# Rutten Lab B1

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## Stratification of the solar atmosphere

In this exercise we study the radial stratification of the solar atmosphere on the basis of the standard model FALC by Fontenla et al. (1993). They derived this description of the solar photosphere and chromosphere empirically, assuming that the solar atmosphere is horizontally homogeneous ("plane parallel layers") and in hydrostatic equilibrium ("time independent").

We will use the FLAC data file, `falc.dat`, which looks like:

```
In [1]: ! head falc.dat
```

```
FALC solar model atmosphere of Fontenla, Avrett & Loeser 1993ApJ...406..319F; 82 heights top-to-bottom
 height   tau_500    colmass    temp    v_turb n_Htotal   n_proton   n_electron pressure  p_gas/p  density
 [km]     dimless    [g/cm^2]   [K]     [km/s] [cm^-3]    [cm^-3]    [cm^-3]    [dyn/cm2] ratio    [g/cm^3]

 2218.20  0.000E+00  6.777E-06  100000  11.73  5.575E+09  5.575E+09  6.665E+09  1.857E-01  0.952  1.306E-14
 2216.50  7.696E-10  6.779E-06   95600  11.65  5.838E+09  5.837E+09  6.947E+09  1.857E-01  0.950  1.368E-14
 2214.89  1.531E-09  6.781E-06   90816  11.56  6.151E+09  6.150E+09  7.284E+09  1.858E-01  0.948  1.441E-14
 2212.77  2.597E-09  6.785E-06   83891  11.42  6.668E+09  6.667E+09  7.834E+09  1.859E-01  0.945  1.562E-14
 2210.64  3.754E-09  6.788E-06   75934  11.25  7.381E+09  7.378E+09  8.576E+09  1.860E-01  0.941  1.729E-14
 2209.57  4.384E-09  6.790E-06   71336  11.14  7.864E+09  7.858E+09  9.076E+09  1.860E-01  0.938  1.843E-14
```

We'll get started by importing some necessary python packages and defining some astrophysical quantities.

```
In [2]: # Import some fundamental python packages
        %matplotlib inline
        import numpy as np; import scipy as sp; import matplotlib as mpl
        import matplotlib.pyplot as plt; from matplotlib import gridspec
        from matplotlib import rc; from astroML.plotting import setup_text_plots
        setup_text_plots(fontsize=25, usetex=True)
        rc('font', **{'family': 'serif', 'serif': ['Computer Modern']})
        mpl.rcParams['font.size'] = 25.0

        # Define some physical constants in CGS
        h = 6.62607e-27        # Planck constant (erg s)
        c = 2.998e10           # Speed of light (cm / s)
        k = 1.3807e-16         # Boltzmann constant (erg / K)
        mH = 1.67352e-24       # Mass of H (g)
        mHe = 3.97 * mH        # Mass of He (g)
        Rsun = 6.96e10         # Solar Radius (cm)
        Msun = 1.989e33        # Solar Mass (g)
        G = 6.67e-8            # Gravitational constant
```

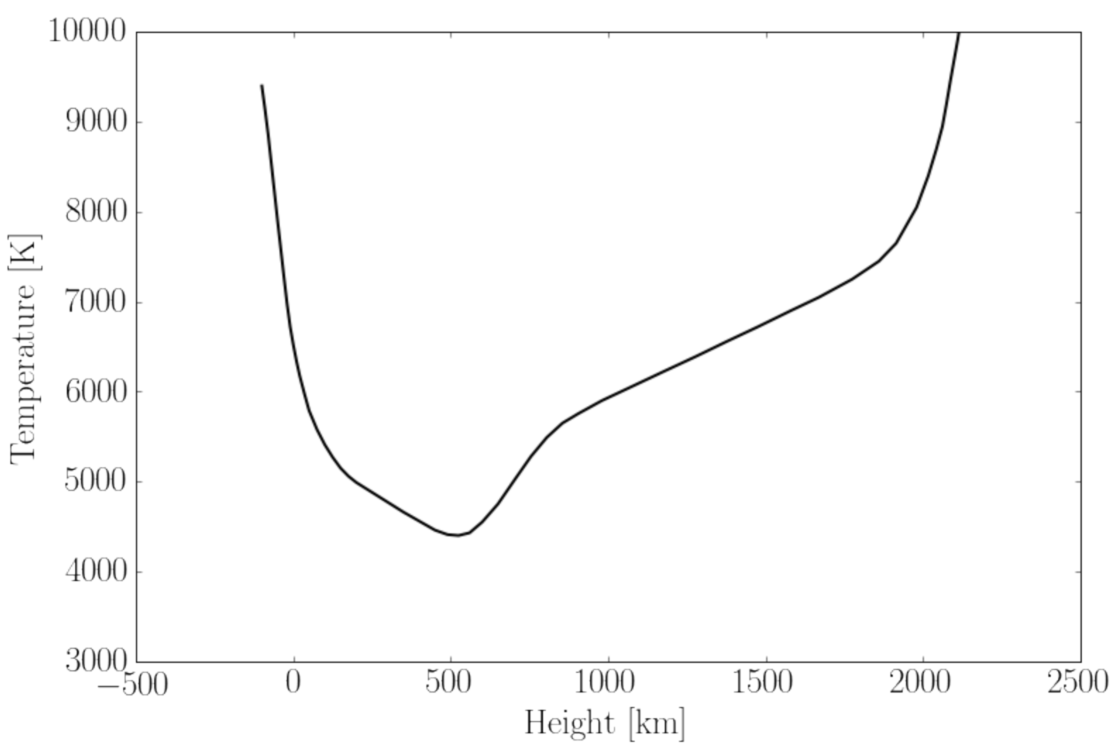
First, let's read-in the `falc.dat` data, and store the columns in sensibly named arrays.

```
In [3]: # Read-in FALC model
        falc = np.genfromtxt('falc.dat', skip_header=4)
        height = falc[:,0]
        tau500 = falc[:,1]
        colm = falc[:,2]
        temp = falc[:,3]
        vturb = falc[:,4]
        nhyd = falc[:,5]
        nprot = falc[:,6]
        nel = falc[:,7]
        ptot = falc[:,8]
        pgasptot = falc[:,9]
        dens = falc[:,10]
```

We'll make sure that the data we're dealing with are the same as those used by Rutten by plotting temperature vs. height as in Fig. 2 of Lab B1.

```
In [4]: fig = plt.figure(figsize=(12,8))
        gs = gridspec.GridSpec(1,1)
        ax0 = plt.subplot(gs[0])
        ax0.plot(height,temp, lw=2.0, c='k')
        ax0.legend()
        ax0.set_xlabel(r"Height [km]")
        ax0.set_ylabel(r"Temperature [K]")
        ax0.set_ylim([3000,10000])
        plt.show()
```

