1. **Homogeneous Transforms** (20 points)
Consider the robot arm in Figure 1.

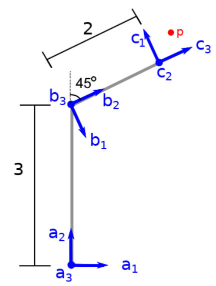

Figure 1: Robot arm diagram. Note that a_3, b_3, and c_2 are pointed out of the page.

There are three coordinate frames A, B, C with their corresponding basis vectors. Let ${}^Ap$ denote the coordinates of the point p in frame A, ${}^Bp$ be the same point in frame B, and ${}^Cp$ the same point in frame C. Define the homogeneous transforms T_1, T_2 , T_3 as follows:

$$\begin{bmatrix} {}^Ap \\ 1 \end{bmatrix} = T_1 \begin{bmatrix} {}^Bp \\ 1 \end{bmatrix} \qquad \begin{bmatrix} {}^Bp \\ 1 \end{bmatrix} = T_2 \begin{bmatrix} {}^Cp \\ 1 \end{bmatrix} \qquad \begin{bmatrix} {}^Ap \\ 1 \end{bmatrix} = T_3 \begin{bmatrix} {}^Cp \\ 1 \end{bmatrix}$$

(a) Find the homogeneous transform T_1. (6 points)

(b) Find the homogeneous transform T_2. (6 points)

(c) Find the homogeneous transform T_3. (Hint: Use T_1 and T_2). (8 points)

2. **Rotation Matrix Sudoku** (20 points)
Only three elements of the rotation matrix below are known (entries with $*$ are unknown). Use the properties of rotation matrices to determine the missing elements. Either prove your solution is unique, or provide an alternative solution. (*Note: Show your work to receive partial credit. You may show your math or include a code segment.) [1]

$$\begin{bmatrix} * & 0.892 & 0.423 \\ * & * & * \\ -0.186 & * & * \end{bmatrix}$$

3. **Rodrigues' Formula** (20 points)

(a) Are the axis and angle always uniquely defined for a rotation? If not, explain the conditions under which the axis and angle are not uniquely defined. (7 points)

(b) Write the axis-angle representation and the quaternion corresponding to the rotation matrix. (13 points)

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\cos(\pi/6) & \sin(\pi/6) \\ 0 & \sin(\pi/6) & \cos(\pi/6) \end{pmatrix}$$

[1] Given the computational nature of this problem, you should use this opportunity to get comfortable doing calculations in Python 3 using PyCharm's interactive console. You'll find Numpy arrays invaluable, including the associated Numpy functions for 'inv', 'cross,' 'dot,' and matrix multiplication using the '@' symbol.

4. **Signal Processing** (20 points)

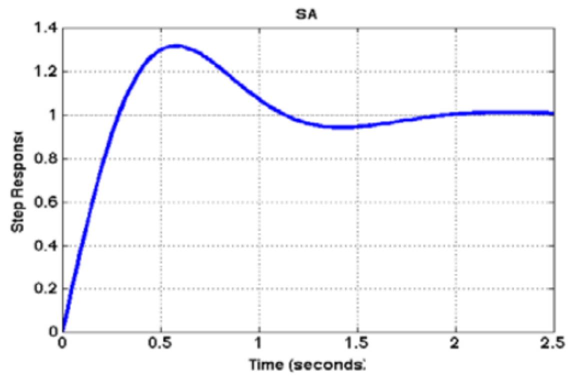

Figure 2: A 2nd-order step response

(a) Given the signal in Fig. 2, is this signal under-damped, critically damped, or over-damped? (5 points)

(b) Calculate the percentage overshoot, damping ratio, and 2% settling time. (15 points)

5. **Transforms and and Python Plotting** (20 points)
Figure 3 shows a robot coordinate system (B) and a world coordinate system (A).

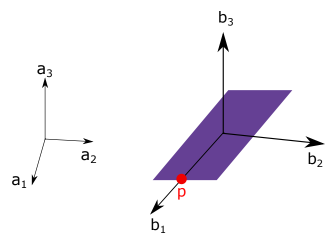

Figure 3: Robot frame B (with basis vectors b_1, b_2, b_3) in the world frame A (with basis vectors a_1, a_2, a_3).

Assume that the position of a robot in meters $d(t)$ and the orientation $R(t)$ are given (both with respect to the world frame) as

$$d(t) = \begin{bmatrix} \cos(0.1 * t) \\ \sin(0.12 * t) \\ \sin(0.08 * t) \end{bmatrix} \qquad R(t) = \begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

*Note: All angles in this problem are in radians!

(a) Compute, as a function of time, the linear velocity $v(t)$ of the robot in the world frame. (5 points)

(b) Suppose there is a point on the robot we are interested in, e.g. a camera that is offset on the x axis. Say the position of this point is given by

$$^{B}p(t) =^{B} p = \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix}$$

for some constant c. Note that the above is represented in the robot frame. What is the position of this point in the world frame as a function of time? In other words, find $^{A}p(t)$. Your expression should include c. (5 points)

(c) Using Python, graph both robot position and ${}^{A}p(t)$ over time for $c = 0.25$ on the same plot. Include a title, axes labels, and legend. You may find the online documentation for matplotlib helpful (e.g. https://matplotlib.org/mpl_toolkits/mplot3d/tutorial.html#line-plots).
An example is shown in Figure 4. (5 points)

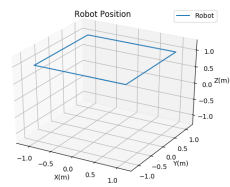

Figure 4: An example plot of robot position.

(d) Suppose at time $t = 1$ the gravity vector measured in body coordinates is $[1.1, 0.9, -9.7]^T$. What is the expected gravity measured in body coordinates at time $t = 5$? (5 points)