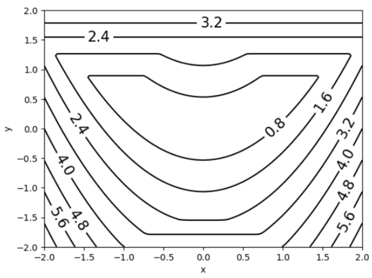
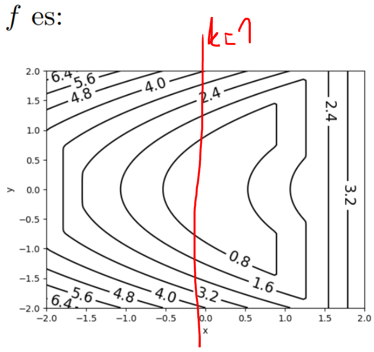
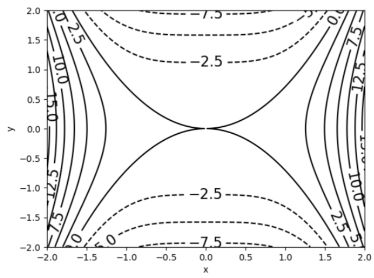
1. Sea $f : \mathbb{R}^2 \to \mathbb{R}$ la función dada por $f(x,y) = e^{\text{máx}\{|xy|,x^2\}}$. Entonces, la figura que más se asemeja a las curvas de nivel de f es:



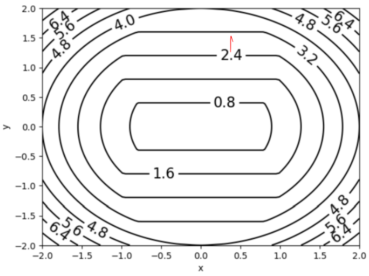
(a)



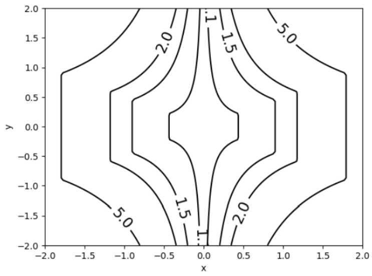
(b)



(c)



(d)



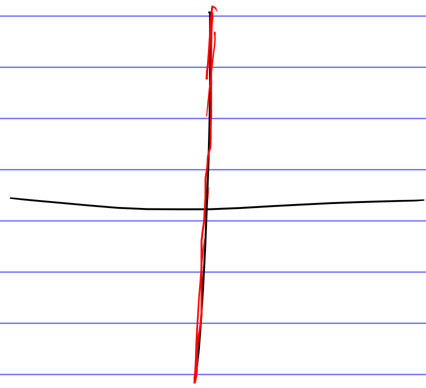
(e)

$C_k = \{(x,y) : f(x,y) = k\}$. $\underline{k=1}$

$e^{\max\{|x,y|, x^2\}} = 1 \iff$

$\max\{\underbrace{|x,y|}_{0}, \underbrace{x^2}_{0}\} = 0$

$\boxed{x=0}$ $\underline{y \in \mathbb{R}}$.



(10) Sea $\alpha \in \mathbb{R}$. La integral $\int_0^2 \left(\frac{x}{2-x}\right)^\alpha dx$:

A. converge para todo $\alpha > 1$.
B. converge para todo $\alpha \in (-1, 1)$.
C. converge para todo $\alpha < 0$.
D. converge para todo $\alpha \in (-1, 0)$.

