

Language Modeling, N-gram Models And Analysis

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**ADOPTED SOME MATERIALS DEVELOPED IN PREVIOUS COURSES BY NANCY
MCCRACKEN, LIZ LIDDY AND OTHERS; AND SOME INSTRUCTOR RESOURCES FOR
THE BOOK "SPEECH AND LANGUAGE PROCESSING" BY DANIEL JURAFSKY AND
JAMES H. MARTIN**

But why do we bother to construct language models?

Word Prediction!

- Guess the next word...
 - *... I notice three guys standing on the ???*
- There are many sources of knowledge that can be used to inform this task, including arbitrary world knowledge.
- But it turns out that you can do pretty well by simply looking at the preceding words and keeping track of some fairly simple counts.
- Probability of an upcoming word:
 $P(w_5 | w_1, w_2, w_3, w_4)$
conditional probability that w_5 occurs, given that we know that w_1, w_2, w_3, w_4 already occurred.

What Is A Language Model?

1. Assume that we have a **corpus** (what is a corpus?)
2. V : the set of all words in the language
 - For example, when building a language model for English we might have $V = \{\text{the, dog, laughs, saw, barks, cat, ...}\}$ (finite)
3. A sentence in the language is a sequence of words $W_1 W_2 \dots W_n$, where $n \geq 1, W_i \in V$ for $i \in \{1 \dots (n - 1)\}$ (Why $n-1$?)
4. W_n is a special symbol, STOP (we assume that STOP is not a member of V).
the dog barks STOP
the cat laughs STOP
5. V^+ : the set of all sentences with the vocabulary V (Is this set finite or infinite?)

WHAT IS A LANGUAGE MODEL?

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3 Definition 1 (Language Model) A language model consists of
4 a finite set V , and a function $P(W_1, W_2, \dots, W_n)$ such that:

1. For any $(W_1 \dots W_n) \in V^+$, $P(W_1, W_2, \dots, W_n) \geq 0$
2. In addition,

$$\sum_{(W_1 \dots W_n) \in V^+} P(W_1, W_2, \dots, W_n) = 1$$

Hence $P(W_1, W_2, \dots, W_n)$ is a probability distribution over the sentences in V^+ .

Language Models

- The goal of a Language Model is to assign a probability that a sentence (or phrase) will occur in natural uses of the language
- Applications
 - **Machine Translation:**
 - $P(\text{high winds tonight}) > P(\text{large winds tonight})$
 - **Spell Correction**
 - The office is about fifteen **minuets** from my house
 - $P(\text{about fifteen minutes from}) > P(\text{about fifteen minuets from})$
 - **Speech Recognition**
 - $P(\text{I saw a van}) \gg P(\text{eyes awe of an})$
 - + **Summarization, question-answering, and many other NLP applications**



How To Compute The Probabilities Of A Sentence?

- **Chain Rules:** computing the joint probability of all the words conditioned by the previous words

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i \mid w_1 w_2 \dots w_{i-1})$$

$P(\text{"its water is so transparent"}) =$

$P(\text{its}) \times P(\text{water} \mid \text{its}) \times P(\text{is} \mid \text{its water})$

$\times P(\text{so} \mid \text{its water is}) \times P(\text{transparent} \mid \text{its water is so})$

- With our training corpus, we can compute the probability that something occurred by counting its occurrences and dividing by the total number

$P(\text{the} \mid \text{its water is so transparent that}) =$

$\frac{\text{Count}(\text{its water is so transparent that the})}{\text{Count}(\text{its water is so transparent that})}$

But, what's the problem with this approach (language modeling)?

Markov Assumption



Andrei Markov

- Instead we make the simplifying Markov assumption that we can predict the next word based on only one word previous:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

- Or perhaps two words previous:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

This particular kind of independence assumption is called a Markov assumption after the Russian mathematician Andrei Markov. For more details, please refer to the posted reading in Blackboard – [“Language Modeling”](#)

N-gram Models

- **Unigram Model:** (word frequencies)
The simplest case is that we predict a sentence probability just based on the probabilities of the words with no preceding words

$$P(w_1 w_2 \dots w_n) \approx \prod P(w_i)$$

- **Bigram Model:** (two word frequencies)
Prediction based on one previous word:

