priors; that is, if an attribute has k possible values we set $p = \frac{1}{k}$. For example, in estimating $P(Wind = strong|PlayTennis = no)$ we note the attribute $Wind$ has two possible values, so uniform priors would correspond to choosing $p = .5$. Note that if m is zero, the m-estimate is equivalent to the simple fraction $\frac{n_c}{n}$. If both n and m are nonzero, then the observed fraction $\frac{n_c}{n}$ and prior p will be combined according to the weight m. The reason m is called the equivalent sample size is that Equation (6.22) can be interpreted as augmenting the n actual observations by an additional m virtual samples distributed according to p.

6.10 AN EXAMPLE: LEARNING TO CLASSIFY TEXT

To illustrate the practical importance of Bayesian learning methods, consider learning problems in which the instances are text documents. For example, we might wish to learn the target concept "electronic news articles that I find interesting," or "pages on the World Wide Web that discuss machine learning topics." In both cases, if a computer could learn the target concept accurately, it could automatically filter the large volume of online text documents to present only the most relevant documents to the user.

We present here a general algorithm for learning to classify text, based on the naive Bayes classifier. Interestingly, probabilistic approaches such as the one described here are among the most effective algorithms currently known for learning to classify text documents. Examples of such systems are described by Lewis (1991), Lang (1995), and Joachims (1996).

The naive Bayes algorithm that we shall present applies in the following general setting. Consider an instance space X consisting of all possible *text documents* (i.e., all possible strings of words and punctuation of all possible lengths). We are given training examples of some unknown target function $f(x)$, which can take on any value from some finite set V. The task is to learn from these training examples to predict the target value for subsequent text documents. For illustration, we will consider the target function classifying documents as interesting or uninteresting to a particular person, using the target values *like* and *dislike* to indicate these two classes.

The two main design issues involved in applying the naive Bayes classifier to such text classification problems are first to decide how to represent an arbitrary text document in terms of attribute values, and second to decide how to estimate the probabilities required by the naive Bayes classifier.

Our approach to representing arbitrary text documents is disturbingly simple: Given a text document, such as this paragraph, we define an attribute for each word position in the document and define the value of that attribute to be the English word found in that position. Thus, the current paragraph would be described by 111 attribute values, corresponding to the 111 word positions. The value of the first attribute is the word "our," the value of the second attribute is the word "approach," and so on. Notice that long text documents will require a larger number of attributes than short documents. As we shall see, this will not cause us any trouble.

Given this representation for text documents, we can now apply the naive Bayes classifier. For the sake of concreteness, let us assume we are given a set of 700 training documents that a friend has classified as *dislike* and another 300 she has classified as *like*. We are now given a new document and asked to classify it. Again, for concreteness let us assume the new text document is the preceding paragraph. In this case, we instantiate Equation (6.20) to calculate the naive Bayes classification as

$$\begin{aligned} v_{NB} &= \underset{v_j \in \{like, dislike\}}{\operatorname{argmax}} P(v_j) \prod_{i=1}^{111} P(a_i|v_j) \\ &= \underset{v_j \in \{like, dislike\}}{\operatorname{argmax}} P(v_j)\ P(a_1 = \text{"}our\text{"}|v_j) P(a_2 = \text{"}approach\text{"}|v_j) \\ &\qquad \ldots\ P(a_{111} = \text{"}trouble\text{"}|v_j) \end{aligned}$$

To summarize, the naive Bayes classification v_{NB} is the classification that maximizes the probability of observing the words that were actually found in the document, subject to the usual naive Bayes independence assumption. The independence assumption $P(a_1, \ldots a_{111}|v_j) = \prod_1^{111} P(a_i|v_j)$ states in this setting that the word probabilities for one text position are independent of the words that occur in other positions, given the document classification v_j. Note this assumption is clearly incorrect. For example, the probability of observing the word "learning" in some position may be greater if the preceding word is "machine." Despite the obvious inaccuracy of this independence assumption, we have little choice but to make it—without it, the number of probability terms that must be computed is prohibitive. Fortunately, in practice the naive Bayes learner performs remarkably well in many text classification problems despite the incorrectness of this independence assumption. Domingos and Pazzani (1996) provide an interesting analysis of this fortunate phenomenon.

