- DO NOT START UNTIL INSTRUCTED TO DO SO.
- Please do not write your name anywhere. We want to grade these anonymously (aside from the final grade entry of course).
- Turn your cellphones to do not disturb. Calculators are not allowed.
- You can leave you answer in unsimplified form. Even 2 + 2 is not something we are grading you for. This is not a test of mental arithmetic.
- Answer the questions in the space provided. We are NOT scanning the back of any page.
- There is one blank sheet of scratch paper at the end. You can also use the back of any sheet for extra scratch paper.
- Each question may or may not be tricky. Please do not spend excessive time on any single question.
- Unless the questions says "No explanation needed", you have to provide some reasoning.
- The last sheet is for scratch paper. You can tear if off but please throw it into recycling when you leave the room.

PennID (the 8 **digits** in big font on your penncard): _____

Some formulae

- The number of ways to choose k items out of n is $\binom{n}{k}$.
- The number of ways to pick and arrange k items out of n is $P(n, k)$.
- $|P(A)| = 2^{|A|}$.
- Number of ways to arrange n items out of which k_1 are identical of one kind and k_2 are identical of another kind is.

$$\frac{n!}{k_1!k_2!}$$

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$$P(F|X) = \frac{P(X|F)P(F)}{P(X|F)P(F) + P(X|F^c)P(F^c)}$$

Table 1.5.1: Set identities.

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double Complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

1. (3 points) For this question, please answer with a True or False right next to the question. No explanation is required.

   On the set $\{1, 2, 3, 4, 5\}$

   Is the relation

   $\{(1, 1), (2, 2)\}$ symmetric?

   $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$ transitive?

   $\{(1, 1), (2, 2), (3, 3)\}$ anti-symmetric?

   $\{(3, 4), (4, 1), (1, 5), (3, 1), (1, 1)\}$ reflexive?

   $\{(5, 1), (3, 4), (2, 3)\}$ anti-symmetric?

   $\{(2, 4), (4, 2), (2, 2), (1, 1), (5, 1)\}$ transitive?

2. (2 points) In a standard deck of 52 cards, you draw a single card. What is the probability of drawing a king or a heart?

   (2 pts)

3. (2 points) Winnie has 4 different duck ornaments and 6 different geese ornaments that she wants to place on her mantle. All of the duck ornaments should be consecutive and the geese ornaments should also be consecutive. How many ways can they be arranged?

4. (3 points) Philadelphia sports fans are likely to do crazy things when the Eagles play a knockout match.

   When the Eagles lose they do crazy things with the probability of 0.75.

   When the Eagles win they do crazy things with the probability of 0.9.

   The Eagles have a 20 percent chance of winning a knockout match.

   On Feb 2, 2018 Philadelphia sports fans did crazy things.

   What is the probability that they won the knockout match that took place that day?

   In a knockout match you do not have ties.

5. (3 points) Is $(A \cap B) - C = (A - C) \cap (B - C)$?

   If so, prove it using set identities.

   Please list the name of the identity being used in every step. You have all the identities listed in the second page.

6. (2 points) How many distinct bijections (something that is both one-one and onto) functions can be made from the set of {orange, apple, pineapple, lemon} to the set {Catwoman, Batman, Wonder Woman, Superman}?

7. (4 points) How many non-negative integer solutions exist for the following system of inequations (remember that a system means that all of these need to be satisfied simultaneously)

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &\leq 21 \\ x_2 &\geq 2 \\ x_3 &\geq 3 \\ x_4 &\leq 19 \end{aligned}$$

8. (4 points) The great Ringo Starr of the Beatles is obsessed about wearing rings on his fingers. He goes shopping and buys 3 identical diamond rings, 2 identical sapphire rings, and 2 identical emerald rings.

   In how many ways can he wear these rings on his fingers?

   Assumptions and facts for this question

   - The thumb is not a finger.
   - Ringo has all four fingers.
   - All the rings will be used.
   - A finger is allowed to be ringless (0 rings).

9. (4 points) In order to get 15 students to interact with each other a teacher decides to take them to Amma's restaurant on Chestnut Street.

   There the teacher orders 8 bowls of Butter Chicken and tells the students that they have to pair up and each pair is going to share a bowl. Since there are 15 students, one of them will be left unpaired and will get to eat an entire bowl by themselves.

   In how many ways can this be done. You can assume that the bowls are identical (it is a restaurant and you would expect that!).