Rutten Lab B3

March 11, 2016

Jacob Lustig-Yaeger

Spectral lines from the solar atmosphere

We now turn to the formation of spectral lines in the solar spectrum. We will concentrate on the formation of the Na I D1 line at $\lambda = 589.6$ nm.

1. Observed Na D line profiles

- **Pull the data that went into making Figure 8 (Rutten B labs) over from `ftp://vso.nso.edu/pub/atlas/visatl`. Concatenate the files covering the two Na I D lines (sp16900 and sp16950) into a single file, deleting the overlaps.**

  First, we'll import modules and assign important variables

In [2]:
```
# Import some fundamental python packages
%matplotlib inline
import numpy as np; import scipy as sp; import matplotlib as mpl
import matplotlib.pyplot as plt; from matplotlib import gridspec
from matplotlib import rc; from astroML.plotting import setup_text_plots
setup_text_plots(fontsize=25, usetex=True)
rc('font', **{'family': 'serif', 'serif': ['Computer Modern']}); mpl.rcParams['font.size'] = 25
import jakely

# Define some physical constants in CGS
h = 6.62607e-27       # Planck constant (erg s)
c = 2.998e10          # Speed of light (cm / s)
k = 1.3807e-16        # Boltzmann constant (erg / K)
mH = 1.67352e-24      # Mass of H (g)
mHe = 3.97 * mH       # Mass of He (g)
Rsun = 6.96e10        # Solar Radius (cm)
Msun = 1.989e33       # Solar Mass (g)
G = 6.67e-8           # Gravitational constant
ec = 4.8032e-10       # Electron charge (esu)
Me = 9.1093e-28       # Mass of electron (g)
```

  The file columns, specified in the README on the server, are as follows: first the **frequency**, second the deduced **telluric spectrum**, third the **observed photospheric spectrum before correction** for telluric absorption, and fourth the **photospheric spectrum corrected for telluric absorption**.

In [3]:
```
! head sp16900
```

```
16897.000  0.93269E+00  0.92475E+00  0.99148E+00
  16897.020  0.93925E+00  0.92890E+00  0.98897E+00
  16897.039  0.94966E+00  0.93672E+00  0.98637E+00
  16897.061  0.96153E+00  0.94633E+00  0.98419E+00
  16897.080  0.97269E+00  0.95586E+00  0.98269E+00
  16897.100  0.98188E+00  0.96419E+00  0.98198E+00
  16897.119  0.98792E+00  0.97014E+00  0.98200E+00
  16897.141  0.99109E+00  0.97396E+00  0.98272E+00
  16897.160  0.99217E+00  0.97645E+00  0.98415E+00
  16897.180  0.99209E+00  0.97839E+00  0.98620E+00
```

In [4]:
```
# Read-in first file
NaID1a = np.genfromtxt('sp16900')

nu_a = NaID1a[:,0]
telluric_a = NaID1a[:,1]
obs_nocor_a = NaID1a[:,2]
obs_cor_a = NaID1a[:,3]
```

In [5]:
```
# Read-in second file
NaID1b = np.genfromtxt('sp16950')

nu_b = NaID1b[:,0]
telluric_b = NaID1b[:,1]
obs_nocor_b = NaID1b[:,2]
obs_cor_b = NaID1b[:,3]
```

In [6]:
```
# Concatenate data
print jakely.toolbox.find_nearest(nu_b, nu_a[-1])

nu = np.hstack([nu_a, nu_b[301:]])
telluric = np.hstack([telluric_a, telluric_b[301:]])
obs_ncor = np.hstack([obs_nocor_a, obs_nocor_b[301:]])
obs_cor = np.hstack([obs_cor_a, obs_cor_b[301:]])
```

```
300
```

- **Write IDL code to read these data. You need the first and third columns (see the README).**

  Already did this :)

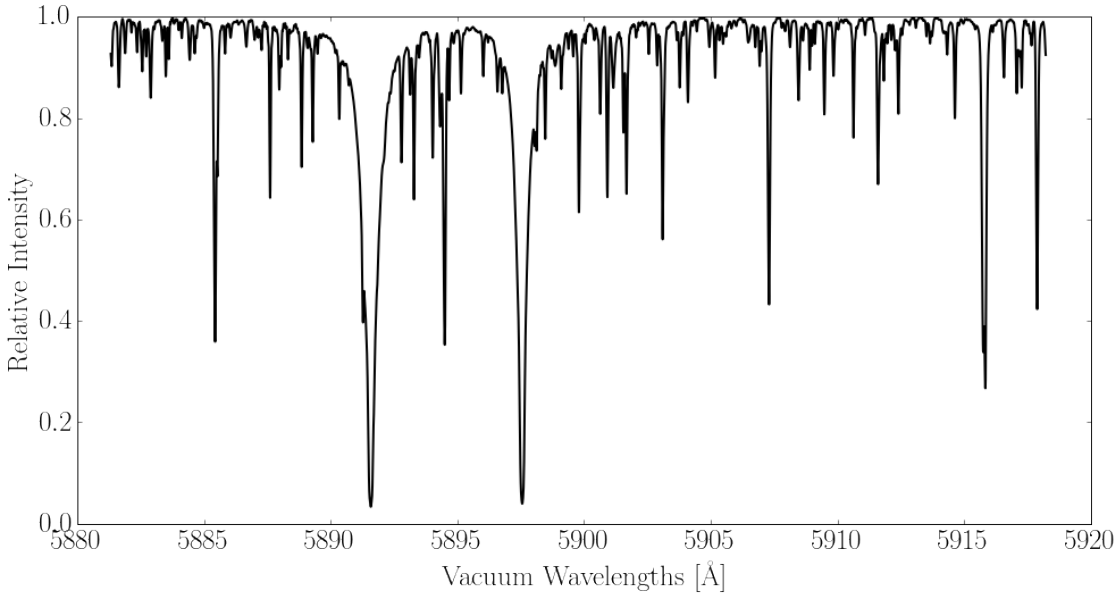
2. Na D wavelengths

- **Plot the solar Na I D lines against vacuum wavelength at various dispersions.**

  The README states: The required multiplicative factors to correct the observed frequencies to the laboratory scale are 1.0000013 for 13,500 to 16,000 cm-1, 0.9999981 for 16,000 to 20,000 cm-1, and 1.0000018 for 20,000 to 28,000 cm-1.

