Robotics 1: Tutorial #1 Submission

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Question 1

My work is written and attached below via the app CamScanner. My final answer is boxed below in RED ink.
For reference, the final answer written should read as follows immediately.

$$H = \left[H_x \; H_y\right]^T$$

$$H_x = \frac{a}{2}(\cos(q_R) + \cos(q_L)) + \frac{d}{2} + c * \cos(\theta) + \sqrt{\frac{b^2 - \left\|\overrightarrow{E_R C}\right\|^2}{\left\|\overrightarrow{E_R C}\right\|^2}} * \left(\frac{a}{2}(\sin(q_L) - \sin(q_R))\right)$$

$$H_y = \frac{a}{2}(\sin(q_R) + \sin(q_L)) + c * \sin(\theta) + \sqrt{\frac{b^2 - \left\|\overrightarrow{E_R C}\right\|^2}{\left\|\overrightarrow{E_R C}\right\|^2}} * \left(\frac{a}{2}(\cos(q_R) - \cos(q_L)) + \frac{d}{2}\right)$$

Where, $\left\|\overrightarrow{E_R C}\right\|^2 = \frac{a^2}{2} - \frac{a^2}{2}\cos(q_L - q_R) + \frac{d^2}{4} + \frac{ad}{2}(cos(q_R) - \cos(q_L))$

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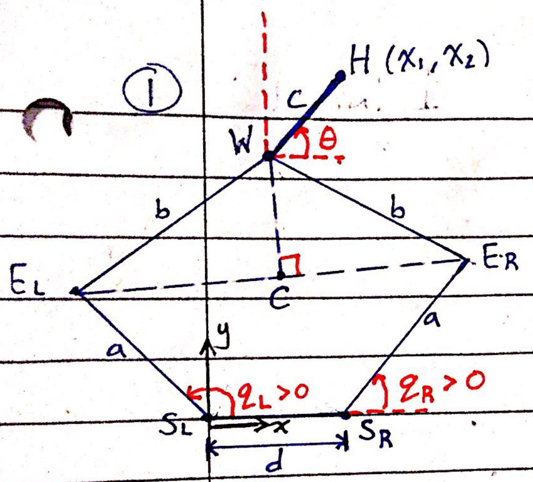

Assumptions: ① Origin at S_L $(0,0) = (x,y)$

② W is $\perp$ to $E_L - E_R$ through midpoint C

$$E_L = (a\cos(q_L))\hat{\imath} + (a\sin(q_L))\hat{\jmath} \quad — ①$$

$$E_R = (a\cos(q_R) + d)\hat{\imath} + (a\sin(q_R))\hat{\jmath} \quad — ②$$

$$C = \left(\frac{E_{Rx} + E_{Lx}}{2}\right)\hat{\imath} + \left(\frac{E_{Ry} + E_{Ly}}{2}\right)\hat{\jmath}$$

$$C = \frac{a}{2}\left(\cos(q_R) + \frac{d}{a} + \cos(q_L)\right)\hat{\imath} + \frac{a}{2}\left(\sin(q_R) + \sin(q_L)\right)\hat{\jmath} \quad — ③$$

- Find vector $\overrightarrow{E_R - C}$

$$\overrightarrow{E_R - C} = \left[\frac{a}{2}\cos q_R + \frac{d}{2} + \frac{a}{2}\cos q_L - a\cos q_R - d\right]\hat{\imath} + \left[\frac{a}{2}\sin q_R + \frac{a}{2}\sin q_L - a\sin q_R\right]\hat{\jmath}$$

$$\overrightarrow{E_R - C} = \left(\frac{a}{2}\cos q_R - \frac{a}{2}\cos q_L + \frac{d}{2}\right)\hat{\imath} + \left[\frac{a}{2}\sin q_R - \frac{a}{2}\sin q_L\right]\hat{\jmath}$$

Continued.

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- Find the normal of vector $\overrightarrow{E_R C}$ $\therefore \|\overrightarrow{E_R C}\| = ?$

$$\|\overrightarrow{E_R C}\| = \sqrt{\left(\frac{a}{2}\cos q_L - \frac{a}{2}\cos q_R - \frac{d}{2}\right)^2 + \left(\frac{a}{2}\sin q_L - \frac{a}{2}\sin q_R\right)^2}$$

$$= \sqrt{\frac{a^2}{4}\cos^2 q_L + \frac{a^2}{4}\cos^2 q_R + \frac{d^2}{4} - \frac{a^2}{2}\cos q_L \cos q_R + \frac{ad}{2}\cos q_R - \frac{ad}{2}\cos q_L + \cdots}$$

$$\cdots \left(\frac{a^2}{4}\sin^2 q_L + \frac{a^2}{4}\sin^2 q_R - \frac{a^2}{2}\sin q_L \sin q_R\right)$$

$$= \frac{a^2}{4}\underbrace{(\cos^2 q_L + \sin^2 q_L)}_{=1} + \frac{a^2}{4}\underbrace{(\cos^2 q_R + \sin^2 q_R)}_{=1} + \frac{d^2}{4} - \frac{a^2}{2}(\cos q_L \cos q_R + \sin q_L \sin q_R) + \cdots$$

$$+ \frac{ad}{2}(\cos q_R - \cos q_L)$$

$$\|\overrightarrow{E_R C}\| = \sqrt{\frac{a^2}{2} + \frac{d^2}{4} - \frac{a^2}{2}\cos(q_L - q_R) + \frac{ad}{2}(\cos q_R - \cos q_L)}$$

