Human Neuromechanical Control and Learning
Tutorial 1: Physiology
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Question 1

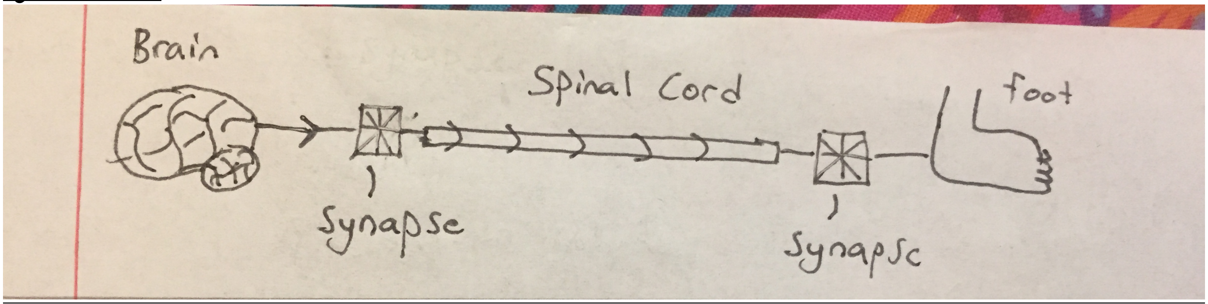

Figure 1: Schematic of the Neural Signal Pathway.

Let the velocity of an action potential in a fast neuron be $v_{AP} = 100m/s$
Let the velocity of an action potential in a muscle be $v_m = 4m/s$
Let the time it takes to diffuse across a membrane be $t_D = 1ms = 0.001s$

a) To calculate the total time it takes for a neural signal to travel from the motor cortex to contract a foot muscle, first determined the correct pathway the signal travels and then added up the time spent in each stage.

Assumed pathway

Firstly, I assumed that the signal travels in a neuron across the motor cortex for 1m, and then it diffuses across a synapse to reach the spine. Next, the signal travels down the spinal cord for 1m, diffuses across a second synapse to reach the foot muscle and lastly travels along the length of the foot fiber for 4cm.

$$t_{total} = t_{motor\ cortex} + t_{synapse} + t_{spine} + t_{synapse} + t_{muscle}$$
$$t_{total} = \frac{d_{motor\ cortex}}{v_{AP}} + \frac{d_{spine}}{v_{AP}} + \frac{d_{muscle}}{v_m} + 2 * t_{synapse}$$
$$t_{total} = \frac{1m}{100m/s} + \frac{1m}{100m/s} + \frac{0.04m}{4m/s} + 2 * 0.001s$$
$$t_{total} = 0.01s + 0.01s + 0.01s + 0.002s$$
$$t_{total} = 0.032s = 32ms$$

The total time it takes for the signal to travel from the motor cortex to the foot is 32ms.

b) I assumed that there are 4 phases the signal spends time in. Those phases are as follows.

Phase 1: Travelling through the motor cortex.
$$\%t_{motor\ cortex} = \frac{t_{motor\ cortex}}{t_{total}} * 100 = \frac{0.01s}{0.032s} * 100 = 31.25\%$$
Phase 2: Travelling through the spinal cord
$$\%t_{spine} = \frac{t_{spine}}{t_{total}} * 100 = \frac{0.01s}{0.032s} * 100 = 31.25\%$$

Phase 3: Travelling through the muscle fiber.
$$\%t_{muscle} = \frac{t_{muscle}}{t_{total}} * 100 = \frac{0.01s}{0.032s} * 100 = 31.25\%$$

Phase 4: Travelling across TWO synapses.
$$\%t_{synapses} = \frac{2 * t_{synapse}}{t_{total}} * 100 = \frac{0.002s}{0.032s} * 100 = 6.25\%$$

The signal spent equivalently the same amount of time in the brain, muscle and spinal cord phases, each respectively spending 31.25% of it's travel time there. The signal spent the least amount of travel time in the synapse phase, 6.25% of the time.

c) The total travel time of the neural signal was 0.032s. In order to compare this value against the speed of an electrical signal, we need to calculate the total travel time of an electrical signal.