Solución. La opción correcta es la (B):

C.V = $2 - x = u$

$$\int_0^2 \frac{(u+2)^\alpha}{u^\alpha} \sim 2^\alpha$$

$$\frac{x}{2-x} = \frac{1}{\frac{2-x}{x}}$$

A

Ejercicio 6

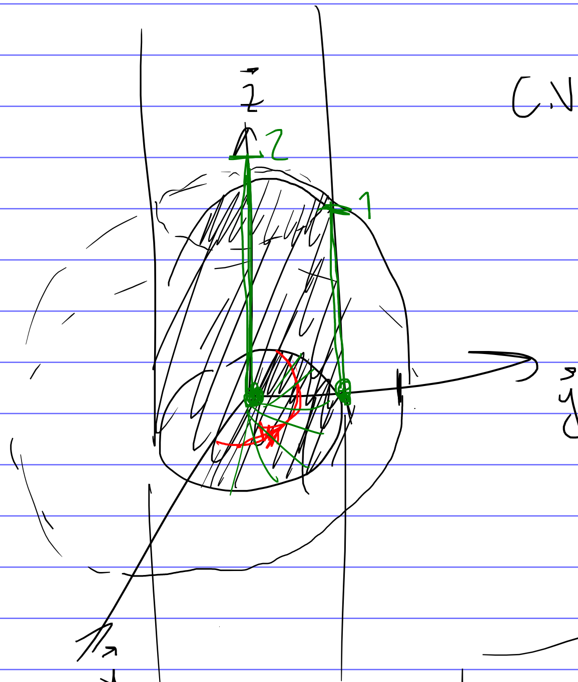
Sea $f(x, y, z) = z$ y $D = \{(x, y, z) \in I\!R^3 : y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 2, x^2 + y^2 \leq 1\}$.
Entonces $\iiint_D f(x, y, z)dxdydz$ vale: (sugerencia: utilizar coordenadas cilíndricas)

(A) $\frac{3\pi}{2}$
(B) $\frac{\pi}{8}$
(C) $\frac{3\pi}{8}$
(D) $\frac{5\pi}{6}$
(E) $\frac{3\pi}{4}$

$y \geq 0$

$x^2 + y^2 + z^2 \leq 2 \rightarrow$ Esfera de radio $\sqrt{2}$

$x^2 + y^2 \leq 1 \rightarrow$ Cilindro de radio 1



C.V $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$

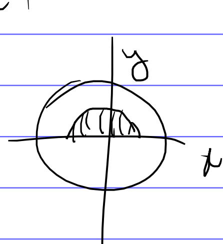
$x^2 + y^2 \leq 1 \rightarrow \boxed{0 \leq r \leq 1}$

$x^2 + y^2 + z^2 \leq 2 \Rightarrow \boxed{r^2 + z^2 \leq 2}$

$z \geq 0$
$z \leq \sqrt{2 - r^2}$

$\boxed{y \geq 0} \rightarrow \sin\theta \geq 0$

$\rightarrow 0 \leq \theta \leq \pi$



$$\int_0^\pi d\pi \int_0^1 dr \int_0^{\sqrt{2-r^2}} |J_g| \, f \cdot dz$$

Ejercicio 4.(20 pts.)Sea $f : \mathbb{R}^2 \to \mathbb{R}$ tal que:

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^4+y^2} & si \quad (x,y) \neq (0,0) \\ 0 & si \quad (x,y) = (0,0) \end{cases}$$

∘ cont en 0 , si no es cont no es diferenciable.

∘ Si f es dif en p $\Rightarrow$

$$f(p_1+h_1, p_2+h_2) = f(p) + \langle \nabla f_p, (h_1,h_2) \rangle + r(h_1,h_2)$$

tal que $\dfrac{r(h_1,h_2)}{\|(h_1,h_2)\|} \longrightarrow 0$ $\quad \|(h_1,h_2)\|$

$$(f_x(p), f_y(p)) = \nabla f(p)$$

2. Sea $f(x,y)$ una función integrable, y considere la siguiente integral doble:

$$\int_0^1 \int_{y^2}^{2-\sqrt{y}} f(x,y)\,dx\,dy$$

Entonces una forma equivalente de escribir la integral es:

