Homework I

MATH 6643
Fall 2016
Prof. Wing Suet Li
Due: September 8, 2016

[TB]: *Numerical Linear Algebra*, L. Trefethen and D. Bau, III, published by SIAM.

1. [TB] Exercise 2.6. (p. 16).

2. [TB], Exercise 3.1. (p. 24).

3. [TB], Exercise 3.5. (p. 24).

4. Let A and B be $n \times n$ matrices. Show that

   (1) $\|A\|_2 \leq \|A\|_F \leq \sqrt{n}\|A\|_2$.
   (2) Show that $\|AB\|_2 \leq \|A\|_2\|B\|_2$.

5. Let $\|\cdot\|$ be a norm on $\mathbb{R}^n$ and also the corresponding induced matrix norm. Let X be an $n \times n$ invertible matrix.

   (1) Show that $|||x||| := \|Xx\|$ is a norm on $\mathbb{R}^n$.
   (2) For the induced matrix norm $|||\cdot|||$, show that $|||A||| = \|XAX^{-1}\|$ for every $n \times n$ matrix A.
   (3) Let $J(\lambda)$ be the $k \times k$ Jordan cell

$$J(\lambda) = \begin{bmatrix} \lambda & 1 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \cdots & \lambda & 1 \\ 0 & \cdots & \cdots & \lambda \end{bmatrix}.$$

   Calculate $\|J(\lambda)\|_1$, the matrix norm induced by the 1-norm.

   (4) Let $\eta > 0$ and X be the $k \times k$ diagonal matrix $X = \begin{bmatrix} \eta & 0 & \cdots & 0 \\ 0 & \eta^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \eta^k \end{bmatrix}$

   Calculate $\|XJ(\lambda)X^{-1}\|_1$. Conclude that for every $\epsilon > 0$, there exists a matrix norm so that the norm of $J(\lambda)$ is less than $\lambda + \epsilon$.

6. (You can use Matlab or any other programming language of your choice.)

Let $f(x) = \dfrac{log(x+1)}{x}$. The goal of the exercise is to plot the function near 0. This function can be calculated in (at least) two different ways:

(1)
$$g(x) = \frac{log(x+1)}{x}$$

(2)
$$h(x) = \frac{log(x+1)}{(x+1)-1}$$

Mathematically they are equivalent. Plot both functions around x=0 (use nine points on each side of 0) with step sizes 10^{-16}, 10^{-15}, 10^{-13} (and other values if necessary). Discuss what you observed and explain why.