Tutorial 2: Dynamics and Control of a parallel robot

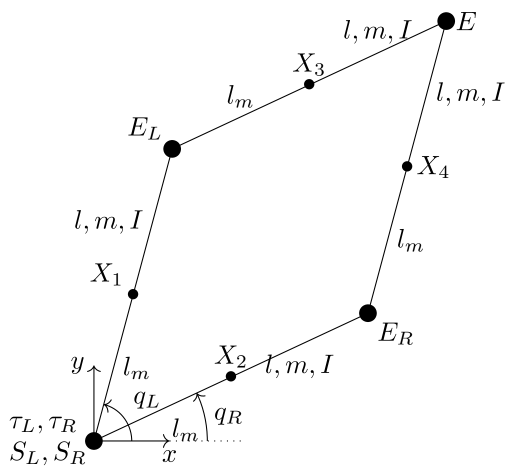

Figure 1: Parallel robot diagram

A planar four-link parallel robot is illustrated in Fig.1. S_L and S_R are the left and right shoulder coordinate vectors, respectively, E_L and E_R the left and right elbow coordinate vectors, and E is the endpoint coordinate vector. q_L and q_R denote the angles of the left and right shoulder joints, respectively, and τ_L and τ_R the motor commands applied to these two joints, respectively. $\{X_i, i = 1, 2, 3, 4\}$ denote the coordinate of the centre of mass of each link. These four links have the same parameters: mass $m = 1.0kg$, length $l = 0.2m$, distance from the joint to the centre of mass $l_m = 0.1m$ and moment of inertia $I = 0.01kg\,m^2$.

Q1: Dynamics [25%]

We have seen in the lectures that the mass matrix of the parallel robot with equal limbs illustrated in Fig.1 is:

$$\mathbf{H} = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}, \; \alpha = 2\,m\,l_m^2 + m\,l^2 + 2I, \quad \beta = 2\,m\,l\,l_m \cos(q_R - q_L)$$

Determine the dynamics $\boldsymbol{\tau} = \mathbf{H}\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q})$ of this robot using the Lagrange formulation.

Q2: Control

a) Feedback control [50%]

For the same parallel robot, you are given the desired trajectory of the endpoint E as:

$$\mathbf{x}_d \equiv \begin{bmatrix} 0.273 - 0.2(6\omega^5 - 15\omega^4 + 10\omega^3) \\ 0.273 - 0.1(6\omega^5 - 15\omega^4 + 10\omega^3) \end{bmatrix}$$

where $\omega \equiv t/T$, t is the time and $T = 2s$ is the movement duration. Use inverse kinematics to obtain the desired joint angles $\mathbf{q}_d = \int_0^t \mathbf{J}^{-1}\dot{\mathbf{x}}_d(t')\,dt'$ where the Jacobian matrix is

$$\mathbf{J} \equiv l \begin{bmatrix} -\sin q_L & -\sin q_R \\ \cos q_L & \cos q_R \end{bmatrix}.$$

The linear feedback controller is defined by:

$$\boldsymbol{\tau}_{FB} \equiv K(\mathbf{e} + \kappa\dot{\mathbf{e}}), \quad \mathbf{e} \equiv \mathbf{q}_d - \mathbf{q}.$$

Set the control gain $K = 0.01Nm$ and $\kappa = 100s$, and use Matlab to program the robot dynamics and control. Please submit your Matlab code and also provide plots of: i) the desired and actual angles against time; ii) the desired and actual endpoint positions in x and y directions against time; and iii) the desired and actual endpoint trajectories in the x-y plane. What do you observe when you change the gain K?

b) Feedforward control [25%]

To compensate for the effect of the dynamics, use the following feedback and feedforward controller:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{FB} + \boldsymbol{\tau}_{FF}, \quad \boldsymbol{\tau}_{FF} = \mathbf{H}\ddot{\mathbf{q}}_d + \mathbf{V}(\mathbf{q}_d, \dot{\mathbf{q}}_d) + \mathbf{G}(\mathbf{q}_d)$$

Please submit your Matlab code and also provide the same plots as in **Q2 a)** and explain how the feedforward controller improves the control performance.