In [7]:
```
# Scale wavenumbers to vacuum
vacfact = 0.9999981
nu_vac = nu * vacfact

# Convert to wavelengths
wlv = (1./nu_vac) * 1e8  # (Angstroms)
```

```
# Plot
fig = plt.figure(figsize=(16,8))
gs = gridspec.GridSpec(1,1)
ax0 = plt.subplot(gs[0])
ax0.plot(wlv,obs_ncor, lw=2.0, c='k')
ax0.set_xlabel(r"Vacuum Wavelengths [\AA]")
ax0.set_ylabel(r"Relative Intensity")
plt.show()
```



- **Find the vacuum wavelengths of their minima (use the IDL where and min functions).**

```
In [8]: min1 = wlv[jakely.toolbox.find_nearest(obs_ncor, np.min(obs_ncor[(wlv>5890) & (wlv<5895)]))]
        print 'vacuum wavelength of first minimum:', min1, 'Angstroms'

        min2 = wlv[jakely.toolbox.find_nearest(obs_ncor, np.min(obs_ncor[(wlv>5895) & (wlv<5900)]))]
        print 'vacuum wavelength of second minimum:', min2, 'Angstroms'
```

```
vacuum wavelength of first minimum: 5891.58930683 Angstroms
vacuum wavelength of second minimum: 5897.56596122 Angstroms
```

- **Check that the Na I D wavelengths tabulated in the solar spectrum line list of Moore et al. (1966) (computer-readable at ftp://vso.nso.edu/pub/atlas/linelist/Moore) are $\lambda = 5895.94$ Angstroms for NaI D1 and $\lambda = 5889.97$ Angstroms for NaI D2, respectively. Check the identification of a few blends (other lines) in Figure 8 with the entries in this table.**

```
5889.973M * 752.  120.SS NA 1(D2) 0.00 1
5895.940M * 564.  91.S NA 1(D1) 0.00 1
5892.883S 66.  11.S NI 1 1.99 68
```

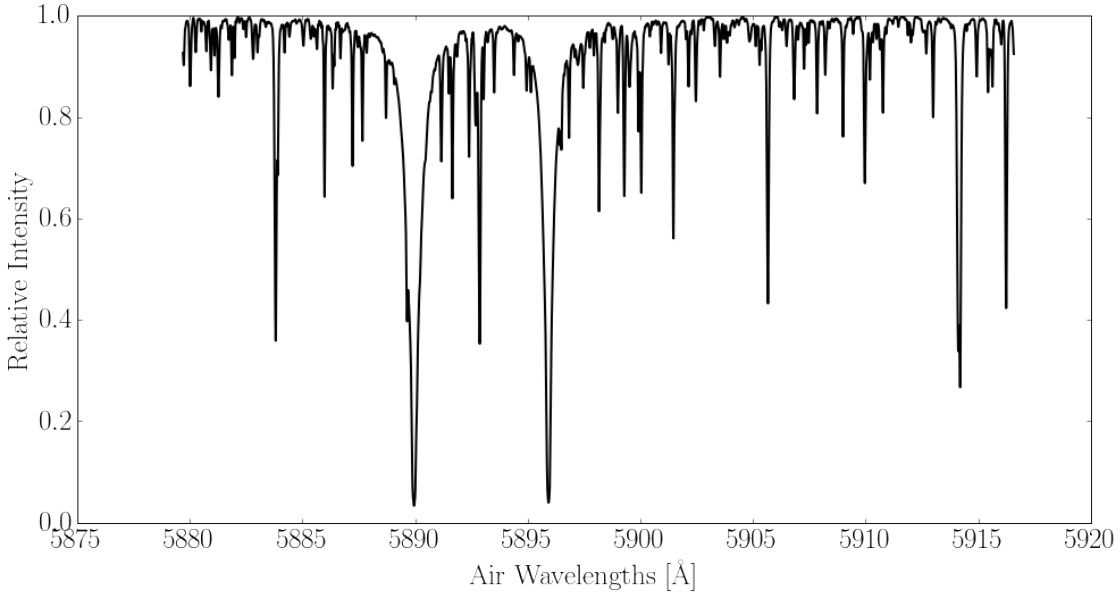
- **The Astrolib routines airtovac and vactoair convert air into vacuum wavelengths and vice versa. A reasonably accurate transformation is also given by**

$$\lambda_{air} = 0.99972683\lambda_{vac} + 0.0107 - \frac{196.25}{\lambda_{vac}}$$

with both wavelength's in angstroms, from Neckel 1999. Use this equation or routine `vactoair` to plot the Na I D lines against air wavelength.

```
In [9]: def vactoair(lamvac):
            return 0.99972683 * lamvac + 0.0107 - 196.25 / lamvac
```

```
In [10]: wlair = vactoair(wlv)

         # Plot
         fig = plt.figure(figsize=(16,8))
         gs = gridspec.GridSpec(1,1)
         ax0 = plt.subplot(gs[0])
         ax0.plot(wlair,obs_ncor, lw=2.0, c='k')
         ax0.set_xlabel(r"Air Wavelengths [\AA]")
         ax0.set_ylabel(r"Relative Intensity")
         plt.show()
```




```
In [11]: lamD2 = wlair[jakely.toolbox.find_nearest(obs_ncor, np.min(obs_ncor[(wlair>5880) & (wlair<5893
         print 'air wavelength of Na D2:', lamD2, 'Angstroms'

         lamD1 = wlair[jakely.toolbox.find_nearest(obs_ncor, np.min(obs_ncor[(wlair>5893) & (wlair<5900
         print 'air wavelength of Na D1:', lamD1, 'Angstroms'
```

```
air wavelength of Na D2: 5889.95729118 Angstroms
air wavelength of Na D1: 5895.93234669 Angstroms
```

3. LTE line formation

We will now compute the solar Na I D1 line assuming the FALC model atmosphere and LTE for the line source function. Since LTE holds already for the continuum processes at these wavelengths (being dominated by H- bound-free transitions), the assumption of LTE line formation implies that you can simply set Sl = Sc = Stotal = B(T).