- Find Unit vector of Segment E_R and C

$$\hat{m} = \frac{\overrightarrow{E_R C}}{\|\overrightarrow{E_R C}\|} = \left(\frac{-\frac{a}{2}\cos q_L + \frac{a}{2}\cos q_R + \frac{d}{2}}{\|\overrightarrow{E_R C}\|}\right)\hat{\imath} + \left(\frac{-\frac{a}{2}\sin q_L + \frac{a}{2}\sin q_R}{\|\overrightarrow{E_R C}\|}\right)\hat{\jmath}$$

- Let Unit Vector of Segment W and C $= \hat{N}$

$$\therefore \hat{N} = \frac{\overrightarrow{WC}}{\|\overrightarrow{WC}\|}$$

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- Let Unit Vector of Segment W and C $= \hat{N}$

$$\therefore \hat{N} = \frac{\overrightarrow{WC}}{\|\overrightarrow{WC}\|}$$

- Assume $\hat{m} \perp \hat{N} \therefore \hat{m}(x,y) = \hat{N}(-y,x)$

$$\therefore \hat{N} = \left(\frac{\frac{a}{2}\sin q_L - \frac{a}{2}\sin q_R}{\|\overrightarrow{E_RC}\|}\right)\hat{i} + \left(\frac{-\frac{a}{2}\cos q_L + \frac{a}{2}\cos q_R + \frac{d}{2}}{\|\overrightarrow{E_RC}\|}\right)\hat{j} = \frac{\overrightarrow{WC}}{\|\overrightarrow{WC}\|}$$

$$\|\overrightarrow{WC}\| = \sqrt{b^2 - \|\overrightarrow{E_RC}\|^2}$$

$$\therefore \overrightarrow{WC} = \sqrt{\frac{b^2 - \|\overrightarrow{E_RC}\|^2}{\|\overrightarrow{E_RC}\|^2}}\left(\frac{a}{2}(\sin q_L - \sin q_R)\hat{i} + \left(\frac{a}{2}(\cos q_R - \cos q_L) + \frac{d}{2}\right)\hat{j}\right)$$

but $\overrightarrow{WC} = W - C \quad \therefore \quad W = C + \overrightarrow{WC}$

and $H = W + c(\cos\theta\hat{i} + \sin\theta\hat{j}) = C + \overrightarrow{WC} + c(\cos\theta\hat{i} + \sin\theta\hat{j})$

$$H_x = \frac{a}{2}(\cos q_R + \cos q_L) + \frac{d}{2} + c\cos\theta + \sqrt{\frac{b^2 - \|\overrightarrow{E_RC}\|^2}{\|\overrightarrow{E_RC}\|^2}}\left(\frac{a}{2}(\sin q_L - \sin q_R)\right)$$

$$H_y = \frac{a}{2}(\sin q_R + \sin q_L) + c\sin\theta + \sqrt{\frac{b^2 - \|\overrightarrow{E_RC}\|^2}{\|\overrightarrow{E_RC}\|^2}}\left(\frac{a}{2}(\cos q_R - \cos q_L) + \frac{d}{2}\right)$$

where $\|\overrightarrow{E_RC}\|^2 = \frac{a^2}{2} - \frac{a^2}{2}\cos(q_L - q_R) + \frac{d^2}{4} + \frac{ad}{2}(\cos q_R - \cos q_L)$.

when $d = 0 \longrightarrow \|\overrightarrow{E_RC}\|^2 = \frac{a^2}{2}(1 - \cos(q_L - q_R))$

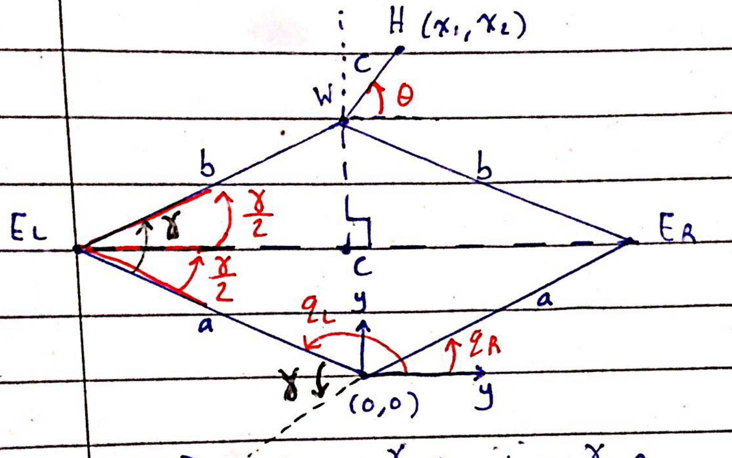
Question 2

a)

