$a = c_n \cdot 10^n + \cdots + c_1 \cdot 10 + c_0$ | $b \mid a$ sse $a \equiv 0 \pmod{b}$

$2 \mid a$ sse $2 \mid c_0$

$\forall k \geq 1,\ 2 \mid 10^k \Rightarrow \forall k \geq 1,\ 10^k \equiv 0 \pmod{2} \Rightarrow$

$$a = c_n \cdot 10^n + \cdots + c_1 \cdot 10 + c_0 \equiv c_n \cdot 0 + \cdots + c_1 \cdot 0 + c_0 \pmod{2} \Rightarrow$$

$$\Rightarrow a \equiv c_0 \pmod{2} \Rightarrow \left(a \equiv 0 \pmod{2} \Leftrightarrow c_0 \equiv 0 \pmod{2}\right)$$

$$\Rightarrow \left(2 \mid a \Leftrightarrow 2 \mid c_0\right).$$

$$a = c_n \cdot 10^n + \cdots + c_1 \cdot 10 + c_0$$

$$3 \mid a \quad \text{sse} \quad 3 \mid c_n + \cdots + c_1 + c_0$$

$\forall k \left(10^k \equiv 1 \pmod 3\right)$ vamos demonstrar por indução em k.

Base: $k=0$ $\quad 10^0 = 1$ e $1 \equiv 1 \pmod 3$ ✓

Hp.: $10^k \equiv 1 \pmod 3 \Rightarrow$ Tese: $10^{k+1} \equiv 1 \pmod 3$

$$10^k \equiv 1 \pmod 3 \quad \text{sse} \quad \exists h \left(10^k - 1 = 3h\right)$$

$$10^{k+1} - 1 = 10 \cdot 10^k - 1 = 10 \cdot 10^k - 10 + 9 = 10\left(10^k - 1\right) + 9 \overset{HP}{=}$$

$$= 10 \cdot 3h + 3 \cdot 3 = 3\left(10h + 3\right) \Rightarrow 10^{k+1} \equiv 1 \pmod 3.$$

$$\forall k \left(10^k \equiv 1 \pmod 3\right) \Rightarrow a = c_n \cdot 10^n + \cdots + c_1 \cdot 10 + c_0 \equiv c_n \cdot 1 + \cdots + c_1 \cdot 1 + c_0 \pmod 3$$

$$\Rightarrow a \equiv c_n + \cdots + c_1 + c_0 \pmod 3 \Rightarrow \left(a \equiv 0 \pmod 3 \text{ sse } c_n + \cdots + c_1 + c_0 \equiv 0 \pmod 3\right)$$

$$\Rightarrow \left(3 \mid a \quad \text{sse} \quad 3 \mid c_n + \cdots + c_1 + c_0\right).$$

$\boxed{a = c_n \cdot 10^n + \cdots + c_1 \cdot 10 + c_0}$ $\Big\}$ $5 \mid a$ sse $5 \mid c_0$ sse $c_0 \in \{0, 5\}$

$\forall k \geq 1 \left(10^k \equiv 0 \pmod 5\right)$ $\quad 10 = 2 \cdot 5 \Rightarrow 10 \equiv 0 \pmod 5 \Rightarrow 10^k \equiv 10 \cdot 10^{k-1} \equiv 0 \cdot 10^{k-1} \equiv 0 \pmod 5$

$a = c_n \cdot 10^n + \cdots + c_1 \cdot 10 + c_0 \equiv c_n \cdot 0 + \cdots + c_1 \cdot 0 + c_0 \pmod 5 \Rightarrow$

$\Rightarrow a \equiv c_0 \pmod 5 \Rightarrow \Big(5 \mid a$ sse $a \equiv 0 \pmod 5$ sse $c_0 \equiv 0 \pmod 5$

sse $5 \mid c_0$ sse $c_0 \in \{0, 5\}\Big)$.

$$\boxed{a = c_m 10^m + \cdots + c_1 10 + c_0}$$

Seja $b = c_m \cdot 10^{m-1} + \cdots + c_2 \cdot 10 + c_1$ (b é obtido "tirando" o último algarismo de a).

$$\boxed{7 \mid a \text{ sse } 7 \mid b - 2c_0 \text{ sse } 7 \mid b + 5c_0}$$

(*) $50 \equiv 1 \pmod 7$, (*') $-20 \equiv 1 \pmod 7$ $\quad$ $50 - 1 = 49 = 7 \cdot 7$

$-20 - 1 = -21 = -3 \cdot 7$

(**) $7 \mid k$ sse $7 \mid 10k$

$$a = c_m \cdot 10^m + \cdots + c_1 \cdot 10 + c_0 \cdot 1 \overset{(*)}{\equiv} c_m \cdot 10^m + \cdots + c_1 \cdot 10 + c_0 \cdot 50 \pmod 7$$

$$c_m \cdot 10^m + \cdots + c_1 \cdot 10 + c_0 \cdot 50 = 10\left(\underbrace{c_m \cdot 10^{m-1} + \cdots + c_1} + 5c_0\right) = 10(b + 5c_0)$$

Pela (**), $7 \mid 10 \cdot (b + 5c_0)$ sse $7 \mid b + 5c_0$

Então $a \equiv 10(b + 5c_0) \pmod 7 \Rightarrow (7 \mid a$ sse $7 \mid 10(b + 5c_0)$ sse $7 \mid b + 5c_0)$

$$a = c_m \cdot 10^m + \cdots + c_1 \cdot 10 + c_0 \cdot 1 \overset{(*')}{\equiv} c_m \cdot 10^m + \cdots + c_1 \cdot 10 + c_0 \cdot (-20) \pmod 7$$

$\Rightarrow a \equiv 10(b - 2c_0) \Rightarrow (7 \mid a$ sse $7 \mid 10 \cdot (b - 2c_0)$ sse$^{(**)}$ $7 \mid b - 2c_0)$.