To calculate v_{NB} using the above expression, we require estimates for the probability terms $P(v_j)$ and $P(a_i = w_k|v_j)$ (here we introduce w_k to indicate the kth word in the English vocabulary). The first of these can easily be estimated based on the fraction of each class in the training data ($P(like) = .3$ and $P(dislike) = .7$ in the current example). As usual, estimating the class conditional probabilities (e.g., $P(a_1 = \text{"}our\text{"}|dislike)$) is more problematic because we must estimate one such probability term for each combination of text position, English word, and target value. Unfortunately, there are approximately 50,000 distinct words in the English vocabulary, 2 possible target values, and 111 text positions in the current example, so we must estimate $2 \cdot 111 \cdot 50,000 \approx 10$ million such terms from the training data.

Fortunately, we can make an additional reasonable assumption that reduces the number of probabilities that must be estimated. In particular, we shall assume the probability of encountering a specific word w_k (e.g., "chocolate") is independent of the specific word position being considered (e.g., a_{23} versus a_{95}). More formally, this amounts to assuming that the attributes are independent and identically distributed, given the target classification; that is, $P(a_i = w_k|v_j) =$

$P(a_m = w_k|v_j)$ for all i, j, k, m. Therefore, we estimate the entire set of probabilities $P(a_1 = w_k|v_j)$, $P(a_2 = w_k|v_j)\ldots$ by the single position-independent probability $P(w_k|v_j)$, which we will use regardless of the word position. The net effect is that we now require only $2 \cdot 50,000$ distinct terms of the form $P(w_k|v_j)$. This is still a large number, but manageable. Notice in cases where training data is limited, the primary advantage of making this assumption is that it increases the number of examples available to estimate each of the required probabilities, thereby increasing the reliability of the estimates.

To complete the design of our learning algorithm, we must still choose a method for estimating the probability terms. We adopt the m-estimate—Equation (6.22)—with uniform priors and with m equal to the size of the word vocabulary. Thus, the estimate for $P(w_k|v_j)$ will be

$$\frac{n_k + 1}{n + |Vocabulary|}$$

where n is the total number of word positions in all training examples whose target value is v_j, n_k is the number of times word w_k is found among these n word positions, and $|Vocabulary|$ is the total number of distinct words (and other tokens) found within the training data.

To summarize, the final algorithm uses a naive Bayes classifier together with the assumption that the probability of word occurrence is independent of position within the text. The final algorithm is shown in Table 6.2. Notice the algorithm is quite simple. During learning, the procedure LEARN_NAIVE_BAYES_TEXT examines all training documents to extract the vocabulary of all words and tokens that appear in the text, then counts their frequencies among the different target classes to obtain the necessary probability estimates. Later, given a new document to be classified, the procedure CLASSIFY_NAIVE_BAYES_TEXT uses these probability estimates to calculate v_{NB} according to Equation (6.20). Note that any words appearing in the new document that were not observed in the training set are simply ignored by CLASSIFY_NAIVE_BAYES_TEXT. Code for this algorithm, as well as training data sets, are available on the World Wide Web at http://www.cs.cmu.edu/~tom/book.html.

6.10.1 Experimental Results

How effective is the learning algorithm of Table 6.2? In one experiment (see Joachims 1996), a minor variant of this algorithm was applied to the problem of classifying usenet news articles. The target classification for an article in this case was the name of the usenet newsgroup in which the article appeared. One can think of the task as creating a newsgroup posting service that learns to assign documents to the appropriate newsgroup. In the experiment described by Joachims (1996), 20 electronic newsgroups were considered (listed in Table 6.3). Then 1,000 articles were collected from each newsgroup, forming a data set of 20,000 documents. The naive Bayes algorithm was then applied using two-thirds of these 20,000 documents as training examples, and performance was measured

LEARN_NAIVE_BAYES_TEXT($Examples$, V)

Examples is a set of text documents along with their target values. V is the set of all possible target values. This function learns the probability terms $P(w_k|v_j)$, describing the probability that a randomly drawn word from a document in class v_j will be the English word w_k. It also learns the class prior probabilities $P(v_j)$.

