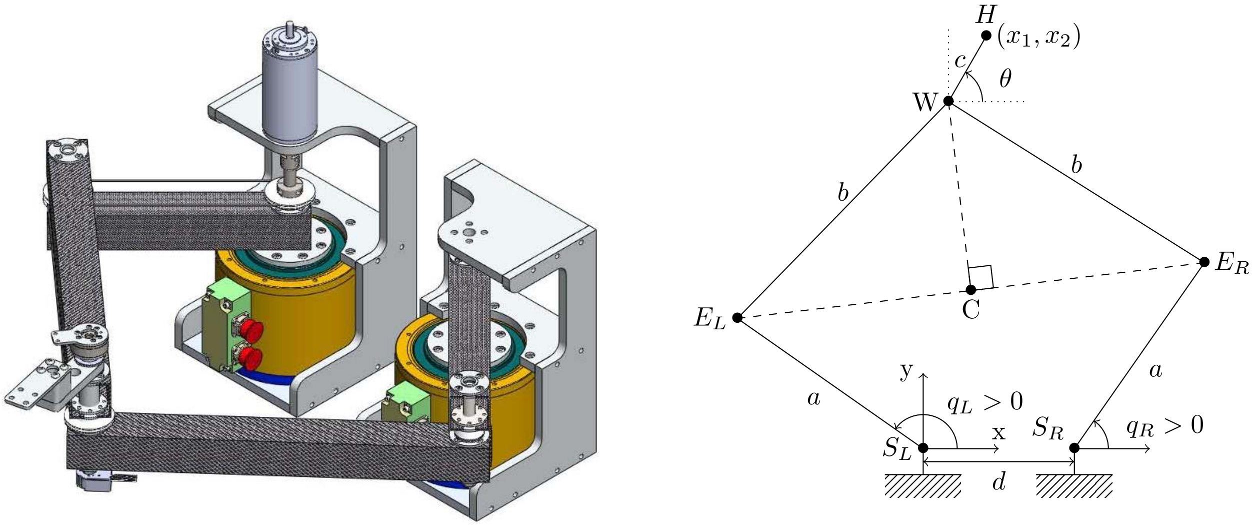
Tutorial 1: Kinematics of a 3-DOF Robot



(a) (b)

Figure 1: The 3-dof parallel robot photo and diagram

A 3-degree-of-freedom parallel robot is illustrated in Fig.1. S_L and S_R are the left and right shoulder coordinate vectors, respectively, E_L and E_R the left and right elbow coordinate vectors, W and H are the wrist and hand coordinate vectors, respectively. Let a, b and c be the lengths of the upper arm, forearm and hand, respectively. θ is the hand's orientation. We place a coordinate origin at the left shoulder, and let d be the distance between the shoulders.

Q1: Direct kinematics [25%]

Determine the direct kinematics transformation of the robot. Compute the position of H as a function of the two motors angles q_L, q_R, the angle θ, and the parameters a, b, c, d. Hint: The wrist W is on the perpendicular to $E_L - E_R$ through the midpoint C of the elbows E_L and E_R.

Q2: Redundancy resolution

If $d = 0\ m$, $a = b = 0.3\ m$ and $c = 0.2\ m$ then the kinematics is simplified to this of a 3-link mechanism as studied in the lectures. In this case:

a) Jacobian [25%]

Plot the robot with $d = 0\ m$, and calculate the Jacobian matrix relating the joint velocities $\dot{q}_L$, $\dot{q}_R$, $\dot{\theta}$ and the hand velocity $\dot{H} = [\dot{x}_1, \dot{x}_2]^T$.

b) Movement integration [25%]

Use Matlab or Python to program a straight hand movement with duration $T = 2s$ from position $[0.141, 0.441]^T m$ to position $[0.241, 0.641]^T m$ with velocity profile $\sigma(\tau) = 30\tau^2(\tau^2 - 2\tau + 1), \tau = t/T$, where $t \in [0, T]$. Provide plots of the hand velocity $\sigma(\tau(t))$, hand position profile $\int_0^t \sigma(\tau(t'))\, dt'$ and hand trajectory $H(t) = [x_1(t), x_2(t)]^T$.

c) Movement integration [25%]

Plot the profiles of $q_L(t)$, $q_R(t)$, $\theta(t)$, against time, of the trajectory minimising the joint velocity norm $\sqrt{\dot{q_L}^2 + \dot{q_R}^2 + \dot{\theta}^2}$. For the starting configuration, use $q_L = 150°$, $q_R = 30°$, and $\theta = 45°$.