$71241 \to 7124 - 2 \cdot 1 = 7122 \to 712 - 2 \cdot 2 = 708 \to 70 - 2 \cdot 8 = 54$

$7 \nmid 54 \Rightarrow 7 \nmid 71241$

$71241 \to 7124 + 5 \cdot 1 = 7129 \to 712 + 5 \cdot 9 = 757 \to 75 + 5 \cdot 7 = 110 \to 11 + 5 \cdot 0 = 11$

$7 \nmid 11 \Rightarrow 7 \nmid 71241$

$14763 \to 1476 - 2 \cdot 3 = 1470 \to 147 \to 14 - 2 \cdot 7 = 0$ e $7 \mid 0 \Rightarrow 7 \mid 14763$

$14763 \to 1476 + 5 \cdot 3 = 1491 \to 149 + 5 \cdot 1 = 154 \to 15 + 5 \cdot 4 = 35 \to 3 + 5 \cdot 5 = 28$

$\to 2 + 5 \cdot 8 = 42 \to 4 + 5 \cdot 2 = 14 \to 1 + 5 \cdot 4 = 21 \to 2 + 5 \cdot 1 = 7$

$$\boxed{Q = c_n \cdot 10^n + \dots + c_1 \cdot 10 + c_0}$$

$11 \mid Q$ sse $11 \mid (c_0 + c_2 + c_4 + \dots) - (c_1 + c_3 + c_5 + \dots)$

$\forall k \left(10^{2k} \equiv 1 \pmod{11}\right)$ e $\forall k \left(10^{2k+1} \equiv -1 \pmod{11}\right)$

<u>Base</u>: $k=0$ $\quad 10^{2\cdot 0} = 10^0 = 1 \equiv 1 \pmod{11}$ ✓

<u>Hp</u>: $\exists h \left(10^{2k} - 1 = 11h\right)$

$\Downarrow$

Tese: $\exists h' \left(10^{2(k+1)} - 1 = 11h'\right)$

$$10^{2(k+1)} - 1 = 10^{2k+2} - 1 = 100 \cdot 10^{2k} - 1 =$$
$$= 100 \cdot 10^{2k} - 100 + 99 = 100\left(10^{2k} - 1\right) + 99$$
$$\overset{HP}{=} 100(11h) + 9 \cdot 11 = 11 \underbrace{(100h + 9)}_{h'}$$

<u>Base</u>: $k=0$ $\quad 10^{2\cdot 0+1} = 10$ e $10 \equiv -1 \pmod{11}$

poiché $10 - (-1) = 10 + 1 = 11$

<u>Hp</u>: $\exists h \left(10^{2k+1} + 1 = 11h\right)$

<u>Tese</u>: $\exists h' \left(10^{2(k+1)+1} + 1 = 11h'\right)$

$$10^{2(k+1)+1} + 1 = 10^{2k+3} + 1 = 100 \cdot 10^{2k+1} + 1 =$$
$$= 100 \cdot 10^{2k+1} + 100 - 99 = 100\left(10^{2k+1} + 1\right) - 99$$
$$\overset{HP}{=} 100 \cdot 11h - 9 \cdot 11 = 11 \underbrace{(100h - 9)}_{h'}$$

$$Q = c_n \cdot 10^n + \dots + c_1 \cdot 10 + c_0 = \sum_{i=0}^{n} c_i \cdot 10^i = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} c_{2i} \cdot 10^{2i} + \sum_{i=0}^{\lfloor \frac{n-1}{2} \rfloor} c_{2i+1} \cdot 10^{2i+1} \Rightarrow$$

$$\Rightarrow \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} c_{2i} \cdot 10^{2i} + \sum_{i=0}^{\lfloor \frac{n-1}{2} \rfloor} c_{2i+1} \cdot 10^{2i+1} \equiv \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} c_{2i} \cdot 1 + \sum_{i=0}^{\lfloor \frac{n-1}{2} \rfloor} c_{2i+1} \cdot (-1) =$$

$$= \underbrace{\sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} c_{2i} - \sum_{i=0}^{\lfloor \frac{n-1}{2} \rfloor} c_{2i+1}}_{b} \Rightarrow 11 \mid Q \text{ sse } 11 \mid b$$

$11 \mid 7243721$? $\qquad 1 + 7 + 4 + 7 - (2 + 3 + 2) = 19 - 7 = 12$

$11 \nmid 12 \Rightarrow 11 \nmid 7243721$

$$\boxed{3 \mid Q \text{ sse } 3 \mid 22 + 31 = 53}$$

$11 \mid \overbrace{6243721431222761}^{Q}$?

$1+7+2+1+1+4+2+3+2 - (6+2+2+3+1+7+4+6) = 22 - 31 = -9$

$11 \nmid -9 \Rightarrow 11 \nmid Q$

$7 \mid Q$?

$Q \to 624372143122274 \to 62437214312219 \to 6243721431203 \to$
$624372143114 \to 62437214303 \to 6243721424 \to 624372134 \to$
$\to 62437205 \to 6243710 \to 624371 \to 62435 \to 6233 \to$
$\to 617 \to 47$ e $7 \nmid 47$. Logo $7 \nmid Q$