

Summary of NPV Rule¹

Before defining the NPV rule, we need several present value concepts:

- Future Value:

Suppose you deposit \$1000 in a bank account, paying 10% interest.

What is the value of the deposit in 1 year

$$\begin{aligned}\text{FV} &= \text{Principal} + \text{Interest} \\ &= \$1000 + \$1000(.10) \\ &= \$1100\end{aligned}$$

In general, suppose r is the interest rate:

$$\begin{aligned}&= \$1000 + \$1000r \\ &= \$1000(1 + r)\end{aligned}$$

The future value of your investment at the interest rate r is $\$1000(1 + r)$.

Definition *The future value of C_0 at interest rate r in 1 year is*

$$\text{FV} = C_0(1 + r)$$

- Present Value:

Suppose you need \$1000 in one year. What amount of money do you need to put aside today?

$$\$1000 = \text{PV} \times (1 + r)$$

¹Notes for Finance 604 & 612 prepared by Jessica A. Wachter.

Rearranging,

$$PV = \frac{\$1000}{1 + r}$$

when $r = 10\%$, $PV = \$1000/(1.10) = \909.90 . $\$909.90$ is the *Present value* of $\$1000$, at 10% . Note that we are bringing the $\$1000$ backward in time.

In general, suppose you will have an *cash flow* of C_1 in one year.

Definition *The Present value of C_1 at interest rate r is*

$$PV = \frac{C_1}{1 + r}$$

- Net Present Value:

Consider an investment that pays a cash flow of C_1 in one year and costs $-C_0$ today. The interest rate is r .

Definition *The NPV of the investment is*

$$NPV = C_0 + \frac{C_1}{1 + r}$$

Basically NPV is really just present value. The “net” emphasizes that we have a negative cash flow at time zero.

We are now ready to define the NPV rule:

Definition *NPV rule: Accept projects with $NPV > 0$ and reject projects $NPV < 0$.*

We are going to establish:

Result. *Following the NPV rule maximizes the value of the corporation.*

To give some intuition for this result, we will look at a very simple corporation, consisting of a single person (Suzy), who has access to 1M and can borrow and lend from a bank at 20%.

Suzy allocates wealth between youth and old age. Assuming this bank represents the only opportunity for Suzy, what should she do?

1. She could go on a trip around the world, and then live in poverty in old age
2. She could go on a smaller trip have a moderate lifestyle in her youth (spending .5M) and still have $.5(1.2) = .6M$ for her old age.
3. She could put all her money in the bank and go for an even better trip around the world in old age. (\$1.2M)

Note: all these possibilities are equally “correct”.

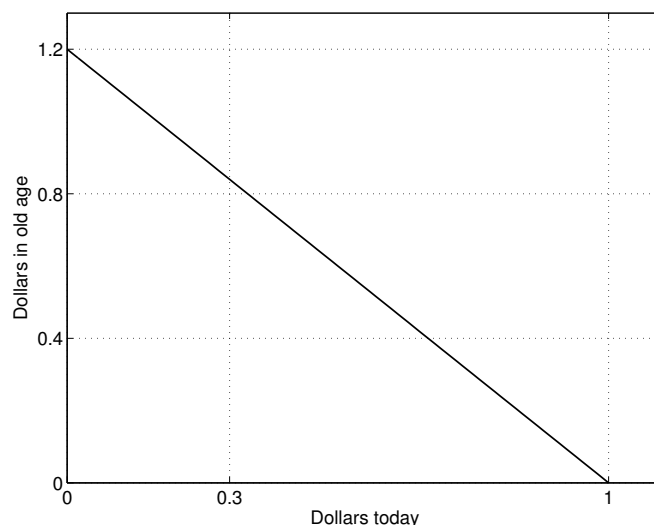
- Derivation of line: Let $y = \$$ in old age, $x = \$$ today. Then FV reasoning tells us

$$y = (1 - x)(1 + r)$$

So the relation between consumption today and consumption in old age is a straight line with a negative slope of $-(1 + r)$. For example, if she consumes everything today (1 M) she will have nothing in old age. If she saves everything, she will have 1.2M in old age.

- Suzy’s preferences between now and old age determine where she is on the line.

Figure 1: Suzy's possibilities provided by the bank



- This figure represents the trade-off between spending in now and spending in old age. Its slope is negative, and equal to $-(1 + r)$. For every \$1 you give up in spending now, you get $\$1(1 + r)$ in old age.
- Now suppose Suzy considers opening a restaurant. The start-up costs would be .70M. She could invest in this restaurant and have .80M in her old age. Should she invest? *No*.
- Why? If she puts the .70M in the bank, she has $.70(1.2) = .84$ M. Don't need to know anything about Suzy's preferences to make this decision.
- Now suppose Suzy spies a plot of land for a vineyard that costs .70M and will yield .91 in old age. Should she make the investment? It seems like the answer should be yes.
- But what if she is planning to spend money on college and an MBA? This will cost her .2M. Now she only has .1M left. Should she still make the investment?

Do we need more information to determine whether Suzy should invest?