Let the velocity of an electrical signal be v_E and assume that it is equivalent to 68% of the speed of light (299792458m/s).

$$v_E = 0.68 * 299{,}792{,}458 m/s = 203{,}858{,}871.4 m/s$$

Assume that the electrical signal travels the same total distance as the neural signal.

$$d_{total} = d_{motor\ cortex} + d_{spine} + d_{muscle} = 1m + 1m + 0.04m = 2.04m$$

$$t_{total\ Electrical} = \frac{d_{total}}{v_E} = \frac{2.04m}{\frac{203{,}858{,}871m}{s}}$$

$$t_{total\ Electrical} = 1.0 * 10^{-8}s = 10ns$$

The electrical signals travel time is 10ns. Now, I will compare this time with the neural signals travel time, calculated in part A of this question.

$$\frac{t_{Neural\ signal}}{t_{Electrical\ signal}} = \frac{0.032s}{1 * 10^{-8}s} = 3{,}200{,}000$$

The total travel time of an electrical signal is 3,200,000 times faster than a neural signal. An electrical signal is quicker than a neural signal.

Question 2

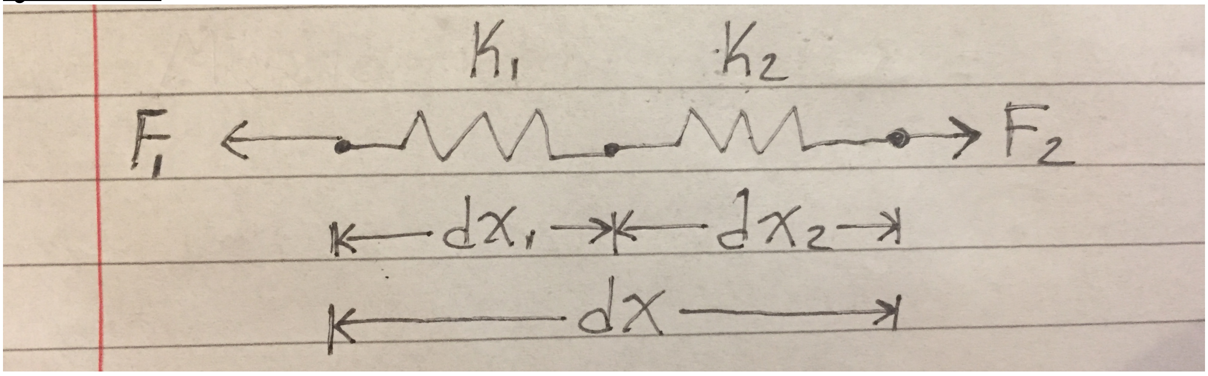

Figure 2: Free Body Diagram of a Muscle-Tendon System

A)
Based on the free body diagram shown in Figure 2 above.
Let the forces applied to the muscle, F_1 and tendon, F_2 be equivalent.

$$F_1 = F_2 = F$$

Let the displacements observed at the muscle, dx_1 and the displacement observed at the tendon, dx_2 sum to be equivalent to the total displacement of the system.

$$dx = dx_1 + dx_2$$

Let stiffness and compliance be relationships of force applied and displacement of the muscle, tendon system.

Stiffness: $k = \frac{dF}{dx}$ Compliance: $C = \frac{dx}{dF}$

$$C = \frac{dx}{dF} = \frac{d(x_1 + x_2)}{dF} = \frac{dx_1}{dF} + \frac{dx_2}{dF} = \frac{dx_1}{dF_1} + \frac{dx_2}{dF_2} = C_1 + C_2$$

Since, $C = \frac{1}{k}$

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Rearranging,

$$k_{total} = \frac{k_1 k_2}{k_1 + k_2}$$

Based on the derivation of the total stiffness, lets consider two extreme cases.
Case #1: Tendon stiffness is very small compared to muscle stiffness.

$$k_2 \ll k_1$$

$$\lim_{k_2 \to 0} \frac{k_1 k_2}{k_1 + k_2} \sim \frac{k_1 k_2}{k_1} = k_2 = k_{total}$$

When tendon stiffness is very small, the effective total stiffness is only dependent on tendon stiffness.
Case #2: Tendon stiffness is very large compared to muscle stiffness.

$$k_2 \gg k_1$$

$$\lim_{k_1 \to 0} \frac{k_1 k_2}{k_1 + k_2} \sim \frac{k_1 k_2}{k_2} = k_1 = k_{total}$$

When tendon stiffness is very large, the effective total stiffness is only dependent on muscle stiffness.