1. collect all words, punctuation, and other tokens that occur in Examples

- $Vocabulary \leftarrow$ the set of all distinct words and other tokens occurring in any text document from $Examples$

2. calculate the required $P(v_j)$ and $P(w_k|v_j)$ probability terms

- For each target value v_j in V do
  - $docs_j \leftarrow$ the subset of documents from $Examples$ for which the target value is v_j
  - $P(v_j) \leftarrow \frac{|docs_j|}{|Examples|}$
  - $Text_j \leftarrow$ a single document created by concatenating all members of $docs_j$
  - $n \leftarrow$ total number of distinct word positions in $Text_j$
  - for each word w_k in $Vocabulary$
    - $n_k \leftarrow$ number of times word w_k occurs in $Text_j$
    - $P(w_k|v_j) \leftarrow \frac{n_k+1}{n+|Vocabulary|}$

CLASSIFY_NAIVE_BAYES_TEXT(Doc)

Return the estimated target value for the document Doc. a_i denotes the word found in the ith position within Doc.

- *positions* $\leftarrow$ all word positions in *Doc* that contain tokens found in *Vocabulary*
- Return v_{NB}, where

$$v_{NB} = \underset{v_j \in V}{\text{argmax}}\ P(v_j) \prod_{i \in positions} P(a_i|v_j)$$

TABLE 6.2
Naive Bayes algorithms for learning and classifying text. In addition to the usual naive Bayes assumptions, these algorithms assume the probability of a word occurring is independent of its position within the text.

over the remaining third. Given 20 possible newsgroups, we would expect random guessing to achieve a classification accuracy of approximately 5%. The accuracy achieved by the program was 89%. The algorithm used in these experiments was exactly the algorithm of Table 6.2, with one exception: Only a subset of the words occurring in the documents were included as the value of the $Vocabulary$ variable in the algorithm. In particular, the 100 most frequent words were removed (these include words such as "the" and "of"), and any word occurring fewer than three times was also removed. The resulting vocabulary contained approximately 38,500 words.

Similarly impressive results have been achieved by others applying similar statistical learning approaches to text classification. For example, Lang (1995) describes another variant of the naive Bayes algorithm and its application to learning the target concept "usenet articles that I find interesting." He describes the NEWSWEEDER system—a program for reading netnews that allows the user to rate articles as he or she reads them. NEWSWEEDER then uses these rated articles as

comp.graphics	misc.forsale	soc.religion.christian	sci.space
comp.os.ms-windows.misc	rec.autos	talk.politics.guns	sci.crypt
comp.sys.ibm.pc.hardware	rec.motorcycles	talk.politics.mideast	sci.electronics
comp.sys.mac.hardware	rec.sport.baseball	talk.politics.misc	sci.med
comp.windows.x	rec.sport.hockey	talk.religion.misc	
		alt.atheism	

TABLE 6.3
Twenty usenet newsgroups used in the text classification experiment. After training on 667 articles from each newsgroup, a naive Bayes classifier achieved an accuracy of 89% predicting to which newsgroup subsequent articles belonged. Random guessing would produce an accuracy of only 5%.

training examples to learn to predict which subsequent articles will be of interest to the user, so that it can bring these to the user's attention. Lang (1995) reports experiments in which NEWSWEEDER used its learned profile of user interests to suggest the most highly rated new articles each day. By presenting the user with the top 10% of its automatically rated new articles each day, it created a pool of articles containing three to four times as many interesting articles as the general pool of articles read by the user. For example, for one user the fraction of articles rated "interesting" was 16% overall, but was 59% among the articles recommended by NEWSWEEDER.

