) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $W_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$   $\forall A \in \mathcal{W}_1$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $W_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$   $\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $W_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$   $\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$
  
 $\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$
  
 $\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$
  
 $\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ 

$$\Rightarrow W_1 =$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix};$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{V_1}; \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{V_2};$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_{1} = \{A \in \mathcal{M}_{2}(\mathbb{R}) | A = A^{t}\}$$
 e  $\mathcal{W}_{2} = \{A \in \mathcal{M}_{2}(\mathbb{R}) | A = -A^{t}\}$   $\forall A \in \mathcal{W}_{1} \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$   $\Rightarrow \mathcal{W}_{1} = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$

$$e$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$
e

$$\forall A \in \mathcal{W}_2$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$
e

$$\forall A \in \mathcal{W}_2 \Rightarrow A = \begin{pmatrix} 0 & d \\ -d & 0 \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$

$$e$$

$$\forall A \in \mathcal{W}_2 \Rightarrow A = \begin{pmatrix} 0 & d \\ -d & 0 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$

$$e$$

$$\forall A \in \mathcal{W}_2 \Rightarrow A = \begin{pmatrix} 0 & d \\ -d & 0 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow \mathcal{W}_2 = \left[ \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{y_1} \right]$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$
e

$$\forall A \in \mathcal{W}_2 \Rightarrow A = \begin{pmatrix} 0 & d \\ -d & 0 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow \mathcal{W}_2 = \left[ \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{} \right].$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\forall A \in \mathcal{W}_1 \Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$
e

$$\forall A \in \mathcal{W}_2 \Rightarrow A = \begin{pmatrix} 0 & d \\ -d & 0 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow \mathcal{W}_2 = \left[ \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{} \right].$$

Subespaço Gerado

Exercícios: (respostas)

Exercícios: (Respostas) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $W_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$ 

$$\text{Exercícios:} \big( \text{Respostas} \big) \ \, \mathcal{W}_1 = \{ A \in \mathcal{M}_2(\mathbb{R}) | A = A^t \} \, \, \text{e} \, \, \mathcal{W}_2 = \{ A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t \}$$

$$\mathcal{W}_1 =$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $\mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$  
$$\mathcal{W}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix};$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t \}$$
 e  $W_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t \}$   $W_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};$ 

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t \}$$
 e  $W_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t \}$   $W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}$ ; e

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; e$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \left[ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{v_1}; \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{v_2}; \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{v_3} \right]; e \mathcal{W}_2 = \left[ \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{u_1} \right].$$

Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2)$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow a = 0;$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow a = 0; b = d;$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow a = 0; b = d; b = -d;$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow a = 0; b = d; b = -d; c = 0$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow a = 0; b = d; b = -d; c = 0 \Rightarrow$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \ \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ 

$$a = 0; b = d; b = -d; c = 0 \Rightarrow a = b = c = d = 0$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ 

$$a = 0; b = d; b = -d; c = 0 \Rightarrow a = b = c = d = 0 \Rightarrow A = 0_2$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ 

$$a = 0; b = d; b = -d; c = 0 \Rightarrow a = b = c = d = 0 \Rightarrow A = 0_2 \Rightarrow \mathcal{W}_1 \cap \mathcal{W}_2 = \{0\}$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; \text{ e } \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ 

$$a = 0; b = d; b = -d; c = 0 \Rightarrow a = b = c = d = 0 \Rightarrow A = 0_2 \Rightarrow \mathcal{W}_1 \cap \mathcal{W}_2 = \{0\}$$
por definição matemática:  $\{0\} := [\emptyset] \Rightarrow$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}.$$
Então,  $\forall A \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ 

$$a = 0; b = d; b = -d; c = 0 \Rightarrow a = b = c = d = 0 \Rightarrow A = 0_2 \Rightarrow \mathcal{W}_1 \cap \mathcal{W}_2 = \{0\}$$
por definição matemática:  $\{0\} := [\emptyset] \Rightarrow \mathcal{W}_1 \cap \mathcal{W}_2 = [\emptyset]$ 

Subespaço Gerado

Exercícios: (respostas)

Exercícios: (respostas) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $W_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$ 

Exercícios: (respostas) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $\mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$ 

$$\mathcal{W}_1 =$$

Exercícios: (respostas) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $\mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$  
$$\mathcal{W}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix};$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \\ \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; \text{ e}$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix};$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \left[ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{v_1}; \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{v_2}; \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{v_3} \right]; \text{ e } \mathcal{W}_2 = \left[ \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{u_1} \right]; \text{ então,}$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2)$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $\mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$   $\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix};$  e  $\mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix};$  então, 
$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\}$$
 e  $\mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$   $\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix};$  e  $\mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix};$  então,  $\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ \mathcal{W}_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; então,$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}; então,$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \ \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}; então,$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} a & b + d \\ b - d & c \end{pmatrix}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \ \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}; então,$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} a & b + d \\ b - d & c \end{pmatrix} \Rightarrow \mathcal{W}_1 + \mathcal{W}_2 = [v_1; v_2; v_3]$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{bmatrix}; então,$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} a & b+d \\ b-d & c \end{pmatrix} \Rightarrow \mathcal{W}_1 + \mathcal{W}_2 = [v_1; v_2; v_3; u_1]$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \in \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$W_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; e \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; então,$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} a & b+d \\ b-d & c \end{pmatrix} \Rightarrow \mathcal{W}_1 + \mathcal{W}_2 = [v_1; v_2; v_3; u_1]$$
E, como  $\mathcal{V} = \mathcal{M}_2(\mathbb{R}) = \mathcal{W}_1 + \mathcal{W}_2$ , temos que:

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \text{ e } \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \text{ então,} \end{bmatrix}; \text{ então,}$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} a & b + d \\ b - d & c \end{pmatrix} \Rightarrow \mathcal{W}_1 + \mathcal{W}_2 = [v_1; v_2; v_3; u_1]$$
E, como  $\mathcal{V} = \mathcal{M}_2(\mathbb{R}) = \mathcal{W}_1 + \mathcal{W}_2$ , temos que :

$$\mathcal{V}=\mathcal{M}_2(\mathbb{R})=[v_1;v_2;v_3;u_1]$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = A^t\} \text{ e } \mathcal{W}_2 = \{A \in \mathcal{M}_2(\mathbb{R}) | A = -A^t\}$$

$$\mathcal{W}_1 = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \text{ e } \mathcal{W}_2 = \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \text{ então,}$$

$$\forall A \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow A = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} a & b+d \\ b-d & c \end{pmatrix} \Rightarrow \mathcal{W}_1 + \mathcal{W}_2 = [v_1; v_2; v_3; u_1]$$
E, como  $\mathcal{V} = \mathcal{M}_2(\mathbb{R}) = \mathcal{W}_1 + \mathcal{W}_2$ , temos que:

$$\mathcal{V}=\mathcal{M}_2(\mathbb{R})=[v_1;v_2;v_3;u_1]$$

Subespaço Gerado

Exercícios:

Subespaço Gerado

#### Exercícios:

Sejam  $\mathcal{V}=\mathcal{P}_2(\mathbb{R})$  um espaço vetorial sobre o corpo  $\mathbb{K}=\mathbb{R}$ ,  $\mathcal{W}_1=\{p(t)\in\mathcal{P}_2(\mathbb{R})|a_0=a_1+a_2\}$ 

Subespaço Gerado

#### Exercícios:

#### Exercícios:

Sejam  $\mathcal{V} = \mathcal{P}_2(\mathbb{R})$  um espaço vetorial sobre o corpo  $\mathbb{K} = \mathbb{R}$ ,  $\mathcal{W}_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\}$  e  $\mathcal{W}_2 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0\}$  subespaços vetoriais de  $\mathcal{V}$ .

1. Determine um conjunto de geradores para  $(W_1 \cap W_2) \subseteq V$ .

#### EXERCÍCIOS:

- 1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ .

#### EXERCÍCIOS:

- 1. Determine um conjunto de geradores para  $(W_1 \cap W_2) \subseteq V$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ .
- 3. Determine um conjunto de geradores para  $\mathcal{V} = \mathcal{P}_2(\mathbb{R})$ .

#### Exercícios:

- 1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ .
- 3. Determine um conjunto de geradores para  $\mathcal{V} = \mathcal{P}_2(\mathbb{R})$ .
- 4. Determine um subespaço  $W_3$  de V tal que  $V = W_2 \oplus W_3$  onde,  $W_3 \neq W_1$ .

#### Exercícios:

- 1. Determine um conjunto de geradores para  $(\mathcal{W}_1 \cap \mathcal{W}_2) \subseteq \mathcal{V}$ .
- 2. Determine um conjunto de geradores para  $(W_1 + W_2) \subseteq V$ .
- 3. Determine um conjunto de geradores para  $\mathcal{V} = \mathcal{P}_2(\mathbb{R})$ .
- 4. Determine um subespaço  $W_3$  de V tal que  $V = W_2 \oplus W_3$  onde,  $W_3 \neq W_1$ .

Subespaço Gerado

Exercícios:(respostas)

Exercícios:(Respostas)  

$$W_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\}$$

EXERCÍCIOS:(RESPOSTAS) 
$$\mathcal{W}_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\} \text{ e } \mathcal{W}_2 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0\}.$$

EXERCÍCIOS: (RESPOSTAS)
$$\mathcal{W}_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\} \text{ e } \mathcal{W}_2 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0\}.$$

$$\forall p(t) \in \mathcal{W}_1 \Rightarrow$$

$$\mathcal{W}_1 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2 \} \text{ e } \mathcal{W}_2 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0 \}.$$

$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\} \text{ e } \mathcal{W}_2 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0\}.$$

$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2$$

EXERCÍCIOS: (RESPOSTAS)
$$W_1 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2 \} \text{ e } W_2 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0 \}.$$

$$\forall p(t) \in W_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1 + a_2).$$

EXERCÍCIOS: (RESPOSTAS)
$$W_1 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2 \} \text{ e } W_2 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0 \}.$$

$$\forall p(t) \in W_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2t^2 = a_1(1+t$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\} \text{ e } \mathcal{W}_2 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0\}.$$

$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2(1+t)$$

EXERCÍCIOS: (RESPOSTAS)
$$W_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\} \in W_2 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \in a_2 = 0\}.$$

$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2(1+t^2).$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2 \} \text{ e } \mathcal{W}_2 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0 \}.$$
 
$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2(1+t^2)$$
 
$$\Rightarrow \mathcal{W}_1 = [(1+t); (1+t^2)]; \forall \lambda_i \in \mathbb{R}; i = 1, 2$$

#### Subespaços Vetoriais Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_{1} = \{ p(t) \in \mathcal{P}_{2}(\mathbb{R}) | a_{0} = a_{1} + a_{2} \} \text{ e } \mathcal{W}_{2} = \{ p(t) \in \mathcal{P}_{2}(\mathbb{R}) | a_{0} + a_{1} = 0 \text{ e } a_{2} = 0 \}.$$
 
$$\forall p(t) \in \mathcal{W}_{1} \Rightarrow p(t) = (a_{1} + a_{2}).1 + a_{1}t + a_{2}t^{2} = a_{1}(1+t) + a_{2}(1+t^{2})$$
 
$$\Rightarrow \mathcal{W}_{1} = [(1+t); (1+t^{2})]; \forall \lambda_{i} \in \mathbb{R}; i = 1, 2$$

$$\forall p(t) \in \mathcal{W}_2 \Rightarrow$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2 \} \text{ e } \mathcal{W}_2 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0 \}.$$
 
$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2(1+t^2)$$
 
$$\Rightarrow \mathcal{W}_1 = [(1+t); (1+t^2)]; \forall \lambda_i \in \mathbb{R}; i = 1, 2$$

$$orall p(t) \in \mathcal{W}_2 \Rightarrow p(t) = (-a_1).1 +$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2 \} \text{ e } \mathcal{W}_2 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0 \}.$$
 