My work is attached below via the app CamScanner. For reference, my Jacobian Matrix is as follows immediately.

$$J = \begin{bmatrix} -\mathrm{a} * \sin{(q_L)} & -b * sin(q_R) & -c * sin(\theta) \\ a * \cos(q_L) & b * \cos(q_R) & c * \cos(\theta) \end{bmatrix}$$

(2.) let $d=0$ $\quad a=b=0.3\,m \quad c=0.2\,m$



$$\vec{E_R} = a(\cos q_R\,\hat{i} + \sin q_R\,\hat{j})$$
$$\vec{E_L} = a(\cos q_L\,\hat{i} + \sin q_L\,\hat{j})$$

Following the rule that $\cancel{\frac{\pi}{2}}$ $\angle 1 = \angle 2$

$$180^\circ = \pi = q_L - q_R + \gamma$$
$$\therefore \gamma = \pi + q_R - q_L$$

$$\vec{WE_L} = b\cos\frac{\gamma}{2}\,\hat{i} + b\sin\frac{\gamma}{2}\,\hat{j}$$
$$\vec{HW} = c\cos\theta\,\hat{i} + c\sin\theta\,\hat{j}$$
$$\therefore H = \vec{E_L} + \vec{WE_L} + \vec{HW} = (a\cos q_L + \underbrace{b\cos\tfrac{\gamma}{2}}_{①} + c\cos\theta)\,\hat{i} + (a\sin q_L + \underbrace{b\sin\tfrac{\gamma}{2}}_{②} + c\sin\theta)\,\hat{j}$$

Assume $\frac{\gamma}{2} = q_R$ because of the rules of a rombus!

test: $q_L = 150^\circ \quad q_R = 30^\circ \longrightarrow \frac{\gamma}{2} = (90^\circ + \frac{30^\circ - 150^\circ}{2} = 30^\circ$

$\therefore q_R = \frac{\gamma}{2} \longrightarrow$ H can be simplified further.

$$\rightarrow H = (a\cos q_L + b\cos q_R + c\cos\theta)\,\hat{i} + (a\sin q_L + b\sin q_R + c\sin\theta)\,\hat{j}$$

- Now, take derivative of H as follows $\dot{H} = \frac{dH}{dt} = \frac{\partial H}{\partial q_R}\cdot\frac{\partial q_R}{\partial t} + \frac{\partial H}{\partial q_L}\cdot\frac{\partial q_L}{\partial t} + \frac{\partial H}{\partial \theta}\frac{\partial \theta}{\partial t}$

$$\rightarrow H = (a\cos q_L + b\cos q_R + c\cos\theta)\,\hat{i} + (a\sin q_L + b\sin q_R + c\sin\theta)\,\hat{j}$$

- Now, take derivative of H as follows $\dot{H} = \frac{dH}{dt} = \frac{\partial H}{\partial q_R}\cdot\frac{\partial q_R}{\partial t} + \frac{\partial H}{\partial q_L}\cdot\frac{\partial q_L}{\partial t} + \frac{\partial H}{\partial \theta}\frac{\partial \theta}{\partial t}$

$$\dot{H}_x = -a\sin q_L(\dot{q}_L) - b\sin q_R(\dot{q}_R) - c\sin\theta(\dot{\theta})$$

$$\dot{H}_y = a\cos q_L(\dot{q}_L) + b\cos q_R(\dot{q}_R) + c\cos\theta(\dot{\theta})$$

let Jacobian Matrix $= J$ $\quad\therefore\quad \dot{H} = \begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = [J]\begin{bmatrix} q_L \\ q_R \\ \theta \end{bmatrix}$

$$\boxed{\dot{H} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix} -a\sin(q_L) & -b\sin(q_R) & -c\sin\theta \\ a\cos(q_L) & b\cos(q_R) & c\cos\theta \end{bmatrix}}_{\text{Jacobian Matrix}} \begin{bmatrix}\dot{q}_L \\ \dot{q}_R \\ \dot{\theta}\end{bmatrix}}$$