(A) $\int_0^2 \int_{\sqrt{x}}^{(x-2)^2} f(x,y)dydx$

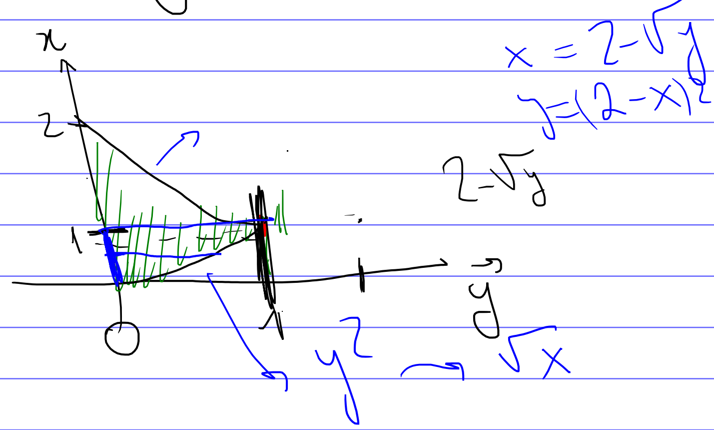
(B) $\int_0^1 \int_0^{\sqrt{x}} f(x,y)dydx + \int_1^2 \int_0^{(x-2)^2} f(x,y)dydx$

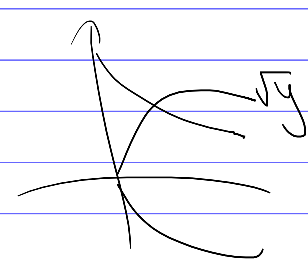
(C) $\int_0^1 \int_0^{x^2} f(x,y)dydx + \int_1^2 \int_0^{\sqrt{x-2}} f(x,y)dydx$

(D) $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)dydx + \int_1^2 \int_{(x-2)^2}^{\sqrt{x-2}} f(x,y)dydx$

(E) $\int_0^2 \int_{x^2}^{\sqrt{2-x}} f(x,y)dydx$

$0 \leq y \leq 1$, $\quad y^2 \leq x \leq 2-\sqrt{y}$





Ejercicio 4

Dados α y β reales positivos, se considera la integral impropia

$$\int_0^{\infty} \frac{dx}{x^{\alpha}(1+e^{x^2}x^{\beta})}$$

Indicar la opción correcta:

(A) La integral es convergente si $\boxed{\alpha < 1}$ para todo β.

(B) La integral es convergente si y solo si $\alpha < 1$, y $\beta > 1-\alpha$.

(C) La integral es convergente si y solo si $\alpha < 1$, y $\boxed{\beta < 1-\alpha.}$

(D) La integral es convergente si y solo si $\alpha > 1$, y $\beta > 1-\alpha$. $\times$

(E) La integral es convergente si y solo si $\alpha > 1$, y $\beta < 1-\alpha$. $\times$

$$\int_0^{+\infty} \frac{1}{x^{\alpha}(\underbrace{1+e^{x^2}x^{\beta}}_{1})} = \int_0^{+\infty} \frac{1}{x^{\alpha} + e^{x^2}x^{\beta+\alpha}} < \int_0^{\infty} \frac{1}{e^{x^2}x^{\beta+\alpha}}$$

$$\int_0^{1} \frac{1}{e^{x^2}x^{\beta+\alpha}} + \int_1^{\infty} \frac{1}{e^{x^2}x^{\beta+\alpha}} < \int_1^{\infty} \frac{1}{e^{x^2}}$$

$$\int_0^{1} \frac{1}{e^{x^2}x^{\beta+\alpha}} \sim \int_0^{1} \frac{1}{x^{\beta+\alpha}} \Rightarrow \boxed{\beta+\alpha<1} \checkmark$$

$$\int_0^{\infty} \frac{1}{x^{\alpha}(1+e^{x^2}x^{\beta})} = \underbrace{\int_0^{1} \frac{1}{x^{\alpha}(1+e^{x^2}x^{\beta})}}_{} + \int_1^{+\infty} \frac{1}{x^{\alpha}(1+e^{x^2}x^{\beta})} \rightarrow \boxed{\alpha<1}$$

$$i\ \frac{1}{x^{\alpha}(1+e^{x^2}x^{\beta})} < \frac{1}{x^{\alpha}} \checkmark < \frac{1}{x^{\alpha}}$$