Teorema Seja $a \in \mathbb{N}$. $\forall b \in \mathbb{N} \setminus \{0,1\}$ $\exists\, n, c_0, \dots, c_n \in \mathbb{N}$ t.q. $a = c_0 + c_1 b + \dots + c_n b^n$ e $0 \leq c_i < b$ $\forall i = 0, \dots, n$.

Além disso, tais $n, c_0, \dots, c_n$ são unicamente determinados.

Exemplo: $11435 = 1 \cdot 10^4 + 1 \cdot 10^3 + 4 \cdot 10^2 + 3 \cdot 10 + 5$

11435 na base 7

$(45224)_7 = (11435)_{10}$

$11435 \div 7 = 1633$, resto 4
$1633 \div 7 = 233$, resto 2
$233 \div 7 = 33$, resto 2
$33 \div 7 = 4$, resto 5
$4 \div 7 = 0$, resto 4

$4 + 2 \cdot 7 + 2 \cdot 7^2 + 5 \cdot 7^3 + 4 \cdot 7^4 =$

$= 4 + 14 + 98 + 1715 + 9604 = 11435$

$(11435)_{10} = (45224)_7 = (26253)_8$

$11435 \div 8 = 1429$, resto 3
$1429 \div 8 = 178$, resto 5
$178 \div 8 = 22$, resto 2
$22 \div 8 = 2$, resto 6
$2 \div 8 = 0$, resto 2

base 7 → base 8

$(45224)_7 \div (11)_7 = 4111$, resto 3
$4111 \div 11 = 343$, resto 5
$343 \div 11 = 31$, resto 2
$31 \div 11 = 2$, resto 6
$2 \div 11 = 0$, resto 2

base 8 → base 7

$(26253)_8 \div (7)_8 = 3141$, resto 4
$3141 \div 7 = 351$, resto 2
$351 \div 7 = 41$, resto 2
$41 \div 7 = 4$, resto 5
$4 \div 7 = 0$, resto 4

base 8 → base 10

$(26253)_8 = 2 \cdot 8^4 + 6 \cdot 8^3 + 2 \cdot 8^2 + 5 \cdot 8 + 3 = \dots$

b na base b é $(10)_b$

base b → base 10

$(c_n \cdots c_0)_b = c_n \cdot b^n + \dots + c_1 b^1 + c_0$

$$\begin{array}{r} (137)_8 \; + \\ (241)_8 \; = \\ \hline (400)_8 \end{array}$$

base 16 : $0, \dots, 9, A, B, C, D, E, F$

$(1F)_{16} = 16 + 15 = 31$

$$\begin{array}{r|l} 21348 & 16 \\ 53 & 1334 \\ 54 & \\ 68 & \\ 4 & \end{array} \quad \begin{array}{r|l} 1334 & 16 \\ 54 & 83 \\ 6 & \end{array} \quad \begin{array}{r|l} 83 & 16 \\ 3 & 5 \end{array} \quad \begin{array}{r|l} 5 & 16 \\ 5 & 0 \end{array}$$

$(5364)_{16} = 21348$

$F = 15$

$(10)_{16} = 16$

$$\begin{array}{r|l} 21348 & 12 \\ 93 & 1779 \\ 94 & \\ 108 & \\ 0 & \end{array} \quad \begin{array}{r|l} 1779 & 12 \\ 57 & 148 \\ 99 & \\ 3 & \end{array} \quad \begin{array}{r|l} 148 & 12 \\ 28 & 12 \\ 4 & \end{array} \quad \begin{array}{r|l} 12 & 12 \\ 0 & 1 \end{array} \quad \begin{array}{r|l} 1 & 12 \\ 1 & 0 \end{array}$$

$$\begin{array}{r|l} 21348 & 14 \\ 73 & 1524 \\ 34 & \\ 68 & \\ 12 & \\ C & \end{array} \quad \begin{array}{r|l} 1524 & 14 \\ 12 & 108 \\ 124 & \\ 12 & \\ C & \end{array} \quad \begin{array}{r|l} 108 & 14 \\ 10 & 7 \\ A & \end{array} \quad \begin{array}{r|l} 7 & 14 \\ 7 & 0 \end{array}$$

$$(7ACC)_{14} = 21348 = (10430)_{12} = (5364)_{16}$$

↓

base 10

$$7 \cdot 14^3 + A \cdot 14^2 + C \cdot 14 + C = 7 \cdot 14^3 + 10 \cdot 14^2 + 12 \cdot 14 + 12 = 21348$$