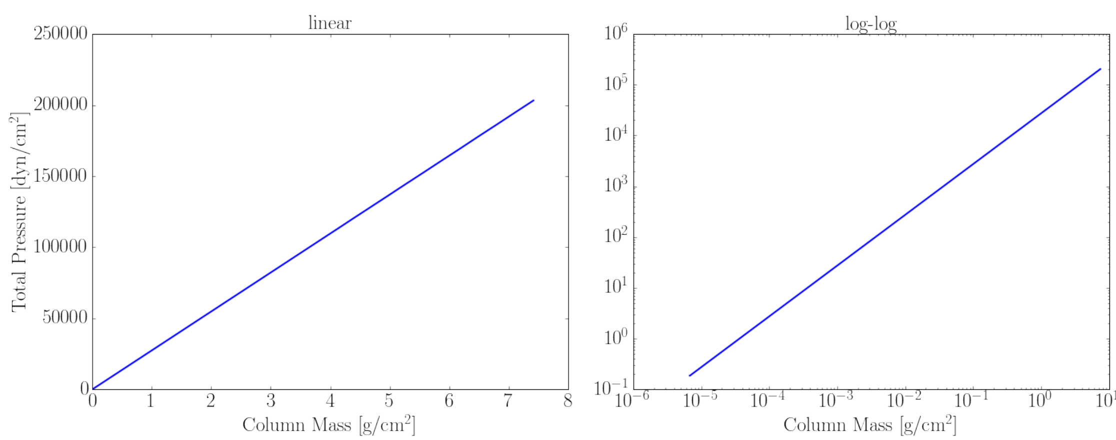
```
/astro/users/jlustigy/.conda/envs/my_root/lib/python2.7/site-packages/matplotlib/axes/_axes.py:519: User
  warnings.warn("No labelled objects found. "
```



## 1.2 FALC density stratification

**Plot the total pressure $p_{tot}$ against the column mass $m$, both linearly and logarithmically. You will find that they scale linearly. Explain what assumption has caused $p_{tot} = cm$ and determine the value of the solar surface gravity $g_{surf} = c$ that went into the FALC-producing code.**

In [5]:
```
fig = plt.figure(figsize=(20,8))
gs = gridspec.GridSpec(1,2)
ax0 = plt.subplot(gs[0])
ax0.plot(colm,ptot, lw=2.0)
ax0.set_xlabel(r"Column Mass [g/cm$^2$]")
ax0.set_ylabel(r"Total Pressure [dyn/cm$^2$]")
#ax0.set_xlim([1000,21000])
ax0.set_title('linear')

ax1 = plt.subplot(gs[1])
ax1.plot(colm,ptot,lw=2.0)
ax1.set_xlabel(r"Column Mass [g/cm$^2$]")
ax1.loglog()
ax1.set_title('log-log')
#ax1.set_xlim([1000,21000])

fig.tight_layout()
plt.show()
```

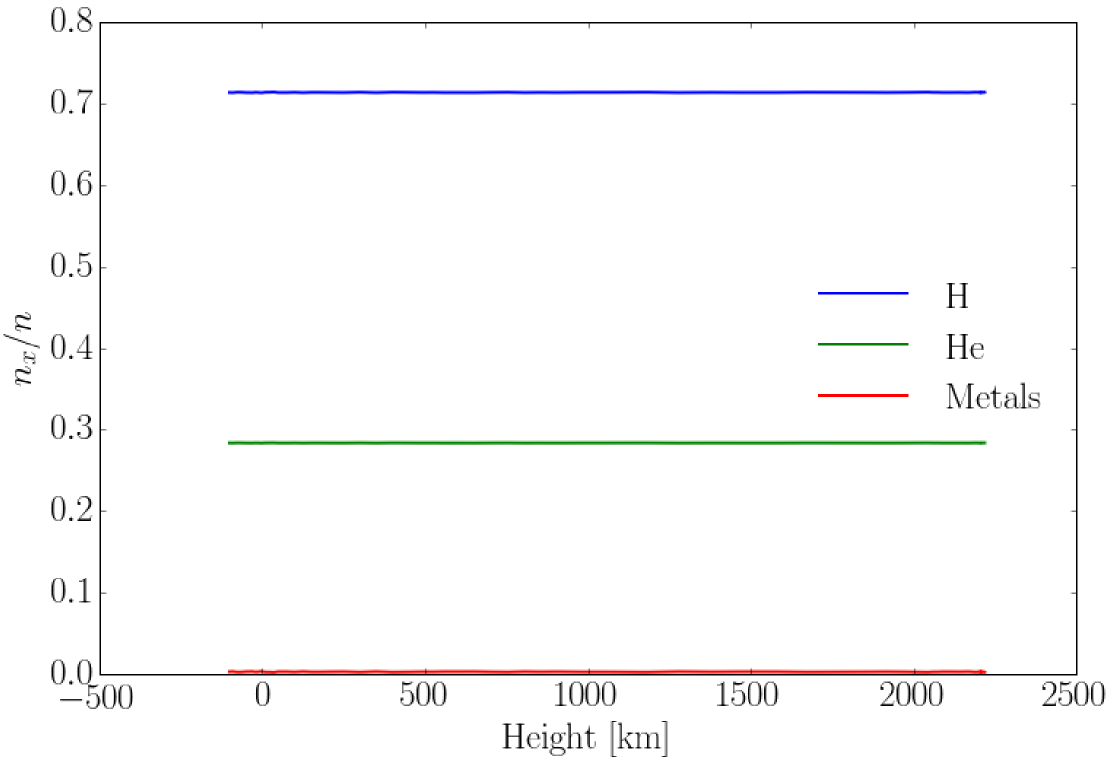



As we can see in the above plots, the pressure scales linearly with column mass. This is because hydrostatic equilibrium is assumed. Thus if $p_{tot} = cm$, then $g_{surf} = c \approx 25,000$ dyn/g.

**Fontenla et al. (1993) also assumed complete mixing, i.e., the same element mix at all heights. Check this by plotting the ratio of the hydrogen mass density to the total mass density against height (the hydrogen atom mass is $m_H = 1.67352 \times 10^{-24}$ g, e.g., Allen 1976). Then add helium to hydrogen using their abundance and mass ratios ($N_{He}/N_H = 0.1, m_{He} = 3.97m_H$), and estimate the fraction of the total mass density made up by the remaining elements in the model mix (the "metals").**

In [6]:
```
fH = nhyd*mH/dens
fHe = 0.1*nhyd*mHe/dens
nHe = 0.1*nhyd
```

```
fZ = 1.0 - fH -fHe

fig = plt.figure(figsize=(12,8))
gs = gridspec.GridSpec(1,1)
ax0 = plt.subplot(gs[0])
ax0.plot(height,fH, lw=2.0, label="H")
ax0.plot(height,fHe, lw=2.0, label="He")
ax0.plot(height,fZ, lw=2.0, label="Metals")
leg = ax0.legend(loc=5)
leg.get_frame().set_alpha(0.0)
ax0.set_xlabel(r"Height [km]")
ax0.set_ylabel(r"$n_{x}/n$")
#ax0.semilogy()
plt.show()
```



