Homework 2

MATH 6643
Fall 2016
Prof. Wing Suet Li
Due: September 27, 2016

[TB]: *Numerical Linear Algebra*, L. Trefethen and D. Bau, III, published by SIAM.

1. (10 pts) [TB] Exercise 7.4. (p. 55).

2. (10 pts) Let V be an $m \times n$ matrix, $n \leq m$. Let $M = V^T V$ and $\mathcal{V} = span\{v_1, v_2, \ldots, v_n\}$ where v_i are the i-th column of V.
    (1) Prove that M is non-singular if and only if V is full rank.
    (2) Prove that $P = V M^{-1} V^T$ is an orthogonal projection onto $\mathcal{V}$ if M is non-singular.
    (3) Let $E = I - V^T V$. Suppose $\|E\| < 1$. Show that M is non-singular. In this case, show that

    $$I - V(I + E)V^T = (I - VV^T)^2,$$

    and

    $$I - V(I + E + E^2 + \cdots + E^k)V^T = (I - VV^T)^{k+1}$$

    for $k \geq 1$.

    (This exercise can be viewed as a justification of repeating Gram-Schmidt leads to an orthonormal projection even when the basis is not fully orthonormal, for example, as one applies classical Gram-Schmidt, the resulting vectors are not quite orthonormal due to rounding errors).

3. (30 pts) [TB] Exercise 11.3 (p. 85).
    In addition to the QR factorizations required, include also (d'): QR factorization computed by using repeated classical Gram-Schmidt, with 1 and 2 times of repeat.