What remains is first to evaluate the line extinction as a function of height and wavelength, and then to add that to the continuous extinction in the integration loop of the previous exercise. The outer loop over wavelength then has to sample the Na I D1 profile.

```
In [12]: # Read-in FALC model
         falc = np.genfromtxt('falc.dat', skip_header=4)
         height = falc[:,0]
         tau500 = falc[:,1]
         colm = falc[:,2]
         temp = falc[:,3]
         vturb = falc[:,4] * 1e5 # cm/s
         nhyd = falc[:,5]
         nprot = falc[:,6]
         nel = falc[:,7]
         ptot = falc[:,8]
         pgasptot = falc[:,9]
         dens = falc[:,10]
         pgas = pgasptot * ptot
```

4. Line extinction

The monochromatic line extinction per cm path length for a bound-bound transition between a lower level l and an upper level u is given by:

$$\alpha_\lambda^l = \frac{\sqrt{\pi}e^2}{m_e c}\frac{\lambda^2}{c} b_l \frac{n_l^{LTE}}{N_E} N_H A_E f_{lu} \frac{H(a,v)}{\Delta\lambda_D}\left[1 - \frac{b_u}{b_l} e^{-hc/\lambda kT}\right]$$

which holds generally when the line broadening is described by the Voigt function $H(a, v)$. For LTE the population departure coefficients of the lower and upper levels are $b_l = b_u = 1$. The LTE population fraction n_l^{LTE}/N_E (lower level population scaled by the total element population) is given by the combined Saha and Boltzmann distributions.

```
In [85]: # Sodium ionization energies
         chiion = [5.139, 47.29, 71.64]

         # Statistical weights
         g = [2,2,4]  # [g1, D1, D2]

         # Oscillator strengths
         flu = [0.318, 0.631] # [D1, D2]

         # Partition functions
         def NaI_part(T):
             theta = 5040./T
             c0 =0.30955
             c1 =-0.17778
             c2 = 1.10594
             c3 = 2.42847
             c4 = 1.70721
             logU = c0 + c1*np.log10(theta) + c2*np.log10(theta)**2 + c3*np.log10(theta)**3 + c4*np.log
             return 10.**logU
         def NaII_part():
             return 1.0
         def NaIII_part():
             return 6.0
```

```
        # Sodium Abundance
        ANa = 1.74e-6 # NNa/NH
```

In [14]:
```
def dlamD(lam0, T, vt):
    mNa =  22.99*1.6605e-24
    return (lam0 / c) * np.sqrt(2. * k * T / mNa  + vt**2)

def unitless_lam(lam, lam0, T, vt):
    return (lam-lam0) / dlamD(lam0, T, vt)

def damping_a(lam, gamma, dlam):
    return lam**2 * gamma / (4. * np.pi * c * dlam)
```

In [156]:
```
# Functions from Rutten Lab A2

 def partfunc_Na(temp):
     return np.array([NaI_part(temp), NaII_part(), NaIII_part()])

 def boltz_Na(temp,r,s):
     # compute Boltzmann population for level r,s of Schadee element E
     # input: temp (temperature, K)
     #     r (ionization stage nr, 1 - 4 where 1 = neutral E)
     #     s (level nr, starting at s=1)
     # output: relative level population n_(r,s)/N_r
     u=partfunc_Na(temp)
     keV=8.61734e-5
     # Boltzmann constant in ev/deg
     relnrs = 1./u[r-1]*np.exp(-(s-1)/(keV*temp))
     return relnrs

 def saha_Na(temp,elpress,ionstage):
     # compute Saha population fraction N_r/N for Schadee element E
     # input: temperature, electron pressure, ion stage
     # physics constants
     kerg=1.380658e-16   # Boltzmann constant (erg K; double precision)
     kev=8.61734e-5      # Boltzmann constant (eV/deg)
     h=6.62607e-27       # Planck constant (erg s)
     elmass=9.109390e-28 # electron mass (g)
     # kT and electron density
     kevT=kev*temp
     kergT=kerg*temp
     eldens=elpress/kergT
     chiion=np.array([5.139, 47.29, 71.64]) # ionization energies for element E
     u=partfunc_Na(temp)  # get partition functions U[0]...u[3]
     u = np.append(u,0)
     sahaconst=(2*np.pi*elmass*kergT/(h*h))**1.5 * 2./eldens
     nstage=np.zeros(len(u)+1)     # double-precision float array
     nstage[0]=1.           # relative fractions only (no abundance)
     for r in range(0,len(chiion)):
         nstage[r+1] = nstage[r]*sahaconst*u[r+1]/u[r]*np.exp(-chiion[r]/kevT)
     ntotal=np.sum(nstage)        # sum all stages = element density
     nstagerel=nstage/ntotal      # fractions of element density
     return nstagerel[ionstage-1] # ion stages start at 1, IDL at 0
```

```
def sahabolt_Na(temp,elpress,ion,level):
    # compute Saha-Boltzmann populaton n_(r,s)/N for level r,s of E
    # input: temperature, electron pressure, ionization stage, level nr
    return saha_Na(temp,elpress,ion) * boltz_Na(temp,ion,level)
```