Based on these case results it can be seen that the muscle can only produce a large range of stiffness' when the tendon is stiff. If the tendon is not stiff then the systems total stiffness becomes dependent on tendon stiffness and the muscles stiffness will have no effect on the resultant total stiffness.

B)
Let human Achilles tendon stiffness be $k_{HT} = 188N/mm$
Let kangaroo tendon stiffness be $k_{KT} = 25N/mm$
Let human and kangaroo muscle stiffness be equivalent, $k_{HM} = k_{KM} = k$

Therefore, the total stiffness equations of each species are as follows.

$$k_{human} = \frac{188k}{k+188}$$
$$k_{kangaroo} = \frac{25k}{k+25}$$

In order to compare the stiffness of these two species, lets consider three cases. One where muscle stiffness is very small, very large and when a constant value is selected.

Case#1: Muscle stiffness is very small.

$$\lim_{k\to 0}\frac{188k}{k+188} \sim \frac{188k}{188} = k = k_{human}$$
$$\lim_{k\to 0}\frac{25k}{k+25} \sim \frac{25k}{25} = k = k_{kangaroo}$$

Therefore,

$$\frac{k_{human}}{k_{kangaroo}} \sim \frac{k}{k} = 1$$

When muscle stiffness, k, is small, total stiffness becomes dependent on muscle stiffness. Therefore, since muscle stiffness is similar for both humans and kangaroo's, their muscle-tendon system stiffness will be equivalent and their ratio will be 1.

Case #2: Muscle stiffness is very large.

$$\lim_{k\to\infty}\frac{188k}{k+188} \xrightarrow{l'Hopital} \lim_{k\to\infty}\frac{188k(1)-188(k+188)}{(k+188)^2} = \lim_{k\to\infty}\frac{-188}{(k+188)^2} \sim 0 = k_{human}$$

$$\lim_{k\to\infty}\frac{25k}{k+25} \xrightarrow{l'Hopital} \lim_{k\to\infty}\frac{25k(1)-25(k+25)}{(k+25)^2} = \lim_{k\to\infty}\frac{-25}{(k+25)^2} \sim 0 = k_{kangaroo}$$

When muscle stiffness, k, is large, the total stiffness of both species tends toward zero.

Case#3: Select k=25N/mm

$$k_{human} = \frac{188k}{k+188} = \frac{188\frac{N}{mm} * 25\frac{N}{mm}}{25\frac{N}{mm} + 188\frac{N}{mm}} = 41.59\frac{N}{mm}$$

$$k_{kangaroo} = \frac{25k}{k+25} = \frac{25\frac{N}{mm} * 25\frac{N}{mm}}{25\frac{N}{mm} + 25\frac{N}{mm}} = 12.5\frac{N}{mm}$$

$$\frac{k_{human}}{k_{kangaroo}} \sim \frac{188(k+25)}{25(k+188)} > 1$$

When muscle stiffness, k, is selected as a reasonable constant, human total stiffness is larger than kangaroo total stiffness., therefore their ratio will always be larger than 1.

Human gait is more versatile than kangaroos. Humans can choose to move from point A to point B by running, walking, jumping or a mixture of the two. Kangaroos move by exclusively jumping, not walking. Therefore, it is expected that humans will have a stiffer tendon because they need to produce a variety of muscular stiffness' to accomplish their various forms of gaits. Kangaroos do not need as larger of tendon stiffness because they do not need to produce as many movements. Kangaroos need a less stiff tendon to allow them to jump.