Question 2 continued…

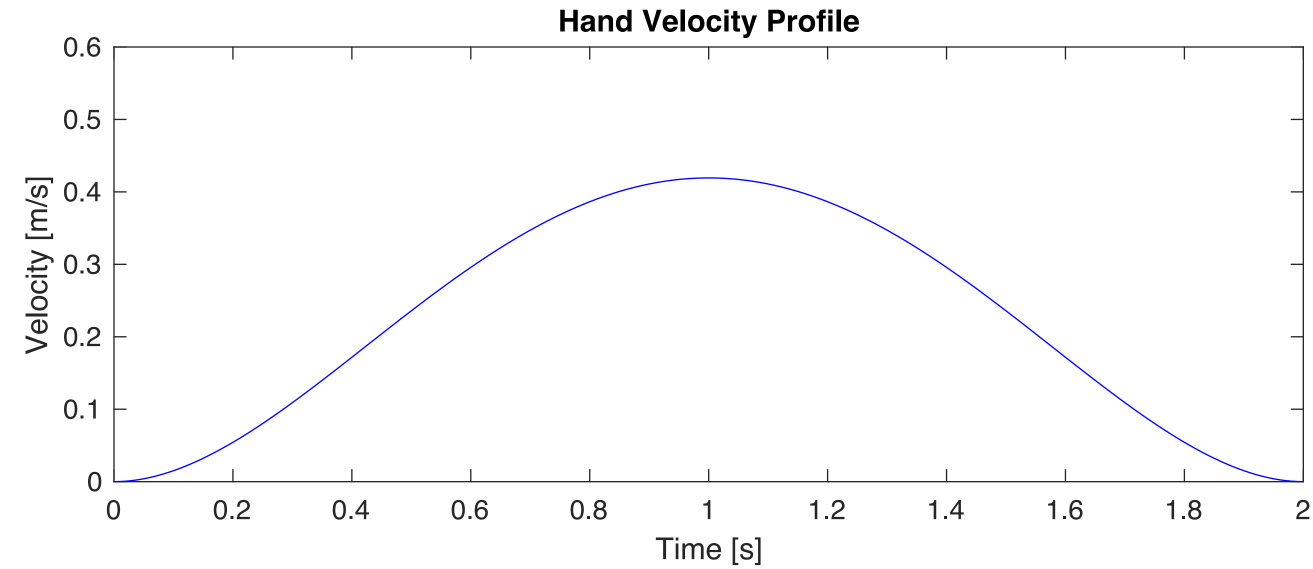
b) /c)

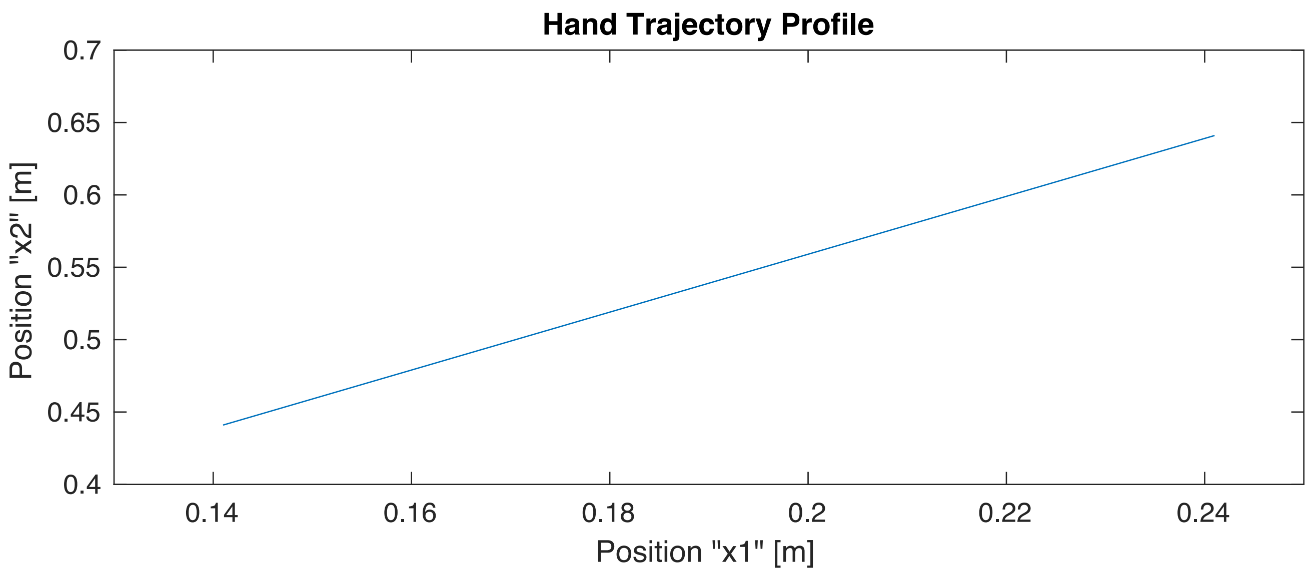
Please note that the same Matlab script was used to compute the answers for question 2.B and 2.C. The Matlab script, then the Matlab function, and then the resulting plots for 2B and 2C are as follows respectively.