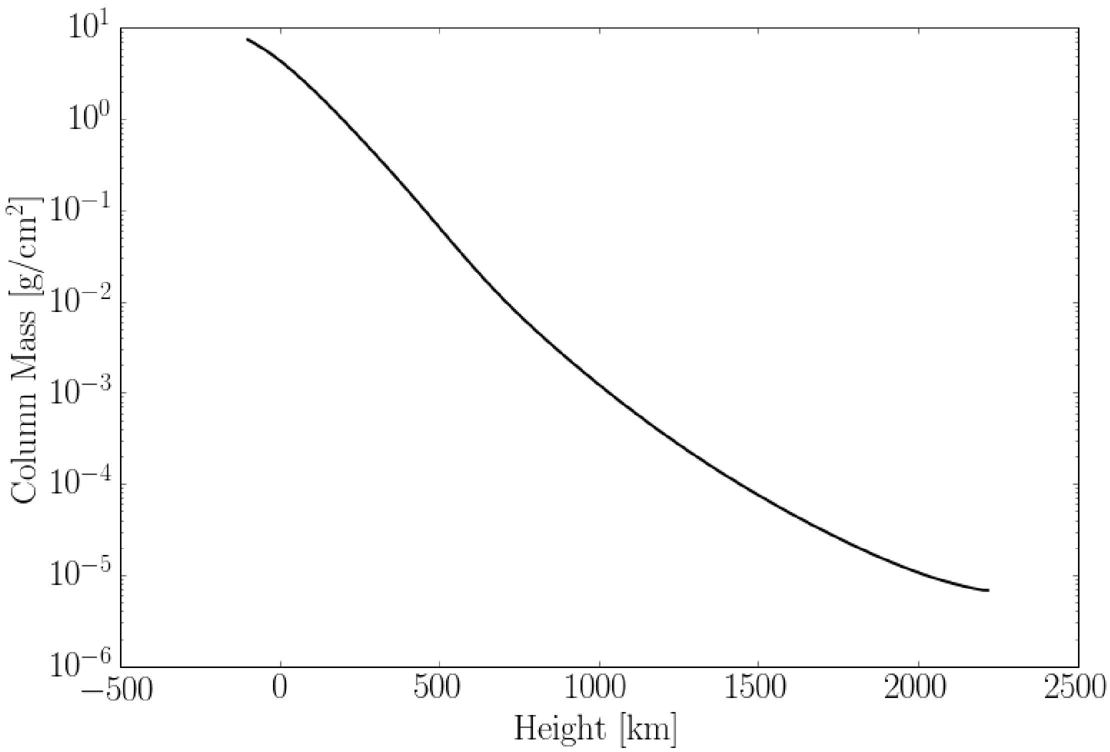

```
In [29]: print "Fraction of total mass density made up by metals: Z =", np.mean(fZ)
```

```
Fraction of total mass density made up by metals: Z = 0.00222580708084
```

**Plot the column mass against height.**

```
In [8]: fig = plt.figure(figsize=(12,8))
        gs = gridspec.GridSpec(1,1)
        ax0 = plt.subplot(gs[0])
        ax0.plot(height,colm, lw=2.0, c='k')
        ax0.set_xlabel(r"Height [km]")
        ax0.set_ylabel(r"Column Mass [g/cm$^2$]")
```

```
ax0.semilogy()
plt.show()
```



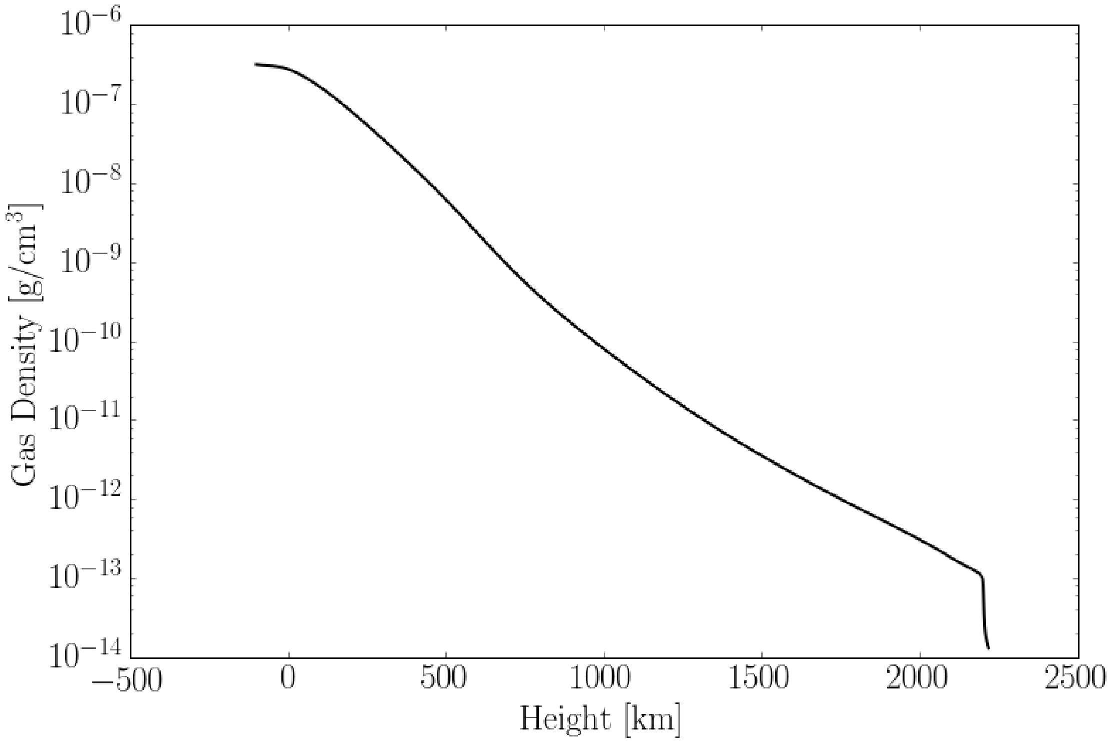

**The curve becomes nearly straight when you make the y-axis logarithmic. Why is that? Why isn't it exactly straight?**

Like the pressure, the column mass decreases with the scale height. Therefore, log(column mass) $\sim -z/H$ where z is the height and H is the scale height. So the relationship *should* be linear. However, the mean molecular weight decreases at high levels which makes the curve deviate from a straight line.

