- DO NOT START UNTIL INSTRUCTED TO DO SO.
- Please do not write your name anywhere. We want to grade these anonymously (aside from the final grade entry of course).
- Turn your cellphones to do not disturb. Calculators are not allowed.
- You can leave you answer in unsimplified form. Even 2 + 2 is not something we are grading you for. This is not a test of mental arithmetic.
- Answer the questions in the space provided. We are NOT scanning the back of any page. We recommend writing in pencil. If you do not have a pencil, we can get you one.
- Each question may or may not be tricky. Please do not spend excessive time on any single question. The questions might not be in increasing order of difficulty.
- Proof by illegibility will be frowned upon. Take your time and please write as neatly as you can.
- Unless the question says "No explanation needed", you have to provide some reasoning. Lack of explanation will result in loss of points.
- The last sheet is for scratch paper. You can tear if off but please throw it into recycling when you leave the room.

PennID (the 8 **digits** in big font on your penncard): ____________________

Definitions

- For integers a and b, $a|b$ (a divides b) iff $\exists k \in \mathbb{Z}$ such that $b = ak$.
- An integer k is even iff $2|k$.
- An integer b is odd iff $\exists k \in \mathbb{Z}$ such that $b = 2k + 1$.
- $\forall x \in \mathbb{Z}$ x is even iff x is not odd.
- A positive integer $n > 1$ is prime iff the only factors of n are 1 and n.
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- The emptyset $\emptyset$ is a subset of every set.

1. (3 points) Prove that if $3n + 2$ is odd then n is odd. Assume $n \in \mathbb{Z}$.

2. (5 points) Assume x, y, z all come from the domain of **even** integers
   Are the following True or False. No explanation needed.
   1. $\exists x \forall y, xy = y$
   2. $\forall x \forall y \forall z, xy > yz$
   3. $\exists x \exists y \forall z, xy \leq z^2$
   4. $\exists x \forall y \forall z, xy < yz$
   5. $\exists x \forall y \exists z, y | xz$

3. (3 points) In the country of C, there are truth tellers and liars. Truth tellers always tell the truth. Liars can never stop lying.

   You meet two people A and B. A says

   "Exactly one of us is a liar"

   What do you think B is (truth teller or liar)? Prove it.

4. (5 points) Express the following statements using logical symbols, predicates etc. Then show how statement 6 can be derived from statements 1 through 5. For the purposes of this question, the opposite of ancient is modern. The domain for your statements must be the set of all poems.

   1. No interesting poems are unpopular among people of real taste.
   2. No modern poetry is free from nonsense.
   3. All your poems are on the subject of dugongs.
   4. No nonsense poetry is popular among people of real taste.
   5. No ancient(not modern) poem is on the subject of dugongs.
   6. Your poems are not interesting.

5. (5 points) Prove by induction that $2^n < (n+2)!$ for all integers $n \geq 0$.

6. (5 points) Let $A[1...n]$ be an array of n distinct integers. If $i < j$ but $A[i] > A[j]$ then the pair (i, j) is called an inversion of A.

   Suppose the elements of A are a random permutation of the numbers from 1 to n.

   What is the expected value for the number of inversions?

7. (5 points) Prove by induction that the number of subsets of an n element set is 2^n. $n \geq 0$, $n \in \mathbb{Z}$.