$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2(1+t^2)$$
 
$$\Rightarrow \mathcal{W}_1 = [(1+t); (1+t^2)]; \forall \lambda_i \in \mathbb{R}; i = 1, 2$$

$$orall p(t) \in \mathcal{W}_2 \Rightarrow p(t) = (-a_1).1 + a_1 t +$$

EXERCÍCIOS: (RESPOSTAS)
$$W_{1} = \{p(t) \in \mathcal{P}_{2}(\mathbb{R}) | a_{0} = a_{1} + a_{2}\} \text{ e } \mathcal{W}_{2} = \{p(t) \in \mathcal{P}_{2}(\mathbb{R}) | a_{0} + a_{1} = 0 \text{ e } a_{2} = 0\}.$$

$$\forall p(t) \in \mathcal{W}_{1} \Rightarrow p(t) = (a_{1} + a_{2}).1 + a_{1}t + a_{2}t^{2} = a_{1}(1 + t) + a_{2}(1 + t^{2})$$

$$\Rightarrow \mathcal{W}_{1} = [(1 + t); (1 + t^{2})]; \forall \lambda_{i} \in \mathbb{R}; i = 1, 2$$

$$\forall p(t) \in \mathcal{W}_2 \Rightarrow p(t) = (-a_1).1 + a_1t + 0.t^2$$

EXERCÍCIOS: (RESPOSTAS)
$$\mathcal{W}_{1} = \{p(t) \in \mathcal{P}_{2}(\mathbb{R}) | a_{0} = a_{1} + a_{2}\} \text{ e } \mathcal{W}_{2} = \{p(t) \in \mathcal{P}_{2}(\mathbb{R}) | a_{0} + a_{1} = 0 \text{ e } a_{2} = 0\}.$$

$$\forall p(t) \in \mathcal{W}_{1} \Rightarrow p(t) = (a_{1} + a_{2}).1 + a_{1}t + a_{2}t^{2} = a_{1}(1 + t) + a_{2}(1 + t^{2})$$

$$\Rightarrow \mathcal{W}_{1} = [(1 + t); (1 + t^{2})]; \forall \lambda_{i} \in \mathbb{R}; i = 1, 2$$

$$\forall p(t) \in \mathcal{W}_2 \Rightarrow p(t) = (-a_1).1 + a_1t + 0.t^2 = a_1(-1+t)$$

#### Subespaços Vetoriais Subespaço Gerado

$$\mathcal{W}_1 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2\} \text{ e } \mathcal{W}_2 = \{p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0\}.$$

$$orall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2(1+t^2)$$

$$\Rightarrow \mathcal{W}_1 = [(1+t); (1+t^2)]; \forall \lambda_i \in \mathbb{R}; i=1,2$$

$$\forall p(t) \in \mathcal{W}_2 \Rightarrow p(t) = (-a_1).1 + a_1t + 0.t^2 = a_1(-1+t)$$

$$\Rightarrow \mathcal{W}_2 = [(-1+t)]; \lambda_1 \in \mathbb{R}$$

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 = a_1 + a_2 \} \text{ e } \mathcal{W}_2 = \{ p(t) \in \mathcal{P}_2(\mathbb{R}) | a_0 + a_1 = 0 \text{ e } a_2 = 0 \}.$$
 
$$\forall p(t) \in \mathcal{W}_1 \Rightarrow p(t) = (a_1 + a_2).1 + a_1t + a_2t^2 = a_1(1+t) + a_2(1+t^2)$$
 
$$\Rightarrow \mathcal{W}_1 = [(1+t); (1+t^2)]; \forall \lambda_i \in \mathbb{R}; i = 1, 2$$

$$\forall p(t) \in \mathcal{W}_2 \Rightarrow p(t) = (-a_1).1 + a_1t + 0.t^2 = a_1(-1+t)$$

$$\Rightarrow \mathcal{W}_2 = [(-1+t)]; \lambda_1 \in \mathbb{R}$$

Subespaço Gerado

Exercícios: (respostas)

Exercícios:(respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)];$$
 e,

Exercícios:(Respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

Subespaço Gerado

Exercícios:(Respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; e, \mathcal{W}_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2)$$

Subespaço Gerado

Exercícios:(Respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; e, \mathcal{W}_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2)$$

Subespaço Gerado

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

Subespaço Gerado

Exercícios:(Respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; e, \mathcal{W}_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow$$
  $(\lambda_1 + \lambda_2).1 +$ 

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t$$

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2$$

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; \text{ e, } \mathcal{W}_2 = [(-1+t)];$$
  
Então, 
$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$
$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$
$$\Rightarrow \lambda_1 + \lambda_2 = -\lambda_3;$$

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$
$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

$$\Rightarrow \lambda_1 + \lambda_2 = -\lambda_3; \lambda_1 = \lambda_3;$$

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

$$\Rightarrow \lambda_1 + \lambda_2 = -\lambda_3 : \lambda_1 = \lambda_3 : \lambda_2 = 0 \Rightarrow \lambda_1 = 0 :$$

MAT A07 - Álgebra Linear A - Semestre Letivo Suplementar - 2021.1

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$
$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

$$\Rightarrow \lambda_1 + \lambda_2 = -\lambda_3; \lambda_1 = \lambda_3; \lambda_2 = 0 \Rightarrow \lambda_1 = 0; \lambda_2 = 0;$$

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$
$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

$$\Rightarrow \lambda_1+\lambda_2=-\lambda_3; \lambda_1=\lambda_3; \lambda_2=0 \Rightarrow \lambda_1=0; \lambda_2=0; \lambda_3=0$$

Exercícios: (respostas) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$

Então.

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

$$\Rightarrow \lambda_1 + \lambda_2 = -\lambda_3; \lambda_1 = \lambda_3; \lambda_2 = 0 \Rightarrow \lambda_1 = 0; \lambda_2 = 0; \lambda_3 = 0$$

$$\Rightarrow (\mathcal{W}_1 \cap \mathcal{W}_2) = \{0\}$$

$$\Rightarrow (\mathcal{W}_1 \cap \mathcal{W}_2) = \{0\}$$

Exercícios: (respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; e, \mathcal{W}_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

$$\Rightarrow \lambda_1 + \lambda_2 = -\lambda_3; \lambda_1 = \lambda_3; \lambda_2 = 0 \Rightarrow \lambda_1 = 0; \lambda_2 = 0; \lambda_3 = 0$$

$$\Rightarrow (\mathcal{W}_1 \cap \mathcal{W}_2) = \{0\} \Rightarrow (\mathcal{W}_1 \cap \mathcal{W}_2) = [\emptyset].$$

$$\Rightarrow (VV_1 \cap VV_2) = \{0\} \Rightarrow (VV_1 \cap VV_2) = [\emptyset]$$

Exercícios: (respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; e, \mathcal{W}_2 = [(-1+t)];$$

$$\forall p(t) \in (\mathcal{W}_1 \cap \mathcal{W}_2) \Rightarrow p(t) = \lambda_1(1+t) + \lambda_2(1+t^2) = \lambda_3(-1+t)$$

$$\Rightarrow (\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_2).t^2 = (-\lambda_3).1 + \lambda_3.t$$

$$\Rightarrow \lambda_1 + \lambda_2 = -\lambda_3; \lambda_1 = \lambda_3; \lambda_2 = 0 \Rightarrow \lambda_1 = 0; \lambda_2 = 0; \lambda_3 = 0$$

$$\Rightarrow (\mathcal{W}_1 \cap \mathcal{W}_2) = \{0\} \Rightarrow (\mathcal{W}_1 \cap \mathcal{W}_2) = [\emptyset].$$

$$\Rightarrow (VV_1 \cap VV_2) = \{0\} \Rightarrow (VV_1 \cap VV_2) = [\emptyset]$$

Subespaço Gerado

Exercícios: (respostas)

Subespaço Gerado

Exercícios:(Respostas)  $\mathcal{W}_1 = [(1+t); (1+t^2)];$  e,

Subespaço Gerado

Exercícios: (respostas)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$ 

Subespaço Gerado

Exercícios:(Respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)];$$
 e,  $\mathcal{W}_2 = [(-1+t)];$  Então, 
$$(\mathcal{W}_1 + \mathcal{W}_2) = [(1+t); (1+t^2); (-1+t)].$$

Subespaço Gerado

Exercícios:(Respostas) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; \text{ e, } \mathcal{W}_2 = [(-1+t)];$$
 Então, 
$$(\mathcal{W}_1 + \mathcal{W}_2) = [(1+t); (1+t^2); (-1+t)].$$

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2)$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; \text{ e, } \mathcal{W}_2 = [(-1+t)];$$
 Então, 
$$(\mathcal{W}_1 + \mathcal{W}_2) = [(1+t); (1+t^2); (-1+t)].$$

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 =$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2)$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então.

$$(\mathcal{W}_1+\mathcal{W}_2)=[(1+t);(1+t^2);(-1+t)].$$

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(1+t) + \lambda_2(1+t^2) + \lambda_3(-1+t)$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t)$$

$$\Rightarrow p(t) = a_0.1 + a_1.t + a_2.t^2 =$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$egin{aligned} orall 
ho(t) &\in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow 
ho(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t) \ \\ &\Rightarrow 
ho(t) = a_0.1 + a_1.t + a_2.t^2 = (\lambda_1 + \lambda_2 - \lambda_3).1 + \end{aligned}$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,

$$(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$$

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(1+t) + \lambda_2(1+t^2) + \lambda_3(-1+t)$$

$$\Rightarrow p(t) = a_0.1 + a_1.t + a_2.t^2 = (\lambda_1 + \lambda_2 - \lambda_3).1 + (\lambda_1 + \lambda_3).t$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t)$$
$$\Rightarrow p(t) = a_0.1 + a_1.t + a_2.t^2 = (\lambda_1 + \lambda_2 - \lambda_3).1 + (\lambda_1 + \lambda_3).t + (\lambda_2).t^2$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)];$$
 e,  $\mathcal{W}_2 = [(-1+t)];$  Então,

$$(\mathcal{W}_1+\mathcal{W}_2)=[(1+t);(1+t^2);(-1+t)].$$

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t)$$

$$\Rightarrow p(t) = a_0.1 + a_1.t + a_2.t^2 = (\lambda_1 + \lambda_2 - \lambda_3).1 + (\lambda_1 + \lambda_3).t + (\lambda_2).t^2$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\begin{aligned} \forall \rho(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow \rho(t) &= a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t) \\ \Rightarrow \rho(t) &= a_0.1 + a_1.t + a_2.t^2 = (\lambda_1 + \lambda_2 - \lambda_3).1 + (\lambda_1 + \lambda_3).t + (\lambda_2).t^2 \\ \Rightarrow a_0 &= \lambda_1 + \lambda_2 - \lambda_3; \end{aligned}$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t)$$

$$\Rightarrow p(t) = a_0 \cdot 1 + a_1 \cdot t + a_2 \cdot t^2 = (\lambda_1 + \lambda_2 - \lambda_3) \cdot 1 + (\lambda_1 + \lambda_3) \cdot t + (\lambda_2) \cdot t^2$$

$$\Rightarrow a_0 = \lambda_1 + \lambda_2 - \lambda_3; a_1 = \lambda_1 + \lambda_3;$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\begin{aligned} \forall \rho(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow \rho(t) &= a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t) \\ \Rightarrow \rho(t) &= a_0 \cdot 1 + a_1 \cdot t + a_2 \cdot t^2 = (\lambda_1 + \lambda_2 - \lambda_3) \cdot 1 + (\lambda_1 + \lambda_3) \cdot t + (\lambda_2) \cdot t^2 \\ \Rightarrow a_0 &= \lambda_1 + \lambda_2 - \lambda_3; a_1 = \lambda_1 + \lambda_3; a_2 = \lambda_2 \end{aligned}$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall \rho(t) \in (W_1 + W_2) \Rightarrow \rho(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1 + t) + \lambda_2 (1 + t^2) + \lambda_3 (-1 + t)$$

$$\Rightarrow \rho(t) = a_0 \cdot 1 + a_1 \cdot t + a_2 \cdot t^2 = (\lambda_1 + \lambda_2 - \lambda_3) \cdot 1 + (\lambda_1 + \lambda_3) \cdot t + (\lambda_2) \cdot t^2$$

$$\Rightarrow a_0 = \lambda_1 + \lambda_2 - \lambda_3; a_1 = \lambda_1 + \lambda_3; a_2 = \lambda_2$$

$$\Rightarrow a_2 = \lambda_2;$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
  
Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall \rho(t) \in (W_1 + W_2) \Rightarrow \rho(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1 + t) + \lambda_2 (1 + t^2) + \lambda_3 (-1 + t)$$

$$\Rightarrow \rho(t) = a_0 \cdot 1 + a_1 \cdot t + a_2 \cdot t^2 = (\lambda_1 + \lambda_2 - \lambda_3) \cdot 1 + (\lambda_1 + \lambda_3) \cdot t + (\lambda_2) \cdot t^2$$

$$\Rightarrow a_0 = \lambda_1 + \lambda_2 - \lambda_3; a_1 = \lambda_1 + \lambda_3; a_2 = \lambda_2$$

$$\Rightarrow a_2 = \lambda_2; \lambda_1 = a_1 - \lambda_3;$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$\mathcal{W}_1 = [(1+t); (1+t^2)]; \text{ e, } \mathcal{W}_2 = [(-1+t)];$$
 Então,  $(\mathcal{W}_1 + \mathcal{W}_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall \rho(t) \in (W_1 + W_2) \Rightarrow \rho(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1 + t) + \lambda_2 (1 + t^2) + \lambda_3 (-1 + t)$$

$$\Rightarrow \rho(t) = a_0 \cdot 1 + a_1 \cdot t + a_2 \cdot t^2 = (\lambda_1 + \lambda_2 - \lambda_3) \cdot 1 + (\lambda_1 + \lambda_3) \cdot t + (\lambda_2) \cdot t^2$$

$$\Rightarrow a_0 = \lambda_1 + \lambda_2 - \lambda_3; a_1 = \lambda_1 + \lambda_3; a_2 = \lambda_2$$

$$\Rightarrow a_2 = \lambda_2; \lambda_1 = a_1 - \lambda_3; a_0 = a_1 - \lambda_3 + a_2 - \lambda_3$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t)$$

$$\Rightarrow p(t) = a_0.1 + a_1.t + a_2.t^2 = (\lambda_1 + \lambda_2 - \lambda_3).1 + (\lambda_1 + \lambda_3).t + (\lambda_2).t^2$$

$$\Rightarrow a_0 = \lambda_1 + \lambda_2 - \lambda_3; a_1 = \lambda_1 + \lambda_3; a_2 = \lambda_2$$

$$\Rightarrow a_2 = \lambda_2; \lambda_1 = a_1 - \lambda_3; a_0 = a_1 - \lambda_3 + a_2 - \lambda_3 = a_1 + a_2 - 2\lambda_3;$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

seja;  

$$\forall p(t) \in (\mathcal{W}_1 + \mathcal{W}_2) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (1+t) + \lambda_2 (1+t^2) + \lambda_3 (-1+t)$$
  
 $\Rightarrow p(t) = a_0.1 + a_1.t + a_2.t^2 = (\lambda_1 + \lambda_2 - \lambda_3).1 + (\lambda_1 + \lambda_3).t + (\lambda_2).t^2$   
 $\Rightarrow a_0 = \lambda_1 + \lambda_2 - \lambda_3; a_1 = \lambda_1 + \lambda_3; a_2 = \lambda_2$   
 $\Rightarrow a_2 = \lambda_2; \lambda_1 = a_1 - \lambda_3; a_0 = a_1 - \lambda_3 + a_2 - \lambda_3 = a_1 + a_2 - 2\lambda_3;$   
 $\Rightarrow \mathcal{P}_2(\mathbb{R}) = (\mathcal{W}_1 + \mathcal{W}_2)$ 

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall \rho(t) \in (\mathcal{W}_{1} + \mathcal{W}_{2}) \Rightarrow \rho(t) = a_{0} + a_{1}t + a_{2}t^{2} = \lambda_{1}(1+t) + \lambda_{2}(1+t^{2}) + \lambda_{3}(-1+t)$$

$$\Rightarrow \rho(t) = a_{0}.1 + a_{1}.t + a_{2}.t^{2} = (\lambda_{1} + \lambda_{2} - \lambda_{3}).1 + (\lambda_{1} + \lambda_{3}).t + (\lambda_{2}).t^{2}$$

$$\Rightarrow a_{0} = \lambda_{1} + \lambda_{2} - \lambda_{3}; a_{1} = \lambda_{1} + \lambda_{3}; a_{2} = \lambda_{2}$$

$$\Rightarrow a_{2} = \lambda_{2}; \lambda_{1} = a_{1} - \lambda_{3}; a_{0} = a_{1} - \lambda_{3} + a_{2} - \lambda_{3} = a_{1} + a_{2} - 2\lambda_{3};$$

$$\Rightarrow \mathcal{P}_{2}(\mathbb{R}) = (\mathcal{W}_{1} + \mathcal{W}_{2}) \Rightarrow \mathcal{P}_{2}(\mathbb{R}) = [(1+t); (1+t^{2}); (-1+t)].$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$$
 Então,  $(W_1 + W_2) = [(1+t); (1+t^2); (-1+t)].$ 

$$\forall \rho(t) \in (\mathcal{W}_{1} + \mathcal{W}_{2}) \Rightarrow \rho(t) = a_{0} + a_{1}t + a_{2}t^{2} = \lambda_{1}(1+t) + \lambda_{2}(1+t^{2}) + \lambda_{3}(-1+t)$$

$$\Rightarrow \rho(t) = a_{0}.1 + a_{1}.t + a_{2}.t^{2} = (\lambda_{1} + \lambda_{2} - \lambda_{3}).1 + (\lambda_{1} + \lambda_{3}).t + (\lambda_{2}).t^{2}$$

$$\Rightarrow a_{0} = \lambda_{1} + \lambda_{2} - \lambda_{3}; a_{1} = \lambda_{1} + \lambda_{3}; a_{2} = \lambda_{2}$$

$$\Rightarrow a_{2} = \lambda_{2}; \lambda_{1} = a_{1} - \lambda_{3}; a_{0} = a_{1} - \lambda_{3} + a_{2} - \lambda_{3} = a_{1} + a_{2} - 2\lambda_{3};$$

$$\Rightarrow \mathcal{P}_{2}(\mathbb{R}) = (\mathcal{W}_{1} + \mathcal{W}_{2}) \Rightarrow \mathcal{P}_{2}(\mathbb{R}) = [(1+t); (1+t^{2}); (-1+t)].$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e,$ 

#### Subespaços Vetoriais Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$ 

Subespaço Gerado

```
EXERCÍCIOS: (RESPOSTAS) W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];
\mathcal{W}_3 = ? um subespoo de \mathcal{V} tal que \mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3 onde, \mathcal{W}_3 \neq \mathcal{W}_1.
```

Subespaço Gerado

```
EXERCÍCIOS: (RESPOSTAS) W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];
\mathcal{W}_3 = ? um subespoo de \mathcal{V} tal que \mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3 onde, \mathcal{W}_3 \neq \mathcal{W}_1.
Então. \mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3 se, e somente se.
```

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então.  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se.

•  $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então.  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se.

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então.  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se.

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3)$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e. W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespço de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então.  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se.

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 =$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e. W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespço de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então.  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se.

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t)$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e. W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então.  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se.

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + a_2t^2 = \lambda_1(-1+t) + a_2t^2 = \lambda_1(-$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então.  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se.

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com}$$

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e. W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então,  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se,

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com } \mathcal{W}_3 = [v_2, v_3].$$

Subespaco Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então,  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se,

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e

Subespaco Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então,  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se,

- $V = W_2 + W_3$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Subespaco Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então,  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se,

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ .

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_1 = [(1+t); (1+t^2)]; e, W_2 = [(-1+t)];$  $\mathcal{W}_3 = ?$  um subespoo de  $\mathcal{V}$  tal que  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  onde,  $\mathcal{W}_3 \neq \mathcal{W}_1$ . Então,  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se,

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ :

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaco vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaco vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $W_3 = [1:$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\}$ ; temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1; t^2] = [v_2, v_3]$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ . Porém, que complementem  $W_2$  a fim de gerar o espaco vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $W_3 = [1: t^2] = [v_2, v_3] \Rightarrow \forall p(t) \in \mathcal{V} : p(t) = \lambda_1(-1+t)$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ . Porém, que complementem  $W_2$  a fim de gerar o espaco vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1:t^2] = [v_2, v_3] \Rightarrow \forall p(t) \in \mathcal{V} : p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(1) = 0$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaco vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1; t^2] = [v_2, v_3] \Rightarrow \forall p(t) \in \mathcal{V} : p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; i = 1, 2, 3$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com } \mathcal{W}_3 = [v_2, v_3].$  Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\};$  temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V}$ ;  $p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2)$ ;  $\forall \lambda_i \in \mathbb{R}$ ; i=1,2,3  $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 +$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com } \mathcal{W}_3 = [v_2, v_3].$  Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\};$  temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V}$ ;  $p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2)$ ;  $\forall \lambda_i \in \mathbb{R}$ ; i=1,2,3  $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com } \mathcal{W}_3 = [v_2, v_3].$  Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\};$  temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; i=1,2,3 \Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com } \mathcal{W}_3 = [v_2, v_3].$  Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\};$  temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; i=1,2,3 \Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com } \mathcal{W}_3 = [v_2, v_3].$  Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\};$  temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; \; i=1,2,3$   $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2 \; ;$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_2$ : e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 = \lambda_1 (-1 + t) + \lambda_2 v_2 + \lambda_3 v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $W_2 \cap W_3 = \{0\}$ ; temos que estes subespacos  $W_2$  e  $W_3$  são SUPLEMENTARES, e  $W_3 \neq W_1$ .

Portanto, podemos escolher vetores para gerar  $W_3$  que não gerem  $W_1$  e nem  $W_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaco vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1; t^2] = [v_2, v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; i = 1, 2, 3$  $\Rightarrow p(t) = (-\lambda_1 + \lambda_2) \cdot 1 + (\lambda_1) \cdot t + (\lambda_3) \cdot t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2 : a_1 = \lambda_1$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3; \text{ com } \mathcal{W}_3 = [v_2, v_3].$  Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\};$  temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; i=1,2,3$   $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2 \; ; \; a_1 = \lambda_1; \; a_2 = \lambda_3$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\}$ ; temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; i=1,2,3$   $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2 \; ; \; a_1 = \lambda_1; \; a_2 = \lambda_3$   $\Rightarrow a_2 = \lambda_3;$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\}$ ; temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; i=1,2,3$   $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2 \; ; \; a_1 = \lambda_1; \; a_2 = \lambda_3$   $\Rightarrow a_2 = \lambda_3; \; a_1 = \lambda_1;$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\}$ ; temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V}$ ;  $p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2)$ ;  $\forall \lambda_i \in \mathbb{R}$ ; i=1,2,3  $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2$ ;  $a_1 = \lambda_1$ ;  $a_2 = \lambda_3$   $\Rightarrow a_2 = \lambda_3$ ;  $a_1 = \lambda_1$ ;  $a_0 = -a_1 + \lambda_2$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\}$ ; temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; \; i=1,2,3$   $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2 \; ; \; a_1 = \lambda_1; \; a_2 = \lambda_3$   $\Rightarrow a_2 = \lambda_3; \; a_1 = \lambda_1; \; a_0 = -a_1 + \lambda_2 \Rightarrow \mathcal{P}_2(\mathbb{R}) = (\mathcal{W}_2 + \mathcal{W}_3).$ 

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ ; e
- $W_2 \cap W_3 = \{0\}.$

 $\forall p(t) \in (\mathcal{W}_2 + \mathcal{W}_3) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 = \lambda_1(-1+t) + \lambda_2v_2 + \lambda_3v_3$ ; com  $\mathcal{W}_3 = [v_2, v_3]$ . Como,  $\mathcal{W}_2 \cap \mathcal{W}_3 = \{0\}$ ; temos que estes subespaços  $\mathcal{W}_2$  e  $\mathcal{W}_3$  são SUPLEMENTARES, e  $\mathcal{W}_3 \neq \mathcal{W}_1$ .