**Plot the gas density against height. Estimate the density scale height $H_\rho$ in $\rho \approx \rho(0)\exp(^{-h}/_{H_\rho})$ in the photosphere.**

```
In [9]: def find_nearest(array,value):
            idx = (np.abs(array-value)).argmin()
            return idx
```

```
In [10]: fig = plt.figure(figsize=(12,8))
         gs = gridspec.GridSpec(1,1)
         ax0 = plt.subplot(gs[0])
         ax0.plot(height,dens, lw=2.0, c='k')
         ax0.set_xlabel(r"Height [km]")
         ax0.set_ylabel(r"Gas Density [g/cm$^3$]")
         ax0.semilogy()
         plt.show()
```



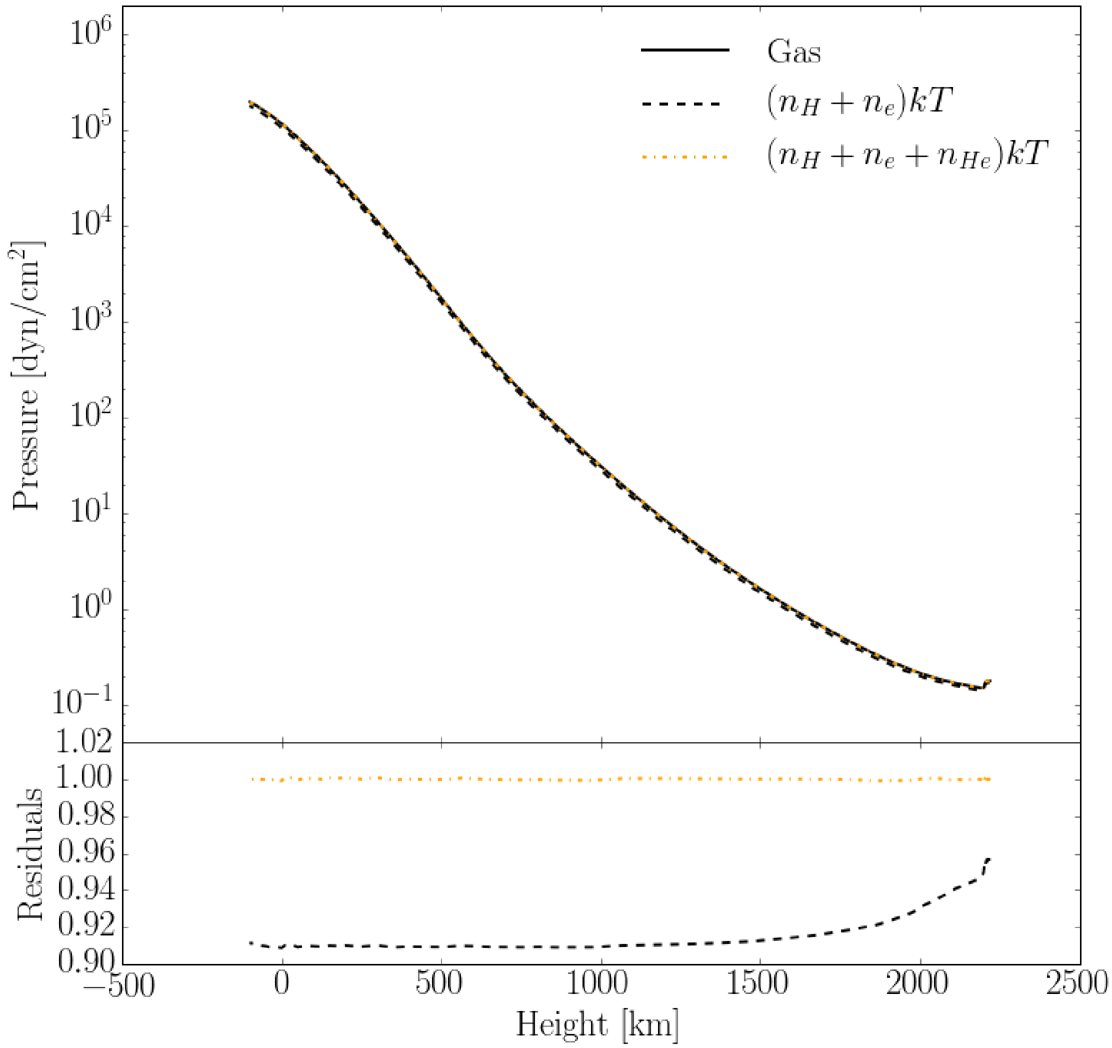

In [11]:
```
Hrho = k * 5770. / 0.7 / mH / (G*Msun/Rsun**2) / 1e5

print "Pressure Scale Height in the solar photosphere: ", Hrho, 'km'
```

```
Pressure Scale Height in the solar photosphere:  248.31562804 km
```

**Compute the gas pressure and plot it against height. Overplot the product $(n_H + n_e)kT$. Plot the ratio of the two curves to show their differences.**

In [12]:
```
# Compute gas pressure
pgas = ptot * pgasptot

Pe = (nhyd + nel)*k*temp
PeHe = (nhyd + nel + nHe)*k*temp

fig = plt.figure(figsize=(12,12))
gs = gridspec.GridSpec(2,1, height_ratios=(1,.3))
ax0 = plt.subplot(gs[0])
ax1 = plt.subplot(gs[1])
ax0.plot(height,pgas, lw=2.0, c='k', label='Gas')
ax0.plot(height,Pe, lw=2.0, c='k', ls='--', label=r'$(n_{H} + n_{e}) kT$')
ax0.plot(height,PeHe, lw=2.0, c='orange', ls='-.', label=r'$(n_{H} + n_{e} + n_{He}) kT$')
ax1.plot(height,Pe/pgas, lw=2.0, c='k', ls='--', label=r'$(n_{H} + n_{e}) kT$')
ax1.plot(height,PeHe/pgas, lw=2.0, c='orange', ls='-.', label=r'$(n_{H} + n_{e} + n_{He}) kT$')
leg = ax0.legend(loc=1)
leg.get_frame().set_alpha(0.0)
plt.setp(ax0.get_xticklabels(), visible=False)
fig.subplots_adjust(hspace=0.0)
```

```
ax1.set_xlabel(r"Height [km]")
ax1.set_ylabel('Residuals')
ax0.set_ylabel(r"Pressure [dyn/cm$^2$]"); ax0.set_ylim([4e-2, 2e6])
ax0.semilogy()
plt.show()
```




**Do the differences measure deviations from the ideal gas law or something else?**

The differences between $(n_H + n_e)kT$ and $P_{gas}$ is clueing us into the fact that we've missed a relevant gas particle... helium.