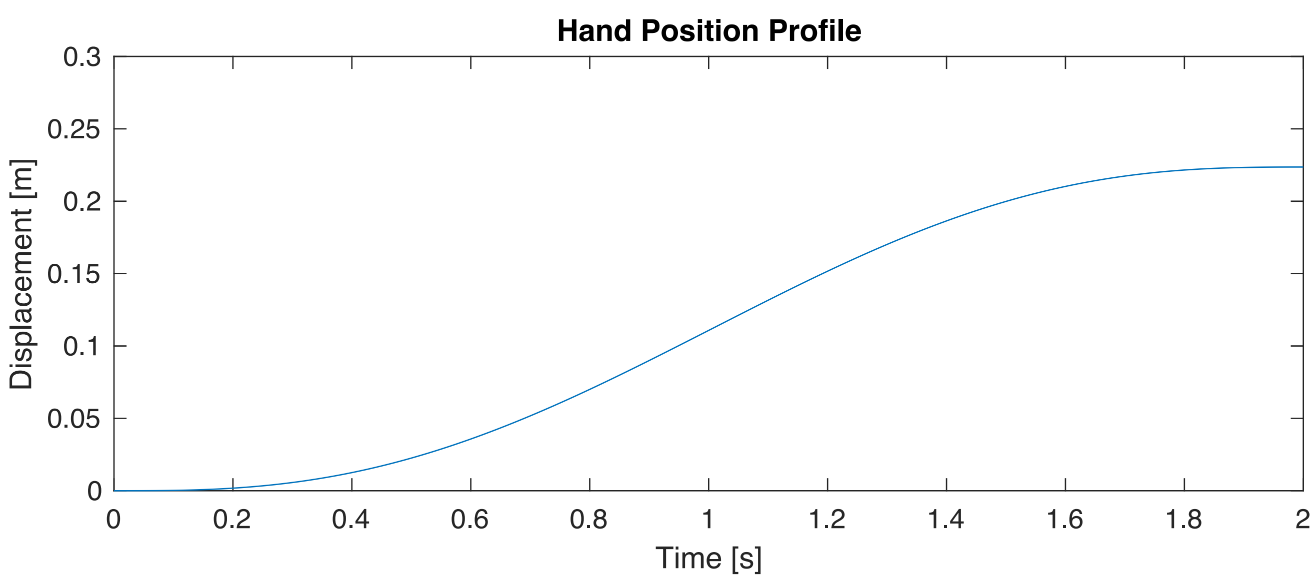
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% File name Tutorial_1_question_2.m
%% Constant Variables
T=2; % Total time duration [s]
ts=0.01; % time step
d=0; % Distance between the shoulders
a=0.3; % Length of upper arm [m]
b=0.3; % Length of forearm [m]
c=0.2; % Length of hand [m]
ql=deg2rad(150); % Initial left motor angle
qr=deg2rad(30); % Initial right motor angle
theta=deg2rad(45); % Initial hand orientation

%% Initiate angular joint matrix
joint_angles=zeros(3,T/ts); % Angle profiles
joint_angles(1,1)=ql;
joint_angles(2,1)=qr;
joint_angles(3,1)=theta;

%% Define initial and final position of point H
x0=[0.141,0.441]'; % Initial position
x1=[0.241,0.641]'; % Final position
length=sqrt((x1(2)-x0(2))^2+(x1(1)-x0(1))^2); % length of vector

%% Define Velocity profile
t=0:ts:T; % time matrix
tau=t/T; % non-dimensional time
sigma=30*tau.^2.*(tau.^2-2.*tau+1); % Velocity Profile
Real_Velocity=sigma*length; % Real Velocity
%% Define Displacement/Position profiles
posx=0:1:T/ts; % Create position x matrix
posy=0:1:T/ts; % Create position y matrix
posx(1,1)=x0(1); % Set starting position x
posy(1,1)=x0(2); % Set starting position y
sum=0:1:T/ts; % Create matrix for displacement

%% Calculate Position and Trajectory
for i=1:1:T/ts
    sum(1,i+1)=sum(1,i)+ts*(length)*sigma(1,i)/T; % calculate
displacements
    posx(1,i+1)=posx(1,i)+ts*(length)*sigma(1,i)/T*(x1(1)-
x0(1))/(length); % calculate position x
    posy(1,i+1)=posy(1,i)+ts*(length)*sigma(1,i)/T*(x1(2)-
x0(2))/(length); % calculate position y
end