↓

altra base

$$\begin{array}{r} (7\dot{A}CC)_{14} + \\ (B1D0)_{14} \\ \hline (14CBC)_{14} \end{array}$$

$11 \cdot 14^3 + 1 \cdot 14^2 + 13 \cdot 14 + 0 = 30562$

$$\begin{array}{r} 21348 + \\ 30562 \\ \hline 51910 \end{array}$$

||

$$1 \cdot 14^4 + 4 \cdot 14^3 + 12 \cdot 14^2 + 11 \cdot 14 + 12 = 51910$$

$$a = \underbrace{c_m \cdot 10^m + \cdots + c_1 \cdot 10 + c_0}_{\text{expansão decimal de } a}$$

$$a = \underbrace{d_k \cdot b^k + \cdots + d_1 \cdot b + d_0}_{\text{expansão } b\text{-ádica de } a}.$$

$2 \mid a$ sse $2 \mid c_0$ sse $c_0 \in \{0,2,4,6,8\}$

$3 \mid a$ sse $3 \mid \sum_{i=0}^{m} c_i$

$5 \mid a$ sse $5 \mid c_0$ sse $c_0 \in \{0,5\}$

$7 \mid a$ sse $7 \mid c_m \cdot 10^{m-1} + \cdots + c_2 \cdot 10 + (c_1 - 2c_0)$ sse $7 \mid c_m \cdot 10^{m-1} + \cdots + c_2 \cdot 10 + (c_1 + 5c_0)$

$11 \mid a$ sse $11 \mid \sum_i c_{2i} - \sum_i c_{2i+1}$

$$\sum_i c_{2i} - \sum_i c_{2i+1} = c_0 + c_2 + \ldots - (c_1 + c_3 + \ldots) \text{ é mult. de } 11$$

$143\,284\,792\,411 = a$

$2 \nmid a$ pois $c_0 = 1$

$1+4+3+2+8+4+7+9+2+4+1+1 = 46 \to 4+6 = 10$ $\quad 3 \nmid 10$ pois $10 \equiv 1 \pmod 3$

$5 \nmid a$ pois $c_0 \notin \{0, 5\}$

$143284792411 \to 14328479241 - 2 \cdot 1 = 14328479239 \to 1432847905 \to$

$\to 143284780 \to 14328478 \to 1432831 \to 143281 \to 14326 \to$

$\to 1420 \to 142 \to 10$ $\quad 7 \nmid 10 \Rightarrow 7 \nmid a$

$11 \mid a$ se $11 \mid 1+4+9+4+2+4 - (1+2+7+8+3+1) = 20 - 22 = -2$

$11 \nmid -2 \Rightarrow 11 \nmid a$

$2 \mid a = c_n \cdot 10^n + \dots + c_1 \cdot 10 + c_0$ sse $2 \mid c_0$

$2 \mid a$ sse $a \equiv 0 \pmod 2$

$\forall i \geq 1,\ 10^i \equiv 0 \pmod 2$ (prove por indução em i)

$c_n \cdot 10^n + \dots + c_1 \cdot 10 + c_0 \equiv_{(\mathrm{mod}\, 2)} c_n \cdot 0 + \dots + c_1 \cdot 0 + c_0 = c_0 \Rightarrow$

$a \equiv 0 \pmod 2$ sse $c_0 \equiv 0 \pmod 2$

$(k \leq n)$

$2^k \mid a$ sse $2^k \mid c_0 + c_1 \cdot 10 + \dots + c_{k-1} \cdot 10^{k-1}$

$4 \mid a$ sse $4 \mid c_0 + c_1 \cdot 10$

$8 \mid a$ sse $8 \mid c_0 + c_1 10 + c_2 \cdot 10^2$

$\forall i \geq k,\ 10^i \equiv 0 \pmod{2^k}$

$c_n \cdot 10^n + \dots + c_1 \cdot 10 + c_0 \equiv_{(\mathrm{mod}\, 2^k)} \underbrace{0 \cdot 10^n + \dots + 0 \cdot 10^k}\ + c_{k-1} \cdot 10^{k-1} + \dots + c_1 \cdot 10 + c_0$