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx \prod P(w_i | w_{i-1})$$

Some Clarifications: Bigrams And Bigram Language Models

- **Bigrams:** any two words that occur together
 - Examples: “two great and powerful groups of nations”, the bigrams are “two great”, “great and”, “and powerful”, etc.
 - The frequency of a bigram: The percentage of times the bigram occurs in all the bigrams of the corpus (e.g., Google N-gram corpus)
 - $\text{Count of bigram } xy / \text{Count of all bigrams in corpus}$
- **Bigram language models:** use the bigram probability, meaning a conditional probability, to predict how likely it is that the second word follows the first

N-gram Models

- We can extend to trigrams, 4-grams, 5-grams
 - Each higher number will get a more accurate model, but will be harder to find examples of the longer word sequences in the corpus
- In general this is an insufficient model of language because language has **long-distance dependencies**:

“The computer which I had just put into the machine room on the fifth floor crashed.”

- the last word *crashed* is not very likely to follow the word *floor*, but it is likely to be the main verb of the word *computer*

N-gram Probabilities

- For N-Grams, we need the **conditional probability**:
 $P(\text{<next word>} \mid \text{<preceding word sequence of length } n\text{>})$
e.g. $P(\text{the} \mid \text{They picnicked by})$
- We define this as
 - the observed frequency (count) of the whole sequence divided by the observed frequency of the preceding, or initial, sequence (sometimes called the **maximum likelihood estimation (MLE)**):

$$\begin{aligned} &P(\text{<next word>} \mid \text{<preceding word sequence of length } n\text{>}) \\ &= \text{Count}(\text{<preceding word sequence> <next word>}) \\ &\quad / \text{Count}(\text{<preceding word sequence>}) \end{aligned}$$

- Example: $\text{Count}(\text{They picnicked by the}) / \text{Count}(\text{They picnicked by})$

BIGRAM PROBABILITIES

- For bigrams, the MLE estimate is:

$$P(w_i | w_{i-1}) = \frac{\text{count}(w_{i-1}, w_i)}{\text{count}(w_{i-1})}$$

- Example mini-corpus of three sentences, where we have sentence detection and we include the sentence tags in order to represent the beginning and end of the sentence.

<S> I am Sam </S>

<S> Sam I am </S>

<S> I do not like green eggs and ham </S>

- Bigram probabilities:

$$P(I | <S>) = \text{Count}(<S>, I) / \text{Count}(<S>) = 2/3 = 0.67$$

(probability that I follows <S>)

What's the probability for $P(</S> | \text{Sam})$?

An Example Of Using Bigram Probabilities To Compute The Probabilities Of Sentences

- Corpus: Berkeley Restaurant Project sentences
 - Example sentences:
 - can you tell me about any good cantonese restaurants close by
 - mid priced thai food is what i'm looking for
 - tell me about chez panisse
 - can you give me a listing of the kinds of food that are available
 - i'm looking for a good place to eat breakfast
 - when is caffe venezia open during the day

An Example of using bigram probabilities to compute the probabilities of sentences (cont'd)

■ Raw Bigram Counts from the corpus

Out of 9222 sentences, showing counts that the word on the left is followed by the word on the top

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

An Example Of Using Bigram Probabilities To Compute The Probabilities Of Sentences (Cont'd)

■ Calculate Bigram Probabilities

1. Unigram counts:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

2. Bigram probabilities = bigram counts / unigram

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

MORE BIGRAMS FROM THE RESTAURANT CORPUS

Eat on	.16	Eat Thai	.03
Eat some	.06	Eat breakfast	.03
Eat lunch	.06	Eat in	.02
Eat dinner	.05	Eat Chinese	.02
Eat at	.04	Eat Mexican	.02
Eat a	.04	Eat tomorrow	.01
Eat Indian	.04	Eat dessert	.007
Eat today	.03	Eat British	.001



ADDITIONAL BIGRAMS

<S> I	.25	Want some	.04
<S> I'd	.06	Want Thai	.01
<S> Tell	.04	To eat	.26
<S> I'm	.02	To have	.14
I want	.32	To spend	.09
I would	.29	To be	.02
I don't	.08	British food	.60
I have	.04	British restaurant	.15
Want to	.65	British cuisine	.01
Want a	.05	British lunch	.01



An Example Of Using Bigram Probabilities To Compute The Probabilities Of Sentences (Cont'd)

