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Exercise 2.6 Write in propositional logic:

- I will only go to school if I get a cookie now.

- John and Mary are running.

- A foreign national is entitled to social security if he has legal employment or if he has had such less than three years ago, unless he is currently also employed abroad.

a) p = I will go to school.
   q = I get a cookie now.
   p → q
b) p = John is running.
   q = Marry is running.
   p ∧ q
c) p = A foreign national is entitled to social security.
   q = A foreign national has legal employment.
   r = A foreign national has had legal employment less than three years ago.
   s = A foreign national is currently also employed abroad.
   ((q ∨ r) ∧ ¬s) → p

Exercise 2.7 Which of the following are formulas in propositional logic:

- $p \to \neg q$

- $\neg\neg \wedge q \vee p$

- $p \neg q$

a) yes
b) no
c) no

Exercise 2.11 Construct truth tables for the following formulas:

- $(p \to q) \vee (q \to p)$,

- $((p \vee \neg q) \wedge r) \leftrightarrow (\neg(p \wedge r) \vee q)$.

p	q	(	p	→	q	)	∨	(	q	→	p	)
0	0	0	1	0	1	0	1	0
0	1	0	1	1	1	1	0	0
1	0	1	0	0	1	0	1	1
1	1	1	1	1	1	1	1	1

p	q	r	((p	V	¬	q)	∧	r)	↔	(¬	(p	∧	r)	V	q)
0	0	0	0	1	1	0	0	0	0	1	0	0	0	1	0
0	0	1	0	1	1	0	1	1	1	1	0	0	1	1	0
0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	1
0	1	1	0	0	0	1	0	1	0	1	0	0	1	1	1
1	0	0	1	1	1	0	0	0	0	1	1	0	0	1	0
1	0	1	1	1	1	0	1	1	0	0	1	1	1	0	0
1	1	0	1	1	0	1	0	0	0	1	1	0	0	1	1
1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1

Exercise 2.22 Which of the following pairs are *logically equivalent*? Confirm your answer using truth tables:

(1) $\varphi \to \psi$ and $\psi \to \varphi$

(2) $\varphi \to \psi$ and $\neg\psi \to \neg\varphi$

(3) $\neg(\varphi \to \psi)$ and $\varphi \lor \neg\psi$

(4) $\neg(\varphi \to \psi)$ and $\varphi \land \neg\psi$

(5) $\neg(\varphi \leftrightarrow \psi)$ and $\neg\varphi \leftrightarrow \neg\psi$

(6) $\neg(\varphi \leftrightarrow \psi)$ and $\neg\varphi \leftrightarrow \psi$

(7) $(\varphi \land \psi) \leftrightarrow (\varphi \lor \psi)$ and $\varphi \leftrightarrow \psi$

1) Not equivalent.

φ	ψ	φ	→	ψ	ψ	→	φ
0	0	0	1	0	0	1	0
0	1	0	1	1	1	0	0
1	0	1	0	0	0	1	1
1	1	1	1	1	1	1	1

2) Equivalent.

φ	ψ	φ	→	ψ	¬	ψ	→	¬	φ
0	0	0	1	0	1	0	1	1	0
0	1	0	1	1	0	1	1	1	0
1	0	1	0	0	1	0	0	0	1
1	1	1	1	1	0	1	1	0	1

3) Not equivalent.

φ	ψ	¬	(φ	→	ψ)	φ	V	¬	ψ
0	0	0	0	1	0	0	1	1	0
0	1	0	0	1	1	0	0	0	1
1	0	1	1	0	0	1	1	1	0
1	1	0	1	1	1	1	1	0	1

4) Equivalent.

φ	ψ	¬	(φ	→	ψ)	φ	∧	¬	ψ
0	0	**0**	0	1	0	0	**0**	1	0
0	1	**0**	0	1	1	0	**0**	0	1
1	0	**1**	1	0	0	1	**1**	1	0
1	1	**0**	1	1	1	1	**0**	0	1

5) Not equivalent.

φ	ψ	¬	(φ	↔	ψ)	¬	φ	↔	¬	ψ
0	0	**0**	0	1	0	1	0	**1**	1	0
0	1	**1**	0	0	1	1	0	**0**	0	1
1	0	**1**	1	0	0	0	1	**0**	1	0
1	1	**0**	1	1	1	0	1	**1**	0	1

6) Equivalent.

φ	ψ	¬	(φ	↔	ψ)	¬	φ	↔	ψ
0	0	**0**	0	1	0	1	0	**0**	0
0	1	**1**	0	0	1	1	0	**1**	1
1	0	**1**	1	0	0	0	1	**1**	0
1	1	**0**	1	1	1	0	1	**0**	1

7) Equivalent.

φ	ψ	(φ	∧	ψ)	↔	(φ	∨	Ψ)	φ	↔	Ψ
0	0	0	0	0	**1**	0	0	0	0	**1**	0
0	1	0	0	1	**0**	0	1	1	0	**0**	1
1	0	1	0	0	**0**	1	1	0	1	**0**	0
1	1	1	1	1	**1**	1	1	1	1	**1**	1