Portanto, podemos escolher vetores para gerar  $\mathcal{W}_3$  que não gerem  $\mathcal{W}_1$  e nem  $\mathcal{W}_2$ . Porém, que complementem  $\mathcal{W}_2$  a fim de gerar o espaço vetorial  $\mathcal{P}_2(\mathbb{R})$ ; por exemplo:  $\mathcal{W}_3 = [1;t^2] = [v_2,v_3] \Rightarrow \forall p(t) \in \mathcal{V} \; ; \; p(t) = \lambda_1(-1+t) + \lambda_2(1) + \lambda_3(t^2); \forall \lambda_i \in \mathbb{R}; \; i=1,2,3$   $\Rightarrow p(t) = (-\lambda_1 + \lambda_2).1 + (\lambda_1).t + (\lambda_3).t^2 \Rightarrow a_0 = -\lambda_1 + \lambda_2 \; ; \; a_1 = \lambda_1; \; a_2 = \lambda_3$   $\Rightarrow a_2 = \lambda_3; \; a_1 = \lambda_1; \; a_0 = -a_1 + \lambda_2 \Rightarrow \mathcal{P}_2(\mathbb{R}) = (\mathcal{W}_2 + \mathcal{W}_3).$ 

Subespaço Gerado

Exercícios: (respostas)

Subespaço Gerado

Exercícios: (respostas)  $W_2 = [(-1 + t)]; e$ 

Subespaço Gerado

Exercícios: (respostas)  $W_2 = [(-1+t)]$ ; e  $W_3 = [1, t^2]$ ;

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_2 = [(-1+t)]; e W_3 = [1, t^2];$  $\mathcal{V} = \mathcal{W}_2 \oplus \mathcal{W}_3$  se, e somente se,

• 
$$\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$$
;

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $\mathcal{V} = \mathcal{W}_2 + \mathcal{W}_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3);$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t)$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \ \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) +$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 +$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$
  
 $\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$ 

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$
  
 $\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$ 

$$\Rightarrow -\lambda_1 = \lambda_2; \lambda_1 = 0;$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$orall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$$

$$\Rightarrow -\lambda_1 = \lambda_2; \lambda_1 = 0; \lambda_2 = 0$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$egin{aligned} orall p(t) &\in (\mathcal{W}_2 \cap \mathcal{W}_3); \ \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2) \ \\ &\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2 \end{aligned}$$

$$\Rightarrow -\lambda_1=\lambda_2; \lambda_1=0; \lambda_3=0 \Rightarrow \lambda_2=\lambda_1=\lambda_3=0$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$$

$$\Rightarrow -\lambda_1 = \lambda_2; \lambda_1 = 0; \lambda_3 = 0 \Rightarrow \lambda_2 = \lambda_1 = \lambda_3 = 0$$

$$\Rightarrow (\mathcal{W}_2 \cap \mathcal{W}_3) = \{0\}$$

EXERCÍCIOS: (RESPOSTAS) 
$$W_2 = [(-1+t)]$$
; e  $W_3 = [1, t^2]$ ;  $V = W_2 \oplus W_3$  se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$$

$$\Rightarrow -\lambda_1 = \lambda_2; \lambda_1 = 0; \lambda_3 = 0 \Rightarrow \lambda_2 = \lambda_1 = \lambda_3 = 0$$

$$\Rightarrow (\mathcal{W}_2 \cap \mathcal{W}_3) = \{0\} \Rightarrow (\mathcal{W}_2 \oplus \mathcal{W}_3).$$

#### Subespacos Vetoriais Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_2 = [(-1+t)]$ ; e  $W_3 = [1, t^2]$ ;

$$\mathcal{V}=\mathcal{W}_2\oplus\mathcal{W}_3$$
 se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$$

$$\Rightarrow -\lambda_1=\lambda_2; \lambda_1=0; \lambda_3=0 \Rightarrow \lambda_2=\lambda_1=\lambda_3=0$$

$$\Rightarrow (\mathcal{W}_2 \cap \mathcal{W}_3) = \{0\} \Rightarrow (\mathcal{W}_2 \oplus \mathcal{W}_3).$$

Concluímos assim, que

$$\mathcal{P}_2(\mathbb{R}) = (\mathcal{W}_2 \oplus \mathcal{W}_3).$$

#### Subespacos Vetoriais Subespaço Gerado

EXERCÍCIOS: (RESPOSTAS)  $W_2 = [(-1+t)]$ ; e  $W_3 = [1, t^2]$ ;

$$\mathcal{V}=\mathcal{W}_2\oplus\mathcal{W}_3$$
 se, e somente se,

- $V = W_2 + W_3$ : OK!
- $W_2 \cap W_3 = \{0\}.$

$$\forall p(t) \in (\mathcal{W}_2 \cap \mathcal{W}_3); \Rightarrow p(t) = \lambda_1(-1+t) = \lambda_2(1) + \lambda_3(t^2)$$

$$\Rightarrow p(t) = (-\lambda_1).1 + (\lambda_1).t = (\lambda_2).1 + (\lambda_3).t^2$$

$$\Rightarrow -\lambda_1=\lambda_2; \lambda_1=0; \lambda_3=0 \Rightarrow \lambda_2=\lambda_1=\lambda_3=0$$

$$\Rightarrow (\mathcal{W}_2 \cap \mathcal{W}_3) = \{0\} \Rightarrow (\mathcal{W}_2 \oplus \mathcal{W}_3).$$

Concluímos assim, que

$$\mathcal{P}_2(\mathbb{R}) = (\mathcal{W}_2 \oplus \mathcal{W}_3).$$

Dependência e Independência Linear

Definição:

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ .

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI)

Dependência e Independência Linear

DEFINICÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

Dependência e Independência Linear

DEFINICÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^{n} \lambda_{i} v$$

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Caso contrário, dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD).

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Caso contrário, dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD).

Ou seia, se existir na COMBINAÇÃO LINEAR NULA.

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se. existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^n \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Caso contrário, dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD).

Ou seia, se existir na COMBINAÇÃO LINEAR NULA, pelo menos um escalar  $\lambda_i \neq 0$ 

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se, existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Caso contrário, dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD).

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se. existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^n \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Caso contrário, dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD).

$$v_i = -\frac{1}{\lambda_i}(\lambda_1v_1 + \lambda_2v_2 + \ldots + \lambda_{i-1}v_{i-1} + \lambda_{i+1}v_{i+1} + \ldots + \lambda_nv_n)$$

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se. existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^n \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Caso contrário, dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD).

$$v_i = -\frac{1}{\lambda_i}(\lambda_1v_1 + \lambda_2v_2 + \ldots + \lambda_{i-1}v_{i-1} + \lambda_{i+1}v_{i+1} + \ldots + \lambda_nv_n)$$

Dependência e Independência Linear

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE INDEPENDENTE (LI) se. existem escalares  $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{K}$  tais que

$$\sum_{i=1}^n \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$$

se, e somente se,  $\lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$ .

Caso contrário, dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD).

$$v_i = -\frac{1}{\lambda_i}(\lambda_1v_1 + \lambda_2v_2 + \ldots + \lambda_{i-1}v_{i-1} + \lambda_{i+1}v_{i+1} + \ldots + \lambda_nv_n)$$

Dependência e Independência Linear

EXEMPLO.1:

Dependência e Independência Linear

### EXEMPLO.1: Seja $\mathcal{V} = \mathbb{R}^2$ e sejam $S_1 = \{(2,0),$

Dependência e Independência Linear

#### EXEMPLO.1:

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{l^2},\underbrace{(0,-1)}_{l^2}\}\subset\mathcal{V}$ , e

Dependência e Independência Linear

#### EXEMPLO.1:

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},$ 

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2$ 

Dependência e Independência Linear

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{V}_1}, \underbrace{(0,-1)}_{\mathsf{V}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{V}_1}, \underbrace{(0,-1)}_{\mathsf{V}_2}, \underbrace{(0,1)}_{\mathsf{V}_3}\} \subset \mathcal{V}.$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_i v_i =$$

Dependência e Independência Linear

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em 
$$S_1$$
, para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1)$$

Dependência e Independência Linear

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em 
$$S_1$$
, para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0)$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_i v_i = \lambda_1(2,0) + \lambda_2(0,-1) = (0,0) \Rightarrow (2\lambda_1,-\lambda_2) = (0,0)$$

Dependência e Independência Linear

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em 
$$S_1$$
, para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,0) + \lambda_2(0,-1) = (0,0) \Rightarrow (2\lambda_1,-\lambda_2) = (0,0) \Rightarrow$$

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ . Então, em  $S_1$ , para  $\forall \lambda_1,\lambda_2\in\mathbb{R}$ ; 
$$\sum_{i=1}^2\lambda_iv_i=\lambda_1(2,0)+\lambda_2(0,-1)=(0,0)\Rightarrow(2\lambda_1,-\lambda_2)=(0,0)\Rightarrow\left\{\begin{array}{c}2\lambda_1=0\Rightarrow\lambda_1=0\\\\2\lambda_1=0\Rightarrow\lambda_1=0\end{array}\right.$$

Dependência e Independência Linear

$$\begin{split} \text{Seja } \mathcal{V} &= \mathbb{R}^2 \text{ e sejam } S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}, \text{ e } S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}. \\ \text{Então, em } S_1, \text{ para } \forall \lambda_1, \lambda_2 \in \mathbb{R}; \\ \sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,0) + \lambda_2(0,-1) = (0,0) \Rightarrow (2\lambda_1,-\lambda_2) = (0,0) \Rightarrow \begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ -\lambda_2 = 0 \Rightarrow \lambda_2 = 0 \end{cases} \end{split}$$

Dependência e Independência Linear

$$\begin{split} \text{Seja } \mathcal{V} &= \mathbb{R}^2 \text{ e sejam } S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}, \text{ e } S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}. \\ \text{Então, em } S_1, \text{ para } \forall \lambda_1, \lambda_2 \in \mathbb{R}; \\ \sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,0) + \lambda_2(0,-1) = (0,0) \Rightarrow (2\lambda_1,-\lambda_2) = (0,0) \Rightarrow \begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ -\lambda_2 = 0 \Rightarrow \lambda_2 = 0 \end{cases} \end{split}$$

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ . Então, em  $S_1$ , para  $\forall\lambda_1,\lambda_2\in\mathbb{R}$ ; 
$$\sum_{i=1}^2\lambda_iv_i=\lambda_1(2,0)+\lambda_2(0,-1)=(0,0)\Rightarrow(2\lambda_1,-\lambda_2)=(0,0)\Rightarrow\left\{\begin{array}{c}2\lambda_1=0\Rightarrow\lambda_1=0\\-\lambda_2=0\Rightarrow\lambda_2=0\end{array}\right.$$
 portanto, fazendo a COMBINAÇÃO LINEAR NULA;

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ . Então, em  $S_1$ , para  $\forall \lambda_1,\lambda_2\in\mathbb{R}$ ; 
$$\sum_{i=1}^2\lambda_iv_i=\lambda_1(2,0)+\lambda_2(0,-1)=(0,0)\Rightarrow(2\lambda_1,-\lambda_2)=(0,0)\Rightarrow\begin{cases}2\lambda_1=0\Rightarrow\lambda_1=0\\-\lambda_2=0\Rightarrow\lambda_2=0\end{cases}$$
 portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1=\lambda_2=0$ ;

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

Enquanto que em  $S_2$ ;

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_i v_i =$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_i v_i = \lambda_1(2,0) +$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1=\lambda_2=0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) +$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1)$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0)$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_i v_i = \lambda_1(2,0) + \lambda_2(0,-1) + \lambda_3(0,1) = (0,0) \Rightarrow (2\lambda_1, -\lambda_2 + \lambda_3) = (0,0)$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_i v_i = \lambda_1(2,0) + \lambda_2(0,-1) + \lambda_3(0,1) = (0,0) \Rightarrow (2\lambda_1, -\lambda_2 + \lambda_3) = (0,0) \Rightarrow$$

Dependência e Independência Linear

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_i v_i = \lambda_1(2,0) + \lambda_2(0,-1) + \lambda_3(0,1) = (0,0) \Rightarrow (2\lambda_1, -\lambda_2 + \lambda_3) = (0,0) \Rightarrow \begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \end{cases}$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}, \underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são linearmente independentes.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$
logo,

Dependência e Independência Linear

#### EXEMPLO.1:

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ .

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são linearmente independentes.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0}, \lambda_{1} = 0$$

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são linearmente independentes.