- For a bigram grammar
 - $P(\text{sentence})$ can be approximated by multiplying all the bigram probabilities in the sequence

$$\prod_{k=1}^n P(w_k | w_{k-1})$$

Example:

a. $P(\text{I want to eat Chinese food}) =$
 $P(\text{I} | <S>) P(\text{want} | \text{I}) P(\text{to} | \text{want}) P(\text{eat} | \text{to})$
 $P(\text{Chinese} | \text{eat}) P(\text{food} | \text{Chinese})$

b. $P(\text{I want to eat British food}) = P(\text{I} | <S>)$
 $P(\text{want} | \text{I}) P(\text{to} | \text{want}) P(\text{eat} | \text{to}) P(\text{British} | \text{eat}) P(\text{food} | \text{British})$

An Example Of Using Bigram Probabilities To Compute The Probabilities Of Sentences (Cont'd)

- $P(\text{I want to eat British food}) = P(\text{I} | \langle S \rangle) P(\text{want} | \text{I}) P(\text{to} | \text{want}) P(\text{eat} | \text{to}) P(\text{British} | \text{eat}) P(\text{food} | \text{British}) = .25 \times .33 \times .66 \times .28 \times .001 \times .60 = .0000092$
- $P(\text{I want to eat Chinese food}) = P(\text{I} | \langle S \rangle) P(\text{want} | \text{I}) P(\text{to} | \text{want}) P(\text{eat} | \text{to}) P(\text{Chinese} | \text{eat}) P(\text{food} | \text{Chinese}) = .25 \times .33 \times .66 \times .28 \times .02 \times .52 = 0.00016$

What do these results tell you?

Probabilities seem to capture “syntactic” facts,
“world knowledge”

eat is often followed by a noun
British food is not too popular



Using N-gram Probabilities In A Language Model: Why Do We Need Smoothing?

- Every N-gram training matrix is sparse, even for very large corpora (remember Zipf's law)
- There are words that don't occur in the training corpus that may occur in future text - known as the **unseen words**
- Whenever a probability is 0, it will multiply the entire sequence to be 0
- **Solution:** estimate the likelihood of unseen N-grams and include a small probability for unseen words



Intuition Of Smoothing (From Dan Klein)

- When we have sparse statistics:

$P(w \mid \text{denied the})$

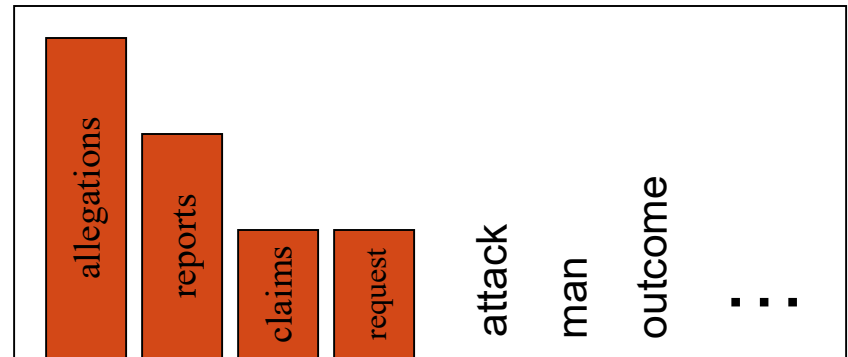
3 allegations

2 reports

1 claims

1 request

7 total



- Steal probability mass to generalize better

$P(w \mid \text{denied the})$

2.5 allegations

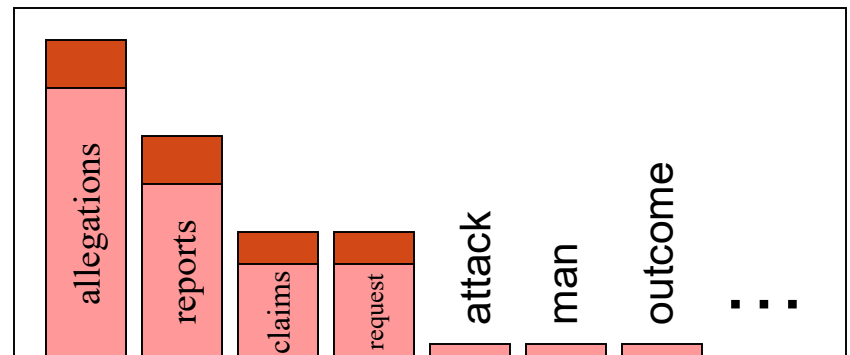
1.5 reports

0.5 claims

0.5 request

2 other

7 total



Smoothing

- Add-one smoothing
- Backoff Smoothing for higher-order N-grams
 - Notice that:
 - N-grams are more precise than (N-1)grams
 - But also, N-grams are more sparse than (N-1) grams
 - How to combine things?
 - Attempt N-grams and back-off to (N-1) if counts are not available
 - E.g. attempt prediction using 4-grams, and back-off to trigrams (or bigrams, or unigrams) if counts are not available
- More complicated techniques exist: in practice, NLP LM use Knesser-Ney smoothing