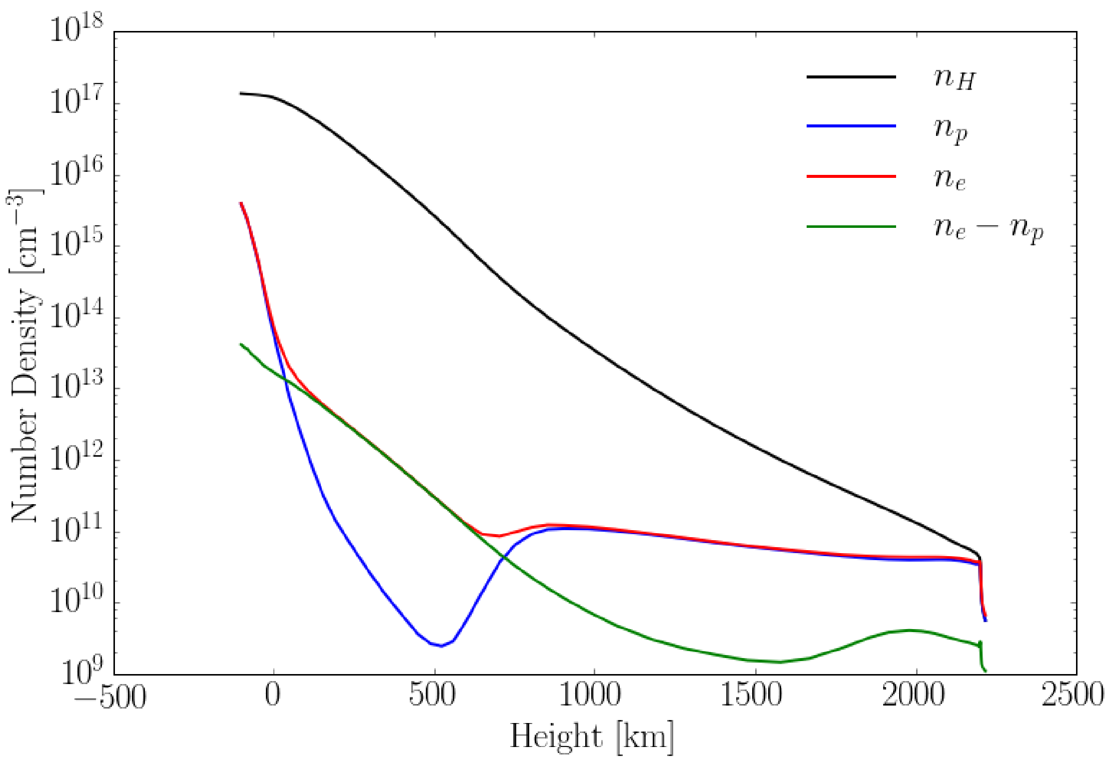
**Now add the helium density $N_{He}$ to the product and enlarge the deviations. Comments?**

With Helium included in the density, it nicely follows the ideal gas law.

**Plot the total hydrogen density against height and overplot curves for the electron density, the proton density, and the density of the electrons that do not result from hydrogen ionization.**

```
In [13]: neNoH = nel - nprot
```

```
fig = plt.figure(figsize=(12,8))
gs = gridspec.GridSpec(1,1)
ax0 = plt.subplot(gs[0])
ax0.plot(height,nhyd, lw=2.0, c='k', label=r'$n_{H}$')
ax0.plot(height,nprot, lw=2.0, c='b', label=r'$n_{p}$')
ax0.plot(height,nel, lw=2.0, c='r', label=r'$n_{e}$')
ax0.plot(height,neNoH, lw=2.0, c='g', label=r'$n_{e} - n_{p}$')
leg = ax0.legend(loc=1)
leg.get_frame().set_alpha(0.0)
ax0.set_xlabel(r"Height [km]")
ax0.set_ylabel(r"Number Density [cm$^{-3}$]")
ax0.semilogy()
plt.show()
```



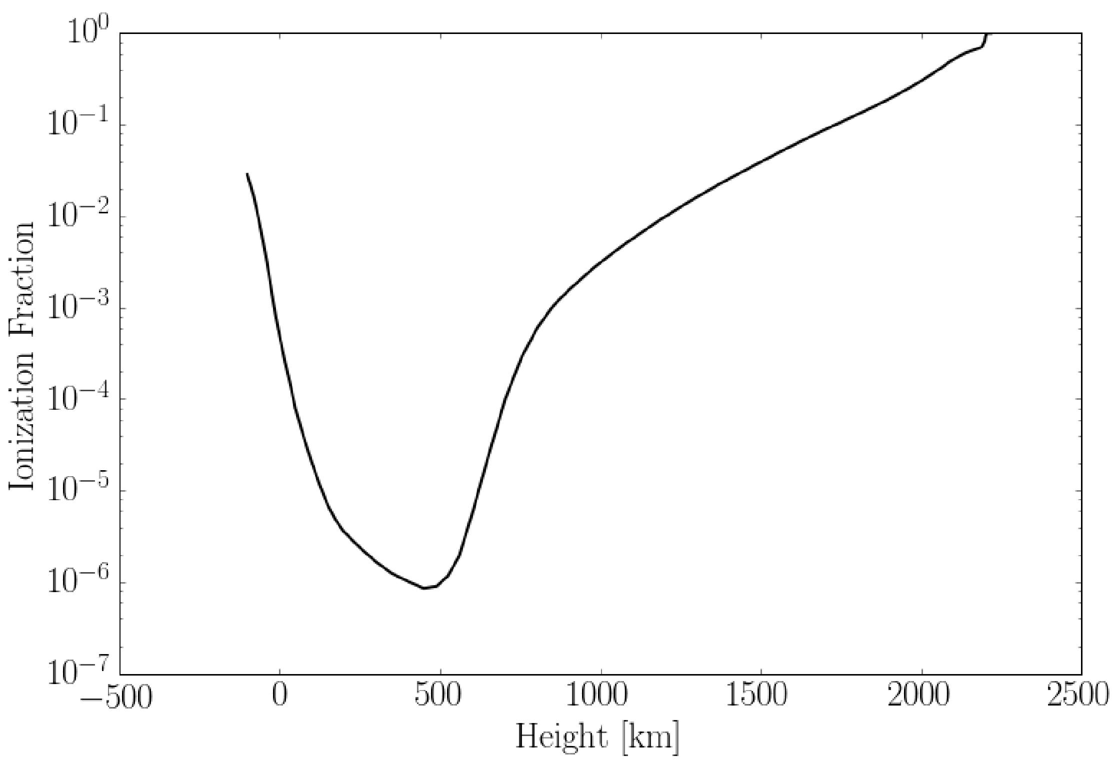

**Explain their behavior. You may find inspiration in Figure 6 on page 13 (Rutten). The last curve is parallel to the hydrogen density over a considerable height range. What does that imply? And what happens at larger height?**

In general, number densities tend to decrease with height. No surprise there. The proton number density falls drastically in the temperature minimum due to the decrease of hydrogen ionization. In this region, the electons are solely due to ionized metals. But as the temperature rises again, hydrogen ionization resumes and the proton and electron number densities increase – protons much more than electrons becuase the electrons due to metals continue to decrease in number density. The electrons due to metals parallels the hydrogen density because the they are a tracer for the number density of metals, which, like hydrogen, exponentially decrease with the scale height. At larger heights, the electons due to metals increase briefly in number density becuase the temperature has become high enough to doubly ionize some metals, adding more electrons to the pool.

**Plot the ionization fraction of hydrogen logarithmically against height.**

Since free protons result from hydrogen ionization, the ionization fraction of hydrogen is just $f = n_p/n_H$

```
In [14]: fion = nprot/nhyd

         fig = plt.figure(figsize=(12,8))
         gs = gridspec.GridSpec(1,1)
         ax0 = plt.subplot(gs[0])
         ax0.plot(height,fion, lw=2.0, c='k')
         ax0.set_xlabel(r"Height [km]")
         ax0.set_ylabel(r"Ionization Fraction")
         ax0.semilogy()
         plt.show()
```




**Why does this curve look like the one in Figure 2? And why is it tilted with respect to that?**

From the Saha eqn we have that the rate of ionization scales as $T^{3/2}e^{-T}$. Thus, because of the $T^{3/2}$ term, the ionization fraction follows the temperature for much of the range shown here, but when the the temperature gets too high the exponential term takes over and leads to a decay in ionization fraction with temperature.