Enquanto que em  $S_2$ ; para  $\forall \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ ;

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0}, \lambda_{1} = 0 \text{ e } \lambda_{2} = \lambda_{3}$$

MAT A07 - Álgebra Linear A - Semestre Letivo Suplementar - 2021.1

Dependência e Independência Linear

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

Enquanto que em  $S_2$ ; para  $\forall \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ ;

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0} \lambda_{1} = 0 = \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) +$$

MAT A07 - Álgebra Linear A - Semestre Letivo Suplementar - 2021.1

Dependência e Independência Linear

### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são linearmente independentes.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0} \lambda_{1} = 0 = \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) + \lambda_{3}(0,-1) +$$

logo, 
$$\lambda_1 = 0$$
 e  $\lambda_2 = \lambda_3 \Rightarrow 0.(2,0) + \lambda_3(0,-1) + \lambda_3(0,-1)$ 

Dependência e Independência Linear

### EXEMPLO.1:

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ .

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são linearmente independentes.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0} \lambda_{1} = 0 \text{ e } \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) + \lambda_{3}(0,-1) + \lambda_{3}(0,1) =$$

logo, 
$$\lambda_1 = 0$$
 e  $\lambda_2 = \lambda_3 \Rightarrow 0.(2,0) + \lambda_3(0,-1) + \lambda_3(0,1) =$ 

Dependência e Independência Linear

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são linearmente independentes.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0} \lambda_{1} = 0 \text{ e } \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) + \lambda_{3}(0,-1) + \lambda_{3}(0,1) = (0,0)$$

logo, 
$$\lambda_1 = 0$$
 e  $\lambda_2 = \lambda_3 \Rightarrow 0.(2,0) + \lambda_3(0,-1) + \lambda_3(0,1) = (0,0)$ 

Dependência e Independência Linear

### EXEMPLO.1:

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ .

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1 = \lambda_2 = 0$ ; ou seja, os vetores em  $S_1$  são linearmente independentes.

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0} \lambda_{1} = 0 \text{ e } \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) + \lambda_{3}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (0,-1) = -(0,1);$$

logo, 
$$\lambda_1 = 0$$
 e  $\lambda_2 = \lambda_3 \Rightarrow 0.(2,0) + \lambda_3(0,-1) + \lambda_3(0,1) = (0,0) \Rightarrow (0,-1) = -(0,1)$ 

Dependência e Independência Linear

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1=\lambda_2=0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

Enquanto que em  $S_2$ ; para  $\forall \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ ;

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0} \lambda_{1} = 0 = \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) + \lambda_{3}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (0,-1) = -(0,1);$$

logo,  $\lambda_1 = 0$  e  $\lambda_2 = \lambda_3 \Rightarrow 0.(2,0) + \lambda_3(0,-1) + \lambda_3(0,1) = (0,0) \Rightarrow (0,-1) = -(0,1)$ 

ou seja, os vetores em  $S_2$  são **linearmente dependentes** 

Dependência e Independência Linear

#### EXEMPLO.1:

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ .

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1},-\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1=\lambda_2=0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

Enquanto que em  $S_2$ ; para  $\forall \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ ;

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0}, \lambda_{1} = 0 \text{ e } \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) + \lambda_{3}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (0,-1) = -(0,1);$$

ou seja, os vetores em  $S_2$  são **linearmente dependentes** pois; o vetor  $v_2$  pode ser escrito como combinação linear do vetor  $v_3$  e vice-versa.

Dependência e Independência Linear

#### EXEMPLO.1:

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\}\subset\mathcal{V}$ .

Então, em  $S_1$ , para  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ ;

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2}) = (0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} = 0 \Rightarrow \lambda_{2} = 0 \end{cases}$$

portanto, fazendo a COMBINAÇÃO LINEAR NULA; obtemos  $\lambda_1=\lambda_2=0$ ; ou seja, os vetores em  $S_1$  são **linearmente independentes**.

Enquanto que em  $S_2$ ; para  $\forall \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ ;

$$\sum_{i=1}^{3} \lambda_{i} v_{i} = \lambda_{1}(2,0) + \lambda_{2}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (2\lambda_{1}, -\lambda_{2} + \lambda_{3}) = (0,0) \Rightarrow$$

$$\begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \\ -\lambda_{2} + \lambda_{3} = 0 \Rightarrow \lambda_{2} = \lambda_{3} \end{cases}$$

$$\log_{0}, \lambda_{1} = 0 \text{ e } \lambda_{2} = \lambda_{3} \Rightarrow 0.(2,0) + \lambda_{3}(0,-1) + \lambda_{3}(0,1) = (0,0) \Rightarrow (0,-1) = -(0,1);$$

logo,  $\lambda_1 = 0$  e  $\lambda_2 = \lambda_3 \Rightarrow 0.(2,0) + \lambda_3(0,-1) + \lambda_3(0,1) = (0,0) \Rightarrow (0,-1) = -(0,1)$ ; ou seja, os vetores em  $S_2$  são **linearmente dependentes** pois; o vetor  $v_2$  pode ser escrito como combinação linear do vetor  $v_3$  e vice-versa.

Dependência e Independência Linear

EXEMPLO.2:

```
EXEMPLO.2:
Seja \mathcal{V} = \mathbb{R}^3 e seja S = \{(2,3,-1),
```

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{v_1},\underbrace{(0,1,1)}_{v_2}\}\subset\mathcal{V}.$ 

Dependência e Independência Linear

#### EXEMPLO.2:

Seja 
$$\mathcal{V} = \mathbb{R}^3$$
 e seja  $S = \{\underbrace{(2,3,-1)}_{1},\underbrace{(0,1,1)}_{1}\} \subset \mathcal{V}.$ 

Então,  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ 

Dependência e Independência Linear

#### EXEMPLO.2:

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\}\subset\mathcal{V}.$ 

Dependência e Independência Linear

Seja 
$$\mathcal{V} = \mathbb{R}^3$$
 e seja  $S = \{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\} \subset \mathcal{V}.$ 

$$\sum_{i=1}^{2} \lambda_i v_i =$$

Dependência e Independência Linear

Seja 
$$\mathcal{V} = \mathbb{R}^3$$
 e seja  $S = \{\underbrace{(2,3,-1)}_{v_1},\underbrace{(0,1,1)}_{v_2}\} \subset \mathcal{V}.$ 

Então,  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ , fazendo a combinação linear nula:  $\sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) =$ 

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,3,-1) + \lambda_{2}(0,1,1) =$$

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{V_1},\underbrace{(0,1,1)}_{V_2}\}\subset\mathcal{V}.$ 

Então,  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ , fazendo a combinação linear nula:  $\sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) = (0,0,0)$ 

$$\sum_{i=1}^{2} \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) = (0,0,0)$$

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{\mathcal{V}_1},\underbrace{(0,1,1)}_{\mathcal{V}_2}\}\subset\mathcal{V}.$ 

Então, 
$$\forall \lambda_1, \lambda_2 \in \mathbb{R}$$
, fazendo a combinação linear nula: 
$$\sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) = (0,0,0) \Rightarrow (2\lambda_1, \lambda_2) = (0,0) \Rightarrow (2\lambda_1, \lambda_$$

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{\mathcal{V}},\underbrace{(0,1,1)}_{\mathcal{V}}\}\subset\mathcal{V}.$ 

Então, 
$$\forall \lambda_1, \lambda_2 \in \mathbb{R}$$
, fazendo a combinação linear nula: 
$$\sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) = (0,0,0) \Rightarrow (2\lambda_1,3\lambda_1+\lambda_2,$$

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{\mathcal{V}},\underbrace{(0,1,1)}_{\mathcal{V}}\}\subset\mathcal{V}.$ 

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,3,-1) + \lambda_{2}(0,1,1) = (0,0,0) \Rightarrow (2\lambda_{1},3\lambda_{1} + \lambda_{2},\lambda_{2} - \lambda_{1})$$

Dependência e Independência Linear

#### EXEMPLO.2:

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\}\subset\mathcal{V}.$ 

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,3,-1) + \lambda_{2}(0,1,1) = (0,0,0) \Rightarrow (2\lambda_{1},3\lambda_{1}+\lambda_{2},\lambda_{2}-\lambda_{1}) =$$

Dependência e Independência Linear

#### EXEMPLO.2:

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{\mathcal{V}},\underbrace{(0,1,1)}_{\mathcal{V}}\}\subset\mathcal{V}.$ 

$$\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,3,-1) + \lambda_{2}(0,1,1) = (0,0,0) \Rightarrow (2\lambda_{1},3\lambda_{1}+\lambda_{2},\lambda_{2}-\lambda_{1}) = (0,0,0) \Rightarrow$$

```
EXEMPLO.2:
Seja \mathcal{V} = \mathbb{R}^3 e seja S = \{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\} \subset \mathcal{V}.
Então, \forall \lambda_1, \lambda_2 \in \mathbb{R}, fazendo a combinação linear nula: \sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) = (0,0,0) \Rightarrow (2\lambda_1,3\lambda_1+\lambda_2,\lambda_2-\lambda_1) = (0,0,0) \Rightarrow
```

```
EXEMPLO.2:
Seja \mathcal{V} = \mathbb{R}^3 e seja S = \{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\} \subset \mathcal{V}.
 Então, \forall \lambda_1, \lambda_2 \in \mathbb{R}, fazendo a combinação linear nula:
\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,3,-1) + \lambda_{2}(0,1,1) = (0,0,0) \Rightarrow (2\lambda_{1},3\lambda_{1}+\lambda_{2},\lambda_{2}-\lambda_{1}) = (0,0,0) \Rightarrow \begin{cases} 2\lambda_{1} = 0 \Rightarrow \lambda_{1} = 0 \end{cases}
```

```
EXEMPLO.2:
Seja \mathcal{V}=\mathbb{R}^3 e seja S=\{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\}\subset\mathcal{V}.
Então. \forall \lambda_1, \lambda_2 \in \mathbb{R}, fazendo a combinação linear nula:
\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2,3,-1) + \lambda_{2}(0,1,1) = (0,0,0) \Rightarrow (2\lambda_{1},3\lambda_{1}+\lambda_{2},\lambda_{2}-\lambda_{1}) = (0,0,0) \Rightarrow
\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \end{cases}
```

```
EXEMPLO.2:
Seja \mathcal{V}=\mathbb{R}^3 e seja S=\{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\}\subset\mathcal{V}.
Então. \forall \lambda_1, \lambda_2 \in \mathbb{R}, fazendo a combinação linear nula:
\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2, 3, -1) + \lambda_{2}(0, 1, 1) = (0, 0, 0) \Rightarrow (2\lambda_{1}, 3\lambda_{1} + \lambda_{2}, \lambda_{2} - \lambda_{1}) = (0, 0, 0) \Rightarrow
\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -3\lambda_1 \Rightarrow \lambda_2 = 0 \end{cases}
```

```
EXEMPLO.2:
Seja \mathcal{V}=\mathbb{R}^3 e seja S=\{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\}\subset\mathcal{V}.
Então, \forall \lambda_1, \lambda_2 \in \mathbb{R}, fazendo a combinação linear nula:
\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2, 3, -1) + \lambda_{2}(0, 1, 1) = (0, 0, 0) \Rightarrow (2\lambda_{1}, 3\lambda_{1} + \lambda_{2}, \lambda_{2} - \lambda_{1}) = (0, 0, 0) \Rightarrow
\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -3\lambda_1 \Rightarrow \lambda_2 = 0 \\ -\lambda_1 + \lambda_2 = 0 \end{cases}
```

$$\begin{split} &\text{Exemplo.2:}\\ &\text{Seja } \mathcal{V} = \mathbb{R}^3 \text{ e seja } S = \{\underbrace{(2,3,-1)}_{v_1}, \underbrace{(0,1,1)}_{v_2}\} \subset \mathcal{V}.\\ &\text{Então, } \forall \lambda_1, \lambda_2 \in \mathbb{R}, \text{ fazendo a combinação linear nula:}\\ &\sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) = (0,0,0) \Rightarrow (2\lambda_1,3\lambda_1+\lambda_2,\lambda_2-\lambda_1) = (0,0,0) \Rightarrow \\ &\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -3\lambda_1 \Rightarrow \lambda_2 = 0 \\ -\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_1 = \lambda_2 = 0 \end{cases} \end{split}$$

$$\begin{split} &\text{Exemplo.2:}\\ &\text{Seja } \mathcal{V} = \mathbb{R}^3 \text{ e seja } S = \{\underbrace{(2,3,-1)}_{v_1}, \underbrace{(0,1,1)}_{v_2}\} \subset \mathcal{V}.\\ &\text{Então, } \forall \lambda_1, \lambda_2 \in \mathbb{R}, \text{ fazendo a combinação linear nula:}\\ &\sum_{i=1}^2 \lambda_i v_i = \lambda_1(2,3,-1) + \lambda_2(0,1,1) = (0,0,0) \Rightarrow (2\lambda_1,3\lambda_1+\lambda_2,\lambda_2-\lambda_1) = (0,0,0) \Rightarrow \\ &\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -3\lambda_1 \Rightarrow \lambda_2 = 0 \\ -\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_1 = \lambda_2 = 0 \end{cases} \end{split}$$

```
Exemplo.2:
Seja \mathcal{V}=\mathbb{R}^3 e seja S=\{\underbrace{(2,3,-1)},\underbrace{(0,1,1)}\}\subset\mathcal{V}.
Então, \forall \lambda_1, \lambda_2 \in \mathbb{R}, fazendo a combinação linear nula:
\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2, 3, -1) + \lambda_{2}(0, 1, 1) = (0, 0, 0) \Rightarrow (2\lambda_{1}, 3\lambda_{1} + \lambda_{2}, \lambda_{2} - \lambda_{1}) = (0, 0, 0) \Rightarrow
\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -3\lambda_1 \Rightarrow \lambda_2 = 0 \\ -\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_1 = \lambda_2 = 0 \end{cases}
Como, \lambda_1 = \lambda_2 = 0 \Rightarrow
```