%% Calculate Joint Angles: ql, qr, theta
```

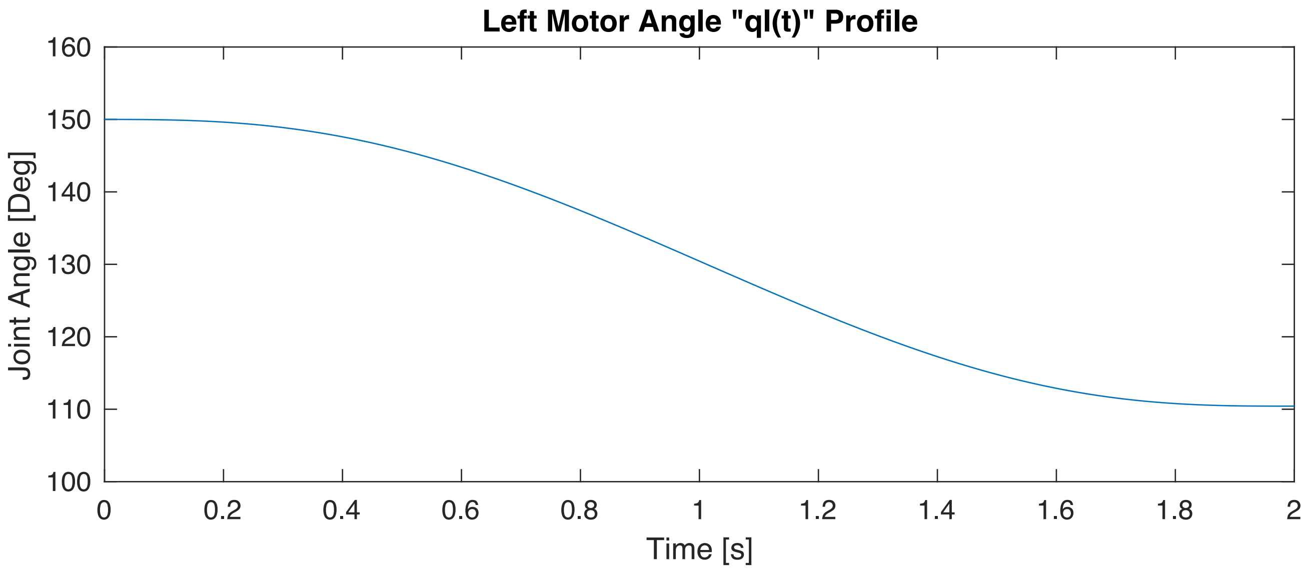
```
joint_velocity_norm=zeros(1,T/ts+1); % Set up an emprty matrix to store
joint veloicty norm values
for i=1:1:T/ts
J=Jacobian(joint_angles(1,i),joint_angles(2,i),joint_angles(3,i)); %
Call the Jacobian Function
J_inv=J.'*inv(J*J.'); % Calculate pseudo inverse Jacobian
velocity=J_inv*[sigma(1,i+1)*length*(x1(1)-
x0(1))/length;sigma(1,i+1)*length*(x1(2)-x0(2))/length];

joint_angles(1,i+1)=joint_angles(1,i)+ts/T*velocity(1,1); % ql
joint_angles(2,i+1)=joint_angles(2,i)+ts/T*velocity(2,1); % qr
joint_angles(3,i+1)=joint_angles(3,i)+ts/T*velocity(3,1); % theta

joint_velocity_norm(i)=norm(velocity); % Store the norm values
end

%% Change joint angles from radians to degrees
Deg_joint_angles=joint_angles*180/pi();

%% Print Plots for Question 2B
figure(1)
hold on
% Hand velocity
subplot(3,2,1:2)
plot(tau*T,Real_Velocity,'b-');
title('Hand Velocity Profile');
xlabel('Time [s]');
ylabel('Velocity [m/s]');
% Hand trajectory
subplot(3,2,3:4)
plot(posx,posy);
title('Hand Trajectory Profile')
xlabel('Position "x1" [m]');
ylabel('Position "x2" [m]');
axis([0.13 0.25 0.4 0.7]);
% Position Profile
subplot(3,2,5:6)
plot(t,sum);
title('Hand Position Profile');
xlabel('Time [s]');
ylabel('Displacement [m]');
hold off