Language Modeling Toolkit

- SRI Language Modeling:
 - <http://www.speech.sri.com/projects/srilm/>
 - <http://www.speech.sri.com/projects/srilm/papers/icslp2002-srilm.pdf> -- a conference paper that gives an overview of the toolkit
 - <http://www.speech.sri.com/projects/srilm/papers/asru2011-srilm.pdf> -- a more recent paper that gives the update of the toolkit

Making tables, probabilities for you

N-gram Model Application - Spell Correction

- Frequency of spelling errors in human typed text varies
 - 0.05% of the words in carefully edited journals
 - 38% in difficult applications like telephone directory lookup
- Word-based spell correction checks each word in a dictionary/lexicon
 - Detecting spelling errors that result in non-words
 - *mesage* -> *message* by looking only at the **word** in isolation
 - May fail to recognize an error (**real-word errors**)
 - Typographical errors e.g. *there* for *three*
 - Homonym or near-homonym e.g. *dessert* for *desert*, or *piece* for *peace*
- Use context of preceding word and language model to choose correct word
 - $P(\text{Japanese Imperial Navy}) > P(\text{Japanese Empirical Navy})$



N-gram Model Application – Analysis of Handwritten Sentence

- Optical character recognition has higher **error** rates than human typists
- Lists of up to top 5 choices of the handwritten word recognizer, with correct choice highlighted
- Using language models with bigram probabilities (*alarm clock*) & syntactic (POS) information, correct sentence is extracted:

my	alarm	clock	did	not
my	alarm	code	soil	rout
		circle	raid	hot
		shute	risk	riot
		clock	visit	not
			did	must

wake	me	up	this	morning
wake	me	up	thai	moving
			taxis	having
			this	running
			tier	morning
				loving



Google N-gram Release

All Our N-gram are Belong to You

By Peter Norvig - 8/03/2006 11:26:00 AM

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word [n-gram models](#) for a variety of R&D projects, such as [statistical machine translation](#), speech recognition, [spelling correction](#), entity detection, information extraction, and others. While such models have usually been estimated from training

to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

Example Data

- Examples of 4-gram frequencies from the Google N-gram release
 - serve as the incoming 92
 - serve as the incubator 99
 - serve as the independent 794
 - serve as the index 223
 - serve as the indication 72
 - serve as the indicator 120
 - serve as the indicators 45
 - serve as the indispensable 111
 - serve as the indispensable 40
 - serve as the individual 234

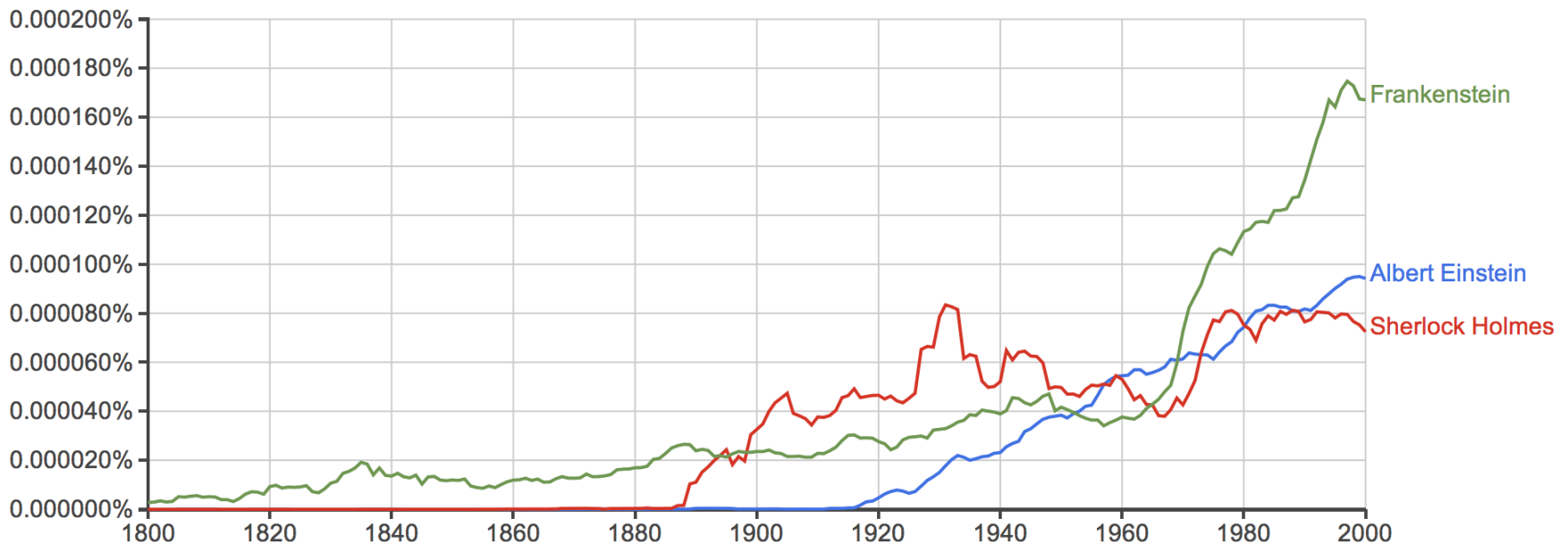
Google N-gram Viewer

- In 2010, Google placed on on-line n-gram viewer that would display graphs of n-gram frequencies of one or more n-grams, based on a corpus defined from Google Books (see the “About NGram Viewer” link)

Google books Ngram Viewer

Graph these comma-separated phrases: ☐ case-insensitive

between and from the corpus with smoothing of [Search lots of books](#)



Additional Corpus Measures

- Recall that so far, we have looked at two measures involving bigrams (and these definitions can be extended to n-grams):
 - **Bigram probability** – conditional probability that the second word follows the first word in the corpus
 - Used to define Language Models
 - **Bigram frequency** – percentage occurrence of the bigram in the corpus (e.g., Google N-gram data)
- Other measures can be defined about the occurrences of bigrams in a corpus
 - **Mutual information**, ...
 - More of these can be found in the NLTK

Corpus Statistics: Mutual Information (MI)

- N-Gram probabilities predict the next word
- Mutual Information computes probability of two words occurring in sequence
- Given a pair of words, compares probability that the two occur together as a joint event to the probability they occur individually & that their co-occurrences are simply the result of chance
 - The more strongly connected 2 items are, the higher will be their MI value



Mutual Information

- Based on work of Church & Hanks (1990), generalizing MI from information theory to apply to words in sequence
 - They used terminology *Association Ratio*
- $P(x)$ and $P(y)$ are estimated by the number of observations of x and y in a corpus and normalized by N , the size of the corpus
- $P(x,y)$ is the number of times that x is followed by y in a window of w words in a corpus and normalized by N , the size of the corpus
- Mutual Information score (also sometimes called PMI, Pointwise Mutual Information):

$$\text{PMI}(x,y) = \log_2 (P(x,y) / P(x) P(y))$$



MI Values Based On 145 WSJ Articles

X	Freq (x)	Y	Freq (y)	Freq (x,y)	MI
Gaza	3	Strip	3	3	14.42
joint	8	venture	4	4	13.00
Chapter	3	11	14	3	12.20
credit	15	card	11	7	11.44
average	22	yield	7	5	11.06
appeals	4	court	47	4	10.45
.....					
said	444	it	346	76	5.02



Uses Of Mutual Information

- Used in similar NLP applications as Language Models
 - Idiomatic phrases for MT
 - Sense disambiguation (both statistical and symbolic approaches)
 - Error detection & correction in speech analysis and spell-checking
- Used for distributional semantics in “deep learning”
- Used in non-text applications such as comparing features in machine learning
- More discussion available at:
<http://nltk.googlecode.com/svn/trunk/doc/howto/collocations.html>



Potential Problems With PMI

$$\text{PMI}(x,y) = \log_2 (P(x,y) / P(x) P(y))$$

With sparse data:

a word pair occurs once, together – Get very high score PMI

High PMI score might not necessarily indicate importance of bigram

With word dependence:

a perfect word pair (bigram): whenever one occurs, the other occurs, then ??

thus the formula becomes $\text{PMI}(x, y) = \log (1 / P(y))$. The rarer the word is, the higher the PMI is High PMI score doesn't mean high word dependence (could just mean rarer words)