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{v_1},\underbrace{(0,1,1)}_{v_2}\}\subset\mathcal{V}.$ 

Então,  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ , fazendo a combinação linear nula:  $\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2, 3, -1) + \lambda_{2}(0, 1, 1) = (0, 0, 0) \Rightarrow (2\lambda_{1}, 3\lambda_{1} + \lambda_{2}, \lambda_{2} - \lambda_{1}) = (0, 0, 0) \Rightarrow$ 

$$\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -3\lambda_1 \Rightarrow \lambda_2 = 0 \\ -\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_1 = \lambda_2 = 0 \end{cases}$$

Como,  $\lambda_1 = \lambda_2 = 0 \Rightarrow$  os vetores em  $S_1$  são LI.

Dependência e Independência Linear

Seja 
$$\mathcal{V}=\mathbb{R}^3$$
 e seja  $S=\{\underbrace{(2,3,-1)}_{v_1},\underbrace{(0,1,1)}_{v_2}\}\subset\mathcal{V}.$ 

Então,  $\forall \lambda_1, \lambda_2 \in \mathbb{R}$ , fazendo a combinação linear nula:  $\sum_{i=1}^{2} \lambda_{i} v_{i} = \lambda_{1}(2, 3, -1) + \lambda_{2}(0, 1, 1) = (0, 0, 0) \Rightarrow (2\lambda_{1}, 3\lambda_{1} + \lambda_{2}, \lambda_{2} - \lambda_{1}) = (0, 0, 0) \Rightarrow$ 

$$\begin{cases} 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0 \\ 3\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -3\lambda_1 \Rightarrow \lambda_2 = 0 \\ -\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_1 = \lambda_2 = 0 \end{cases}$$

Como,  $\lambda_1 = \lambda_2 = 0 \Rightarrow$  os vetores em  $S_1$  são LI.

Dependência e Independência Linear

OBSERVAÇÕES:

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ .

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ .

Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$ 

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um sistema de equações lineares HOMOGÊNEO:

Dependência e Independência Linear

#### OBSERVAÇÕES:

HOMOGÊNEO: então.

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^n \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \dots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES

15 MAT A07 - Álgebra Linear A - Semestre Letivo Suplementar - 2021.1

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO: então.

• Dizemos que *S* é LINEARMENTE INDEPENDENTE (LI)

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO: então.

• Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO: então.

• Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é possível e determinado,

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO; então,

• Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é **possível e determinado**, isto é, possui apenas a solução TRIVIAL:

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO: então.

• Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é possível e determinado, isto é, possui apenas a solução TRIVIAL:

$$\lambda_1 = \lambda_2 = \ldots = \lambda_i = \ldots = \lambda_n = 0; \forall i = 1, \ldots, n.$$

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO: então.

- Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é possível e determinado, isto é, possui apenas a solução TRIVIAL:  $\lambda_1 = \lambda_2 = \ldots = \lambda_i = \ldots = \lambda_n = 0; \forall i = 1, \ldots, n.$
- Dizemos que  $S \subset V$  é LINEARMENTE DEPENDENTE (LD)

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO: então.

- Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é possível e determinado, isto é, possui apenas a solução TRIVIAL:  $\lambda_1 = \lambda_2 = \ldots = \lambda_i = \ldots = \lambda_n = 0; \forall i = 1, \ldots, n.$
- Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE DEPENDENTE (LD) se, e somente se, o SISTEMA HOMOGÊNEO

Dependência e Independência Linear

#### OBSERVAÇÕES:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^{n} \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO: então.

- Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é possível e determinado, isto é, possui apenas a solução TRIVIAL:  $\lambda_1 = \lambda_2 = \ldots = \lambda_i = \ldots = \lambda_n = 0; \forall i = 1, \ldots, n.$
- Dizemos que  $S \subset \mathcal{V}$  é LINEARMENTE DEPENDENTE (LD) se, e somente se, o SISTEMA HOMOGÊNEO é possível e indeterminado.

Dependência e Independência Linear

#### Observações:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^n \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \dots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO; então,

- Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é **possível e determinado**, isto é, <u>possui apenas a solução TRIVIAL</u>:  $\lambda_1 = \lambda_2 = \ldots = \lambda_i = \ldots = \lambda_n = 0; \forall i = 1, \ldots, n.$
- Dizemos que *S* ⊂ *V* é LINEARMENTE DEPENDENTE (LD) se, e somente se, o SISTEMA HOMOGÊNEO é **possível e indeterminado**, isto é, <u>possui infinitas soluções</u>, incluindo a <u>TRIVIAL</u>.

Dependência e Independência Linear

#### Observações:

Seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** do espaço vetorial  $\mathcal{V}$ . Note que  $\sum_{i=1}^n \lambda_i v_i = \lambda_1 v_1 + \lambda_2 v_2 + \dots + \lambda_n v_n = 0$  é um SISTEMA DE EQUAÇÕES LINEARES HOMOGÊNEO; então,

- Dizemos que S é LINEARMENTE INDEPENDENTE (LI) se, e somente se, o SISTEMA HOMOGÊNEO é **possível e determinado**, isto é, <u>possui apenas a solução TRIVIAL</u>:  $\lambda_1 = \lambda_2 = \ldots = \lambda_i = \ldots = \lambda_n = 0; \forall i = 1, \ldots, n.$
- Dizemos que *S* ⊂ *V* é LINEARMENTE DEPENDENTE (LD) se, e somente se, o SISTEMA HOMOGÊNEO é **possível e indeterminado**, isto é, <u>possui infinitas soluções</u>, incluindo a <u>TRIVIAL</u>.

Base

Definição:

Base

DEFINIÇÃO: Seja  $\mathcal V$  um espaço vetorial, **finitamente gerado**, sobre o corpo  $\mathbb K$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um subconjunto finito de  $\mathcal{V}$ .

Base

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial, finitamente gerado, sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  forma uma BASE

Base

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial, finitamente gerado, sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  forma uma BASE para o espaço vetorial  $\mathcal{V}$ , se, e somente se,

Base

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial, finitamente gerado, sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  forma uma BASE para o espaço vetorial  $\mathcal{V}$ , se, e somente se,

(i) S GERA  $\mathcal{V}$ : e

Base

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial, **finitamente gerado**, sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  forma uma BASE para o espaço vetorial  $\mathcal{V}$ , se, e somente se,

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é LINEARMENTE INDEPENDENTE.

Base

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial, **finitamente gerado**, sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  forma uma BASE para o espaço vetorial  $\mathcal{V}$ , se, e somente se,

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é LINEARMENTE INDEPENDENTE.

Base

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial, **finitamente gerado**, sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  forma uma BASE para o espaço vetorial  $\mathcal{V}$ , se, e somente se,

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é LINEARMENTE INDEPENDENTE.

Ou seja:

(i)  $\forall u \in \mathcal{V}$ 

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é linearmente independente.

(i) 
$$\forall u \in \mathcal{V} \Rightarrow u = \sum_{i=1}^{n} \lambda_i v_i$$
;

Base

DEFINIÇÃO: Seja  $\mathcal{V}$  um espaço vetorial, **finitamente gerado**, sobre o corpo  $\mathbb{K}$ , e seja  $S = \{v_1, v_2, \dots, v_n\}; n \in \mathbb{N}^*$ , um **subconjunto finito** de  $\mathcal{V}$ . Dizemos que  $S \subset \mathcal{V}$  forma uma BASE para o espaço vetorial  $\mathcal{V}$ , se, e somente se,

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é linearmente independente.

(i) 
$$\forall u \in \mathcal{V} \Rightarrow u = \sum_{i=1}^{n} \lambda_i v_i; \forall \lambda_i \in \mathbb{K}; e$$

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é linearmente independente.

- (i)  $\forall u \in \mathcal{V} \Rightarrow u = \sum_{i=1}^{n} \lambda_i v_i; \forall \lambda_i \in \mathbb{K}$ ; e
- (ii)  $\sum_{i=1}^{n} \lambda_i v_i = 0$

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é linearmente independente.

- (i)  $\forall u \in \mathcal{V} \Rightarrow u = \sum_{i=1}^{n} \lambda_i v_i; \forall \lambda_i \in \mathbb{K}; e$
- (ii)  $\sum_{i=1}^{n} \lambda_i v_i = 0 \Leftrightarrow \lambda_i = 0; \forall i = 1, \dots, n.$

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é linearmente independente.

#### Ou seja:

- (i)  $\forall u \in \mathcal{V} \Rightarrow u = \sum_{i=1}^{n} \lambda_i v_i; \forall \lambda_i \in \mathbb{K}; e$
- (ii)  $\sum_{i=1}^{n} \lambda_i v_i = 0 \Leftrightarrow \lambda_i = 0; \forall i = 1, \dots, n.$

### Notação:

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é linearmente independente.

#### Ou seja:

- (i)  $\forall u \in \mathcal{V} \Rightarrow u = \sum_{i=1}^{n} \lambda_i v_i; \forall \lambda_i \in \mathbb{K}; e$
- (ii)  $\sum_{i=1}^{n} \lambda_i v_i = 0 \Leftrightarrow \lambda_i = 0; \forall i = 1, \dots, n.$

### Notação:

$$\beta_{\mathcal{V}} = \{v_1, v_2, \dots, v_n\}$$

- (i) S GERA  $\mathcal{V}$ : e
- (ii) S é linearmente independente.

#### Ou seja:

- (i)  $\forall u \in \mathcal{V} \Rightarrow u = \sum_{i=1}^{n} \lambda_i v_i; \forall \lambda_i \in \mathbb{K}; e$
- (ii)  $\sum_{i=1}^{n} \lambda_i v_i = 0 \Leftrightarrow \lambda_i = 0; \forall i = 1, \dots, n.$

### Notação:

$$\beta_{\mathcal{V}} = \{v_1, v_2, \dots, v_n\}$$

Base

Base

#### EXEMPLO.1:

Seja  $\mathcal{V} = \mathbb{R}^2$  e sejam  $S_1 = \{(2,0),$ 

Base

#### EXEMPLO.1:

Seja  $\mathcal{V}=\mathbb{R}^2$  e sejam  $S_1=\{\underbrace{(2,0)},\underbrace{(0,-1)}\}\subset\mathcal{V}$ , e

Base

Seja 
$$V = \mathbb{R}^2$$
 e sejam  $S_1 = \{(2,0), (0,-1)\} \subset V$ , e  $S_2 = \{(2,0), (2,0)\}$ 

Base

Seja 
$$\mathcal{V}=\mathbb{R}^2$$
 e sejam  $S_1=\{\underbrace{(2,0)}_{\mathcal{V}_1},\underbrace{(0,-1)}_{\mathcal{V}_2}\}\subset\mathcal{V}$ , e  $S_2=\{\underbrace{(2,0)}_{\mathcal{V}_1},\underbrace{(0,-1)}_{\mathcal{V}_2},\underbrace{($ 

Base

#### EXEMPLO.1:

Seja  $\mathcal{V} = \mathbb{R}^2$  e sejam  $S_1 = \{(2,0), (0,-1)\} \subset \mathcal{V}$ , e  $S_2 = \{(2,0), (0,-1), (0,1)\} \subset \mathcal{V}$ .

Base

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

(i)  $S_1$  GERA  $\mathcal{V}$ ; pois

Base

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

(i)  $S_1$  GERA  $\mathcal{V}$ : pois  $\forall u \in \mathbb{R}^2$ 

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

(i) 
$$S_1$$
 GERA  $V$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i$ 

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

(i) 
$$S_1$$
 GERA  $V$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2)$ ;

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1}, \underbrace{(0,-1)}_{\mathsf{v}_2}, \underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

(i) 
$$S_1$$
 GERA  $\mathcal{V}$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V}\text{, e}\ S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é linearmente independente; pois

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{v_1},\underbrace{(0,-1)}_{v_2},\underbrace{(0,1)}_{v_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_{i} v_{i} = (0,0)$

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2},\underbrace{(0,1)}_{\mathsf{V}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Base

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2},\underbrace{(0,1)}_{\mathsf{V}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto,  $S_1$  forma uma BASE para  $\mathcal{V}$ :

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{\mathcal{V}},$ 

Base

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{\mathcal{V}_2}, \underbrace{(0,-1)}_{\mathcal{V}_2}\}.$ 

Base

#### EXEMPLO.1:

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2},\underbrace{(0,1)}_{\mathsf{V}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{\mathcal{V}}, \underbrace{(0,-1)}_{\mathcal{V}}\}.$ 

Enquanto que  $S_2$  NÃO forma uma BASE para  $\mathcal{V}$ :

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{\mathcal{V}}, \underbrace{(0,-1)}_{\mathcal{V}}\}.$ 

Enquanto que  $S_2$  NÃO forma uma BASE para  $\mathcal{V}$ :

(i)  $S_2$  GERA  $\mathcal{V}$ ; pois

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2},\underbrace{(0,1)}_{\mathsf{V}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{\mathcal{V}_2}, \underbrace{(0,-1)}_{\mathcal{V}_2}\}.$ 

Enquanto que  $S_2$  NÃO forma uma BASE para  $\mathcal{V}$ :

(i)  $S_2$  GERA  $\mathcal{V}$ : pois  $\forall \mu \in \mathbb{R}^2$ 

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\}.$ 

(i) 
$$S_2$$
 GERA  $V$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^3 \lambda_i v_i$ 

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{v_1}, \underbrace{(0,-1)}_{v_2}\}.$ 

(i) 
$$S_2$$
 GERA  $V$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^3 \lambda_i v_i = (2\lambda_1, -\lambda_2 + \lambda_3)$ ;

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2},\underbrace{(0,1)}_{\mathsf{V}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{V_1}, \underbrace{(0,-1)}_{V_2}\}.$ 

(i) 
$$S_2$$
 GERA  $\mathcal{V}$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^3 \lambda_i v_i = (2\lambda_1, -\lambda_2 + \lambda_3); \forall \lambda_i \in \mathbb{R}$ ; mas,

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2}\} \subset \mathcal{V}\text{, e } S_2 = \{\underbrace{(2,0)}_{\mathsf{V}_1},\underbrace{(0,-1)}_{\mathsf{V}_2},\underbrace{(0,1)}_{\mathsf{V}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA  $\mathcal{V}$ : pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ : e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{V_1}, \underbrace{(0,-1)}_{V_2}\}.$ 

- (i)  $S_2$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^3 \lambda_i v_i = (2\lambda_1, -\lambda_2 + \lambda_3); \forall \lambda_i \in \mathbb{R}$ ; mas,
- (ii)  $S_1$  NÃO É LINEARMENTE INDEPENDENTE; pois

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA  $\mathcal{V}$ : pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ : e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i = 1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{V_1}, \underbrace{(0,-1)}_{V_2}\}.$ 

- (i)  $S_2$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^3 \lambda_i v_i = (2\lambda_1, -\lambda_2 + \lambda_3); \forall \lambda_i \in \mathbb{R}$ ; mas,
- (ii)  $S_1$  NÃO É LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{3} \lambda_i v_i = (0,0)$ ; NÃO é apenas a TRIVIAL;

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i=1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{\mathcal{V}_2}, \underbrace{(0,-1)}_{\mathcal{V}_2}\}.$ 

Enquanto que  $S_2$  NÃO forma uma BASE para V:

- (i)  $S_2$  GERA  $\mathcal{V}$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^3 \lambda_i v_i = (2\lambda_1, -\lambda_2 + \lambda_3); \forall \lambda_i \in \mathbb{R}$ ; mas,
- (ii)  $S_1$  NÃO É LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{3} \lambda_i v_i = (0,0)$ ; NÃO É apenas a TRIVIAL; o sistema homogêneo obtido pela combinação linear nula possui infinitas soluções.

$$\mathsf{Seja}\ \mathcal{V} = \mathbb{R}^2\ \mathsf{e}\ \mathsf{sejam}\ S_1 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2}\} \subset \mathcal{V},\ \mathsf{e}\ S_2 = \{\underbrace{(2,0)}_{\mathsf{v}_1},\underbrace{(0,-1)}_{\mathsf{v}_2},\underbrace{(0,1)}_{\mathsf{v}_3}\} \subset \mathcal{V}.$$

- (i)  $S_1$  GERA V; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^2 \lambda_i v_i = (2\lambda_1, -\lambda_2); \forall \lambda_i \in \mathbb{R}$ ; e
- (ii)  $S_1$  é LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{2} \lambda_i v_i = (0,0) \Leftrightarrow \lambda_i = 0; \forall i=1,2$ ; é apenas a TRIVIAL.

Portanto, 
$$S_1$$
 forma uma BASE para  $\mathcal{V}$ :  $\beta_{\mathbb{R}^2} = \{\underbrace{(2,0)}_{\mathcal{V}_2}, \underbrace{(0,-1)}_{\mathcal{V}_2}\}.$ 

Enquanto que  $S_2$  NÃO forma uma BASE para V:

- (i)  $S_2$  GERA  $\mathcal{V}$ ; pois  $\forall u \in \mathbb{R}^2 \Rightarrow u = \sum_{i=1}^3 \lambda_i v_i = (2\lambda_1, -\lambda_2 + \lambda_3); \forall \lambda_i \in \mathbb{R}$ ; mas,
- (ii)  $S_1$  NÃO É LINEARMENTE INDEPENDENTE; pois a solução do sistema homogêneo :  $\sum_{i=1}^{3} \lambda_i v_i = (0,0)$ ; NÃO É apenas a TRIVIAL; o sistema homogêneo obtido pela combinação linear nula possui infinitas soluções.

Base

Exercícios:

Base

### Exercícios:

Sejam  $\mathcal{V}$  um espaço vetorial finitamente gerado sobre um corpo  $\mathbb{K}$ ; e  $\beta_{\mathcal{V}}$  uma base de  $\mathcal{V}$ .

Base

#### Exercícios:

Base

#### Exercícios:

1. 
$$\mathcal{V} = \mathbb{R}^3$$

Base

#### Exercícios:

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$
- 5.  $\mathcal{V} = \mathcal{P}_3(\mathbb{R})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$
- 5.  $\mathcal{V} = \mathcal{P}_3(\mathbb{R})$
- 6.  $\mathcal{V} = \mathcal{P}_3(\mathbb{C})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$
- 5.  $\mathcal{V} = \mathcal{P}_3(\mathbb{R})$
- 6.  $\mathcal{V} = \mathcal{P}_3(\mathbb{C})$

## Espaços Vetoriais Base

Exercícios: (respostas)

Base

## Exercícios: (respostas)

1.  $\mathcal{V} = \mathbb{R}^3$ 

Base

# EXERCÍCIOS: (RESPOSTAS)

1.  $\mathcal{V} = \mathbb{R}^3$  $\forall u \in \mathbb{R}^3$ 

Base

1. 
$$V = \mathbb{R}^3$$
  
 $\forall u \in \mathbb{R}^3 \Rightarrow u = (x, y, z) =$ 

1. 
$$\mathcal{V} = \mathbb{R}^3$$
  
 $\forall u \in \mathbb{R}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0)$ 

1. 
$$V = \mathbb{R}^3$$
  
 $\forall u \in \mathbb{R}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0)$ 

1. 
$$V = \mathbb{R}^3$$
  
 $\forall u \in \mathbb{R}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$ 

1. 
$$\mathcal{V} = \mathbb{R}^3$$
  
 $\forall u \in \mathbb{R}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^3 = [\underbrace{(1, 0, 0)}_{v_1}; \underbrace{(0, 1, 0)}_{v_2}; \underbrace{(0, 0, 1)}_{v_3}]$ 

1. 
$$\mathcal{V} = \mathbb{R}^3$$
  
 $\forall u \in \mathbb{R}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^3 = [\underbrace{(1, 0, 0)}_{v_1}; \underbrace{(0, 1, 0)}_{v_2}; \underbrace{(0, 0, 1)}_{v_3}] \in \sum_{i=1}^3 \lambda_i v_i = 0 = (0, 0, 0) \Leftrightarrow$ 

1. 
$$\mathcal{V} = \mathbb{R}^3$$
  
 $\forall u \in \mathbb{R}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^3 = [\underbrace{(1, 0, 0)}_{Y_2}; \underbrace{(0, 1, 0)}_{Y_2}; \underbrace{(0, 0, 1)}_{Y_2}] \in \sum_{i=1}^3 \lambda_i v_i = 0 = (0, 0, 0) \Leftrightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI}$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \text{ e } \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \text{ \'e LI } \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \text{ e } \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \text{ \'e LI } \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \in \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

 $2 \mathcal{V} = \mathbb{C}^3$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

2. 
$$\mathcal{V} = \mathbb{C}^3$$
  $\forall \mu \in \mathbb{C}^3$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

2. 
$$V = \mathbb{C}^3$$
  
 $\forall u \in \mathbb{C}^3 \Rightarrow u = (x, y, z) =$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

2. 
$$V = \mathbb{C}^3$$
  
 $\forall u \in \mathbb{C}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0)$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

2. 
$$V = \mathbb{C}^3$$
  
 $\forall u \in \mathbb{C}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0)$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

2. 
$$V = \mathbb{C}^3$$
  
 $\forall u \in \mathbb{C}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$   
2.  $\mathcal{V} = \mathbb{C}^{3}$   
 $\forall u \in \mathbb{C}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{C}^{3} = [(1, 0, 0); (0, 1, 0); (0, 0, 1)]$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$   
2.  $\mathcal{V} = \mathbb{C}^{3}$   
 $\forall u \in \mathbb{C}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{C}^{3} = [(1, 0, 0); (0, 1, 0); (0, 0, 1)] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0)}_{v_{1}}; \underbrace{(0, 1, 0)}_{v_{2}}; \underbrace{(0, 0, 1)}_{v_{3}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$ 

2. 
$$\mathcal{V} = \mathbb{C}^3$$
  
 $\forall u \in \mathbb{C}^3 \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{C}^3 = [\underbrace{(1, 0, 0)}_{y_1}; \underbrace{(0, 1, 0)}_{y_2}; \underbrace{(0, 0, 1)}_{y_3}] \in \sum_{i=1}^3 \lambda_i v_i = 0 = (0, 0, 0) \Leftrightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0$ 

## Exercícios: (respostas)

1. 
$$V = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$   
2.  $V = \mathbb{C}^{3}$   
 $\forall u \in \mathbb{C}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{C}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{2}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI}$ 

## Exercícios: (respostas)

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$   
2.  $\mathcal{V} = \mathbb{C}^{3}$   
 $\forall u \in \mathbb{C}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{C}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{2}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{C}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$   
2.  $\mathcal{V} = \mathbb{C}^{3}$   
 $\forall u \in \mathbb{C}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{C}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{C}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{C}.$ 