In [16]:
```
# Functions from Rutten Lab A3

# Define planck function
def planck(temp, wav):
    return (2. * h * c**2) / (wav**5) / (np.exp(h * c / (wav * k * temp)) - 1.0)

def integrand(y,a,u):
    return np.exp(-y**2.) / ((u - y)**2. + a**2.)

# Define voigt function
def voigt(a,u):
    integ = sp.integrate.quad(integrand, -np.inf, np.inf, args=(a,u))[0]
    return (a / np.pi) * integ
```

5. Line broadening

The Voigt function $H(a, v)$ describes the extinction profile shape.

$$m_{Na} = 22.99 \times 1.6605 \times 10^{-24} \text{ g}$$

Since the FALC model reaches higher we might also add in natural broadening with:

$$\gamma_{rad} = A_{ul} = 6.67 \times 10^{13} \frac{g_l}{g_u} \frac{f_{lu}}{\lambda^2} \text{ s}^{-1}$$

The classical evaluation recipe of Van der Waals broadening by Unsold 1955 is (cf. Warner 1967)

$$\log \gamma_{vdw} \approx 6.33 + 0.4 \log(\bar{r_u^2} - \bar{r_l^2}) + \log P_g - 0.7 \log T$$

where the mean square radii $\bar{r^2}$ of the upper and lower level are usually estimated from the hydrogenic approximation of Bates and Damgaard (1948):

$$\bar{r^2} = \frac{n^{\star 2}}{2Z^2} \left(5n^{\star 2} + 1 - 3l(l+1)\right)$$

with $\bar{r^2}$ measured in atomic units, l the angular quantum number of the level and $n^\star$ its effective (hydrogen-like) principal quantum number given by

$$n^{\star 2} = R \frac{Z^2}{E_\infty - E_n}$$

in which the Rydberg constant $R = 13.6$ eV $= 2.18 \times 10^{-11}$ erg, Z is the ionization stage (Z = 1 for Na I, Z = 2 for Na II, etc) and $E_\infty - E_n$ is the ionization energy from the level (compute the excitation energy of the upper level from the line-center wavenumber). The common Na I D1 and Na I D2 lower level (3s 2S1/2) has l = 0, the upper levels (3p 2P1/2 and 3p 2P3/2) have l = 1.

In [148]:
```
def n_star2(Z,chi,En):
    ev2erg = 1.60218e-12
    return 2.18e-11 * Z**2 / (np.fabs(chi*ev2erg - En))

def r2_bar(Z,chi,lam,T,l):
    if l > 0.0:
        En = h*c/lam
```

```
        else:
            En = 0.0
        return n_star2(Z,chi,En) / (2.*Z**2) * (5.*n_star2(Z,chi,En) + 1.0 - 3.*l*(l+1.))

    def gamma_vdw(Z,chi,lam,T,Pg):
        ru2 = r2_bar(Z,chi,lam,T,1.0)
        rl2 = r2_bar(Z,chi,lam,T,0.0)
        logg_vdw = 6.33 + 0.4 * np.log10(np.fabs(ru2 - rl2)) + np.log10(Pg) - 0.7 * np.log10(T)
        return 10**logg_vdw

    def gamma_rad(lam, gl, gu, f):
        return 6.67e13 * (gl/gu) * f / (lam * 1e7)**2
```

6. Implementation

- **Implement the above into a function that returns the line extinction at all heights of the FALC atmosphere, for example `NaD1_ext(wav,temp,eldens,nhyd,vmicro)`. It is best to make this a set of subroutines that process the atmospheric arrays in one go, without "`for ih=0,nh-1 do begin`" unrolling because that results in much slower execution.**

In [173]:
```
#def NaD1_ext(wav,temp,eldens,nhyd,vmicro):
#    return

vsahabolt_Na = np.vectorize(sahabolt_Na)
vvoigt = np.vectorize(voigt)
E_fudge = 1.0

def NaD1_ext(lam,temp,eldens,nhyd,vmicro,pgas, line='D1'):

    # Set parameters based on whether the use wants the Na D1 or D2 line
    if line == 'D1':
        # set lam0, flu, gl,gu
        lam0 = 5895.940*1e-8 # cm
        f = 0.318
        gl = 2.0
        gu = 2.0
    elif line == 'D2':
        # set lam0, flu, gl,gu
        lam0 = 5889.973*1e-8 # cm
        f = 0.631
        gl = 2.0
        gu = 4.0
    else:
        print "Error in extline: Only 'D1' and 'D2' can be used with kwarg 'line'."
        return -1

    # Ionization stages
    Z = np.array([1.,2.,3.])
    chiion=np.array([5.139, 47.29, 71.64])

    # Compute coupled Saha-Boltzmann LTE population level fraction
    elpress=eldens*k*temp
    ion = 1
    level = 1
    sahabolt = vsahabolt_Na(temp,elpress,ion,level)
```

```
        # Compute Delta Wavelength & unitless wavelength
        dlam = dlamD(lam0, temp, vmicro)
        u = (lam-lam0) / dlam

        # Compute gamma parameter
        gamma = E_fudge*gamma_vdw(Z[0],chiion[0],lam0,temp,pgas)  + gamma_rad(lam, gl, gu, f)

        # Compute voigt damping parameter
        a = damping_a(lam, gamma, dlam)

        # Compute voigt function
        H = vvoigt(a,np.fabs(u))

        # retrun monochromatic line extinction per cm path length
        return np.sqrt(np.pi) * ec**2 * lam**2 / (Me * c**2) * (sahabolt) * (ANa * nhyd) * f * (H
```

```
In [168]: lam_cm = np.linspace(5890.,5900., num=500) * 1e-8

          # Test function
          NaD1 = NaD1_ext(lam_um*1e-4,temp[0],nel[0],nhyd[0],vturb[0],pgas[0], line='D1')
```

- **You might prefer to generalize this effort to any Fraunhofer line instead of making a special function for this particular line only. This then involves automation of getting the partition functions of all atom stages of interest for use in a more general Saha function. You might check out the ancient IDL routines and atomic data file at** `https://github.com/aasensio/lte/tree/master/idl`**, based on the yet more ancient Fortran routines of Wittmann (1974). I used them in my own ltelib.**

  Interesting... I've written it only for the Na I D1 and Na D2 lines.