%% Print plots for question 2C
% ql versus time
figure(2)
hold on
subplot(3,2,1:2)
plot(t, Deg_joint_angles(1,:));
title('Left Motor Angle "ql(t)" Profile');
xlabel('Time [s]');
ylabel('Joint Angle [Deg]');
% qr versus time
subplot(3,2,3:4)
plot(t, Deg_joint_angles(2,:));
title('Right Motor Angle "qr(t)" Profile');
xlabel('Time [s]');
```

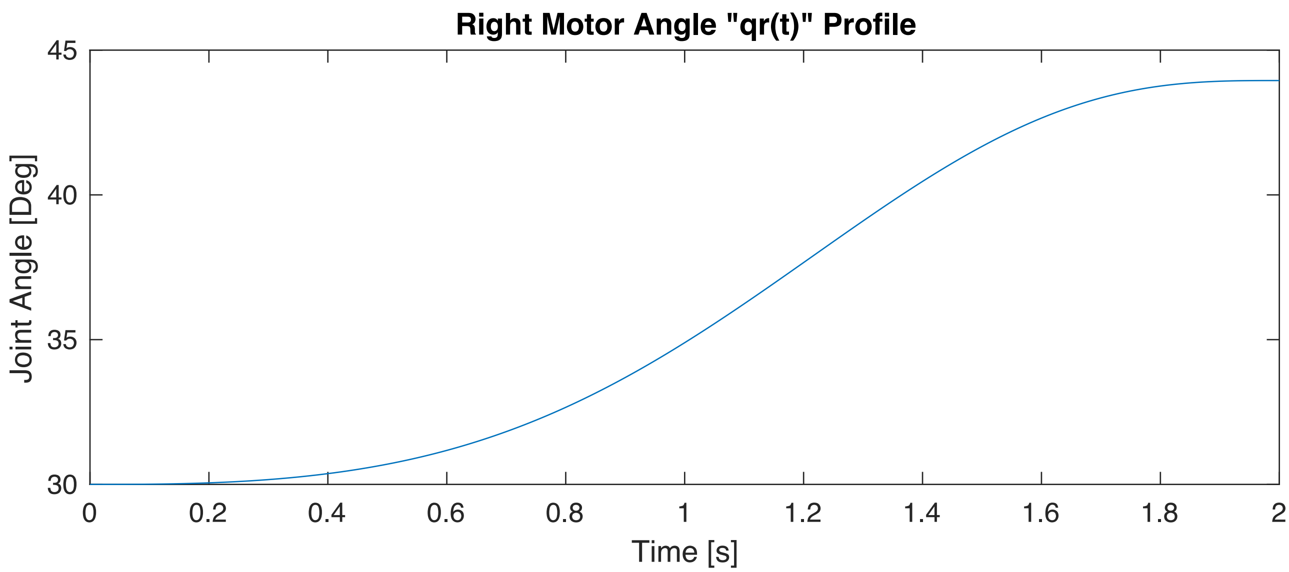
```
ylabel('Joint Angle [Deg]');
% theta versus time
subplot(3,2,5:6)
plot(t, Deg_joint_angles(3,:));
title('Hand Orientation Angle "theta(t)" Profile');
xlabel('Time [s]');
ylabel('Joint Angle [Deg]');
hold off
% Plot the joint velocity norm
figure(3)
plot(t,joint_velocity_norm);
title('Joint Velocity Norm Versus Time');
xlabel('Time [s]');
ylabel('Joint Velocity Norm [deg/s]');

%% End of Script
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% File name Jacobian.m
% This function is to calculate the Jacobian matrix, and will be called
%  in the script Tutorial_1_question_2
function J=Jacobian(ql,qr,theta)
a=0.3;
b=0.3;
c=0.2;
J=[-a*sin(ql) -b*sin(qr) -c*sin(theta); a*cos(ql) b*cos(qr)
c*cos(theta)];
end
% End of function
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

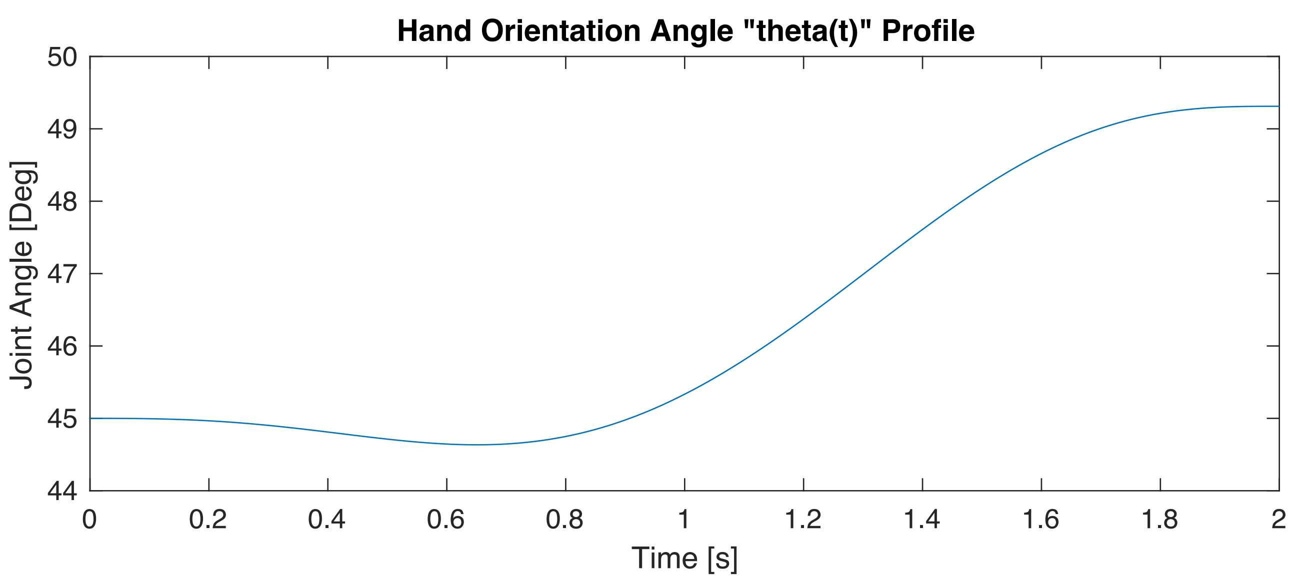

Hand Velocity Profile
Velocity [m/s]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