1. 
$$\mathcal{V} = \mathbb{R}^{3}$$
  
 $\forall u \in \mathbb{R}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{R}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{R}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{R}.$   
2.  $\mathcal{V} = \mathbb{C}^{3}$   
 $\forall u \in \mathbb{C}^{3} \Rightarrow u = (x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$   
 $\Rightarrow \mathbb{C}^{3} = [\underbrace{(1, 0, 0); (0, 1, 0); (0, 0, 1)}_{v_{1}}] \in \sum_{i=1}^{3} \lambda_{i} v_{i} = 0 = (0, 0, 0) \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = 0$   
 $\Rightarrow \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\} \notin \mathbf{LI} \Rightarrow \beta_{\mathbb{C}^{3}} = \{(1, 0, 0); (0, 1, 0); (0, 0, 1)\}; \forall \lambda_{i} \in \mathbb{C}.$ 

Base

Exercícios: (respostas)

Base

EXERCÍCIOS: (RESPOSTAS)

Base

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = egin{pmatrix} a_{11} & a_{12} & a_{13} \ a_{21} & a_{22} & a_{23} \ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Base

3. 
$$V = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Base

3. 
$$V = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Base

3. 
$$V = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Base

# EXERCÍCIOS: (RESPOSTAS)

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{M}_3(\mathbb{R}) =$$

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_3(\mathbb{R}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \end{split}$$

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ;$$

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_3(\mathbb{R}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0$$

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix}$$

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0$$

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\$$

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0$$

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0$$

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0$$

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ &+ a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\mathcal{M}_{3}(\mathbb{R}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0$$

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33$$

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33$$

3. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{R})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{R})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33$$

Base

Base

EXERCÍCIOS: (RESPOSTAS)

4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$ ; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

Base

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Base

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Base

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Base

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Base

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$ ; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4. 
$$\mathcal{V}=\mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A\in\mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{M}_3(\mathbb{C}) =$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{34} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{35} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{36} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 &$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{C}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ;$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{C}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{C}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \\ \begin{pmatrix}$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{C}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \begin{pmatrix}$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{M}_{3}(\mathbb{C}) = \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{C}) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ &+ a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\mathcal{M}_3(\mathbb{C}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{bmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{C}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$\begin{split} A &= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathcal{M}_{3}(\mathbb{C}) &= \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{bmatrix} = [v_{1}; v_{2}; v_{3}; v_{4}; v_{5}; v_{6}; v_{7}; v_{8}; v_{9}] \\ e; \{v_{1}; v_{2}; v_{3}; v_{4}; v_{5}; v_{6}; v_{7}; v_{8}; v_{9}\} \notin \mathbf{LI} \quad : \end{split}$$

MAT A07 - Álgebra Linear A - Semestre Letivo Suplementar - 2021.1

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 &$$

MAT A07 - Álgebra Linear A - Semestre Letivo Suplementar - 2021.1

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{M}_{3}(\mathbb{C}) = \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = [v_{1}; v_{2}; v_{3}; v_{4}; v_{5}; v_{6}; v_{7}; v_{8}; v_{9}]$$

$$\mathbf{e}; \{v_{1}; v_{2}; v_{3}; v_{4}; v_{5}; v_{6}; v_{7}; v_{8}; v_{9}\} \in \mathbf{LI} \quad : \sum_{i=1}^{9} \lambda_{i} v_{i} = 0 = 0_{3} \Leftrightarrow \lambda_{i} = 0, \forall i = 1, \dots, 9.$$

$$\Rightarrow \beta_{\mathcal{M}_{3}}(\mathbb{C}) = \{v_{1}; v_{2}; v_{3}; v_{4}; v_{5}; v_{6}; v_{7}; v_{8}; v_{9}\}$$

MAT A07 - Álgebra Linear A - Semestre Letivo Suplementar - 2021.1

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33$$

4. 
$$\mathcal{V} = \mathcal{M}_3(\mathbb{C})$$
; então,  $\forall A \in \mathcal{M}_3(\mathbb{C})$ 

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{12} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{13} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{33$$

#### Espaços Vetoriais Base

Exercícios: (respostas)

Base

Exercícios: (respostas)

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$

Base

5. 
$$V = \mathcal{P}_3(\mathbb{R})$$
  $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow$ 

Base

5. 
$$V = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) =$ 

Base

5. 
$$V = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_0$ 

# Espaços Vetoriais

Base

5. 
$$V = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t + a_3 t + a_4 t + a_4 t + a_5 t +$ 

## Espaços Vetoriais

Base

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_3$ 

## Espaços Vetoriais

Base

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ 

5. 
$$V = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1 t^2 + a_1 t^2 + a_2 t^2 + a_2 t^2 + a_2 t^2 + a_3 t^2$ 

Base

5. 
$$V = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t) + a_3(t) + a_3$ 

5. 
$$V = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^2) + a_$ 

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$ 

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{R}) = \underbrace{1};$ 

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{R}) = \underbrace{1}; \underbrace{t};$ 

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{R}) = \underbrace{1}_{t}; \underbrace{t}_{t}; \underbrace{t^2}_{t};$ 

5. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{R}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{R}) = \underbrace{1}_{t}; \underbrace{t}_{t}; \underbrace{t^2}_{t}; \underbrace{t^3}_{t}$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{V_{1}}; \underbrace{t}_{V_{2}}; \underbrace{t^{3}}_{V_{3}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}:$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{V_{1}}; \underbrace{t}_{V_{2}}; \underbrace{t}_{V_{3}}; \underbrace{t}_{V_{4}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \in \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i} v_{i}$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3}$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{V_{1}}; \underbrace{t}_{V_{2}}; \underbrace{t^{3}}_{V_{4}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \in \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}] \text{ e};$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}$ 

```
5. V = \mathcal{P}_{3}(\mathbb{R})

\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})

\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t}_{v_{3}}; \underbrace{t}_{v_{4}}] e;

\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.

portanto; \beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.
```

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}} = 0;$   
 $\{1, t, t^{2}, t^{3}\} \in \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t^{3}}_{v_{4}} = 0;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_3(\mathbb{C})$$
  $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t^{3}}_{v_{4}} = 0;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) =$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_0$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t + a_3 t + a_4 t + a_4 t + a_5 t +$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^2 + a_3$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1t + a_2t^2 + a_3t^3$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1 t + a_2 t + a_3 t + a_3$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t) + a_3(t) +$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3) + a_$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t^{3}}_{v_{4}} = 0;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{C}) = \underbrace{1}_{Y_1};$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{C}) = \underbrace{1}_{Y_0}; \underbrace{t}_{Y_0};$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{C}) = \underbrace{1}_{V_1}; \underbrace{t}_{V_2}; \underbrace{t^2}_{V_3};$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$\mathcal{V} = \mathcal{P}_3(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_3(\mathbb{C}) \Rightarrow p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0(1) + a_1(t) + a_2(t^2) + a_3(t^3)$   
 $\Rightarrow \mathcal{P}_3(\mathbb{C}) = \underbrace{1}_{V_1}; \underbrace{t}_{V_2}; \underbrace{t}_{V_3}; \underbrace{t}_{V_4}$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{3}}] \text{ e};$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_{3}(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{C}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{3}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}:$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$   
6.  $V = \mathcal{P}_{3}(\mathbb{C})$   
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{C}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{3}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i}$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{V_{1}}; \underbrace{t}_{V_{2}}; \underbrace{t^{3}}_{V_{4}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \in \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$   
6.  $V = \mathcal{P}_{3}(\mathbb{C})$   
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{C}) = \underbrace{1}_{V_{1}}; \underbrace{t}_{V_{2}}; \underbrace{t}_{V_{3}}; \underbrace{t}_{V_{4}} = 0 = 0 + 0t + 0t^{2} + 0t^{3}$ 

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{4}}; \underbrace{t}_{v_{4}}; \underbrace{t}_{v_{4}} = 0$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$ 

6. 
$$V = \mathcal{P}_{3}(\mathbb{C})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{C}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{3}}] e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$ 

## EXERCÍCIOS: (RESPOSTAS)

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{V_{1}}; \underbrace{t^{2}}_{V_{2}}; \underbrace{t^{3}}_{V_{4}} = 0;$   
 $\{1, t, t^{2}, t^{3}\} \in \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$   
6.  $V = \mathcal{P}_{3}(\mathbb{C})$   
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{C}) = \underbrace{1}_{V_{1}}; \underbrace{t^{2}}_{V_{2}}; \underbrace{t^{3}}_{V_{4}} = 0;$   
 $\{1, t, t^{2}, t^{3}\} \in \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{C})} = \{1; t; t^{2}; t^{3}\}$ 

## EXERCÍCIOS: (RESPOSTAS)

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{3}} = e;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$   
6.  $V = \mathcal{P}_{3}(\mathbb{C})$   
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{C}) = \underbrace{1}; \underbrace{t}; \underbrace{t^{2}}; \underbrace{t^{3}} = e;$ 

 $\{1,t,t^2,t^3\} \notin \mathbf{LI}: \sum_{i=1}^4 \lambda_i v_i = 0 = 0 + 0t + 0t^2 + 0t^3 \Leftrightarrow \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.$  portanto;  $\beta_{\mathcal{P}_3(\mathbb{C})} = \{1;t;t^2;t^3\} \; ; \; \forall \lambda_i \in \mathbb{C}.$ 

## Exercícios: (respostas)

5. 
$$V = \mathcal{P}_{3}(\mathbb{R})$$
  
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{R}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3})$   
 $\Rightarrow \mathcal{P}_{3}(\mathbb{R}) = \underbrace{1}_{V_{1}}; \underbrace{t}_{V_{2}}; \underbrace{t^{3}}_{V_{3}} = 0;$   
 $\{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0.$   
portanto;  $\beta_{\mathcal{P}_{3}(\mathbb{R})} = \{1; t; t^{2}; t^{3}\}; \forall \lambda_{i} \in \mathbb{R}.$   
6.  $V = \mathcal{P}_{3}(\mathbb{C})$   
 $\forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{2} + a_{2}t + a_{3}t^{2} + a_{3}t^{3} = a_{3}(1) + a_{3}(t) + a_{3}(t^{2}) + a_{3}(t^{3})$ 

$$\begin{array}{l} \forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \\ \forall p(t) \in \mathcal{P}_{3}(\mathbb{C}) \Rightarrow p(t) = a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} = a_{0}(1) + a_{1}(t) + a_{2}(t^{2}) + a_{3}(t^{3}) \\ \Rightarrow \mathcal{P}_{3}(\mathbb{C}) = \underbrace{1}_{v_{1}}; \underbrace{t}_{v_{2}}; \underbrace{t^{3}}_{v_{3}}] \text{ e;} \\ \{1, t, t^{2}, t^{3}\} \notin \mathbf{LI}: \sum_{i=1}^{4} \lambda_{i}v_{i} = 0 = 0 + 0t + 0t^{2} + 0t^{3} \Leftrightarrow \lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda_{4} = 0. \\ \text{portanto;} \ \beta_{\mathcal{P}_{3}(\mathbb{C})} = \{1; t; t^{2}; t^{3}\}; \ \forall \lambda_{i} \in \mathbb{C}. \end{array}$$

# Espaços Vetoriais

Base

Exercícios:

## Espaços Vetoriais

Base

#### Exercícios:

Sejam  $\mathcal{V}$  um espaço vetorial finitamente gerado sobre um corpo  $\mathbb{K}$ ; e  $\beta_{\mathcal{V}}$  uma base de  $\mathcal{V}$ .

## Espaços Vetoriais

Base

#### Exercícios:

1. 
$$\mathcal{V}=\mathbb{R}^3$$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- $2 \mathcal{V} = \mathbb{C}^3$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$
- 5.  $\mathcal{V} = \mathcal{P}_3(\mathbb{R})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$
- 5.  $\mathcal{V} = \mathcal{P}_3(\mathbb{R})$
- 6.  $\mathcal{V} = \mathcal{P}_3(\mathbb{C})$

- 1.  $\mathcal{V} = \mathbb{R}^3$
- 2.  $\mathcal{V} = \mathbb{C}^3$
- 3.  $\mathcal{V} = \mathcal{M}_3(\mathbb{R})$
- 4.  $\mathcal{V} = \mathcal{M}_3(\mathbb{C})$
- 5.  $\mathcal{V} = \mathcal{P}_3(\mathbb{R})$
- 6.  $\mathcal{V} = \mathcal{P}_3(\mathbb{C})$