- DO NOT START UNTIL INSTRUCTED TO DO SO.
- Please do not write your name anywhere. We want to grade these anonymously (aside from the final grade entry of course).
- Turn your cellphones to do not disturb. Calculators are not allowed.
- Answer the questions in the space provided. We are NOT scanning the back of any page.
- The word **graph** means an undirected, no self-loop, and no parallel edges graph unless specified otherwise.
- There is one blank sheet of scratch paper at the end. You can also use the back of any sheet for extra scratch paper.
- Each question may or may not be tricky. Please do not spend excessive time on any single question.
- Unless the questions says ”No explanation needed”, you have to provide some reasoning..

PennID (the 8 **digits** in big font on your penncard): ____________________

Some formulae

- A tree with n vertices has $n-1$ edges.
- $$\sum_{v \in V} deg(v) = 2|E|$$
- An integer is either odd or even.
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- The number of permutations of n objects with n_1 identical objects of type 1, n_2 identical objects of type 2, $\ldots$, and n_k identical objects of type k is

$$\frac{n!}{n_1! n_2! \ldots n_k!}$$

1. (10 points) The following 8 questions are all either True or False or fill in the blanks. Each is worth 1 point. For the True or False questions you just need to write T or F. For the others you just need to fill in the blank. No explanation is needed.

   1. True or False $(A \vee \neg A) \rightarrow (A \wedge \neg A)$
   2. The expected value of an indicator random variable can be at most ____________ and will be at least ____________.
   3. A math teacher gave her class two tests. 25% of the class passed both tests and 50% of the class passed the first test. The percent of those who passed the first test that also passed the second test is ____________.
   4. True or False

      On a graph if we define a function mapping vertices to their degrees, this function can never be one to one.
   5. True or False

      If A is the power set of some set we are guaranteed that $|A|$ is even.
   6. The number of ways in which your 6 TAs can eat 10 identical cupcakes such that all of the cupcakes are consumed and each one of them eats at least one is ____________.
   7. Consider the set of integers from 1 to 10. Consider integer a to be related to integer b if and only if $a < b$. Is this relation

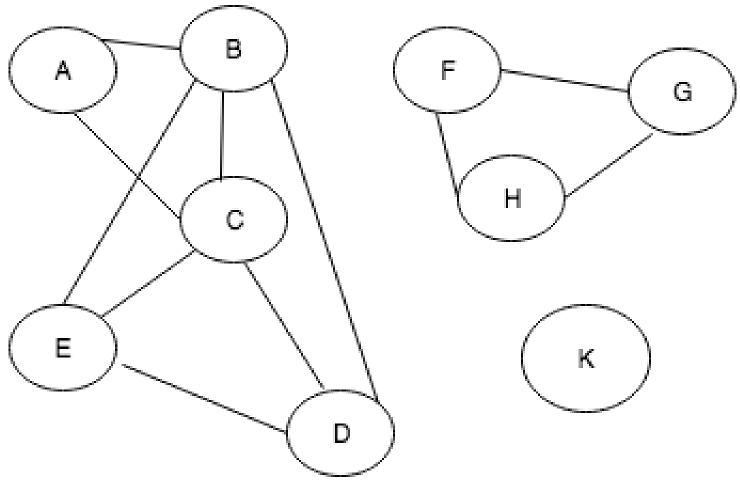
      (0.5 pts each)

      (a) Reflexive ____________

      (b) Symmetric ____________

      (c) Transitive ____________

      (d) Anti-symmetric ____________
   8. If $f(n) = 2 * f(n-2) + f(n-1)$ and $f(1) = 1$ and $f(2) = 2$, then $f(4) =$ ____________.



2. (5 points)

   How many connected components does the above graph have?

   What is the chromatic number of this graph? Show this by colouring the graph. You can do your "colouring" in the graph that we have drawn. You do not need to copy the graph.

   Is this graph bipartite? Yes or no. No explanation needed for this part.

   Is this graph planar? If so, please draw it out in a manner that proves it to be planar.

3. (5 points) Which of these degree sequences are valid degree sequences. If they are not vaild provide a reason. If they are valid, draw a graph that satisfies the sequence

   - 1, 1, 2

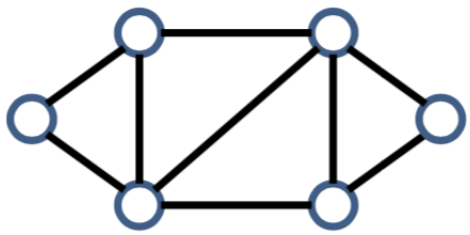
   - 1, 2, 3, 3

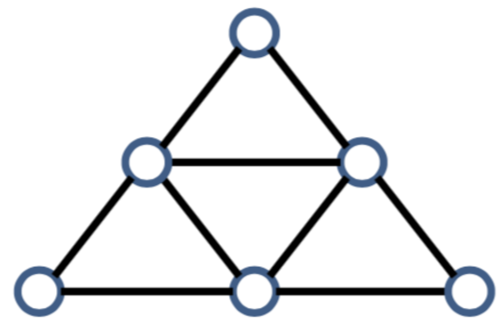
   - 0, 0, 1, 1

   - 0, 1, 2, 3

   - 3,3,1,1

4. (2 points) Are these two graphs isomorphic? If yes provide a mapping. If no provide a reason.





5. (4 points) Prove that any tree with $v \geq 2$ vertices must has at least two leaves.
   You are allowed to use the result that any tree has at least one leaf.

6. (6 points) Here is a variation of the Nim matchstick game. Instead of having 2 piles (like we did in lecture), we have just 1 pile of matches. Two players take turns removing matches, but there is a constraint. In each move, a player can only remove one match, or two matches, or three matches.

   The player removing the last match loses.

   Use induction to prove that if each player plays the best strategy possible then given the initial number of matchsticks it is possible to determine which of the two players will win.

   Your proof should describe the winning strategy as well.