**Let us now compare the photon and particle densities. In thermodynamic equilibrium (TE) the radiation is isotropic with intensity $I_\nu = B_\nu$ and has total energy density (Stefan Boltzmann)**

$$u = \frac{1}{c}\int\int B_\nu d\Omega d\nu = \frac{4\pi}{c}T^4$$

**so that the total photon density for isotropic TE radiation is given, with $u_\nu = du/d\nu$, $T$ in K and $N_{phot}$ in photons per cm$^3$, by**

$$N_{phot} = \int_0^\infty \frac{u_\nu}{h\nu} d\nu \approx 20T^3$$

**This equation gives a reasonable estimate for the photon density at the deepest model location, why? Compute the value there and compare it to the hydrogen density. Why is the equation not valid higher up in the atmosphere?**

> At the deepest model location where there are many gas particle collisions LTE holds, so this equation gives reasonable results. As we can see below, the hydrogen density dominates the photon density at the deepest level. Higher up in the atmosphere where the hydrogen density is much lower, LTE no longer holds and so the above equation breaks down.

```
In [15]: u = 4. * np.pi / c * temp**4.
         Nphot = 20. * temp**3.
         print 'Deepest location: '
         print '-----------------'
         print 'N_phot =', Nphot[-1]
         print 'n_hyd =',  nhyd[-1]
```

```
Deepest location:
-----------------
N_phot = 1.661168e+13
n_hyd = 1.351e+17
```

**The photon density there is $N_{phot} \approx 20T_{eff}^3/2\pi$ with $T_{eff} = 5770$ K the effective solar temperature (since $\pi B(T_{eff}) = \sigma T_{eff}^3 = F+ = \pi I+$ with $F+ $ the emergent flux and $I+$ the diskaveraged emergent intensity). Compare it to the hydrogen density at the highest location in the FALC model. The medium there is insensitive to these photons (except those at the center wavelength of the hydrogen Ly$\alpha$ line), why?**

> Since the photon density at the highest model level is much higher than the hydrogen density, the medium is largely optically thin. With such low particle densities, there is little collisional broadening, and thus only the hydrogen Ly$\alpha$ line center is optically thick.

```
In [16]: Nphoteff = 20. * 5770.**3. / (2*np.pi)
         print 'Highest location: '
         print '-----------------'
         print 'N_phot =', Nphoteff
         print 'n_hyd =',  "%e" %nhyd[0]
```

```
Highest location:
-----------------
N_phot = 6.11473396401e+11
n_hyd = 5.575000e+09
```

## 1.3 Comparison with the earth's atmosphere

We will use the data file: `earth.dat`.

```
In [17]: ! head earth.dat
```

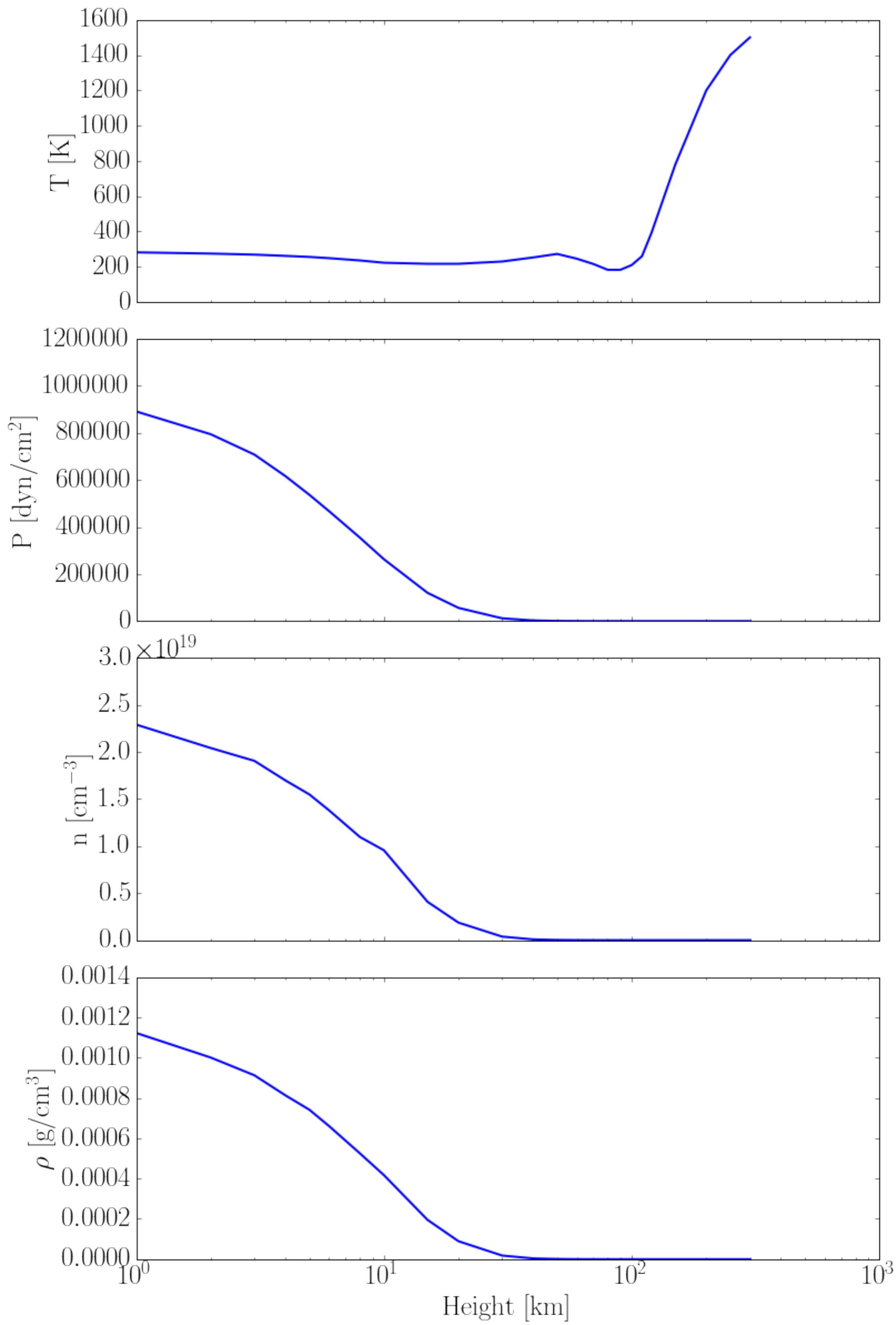
```
0   6.01   288   -2.91   19.41
   1   5.95   282   -2.95   19.36
   2   5.90   275   -3.00   19.31
   3   5.85   269   -3.04   19.28
   4   5.79   262   -3.09   19.23
   5   5.73   256   -3.13   19.19
   6   5.67   249   -3.18   19.14

   8   5.55   236   -3.28   19.04
```