7. Computed Na D1 line profile

- **Add the Na I D1 line extinction to the continuous extinction in your integration code from the previous exercise and compute the disk-center Na I D1 profile.**

```
In [60]: lam_um = np.linspace(5894.,5898., num=500) * 1e-4
```

```
In [87]: # This is the H- extinction from Rutten lab B2

         def exthmin(wav,temp,eldens, check=False):
             # physics constants in cgs (all cm)
             kcgs=k    # Boltzmann constant (erg/K; double precision)
             hcgs=h    # Planck constant (erg s)
             ccgs=c    # velocity of light (cm/s)

             # other parameters
             theta=5040./temp
             elpress=eldens*kcgs*temp

             # evaluate H-min bound-free per H-min ion = Gray (8.11)
             # his alpha = my sigma in NGSB/AFYC (per particle without stimulated)
             sigmabf = 1.99654 -1.18267E-5*wav +2.64243E-6*wav**2 -4.40524E-10*wav**3 \
                     +3.23992E-14*wav**4 -1.39568E-18*wav**5 +2.78701E-23*wav**6
             sigmabf=sigmabf*1e-18  # cm^2 per H-min ion
```

```
    if np.sum((wav > 16444)):
        try:
            sigmabf[wav > 16444] = 0.
        except TypeError:
            sigmabf = 0
    # H-min ionization limit
    # This Krijger trick to permit array input

    # convert into bound-free per neutral H atom assuming Saha = Gray p135
    # units: cm2 per neutral H atom in whatever level (whole stage)
    graysaha=4.158e-10*elpress*theta**2.5*10.**(0.754*theta) # Gray (8.12)
    kappabf=sigmabf*graysaha                                 # per neutral H atom
    kappabf=kappabf*(1.-np.exp(-hcgs*ccgs/(wav*1e-8*kcgs*temp)))
    # correct stimulated emission

    # check Gray's Saha-Boltzmann with AFYC (edition 1999) p168
    if check:
        logratio=-0.1761-np.log10(elpress)+np.log10(2.)+2.5*np.log10(temp)-theta*0.754
        print'Hmin/H ratio=',1/(10.**logratio) # OK, same as Gray factor SB

    # evaluate H-min free-free including stimulated emission = Gray p136
    lwav=np.log10(wav)
    f0 =  -2.2763 -1.6850*lwav +0.76661*lwav**2 -0.0533464*lwav**3
    f1 =  15.2827 -9.2846*lwav +1.99381*lwav**2 -0.142631*lwav**3
    f2 = -197.789 +190.266*lwav -67.9775*lwav**2 +10.6913*lwav**3 -0.625151*lwav**4
    ltheta=np.log10(theta)
    kappaff = 1e-26*elpress*10**(f0+f1*ltheta+f2*ltheta**2)   # Gray (8.13)

    return kappabf+kappaff
```

Now we integrate over height to calculate the emergent intensity and the continuum intensity.

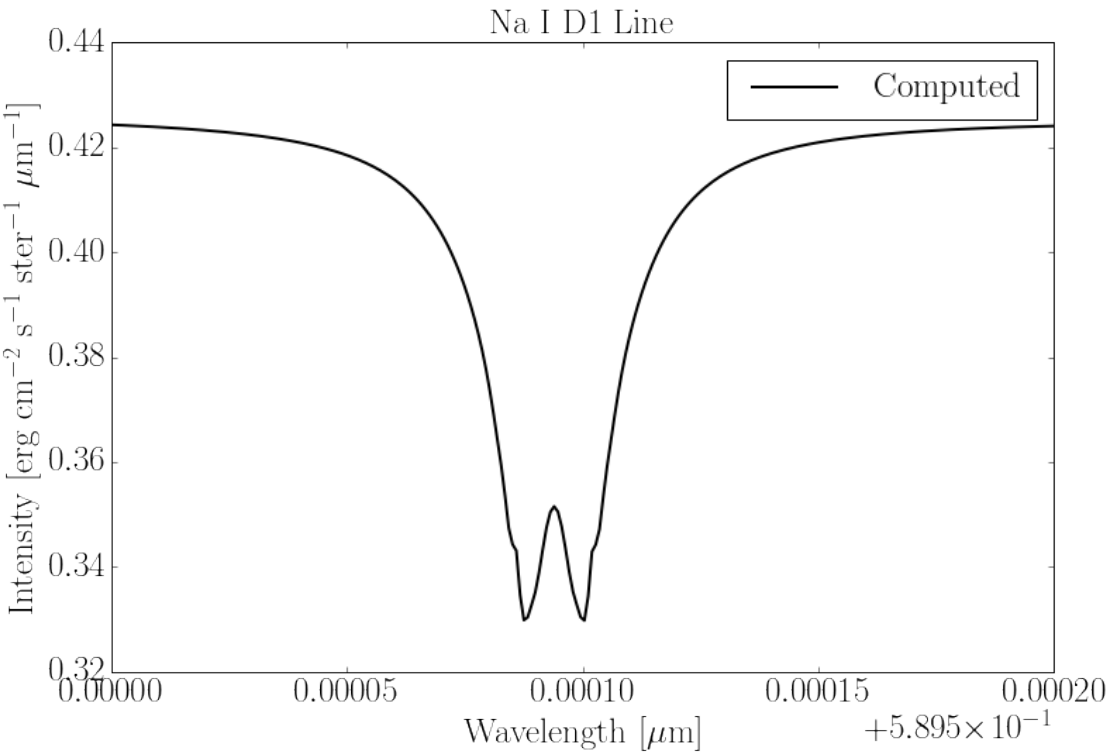
In [174]:
```
vemergent_tot = np.vectorize(emergent_tot)
vemergent_cont = np.vectorize(emergent_cont)

def emergent_tot(wl):
    # Emergent intensity at wavelength wl (micron)
    ext=np.zeros_like(nhyd)
    tau=np.zeros_like(nhyd)
    integrand=np.zeros_like(nhyd)
    contfunc=np.zeros_like(nhyd)
    intg=0.
    hint=0.
    for ih in range(len(nhyd)):
        ext[ih]=exthmin(wl*1e4,temp[ih],nel[ih])*(nhyd[ih]-nprot[ih])+0.664E-24*nel[ih]
        ext[ih]+=NaD1_ext(wl*1e-4,temp[ih],nel[ih],nhyd[ih],vturb[ih],pgas[ih], line='D1')
        tau[ih]=tau[ih-1]+0.5*(ext[ih]+ext[ih-1])*(height[ih-1]-height[ih])*1e5
        integrand[ih]=planck(temp[ih],wl)*np.exp(-tau[ih])
        intg=intg+0.5*(integrand[ih]+integrand[ih-1])*(tau[ih]-tau[ih-1])
        hint=hint+height[ih]*1e5*0.5*(integrand[ih]+integrand[ih-1])*(tau[ih]-tau[ih-1])
        contfunc[ih]=integrand[ih]*ext[ih]
    hmean=hint/intg
    return intg, contfunc, hmean, tau

def emergent_cont(wl):
```

```
    # Continuum intensity at wavelength wl (micron)
    ext=np.zeros_like(nhyd)
    tau=np.zeros_like(nhyd)
    integrand=np.zeros_like(nhyd)
    contfunc=np.zeros_like(nhyd)
    intg=0.
    hint=0.
    for ih in range(len(nhyd)):
        ext[ih]=exthmin(wl*1e4,temp[ih],nel[ih])*(nhyd[ih]-nprot[ih])+0.664E-24*nel[ih]
        tau[ih]=tau[ih-1]+0.5*(ext[ih]+ext[ih-1])*(height[ih-1]-height[ih])*1e5
        integrand[ih]=planck(temp[ih],wl)*np.exp(-tau[ih])
        intg=intg+0.5*(integrand[ih]+integrand[ih-1])*(tau[ih]-tau[ih-1])
        hint=hint+height[ih]*1e5*0.5*(integrand[ih]+integrand[ih-1])*(tau[ih]-tau[ih-1])
        contfunc[ih]=integrand[ih]*ext[ih]
    hmean=hint/intg
    return intg, contfunc, hmean, tau
```

In [175]:
```
E_fudge = 1.0

# Create array to store emergent continuum intensity
int_tot = np.zeros_like(lam_um)
int_cont = np.zeros_like(lam_um)

# Loop over all wavelenghts
for iwl in range(len(lam_um)):

    # Calculate emergent Intensity
    int_cont[iwl], contfunc_tmp, hmean_tmp, tau_tmp = emergent_cont(lam_um[iwl])
    int_tot[iwl], contfunc_tmp, hmean_tmp, tau_tmp = emergent_tot(lam_um[iwl])
```

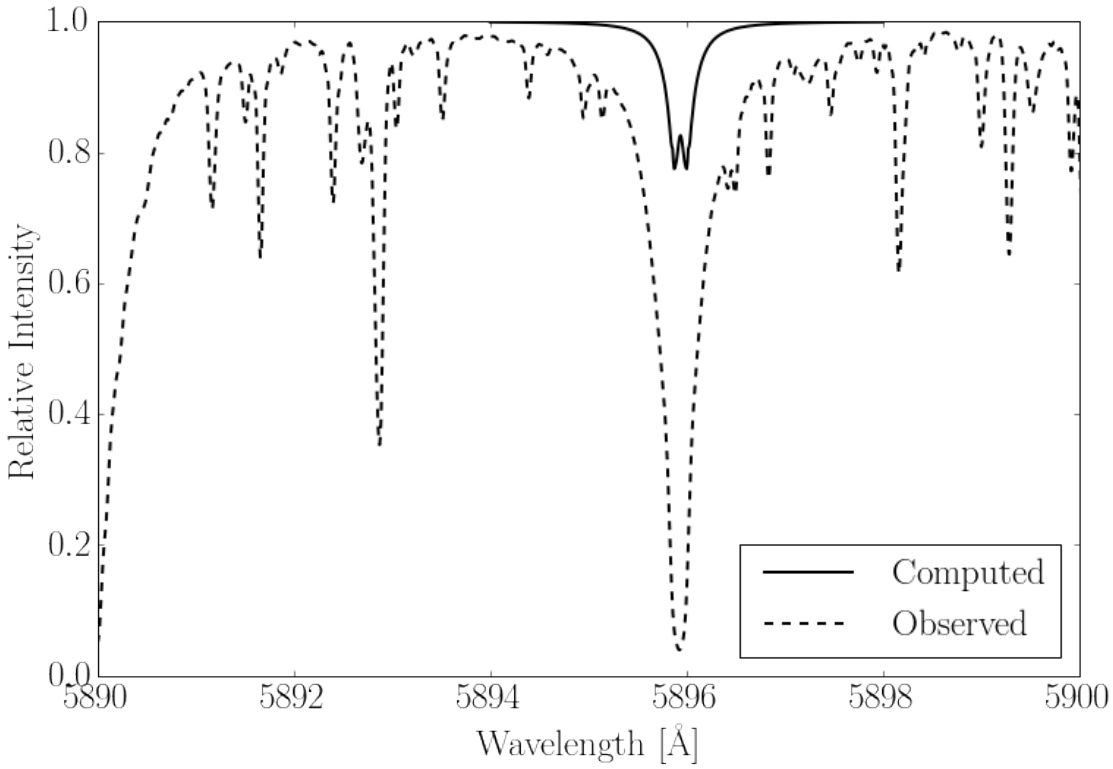
In [176]:
```
# Create array to store emergent continuum intensity
#int_tot = np.zeros_like(lam_um)
#int_cont = np.zeros_like(lam_um)

# Calculate emergent Intensity
#int_cont, contfunc_tmp, hmean_tmp, tau_tmp = vemergent_cont(lam_um)
#int_tot, contfunc_tmp, hmean_tmp, tau_tmp = vemergent_tot(lam_um)
```

In [177]:
```
# Plot
fig = plt.figure(figsize=(12,8))
gs = gridspec.GridSpec(1,1)
ax0 = plt.subplot(gs[0])
ax0.plot(lam_um,int_tot, lw=2.0, c='k', label='Computed')
ax0.set_xlabel(r"Wavelength [$\mu$m]")
ax0.set_ylabel(r"Intensity [erg cm$^{-2}$ s$^{-1}$ ster$^{-1}$ $\mu$m$^{-1}$]")
ax0.set_xlim([.5895, .5897])
ax0.set_title('Na I D1 Line')
ax0.legend()
plt.show()
```



- **Compare the computed line profile to the observed line profile and discuss the differences. Explain why your computed profile has a line-center reversal.**

```
In [178]: # Plot
          fig = plt.figure(figsize=(12,8))
          gs = gridspec.GridSpec(1,1)
          ax0 = plt.subplot(gs[0])
          ax0.plot(lam_um*1e4,int_tot/int_cont, lw=2.0, c='k', label='Computed')
          ax0.plot(wlair,obs_ncor, lw=2.0, c='k', label='Observed', ls='--')
          ax0.set_xlabel(r"Wavelength [\AA]")
          ax0.set_ylabel(r"Relative Intensity")
          ax0.set_xlim([5890, 5900])
          ax0.legend(loc=4)
          plt.show()
```



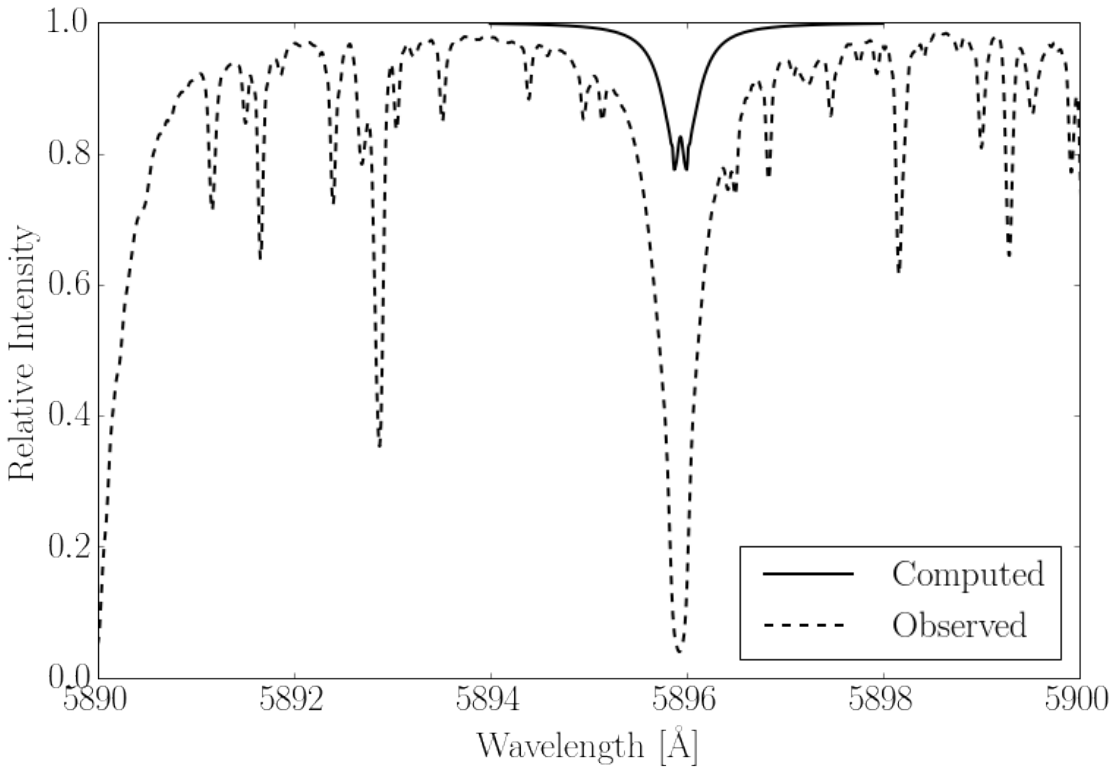
My computed line profile does not have a deep enough core or deep enough wings compared to the observed Na I D1 line. In addition, my line has a reversal at line center, in other wordes, an emission spike at the center of the line. Much of these differences are be attributed to the fact that this is an LTE model, which will tend to over estimate the intensity compared to the non-LTE reality. My computed profile has a line-center reversal because the LTE model generates high altitude emission by using the extremely high local atmospheric temperature in the physically thin, optically thin chromosphere rather than using the radiation from deeper in the atmosphere where it is cooler. As we saw in Rutten Lab B1, in the chromosphere the photon number density dominates the gas number density, so LTE breaks down. This manifests as a chromospheric emission line at line-center for the Na I D1 line, as well as an overestimate of the continuum emission.

- **Traditionally, stellar abundance determiners vary a collisional enhancement factor E by which γ_{vdW} is multiplied in ad-hoc "fudge-factor" fashion in order to obtain a better fit of the line wings. You may try such, or better broadening recipes from Paul Barklem at `http://www.astro.uu.se/~barklem`.**

  Below I try a few fudge factors, all of which seem to make the line profile deviate even more from the observed line profile.

In [193]:
```
E_fudge = 2.0

    # Create array to store emergent continuum intensity
    int_tot = np.zeros_like(lam_um)
    int_cont = np.zeros_like(lam_um)

    # Loop over all wavelenghts
```

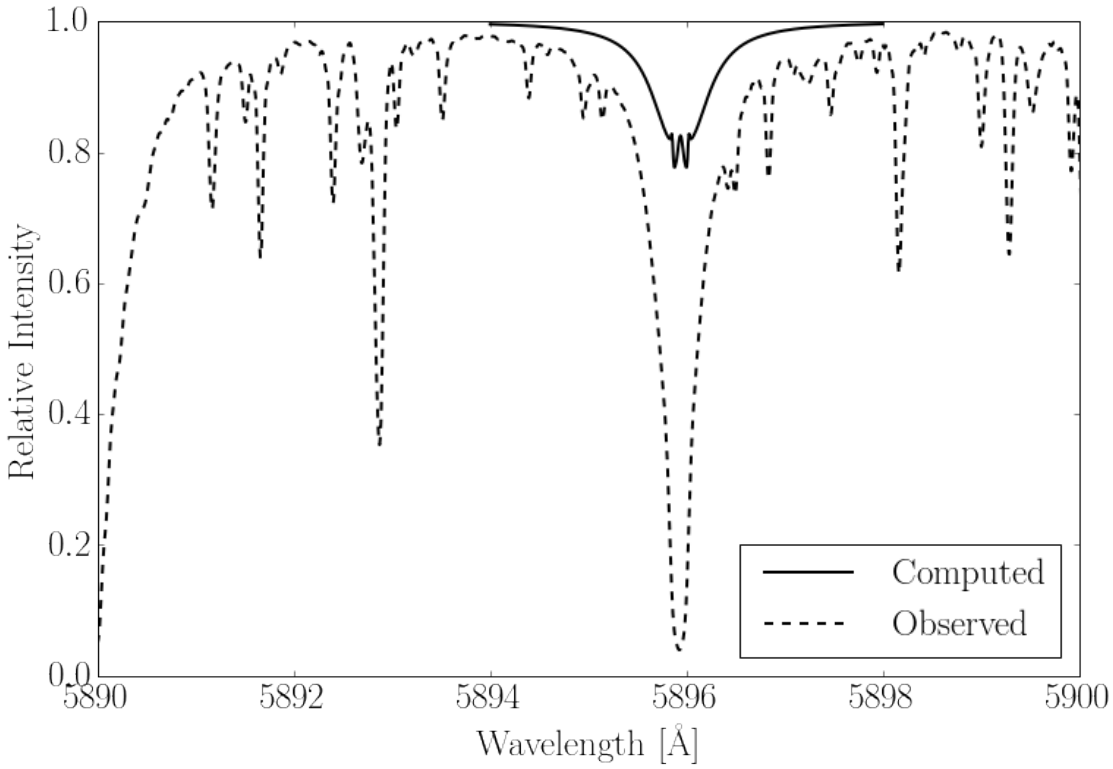
```
for iwl in range(len(lam_um)):

    # Calculate emergent Intensity
    int_cont[iwl], contfunc_tmp, hmean_tmp, tau_tmp = emergent_cont(lam_um[iwl])
    int_tot[iwl], contfunc_tmp, hmean_tmp, tau_tmp = emergent_tot(lam_um[iwl])

# Plot
fig = plt.figure(figsize=(12,8))
gs = gridspec.GridSpec(1,1)
ax0 = plt.subplot(gs[0])
ax0.plot(lam_um*1e4,int_tot/int_cont, lw=2.0, c='k', label='Computed')
ax0.plot(wlair,obs_ncor, lw=2.0, c='k', label='Observed', ls='--')
ax0.set_xlabel(r"Wavelength [\AA]")
ax0.set_ylabel(r"Relative Intensity")
ax0.set_xlim([5890, 5900])
ax0.legend(loc=4)
plt.show()

print 'Fudge-factor:', E_fudge
```




```
Fudge-factor: 2.0
```

In [194]:
```
E_fudge = 5.0

# Create array to store emergent continuum intensity
int_tot = np.zeros_like(lam_um)
int_cont = np.zeros_like(lam_um)
```

```
# Loop over all wavelenghts
for iwl in range(len(lam_um)):

    # Calculate emergent Intensity
    int_cont[iwl], contfunc_tmp, hmean_tmp, tau_tmp = emergent_cont(lam_um[iwl])
    int_tot[iwl], contfunc_tmp, hmean_tmp, tau_tmp = emergent_tot(lam_um[iwl])

# Plot
fig = plt.figure(figsize=(12,8))
gs = gridspec.GridSpec(1,1)
ax0 = plt.subplot(gs[0])
ax0.plot(lam_um*1e4,int_tot/int_cont, lw=2.0, c='k', label='Computed')
ax0.plot(wlair,obs_ncor, lw=2.0, c='k', label='Observed', ls='--')
ax0.set_xlabel(r"Wavelength [\AA]")
ax0.set_ylabel(r"Relative Intensity")
ax0.set_xlim([5890, 5900])
ax0.legend(loc=4)
plt.show()

print 'Fudge-factor:', E_fudge
```




```
Fudge-factor: 5.0
```

In [ ]: