HNCL Tutorial 3: Dynamics and control of planar arm movements

Consider the two-joint arm model of horizontal movement illustrated in Figure 1. Using this model you will learn the effect of coupled and nonlinear dynamics on arm movements, and implement linear feedback control. MATLAB will be required to simulate two-joint planar arm movements and plot various variables during movement.

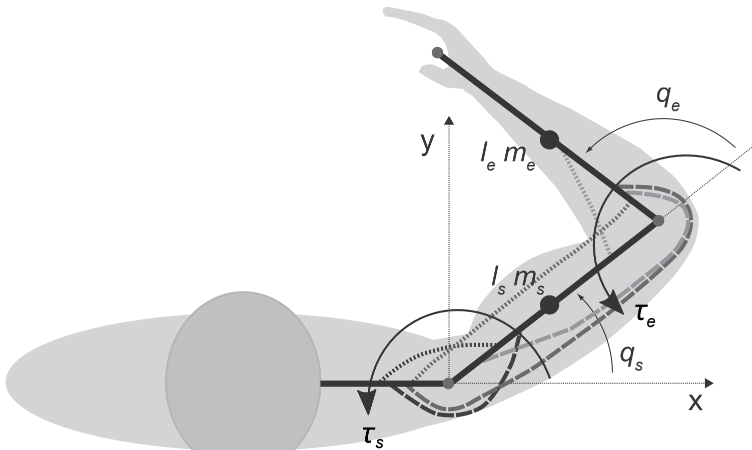

Figure 1: Two-joint model of planar arm movement. q_s and q_e denote the angles of the shoulder and elbow joints, respectively, τ_s and τ_e the torques applied to these joints. The following parameters are used: shoulder and elbow limbs mass: $M_s = 1.93kg$ and $M_e = 2.04kg$, length: $l_s = 0.31m$ and $l_e = 0.34m$, distance to the centre of mass: $l_{ms} = 0.165m$ and $l_{me} = 0.2m$ and moment of inertia: $J_s = 0.0141kg\,m^2$ and $J_e = 0.0188kg\,m^2$.

Question 1: Coupled dynamics

The dynamics of the two-joint arm model are given by the equations

$$\tau = \Psi(q,\dot{q},\ddot{q})\,p\,, \quad \Psi_{11} = \Psi_{21} = \ddot{q}_s + \ddot{q}_e\,, \quad \Psi_{12} = (2\ddot{q}_s + \ddot{q}_e)\cos(q_e) - \dot{q}_e(2\,\dot{q}_s + \dot{q}_e)\sin(q_e)\,,$$
$$\Psi_{22} = \ddot{q}_s\cos(q_e) + \dot{q}_s^2\sin(q_e)\,, \quad \Psi_{13} = \ddot{q}_s\,, \quad \Psi_{23} = 0$$
$$p_1 \equiv J_e + m_e\,l_{me}^2 = 0.1004\,kg\,m^2, \quad p_2 \equiv m_e\,l_s\,l_{me} = 0.12\,kg\,m^2, \quad p_3 \equiv J_s + m_s\,l_{ms}^2 + m_e\,l_s^2 = 0.263\,kg\,m^2$$

(a) First, suppose that the shoulder joint is blocked, and the elbow is moving the forearm horizontally.

- Write the dynamic equation of the elbow single-joint movement. Are these dynamics linear or nonlinear?
  [8 mark]
- Starting at $q_e(0) = 30°$, the elbow is subjected to a stimulus

$$\tau_e(t) = \begin{cases} 0.02Nm - 0.1\dot{q}_e(t) & 0 \le t < 2s \\ -0.1\,\dot{q}_e(t) & 2s \le t \le 20s \end{cases} \tag{1}$$

  Simulate the response of the joint angle $q_e(t)$ in MATLAB and plot the trajectory of $q_e(t)$ with respect to time t using a sampling time $0.01s$.
  [14 marks]

(b) Then, suppose that the shoulder is no longer blocked. Simulate the joint angles $q_s(t)$ and $q_e(t)$, starting at $q_s(0) = 0°$ and $q_e(0) = 30°$, when the elbow is stimulated as in Equ.(1) and $\tau_s = -0.1\dot{q}_s$. Plot the trajectories of $q_s(t)$ and $q_e(t)$ with respect to time t, together with the trajectory of $q_e(t)$ in (a).

Describe the differences in the elbow movement to that in Question 1(b), and explain the reason(s) of the differences.

[21 marks]

Question 2: Nonlinear dynamics and linear feedback control

(a) Use the initial joint angles ($q_s(0) = 90°,\ q_e(0) = 130°$) and the planned hand trajectory

$$\begin{aligned} x^*(t_n) &= -0.2605 + 0.11\, g(t/T)\,, \quad g(t_n) \equiv t_n^3\,(6\,t_n^2 - 15\,t_n + 10) \\ y^*(t_n) &= 0.0915 + 0.5\, g(t/T) \end{aligned} \tag{2}$$

where $t \in [0, T]$ is time and $T = 1s$ is the movement duration. Using the inverse differential kinematics, compute the planned joint angle trajectories $q_s^*(t)$ and $q_e^*(t)$, and plot them.

To apply torques to the dynamic equation of the two-joint arm, use a linear PD controller at each joint. Use the control gains $K_p = 100\ Nm/rad$ and $K_d = 10\ Nms/rad$ for both shoulder and elbow. Plot the actual trajectories $q_s(t)$ and $q_e(t)$ with respect to time t against the planned trajectories $q_s^*(t)$ and $q_e^*(t)$ respectively. Plot also the resultant hand trajectory $x(t)$ and $y(t)$ each with respect to time t, against the desired hand trajectory $x^*(t)$ and $y^*(t)$.

[29 marks]

(b) Change both K_d and K_p values and discuss the resulting control. Explore the effect of large K_d. Compare the results with (a).

[14 marks]

(c) What happens when the arm has to move five times as fast, i.e. when $T \equiv 1s$ changes to $T \equiv 0.2s$ in the planned trajectory (x^*, y^*) of equation (2)? Compare with (a) and explain the reason of the results.

[14 marks]