**Write code to read file earth.dat.**

In [18]:
```
# Read earth.dat
earth = np.genfromtxt('earth.dat')
hE=earth[:,0]
pgasE=10.**earth[:,1]
tempE=earth[:,2]
densE=10.**earth[:,3]
npartE=10.**earth[:,4]
```

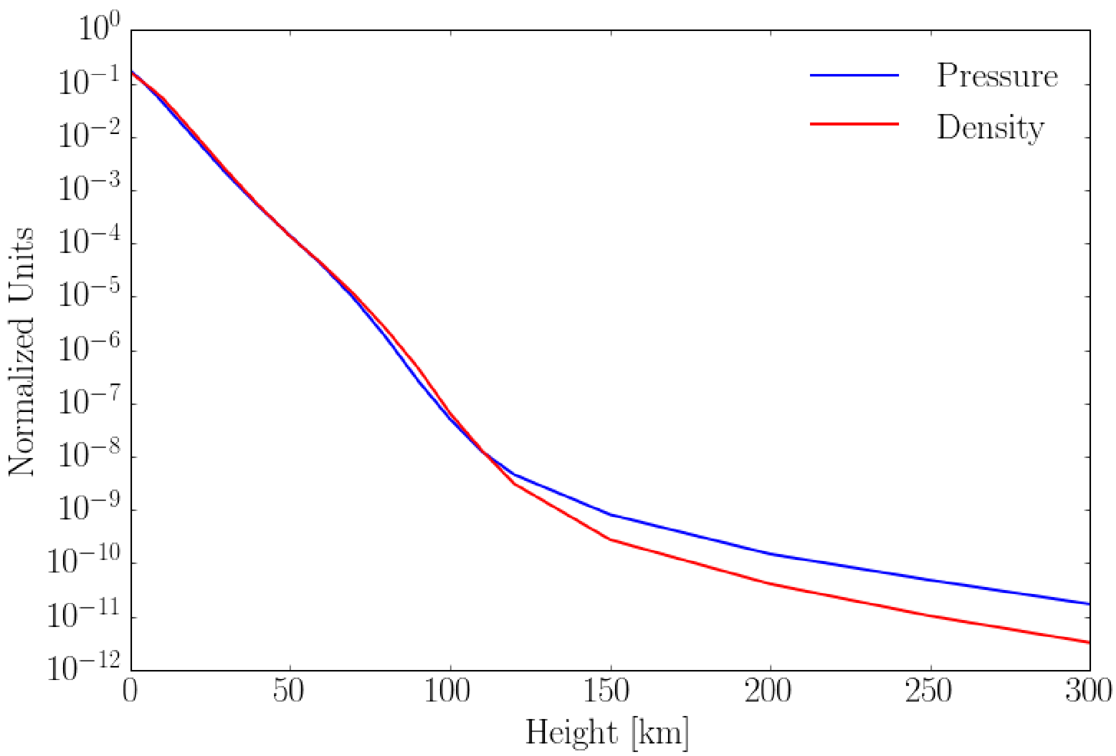
**Plot the temperature, pressure, particle density and gas density against height, logarithmically where appropriate.**

In [19]:
```
fig = plt.figure(figsize=(12,20))
gs = gridspec.GridSpec(4,1)
ax0 = plt.subplot(gs[0])
ax1 = plt.subplot(gs[1])
ax2 = plt.subplot(gs[2])
ax3 = plt.subplot(gs[3])
ax0.plot(hE, tempE, lw=2.0)
ax1.plot(hE, pgasE, lw=2.0)
ax2.plot(hE, npartE, lw=2.0)
ax3.plot(hE, densE, lw=2.0)
plt.setp(ax0.get_xticklabels(), visible=False)
plt.setp(ax1.get_xticklabels(), visible=False)
plt.setp(ax2.get_xticklabels(), visible=False)
fig.subplots_adjust(hspace=0.13)
ax3.set_xlabel(r"Height [km]")
ax0.set_ylabel(r"T [K]")
ax1.set_ylabel(r"P [dyn/cm$^2$]")
ax2.set_ylabel(r"n [cm$^{-3}$]")
ax3.set_ylabel(r"$\rho$ [g/cm$^3$]")
ax0.semilogx()
ax1.semilogx()
ax2.semilogx()
ax3.semilogx()
plt.show()
```


T [K]
P [dyn/cm$^2$]
n [cm$^{-3}$]
$\times 10^{19}$
$\rho$ [g/cm$^3$]
Height [km]

**Plot the pressure and density stratifications together in normalized units in one graph.**

```
In [20]: fig = plt.figure(figsize=(12,8))
         gs = gridspec.GridSpec(1,1)
         ax0 = plt.subplot(gs[0])
         ax0.plot(hE,pgasE/np.sum(pgasE), lw=2.0, c='b', label='Pressure')
         ax0.plot(hE,densE/np.sum(densE), lw=2.0, c='r', label='Density')
         leg = ax0.legend(loc=1); leg.get_frame().set_alpha(0.0)
         ax0.set_xlabel(r"Height [km]")
         ax0.set_ylabel(r"Normalized Units")
         ax0.semilogy()
         plt.show()
```