Several other, non-Bayesian, statistical text learning algorithms are common, many based on similarity metrics initially developed for information retrieval (e.g., see Rocchio 1971; Salton 1991). Additional text learning algorithms are described in Hearst and Hirsh (1996).

6.11 BAYESIAN BELIEF NETWORKS

As discussed in the previous two sections, the naive Bayes classifier makes significant use of the assumption that the values of the attributes $a_1 \ldots a_n$ are conditionally independent given the target value v. This assumption dramatically reduces the complexity of learning the target function. When it is met, the naive Bayes classifier outputs the optimal Bayes classification. However, in many cases this conditional independence assumption is clearly overly restrictive.

A Bayesian belief network describes the probability distribution governing a set of variables by specifying a set of conditional independence assumptions along with a set of conditional probabilities. In contrast to the naive Bayes classifier, which assumes that *all* the variables are conditionally independent given the value of the target variable, Bayesian belief networks allow stating conditional independence assumptions that apply to *subsets* of the variables. Thus, Bayesian belief networks provide an intermediate approach that is less constraining than the global assumption of conditional independence made by the naive Bayes classifier, but more tractable than avoiding conditional independence assumptions altogether. Bayesian belief networks are an active focus of current research, and a variety of algorithms have been proposed for learning them and for using them for inference.

In this section we introduce the key concepts and the representation of Bayesian belief networks. More detailed treatments are given by Pearl (1988), Russell and Norvig (1995), Heckerman et al. (1995), and Jensen (1996).

In general, a Bayesian belief network describes the probability distribution over a set of variables. Consider an arbitrary set of random variables $Y_1 \ldots Y_n$, where each variable Y_i can take on the set of possible values $V(Y_i)$. We define the *joint space* of the set of variables Y to be the cross product $V(Y_1) \times V(Y_2) \times \ldots V(Y_n)$. In other words, each item in the joint space corresponds to one of the possible assignments of values to the tuple of variables $\langle Y_1 \ldots Y_n \rangle$. The probability distribution over this joint space is called the *joint probability distribution.* The joint probability distribution specifies the probability for each of the possible variable bindings for the tuple $\langle Y_1 \ldots Y_n \rangle$. A Bayesian belief network describes the joint probability distribution for a set of variables.

6.11.1 Conditional Independence

Let us begin our discussion of Bayesian belief networks by defining precisely the notion of conditional independence. Let X, Y, and Z be three discrete-valued random variables. We say that X is *conditionally independent* of Y given Z if the probability distribution governing X is independent of the value of Y given a value for Z; that is, if

$$(\forall x_i, y_j, z_k)\ P(X = x_i | Y = y_j, Z = z_k) = P(X = x_i | Z = z_k)$$

where $x_i \in V(X)$, $y_j \in V(Y)$, and $z_k \in V(Z)$. We commonly write the above expression in abbreviated form as $P(X|Y, Z) = P(X|Z)$. This definition of conditional independence can be extended to sets of variables as well. We say that the set of variables $X_1 \ldots X_l$ is conditionally independent of the set of variables $Y_1 \ldots Y_m$ given the set of variables $Z_1 \ldots Z_n$ if

$$P(X_1 \ldots X_l | Y_1 \ldots Y_m, Z_1 \ldots Z_n) = P(X_1 \ldots X_l | Z_1 \ldots Z_n)$$

Note the correspondence between this definition and our use of conditional independence in the definition of the naive Bayes classifier. The naive Bayes classifier assumes that the instance attribute A_1 is conditionally independent of instance attribute A_2 given the target value V. This allows the naive Bayes classifier to calculate $P(A_1, A_2|V)$ in Equation (6.20) as follows

$$P(A_1, A_2|V) = P(A_1|A_2, V)P(A_2|V) \quad (6.23)$$

$$= P(A_1|V)P(A_2|V) \quad (6.24)$$

Equation (6.23) is just the general form of the product rule of probability from Table 6.1. Equation (6.24) follows because if A_1 is conditionally independent of A_2 given V, then by our definition of conditional independence $P(A_1|A_2, V) = P(A_1|V)$.