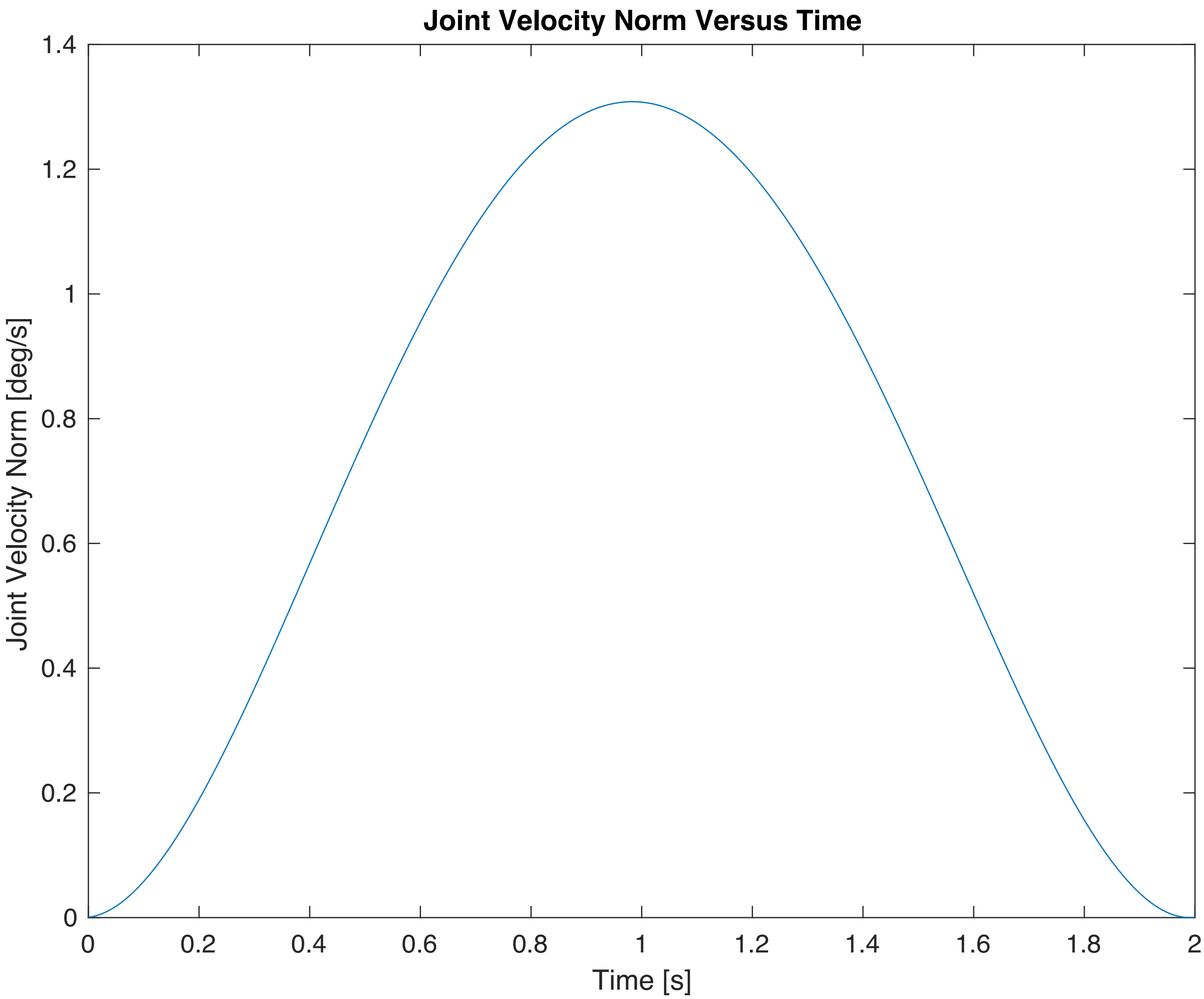

Hand Trajectory Profile
Position "x2" [m]
Position "x1" [m]
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.14
0.16
0.18
0.2
0.22
0.24


Hand Position Profile
Displacement [m]
Time [s]
0
0.05
0.1
0.15
0.2
0.25
0.3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Left Motor Angle "ql(t)" Profile
Joint Angle [Deg]
Time [s]
160
150
140
130
120
110
100
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Right Motor Angle "qr(t)" Profile
Joint Angle [Deg]
Time [s]
45
40
35
30
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Hand Orientation Angle "theta(t)" Profile
Joint Angle [Deg]
Time [s]
50
49
48
47
46
45
44
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Joint Velocity Norm Versus Time
Joint Velocity Norm [deg/s]
Time [s]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2