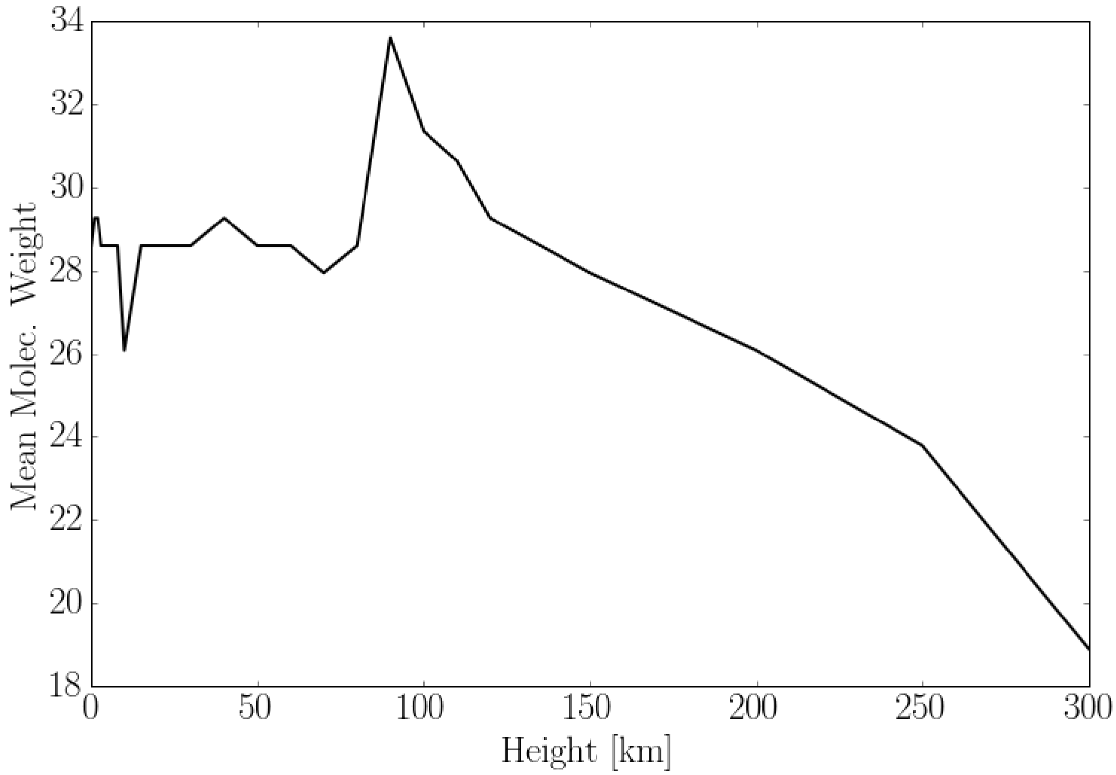

**Comments?**

Both the pressure and density follow the same exponential decay with height. However, about 120 km up they depart from one another, with the pressure remaining higher than the density. Perhaps this is because the molecular weight of the species higher up in the atmosphere is lower than near the surface.

**Plot the mean molecular weight $\mu_E \equiv m/m_H = \rho/(Nm_H)$ against height.**

```
In [21]: muE = densE / (npartE * mH)

         fig = plt.figure(figsize=(12,8))
         gs = gridspec.GridSpec(1,1)
         ax0 = plt.subplot(gs[0])
```

```
ax0.plot(hE,muE, lw=2.0, c='k')
ax0.set_xlabel(r"Height [km]")
ax0.set_ylabel(r"Mean Molec. Weight")
plt.show()
```




**Why does it decrease in the high atmosphere?**

The mean molecular weight decreases high in the atmosphere because only light molecules have enough kinetic energy to climb so high out of the Earth's gravitational well. Note from the above plot of temperature vs height, that the temperature starts to climb rapidly above about 100 km, suggesting that only fast moving particles are there. It's also possible that photolysis by solar UV radiation breaks apart larger molecules, like ozone, into lighter molecules.

**Estimate the density scales height of the lower terrestrial atmosphere. Which quantities make it differ from the solar one? How much harder do you have to breathe on Mount Everest?**

The scale height of the Earth's lower atmosphere is much less than that of the solar photosphere because the temprerature is much lower and the mean molecular weight is much higher. Although the gravitational acceleration on the earth is much less than on the sun, Earth's weak gravity allows the atmohsphere to be more extended and thus acts to increase the scale height.

```
In [22]: HE = k * 290.0 / 29.0 / mH / 980.0
         print "Earth's scale height:", HE / 1e5, 'km'
```

```
Earth's scale height: 8.41864782626 km
```

Mt. Everest is about 8.8 km. This is roughly the pressure scale height of the Earth's atmosphere. Therefore the pressure and density of the atmposphere are decreased by a factor of $e^{-1} \approx 0.37$. I'm certainly no expert in lung physiology, but it would seem like you would have to breathe about $e \approx 2.7$ times harder to make up for the lower density air.

**Compare the terrestrial parameter values to the solar ones, at the base of each atmosphere. What is the ratio of the particle densities at $h = 0$ in the two atmospheres?**

```
In [23]: ihs = find_nearest(height, 0.0)
         ihe = find_nearest(hE, 0.0)
         height[ihs], hE[ihe]

         print 'Solar: T(0)=', temp[ihs]
         print 'Earth: T(0)=', tempE[ihe]
         print 'Solar: n_H + n_He + n_e + n_p =', nhyd[ihs]+nprot[ihs]+nel[ihs]+nHe[ihs]
         print 'Earth: n_part =',npartE[ihe]
         print 'Solar: P_gas =', pgasptot[ihs]
         print 'Earth: P_gas =', pgasE[ihe]
         print 'Solar: rho =', dens[ihs]
         print 'Earth: rho =', densE[ihe]
```

```
Solar: T(0)= 6520.0
Earth: T(0)= 288.0
Solar: n_H + n_He + n_e + n_p = 1.3015711e+17
Earth: n_part = 2.57039578277e+19
Solar: P_gas = 0.971
Earth: P_gas = 1023292.99228
Solar: rho = 2.771e-07
Earth: rho = 0.00123026877081
```

Ratio of paticle densities at h=0,

$$R = \frac{n_{\oplus}}{n_{\odot}} = \frac{n_{\oplus}}{n_H + n_{He} + n_e + n_p}$$

```
In [24]: print 'R =', (npartE[ihe]) / (nhyd+nprot+nel+nHe)[ihs]
```

```
R = 197.484085408
```

**The standard gravity at the earth's surface is $g_E = 980.665$ cm s$^{-2}$. Use this value to estimate the atmospheric column mass (g cm−2) at the earth's surface and compare that also to the value at the base of the solar atmosphere.**

We can calculate the column mass at the Earth;s surface by solving $m = \frac{P}{g}$.

```
In [25]: print "Earth column mass:", pgasE[ihe] / 980.665, "g/cm^2"
```

```
Earth column mass: 1043.46845486 g/cm^2
```

```
In [26]: print "Solar column mass:", colm[ihs], "g/cm^2"
```

```
Solar column mass: 4.404 g/cm^2
```

**Final question: the energy flux of the sunshine reaching our planet ("irradiance") is:**

$$\mathcal{R} = \frac{4\pi R^2}{4\pi D^2}\mathcal{F}_{\odot}^{+}$$

**with $\mathcal{F}_{\odot}^{+} = \pi B(T_{eff}^{\odot})$ the emergent solar flux, $R$ the solar radius and $D$ the distance sun–earth, so that the sunshine photon density at earth is**

$$N_{phot} = \pi \frac{R^2}{D^2} N_{phot}^{top}$$

**with $N_{phot}^{top}$ the photon density at the top of FALC which was determined at the end of the first section. Compare $N_{phot}$ to the particle density in the air around us, and to the local thermal photon production derived from (2). Comments?**

In [27]:
```
D = 1.49598e13 # Astronomical Unit (cm)
NphotE_solar = np.pi * (Rsun / D)**2. * Nphoteff
NphotE_local = 20. * tempE[0]**3.
print 'Sunshine photon desnity at earth:', "%e" %NphotE_solar, 'photons/cm^3'
print 'Earth atmospheric particle density:', npartE[0], 'molecules/cm^3'
print 'Earth surface local photon production:', "%e" %NphotE_local, 'photons/cm^3'
```

```
Sunshine photon desnity at earth: 4.158094e+07 photons/cm^3
Earth atmospheric particle density: 2.57039578277e+19 molecules/cm^3
Earth surface local photon production: 4.777574e+08 photons/cm^3
```

The atmospheric particle density is roughly one trillion times (about 12 orders of magnitude) higher than the solar photon density at the Earth's surface. This makes sense becuase the Earth is in LTE, where particle-particle collisions dominate compared to radiative processes. The locally produced (thermal) photon number density at Earth's surface is also about an order of magnitude greater than the solar photon number density. Although, the energies of solar photons are, on average, much higher than the thermal photons.

In [ ]: