Autonomous Vehicle Planning and Control

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Session 3 Vehicle Lateral control

Outline

1. Lateral vehicle dynamics
    a. Kinematic bicycle model
    b. Ackermann steering geometry
    c. Limitation of bicycle model
2. Modern control review
    a. Problem of classical control
    b. Modern control: State space
    c. Controllability, observability
    d. State space system stability
3. Vehicle lateral control with geometric model
    a. Pure Pursuit and its tuning
    b. Stanley Method and its tuning

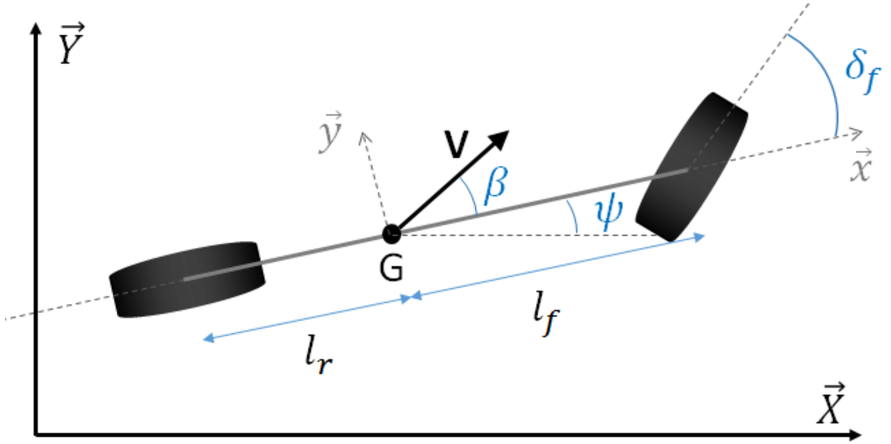
Lateral Vehicle Dynamics

Kinematic Model of Lateral Vehicle Motion

The bicycle model is derived assuming both front and rear wheels can be steered.

Assumption:

- The vehicle's motion is restricted to the X-Y plane i.e. There is no "up & down" movement taken into consideration.
- Same left and right wheel steer motion; the left and right wheels are lumped together into a single wheel.
- Ignore wheel's slip angle. the velocity vectors at points A and B are in the direction of the orientation of the front and rear wheels respectively.
- Ignore the load shift;
- Assume the vehicle is rigid body



Kinematic Model of Lateral Vehicle Motion

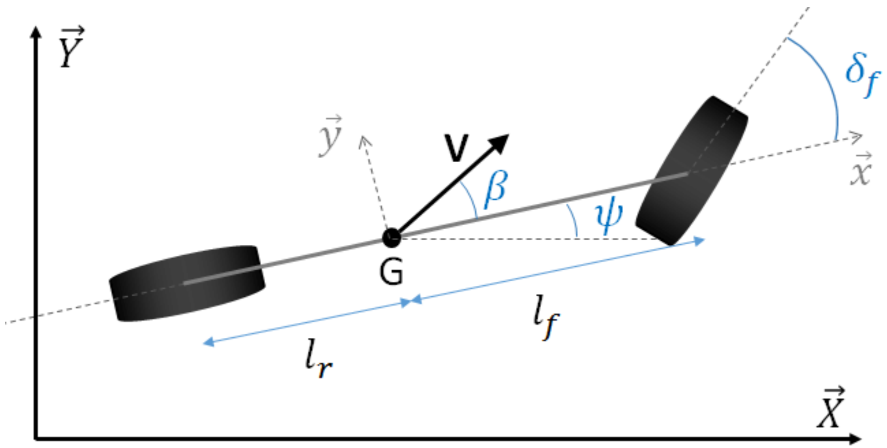
The bicycle model is derived assuming both front and rear wheels can be steered.

Input:

- Steer angle δ
- Throttel/brake: acceleration $a = \dot{v}$

State:

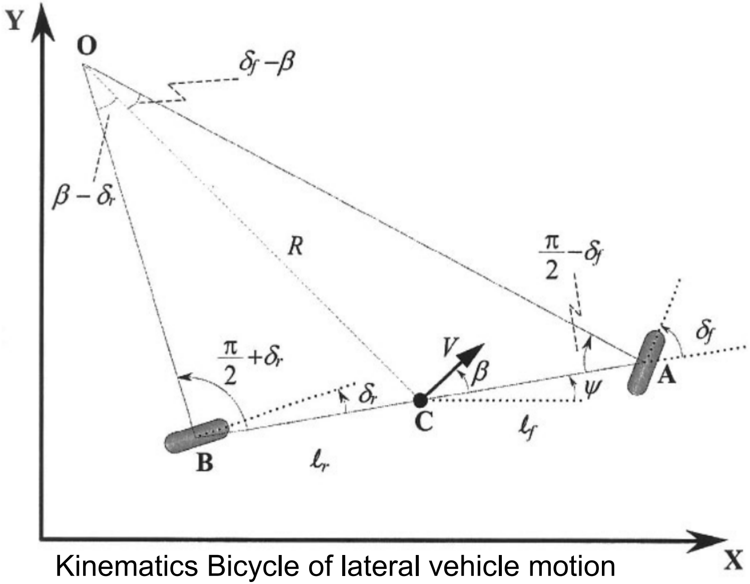
- Vehicle position (x, y);
- Velocity; velocity rate (accel)
- Yaw; yaw rate $\psi, \dot{\psi}$



Kinematic Model of Lateral Vehicle Motion

1. Apply the sine rule to triangle OCA and OCB;

$$\frac{\sin(\delta_f-\beta)}{\ell_f}=\frac{\sin\left(\frac{\pi}{2}-\delta_f\right)}{R} \qquad \frac{\sin(\beta-\delta_r)}{\ell_r}=\frac{\sin\left(\frac{\pi}{2}+\delta_r\right)}{R}$$



Kinematics Bicycle of lateral vehicle motion

2.

$$\frac{\sin(\delta_f)\cos(\beta)-\sin(\beta)\cos(\delta_f)}{\ell_f}=\frac{\cos(\delta_f)}{R}$$

$$\frac{\cos(\delta_r)\sin(\beta)-\cos(\beta)\sin(\delta_r)}{\ell_r}=\frac{\cos(\delta_r)}{R}$$

3.

$$\tan(\delta_f)\cos(\beta)-\sin(\beta)=\frac{l_f}{R}$$

$$\sin(\beta)-\tan(\delta_r)\cos(\beta)=\frac{l_r}{R}$$

Kinematic Model of Lateral Vehicle Motion

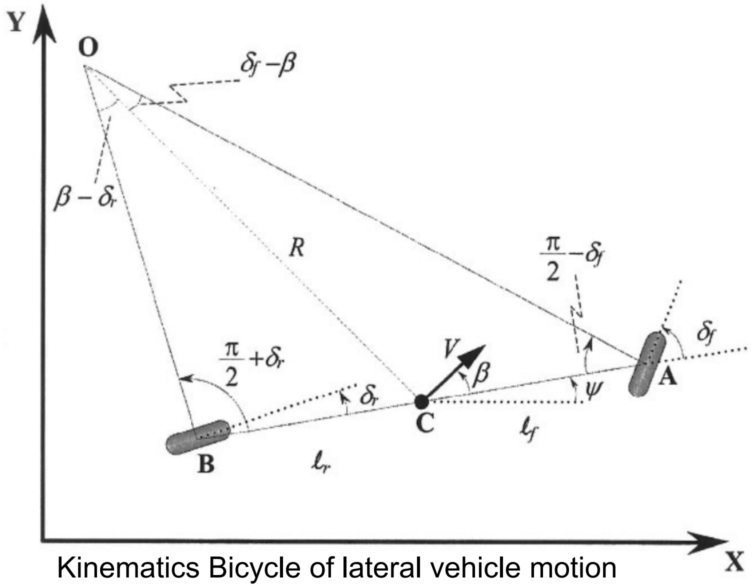
3.
$$\tan(\delta_f)\cos(\beta) - \sin(\beta) = \frac{l_f}{R}$$
$$\sin(\beta) - \tan(\delta_r)\cos(\beta) = \frac{l_r}{R}$$

4. $\left(tan(\delta_f) - tan(\delta_r)\right) cos(\beta) = \frac{l_f + l_r}{R}$

5. For low speed motion, the rate of change of orientation of the vehicle must be equal to the angular velocity of the vehicle.

$$\dot{\psi} \approx r = \frac{V}{R} = V \cdot \frac{1}{R} = V \cdot \frac{\left(tan(\delta_f) - tan(\delta_r)\right) cos(\beta)}{l_f + l_r}$$

$$= \frac{V cos(\beta)}{l_f + l_r}\left(tan(\delta_f) - tan(\delta_r)\right)$$

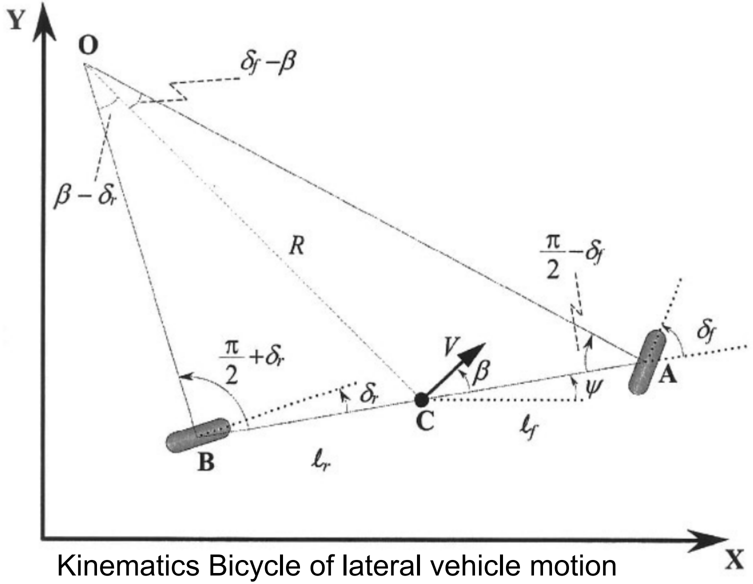


Kinematics Bicycle of lateral vehicle motion

Kinematic Model of Lateral Vehicle Motion

The overall equations of motion:

- Global X axis coordinate: $\dot{X} = V\cos(\psi + \beta)$
- Global Y axis coordinate: $\dot{Y} = V\sin(\psi + \beta)$
- Yaw angle (Orientation): $\dot{\psi} = \frac{V\cos(\beta)}{l_f + l_r}\left(\tan\delta_f - \tan\delta_r\right)$
- Slip angle: $\beta = \tan^{-1}\left(\frac{l_f \tan\delta_r + l_r \tan\delta_f}{l_f + l_r}\right)$



Kinematics Bicycle of lateral vehicle motion

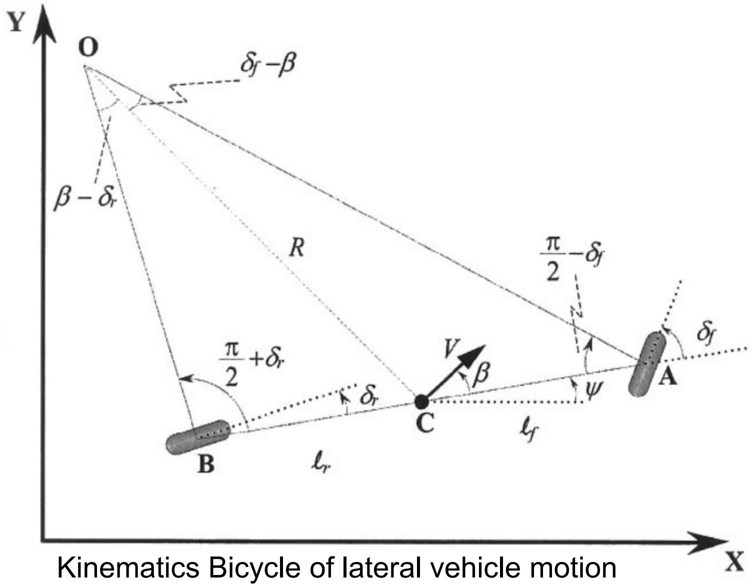
Kinematic Model of Lateral Vehicle Motion

The overall equations of motion:

$$l_r \times (\tan(\delta_f)\cos(\beta) - \sin(\beta)) = \frac{l_f l_r}{R}$$

$$l_f \times (\sin(\beta) - \tan(\delta_r)\cos(\beta)) = \frac{l_f l_r}{R}$$

- Let the above equations equals each other:
- Slip angle: $\beta = \tan^{-1}\left(\dfrac{\ell_f \tan\delta_r + \ell_r \tan\delta_f}{\ell_f + \ell_r}\right)$



Kinematics Bicycle of lateral vehicle motion

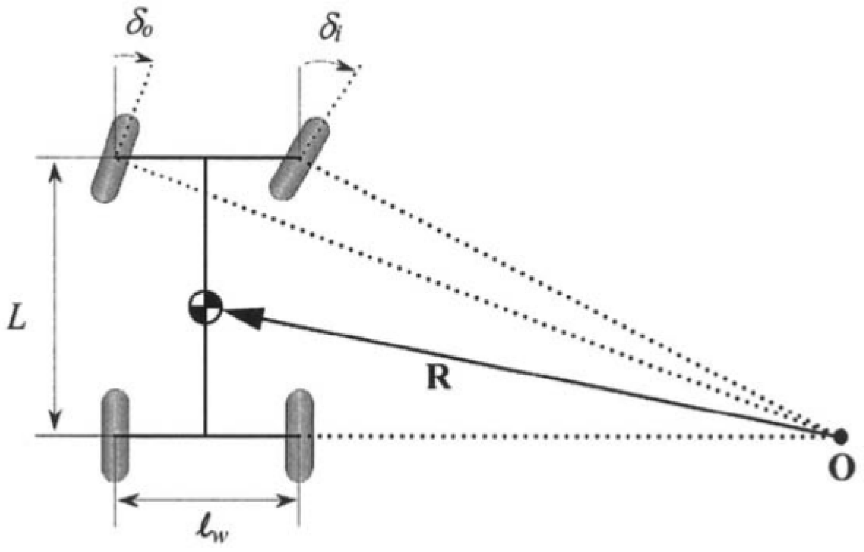
Ackerman turning geometry

Assume the slip angle β is small, the yaw motion can be approximated as

$$\dot{\psi} = r = \frac{V\cos(\beta)}{l_f + l_r}\left(\tan(\delta_f) - \tan(\delta_r)\right)$$

$$L = l_f + l_r$$

- Yaw Motion: $\dot{\psi} = \frac{V}{R} \approx V\frac{\delta}{L}$
- Average Steering: $\delta \approx \frac{L}{R}$
- Outer turning: $\delta_o = \frac{L}{R + l_w/2}$
- Inner turning: $\delta_i = \frac{L}{R - l_w/2}$



Ackerman turning geometry

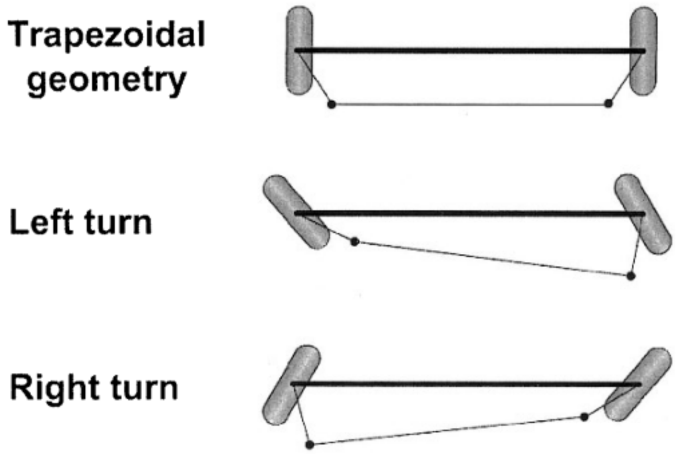
Limitation of "bicycle" model

Assume left and right steering angles are same. But actually no.

- Outer turning: $\delta_o = \frac{L}{R+l_w/2}$
- Inner turning: $\delta_i = \frac{L}{R-l_w/2}$
- The average steer: $\delta = (\delta_i + \delta_o)/2$
- Difference: $\delta_i - \delta_o = \frac{Ll_w}{R^2-l_w^2/4}$

$\because$ l_w^2 is much smaller than R^2

$\therefore \delta_i - \delta_o \approx \frac{L \cdot \ell_w}{R^2} = \delta^2 \frac{\ell_w}{L}$



Differential steer from a trapezoidal tie-rod arrangement

*The difference in the steering angles of the two front wheels is proportional to the **square** of the average steering angle.*

Scenarios to use kinematic bicycle model

Due to the assumption we made for deriving the kinematic bicycle model, it is more suitable for the following situation (like parking in slow speed):

- Slow driving
- Moderate turn

Break down scenarios:

- High speed
- Very tight turn

Modern Control Theory Background

Stability (SISO) based on Transfer Function

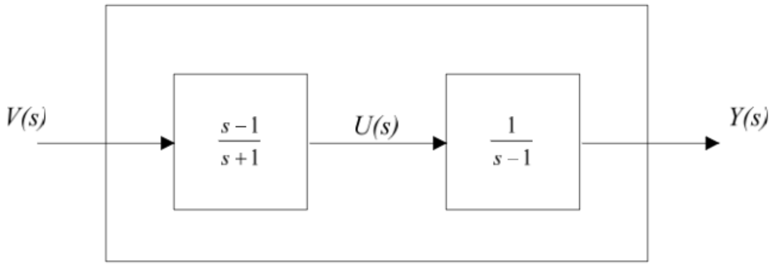
A SISO system with proper rational transfer function G(s) is BIBO stable if and only if all the poles of G(s) are in the open left-half s-plane or, equivalently, all the poles of G(s) have negative real parts.

Proof: If G(s) is a proper transfer function, it can be expressed as

$$G(s) = \gamma + \sum \frac{\beta_{ik}}{(s-\lambda_i)^k}$$

This means that the impulse response $g(t)$ is a sum of finite number of terms $t^{k-1}e^{\lambda_i t}$ and possibly the δ function (corresponding to the inverse Laplace of a constant.) Since terms like $t^{k-1}e^{\lambda_i t}$ is absolutely integrable (i.e., $\int_0^\infty |g(t)|dt \leq k < \infty$) if and only if λ_i has negative real part. Hence, we conclude that the system is BIBO stable if and only all the poles of $G(s)$ have negative real parts. If the transfer function has at least one pole with zero or positive real part, then the system is not BIBO stable.

The problem with Classical Control



Consider the system:

- Where the plant is unstable given by: $H_f(s) = \frac{1}{s-1}$
- Compensator(controller) is chosen to be: $H_c(s) = \frac{s-1}{s+1}$

The system is stable?

The problem with Classical Control

Convert the system back into ODE first: $\frac{Y(s)}{U(s)} = \frac{1}{s-1} \rightarrow \dot{y} - y = u$ and $\frac{U(s)}{V(s)} = \frac{s-1}{s+1} \rightarrow \dot{u} + u = \dot{v} - v$

Then take Laplace transform without assuming zero initial condition

$$y(t) = e^t y(0) + \frac{1}{2}(e^t - e^{-t})u(0) + \int_0^t \mathrm{e}^{-t} v(t-\tau)\,\mathrm{d}\tau$$

Because of the term e^t, we can see that the system is stable only of the initial condition is 0, which what transfer function assumes.

Conclusion:

- The **internal** behavior of a system is more complicated than is indicated by the external behavior.
- It was the state equation analysis in the examples that first clarified these and related questions.

State space equation

Let's consider the system whose transfer function:

$$\frac{Y(s)}{U(s)} = G(s)$$

This system may be represented in state space by the following equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$$

$$y = \boldsymbol{C}\boldsymbol{x} + Du$$

The Laplace Transform are:

$$sX(s) - x(0) = \boldsymbol{A}X(s) + \boldsymbol{B}U(s)$$

$$Y(s) = \boldsymbol{C}X(s) + DU(s)$$

$$\Rightarrow$$

$$G(s) = \boldsymbol{C}(sI - \boldsymbol{A})^{-1}\boldsymbol{B} + D$$

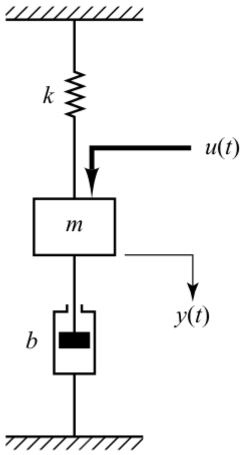
$$G(s) = \frac{Q(s)}{|s\boldsymbol{I} - \boldsymbol{A}|}$$

Notice that is equal to the characteristic polynomial of $G(s)$. In other words, the eigenvalues of $\boldsymbol{A}$ are identical to the poles of $G(s)$.

Linear State Space

Let's look at a classic mass-spring-damper system and apply Newton Second law, we get

$$m\ddot{y} + b\dot{y} + ky = u$$



Rearranging the equation and let $\boldsymbol{x} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

We have:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$$
$$y = \boldsymbol{C}\boldsymbol{x} + Du$$

Where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \qquad \boldsymbol{C} = [1\ 0], \qquad D = 0$$

where $\boldsymbol{A}(t)$ is called the **state matrix**, $\boldsymbol{B}(t)$ the **input matrix**, $\boldsymbol{C}(t)$ the **output matrix**, and $\boldsymbol{D}(t)$ the **direct transmission matrix**

Stability based on State Space Equation

Given that

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u \qquad G(s) = \boldsymbol{C}(sI - \boldsymbol{A})^{-1}\boldsymbol{B} + D$$

$$y = \boldsymbol{C}\boldsymbol{x} + Du \qquad G(s) = \frac{Q(s)}{|s\boldsymbol{I} - \boldsymbol{A}|}$$

If the system is asymptotically stable, **all the eigenvalues of the matrix A have negative real parts**.

We know that the poles of $G(s)$ are eigenvalues of A.

Then asymptotic stability implies BIBO (Bounded-input-bounded-output) stable

Hence, it follows that **all poles of $G(s)$ have negative real parts**. So the system is BIBO stable.

Note that the converse of this theorem is not true in general; i.e., BIBO stability does not always imply asymptotic stability.

Stability based on State Space Equation

e.g. For the following system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [-1\ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

1. Is the system asymptotically stable? Answer: No, since eigenvalues of $\boldsymbol{A}$ are 1 and –2.

2. Is the system BIBO stable? Answer: yes, since the transfer function is

$$G(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} = [-1\ 1] \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s+2}$$

All its poles, i.e. the pole at –2 has negative real part

Once again, it proves the limitation of transfer function

Modern Control vs Classical Control

Classical Control	Modern Control
single-input, single-output system.	multiple-input, multiple-output system
linear	linear or nonlinear
Time invariant	Time invariant or time varying,
a complex frequency-domain approach	essentially time-domain approach and frequency domain approach

In modern control theory, systems are analyzed in state-space, a time domain representation of the system.

What is the state of a system?

State at present: The information needed to predict the future assuming the current and all the future inputs are known.

Controllability and Observability

Controllability and Observability are **unique** features of state-space analysis. These ideas were first introduced by E.G.Gilbert and R.F. Kalman in the early 1960s.

They give a clear explanation as to why cancellation of unstable poles are undesirable even if perfect cancellation is possible. Basically, these 2 concepts tell us that:

- Can we drive x(t) wherever we want using u?
- Can we cause x(t) to follow a given path?
- Can the measurement of y(t) tell us what x(t0) was?
- Can we track x(t) by observing y(t)?

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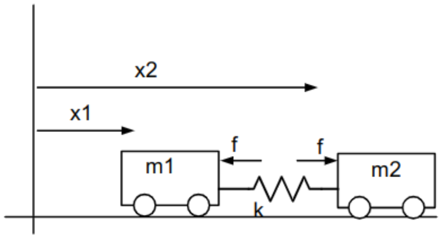
Controllability

Definition : *A Linear Time-invariant system (1) or the pair (A,B) is said to be controllable if there exists an input,* $u(t), 0 \le t \le t_1$ *that drives the system from* ***any*** *initial state* $x(0) = x_0$ *to* ***any*** *other state* $x(t_1) = x_1$ *in a* ***finite*** *time* t_1*. Otherwise, (1) or (A,B) is said to be uncontrollable.*

Theorem: The system is controllable if the controllability matrix has full row rank, where the controllability matrix is defined as

$$\boldsymbol{U} = [\boldsymbol{B}\ \boldsymbol{AB}\ \cdots\ \boldsymbol{A}^{n-1}\boldsymbol{B}]$$



e.g. Let's look at the following system. Assuming there is a force *f* acting on both cars besides the spring, and the cars have initial position x_1 and x_2.

It is well known from the law of physics that we can control the relative position of the 2 cars by controlling the force *f*.

However, can we control x_1 and x_2 Independently?

Controllability

The equations of motion of the system can be shown to be

$$\dot{x}_1 = x_3$$

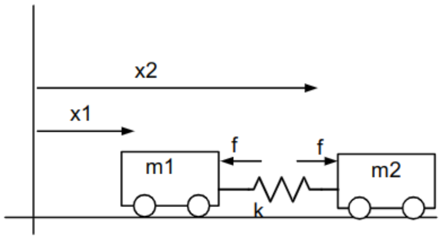
$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = -\frac{k}{m_1}(x_1 - x_2) - \frac{f}{m_1}$$

$$\dot{x}_4 = -\frac{k}{m_2}(x_2 - x_1) + \frac{f}{m_2}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & 0 & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & 0 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{m_1} \\ \frac{1}{m_2} \end{bmatrix}$$



$$\boldsymbol{U} = [\boldsymbol{B}\ \boldsymbol{AB}\ \cdots\ \boldsymbol{A}^{n-1}\boldsymbol{B}]$$

$$U = \begin{bmatrix} 0 & -\frac{1}{m_1} & 0 & \frac{k}{m_1^2} + \frac{k}{m_1 m_2} \\ 0 & \frac{1}{m_2} & 0 & -\frac{k}{m_1 m_2} - \frac{k}{m_2^2} \\ -\frac{1}{m_1} & 0 & \frac{k}{m_1^2} + \frac{k}{m_1 m_2} & 0 \\ \frac{1}{m_2} & 0 & -\frac{k}{m_1 m_2} - \frac{k}{m_2^2} & 0 \end{bmatrix}$$

Clearly it is not a full rank matrix.

Therefore, the system is uncontrollable.

Observability

Definition : A linear time-invariant system is observable if every unknown initial state $x(0)$ can be determined from the knowledge of $u(t)$ and the observation of $y(t)$ over a finite time interval. Otherwise, the LTI system is said to be unobservable.

Theorem: The system is observable if the observability matrix has **full column rank**, where the observability matrix is defined as

$$\boldsymbol{O} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \vdots \\ \boldsymbol{CA}^{n-1} \end{bmatrix}$$

https://link.springer.com/chapter/10.1007/978-94-011-5654-7_9#:~:text=Controllability%20measures%20the%20ability%20of,the%20states%20of%20the%20system.

Observability

Consider this simple system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 0 \\ y = x_2 \end{cases}$$

The above shows that $y(t)$ will always be a constant. Hence, observing $y(t)$ does not tell us what x_1 is doing.

Represent the system in state space equation, we have:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The observability matrix is:

$$O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

which is not a full column rank, and therefore the system is unobservable.

Controllability and Observability

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- Can we drive x(t) wherever we want using u?
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- Can we track x(t) by observing y(t)?

Lateral control based on Vehicle Geometric Model

Pure Pursuit

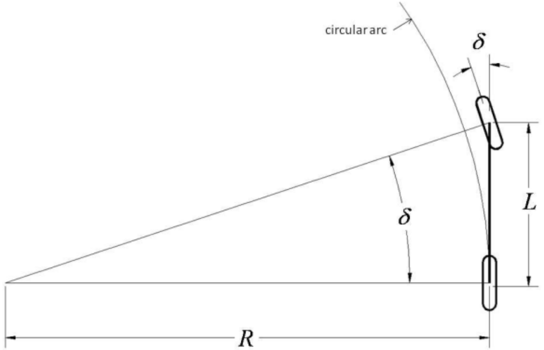
Pure Pursuit

A common simplification of an Ackerman steered vehicle used for geometric path tracking is the bicycle model.

$$\tan\delta = \frac{L}{R}$$

where

- δ is the steering angle of the front wheel,
- L is the distance between the front axle and rear axle (wheelbase),
- R is the radius of the circle that the rear axle will travel along at the given steering angle.



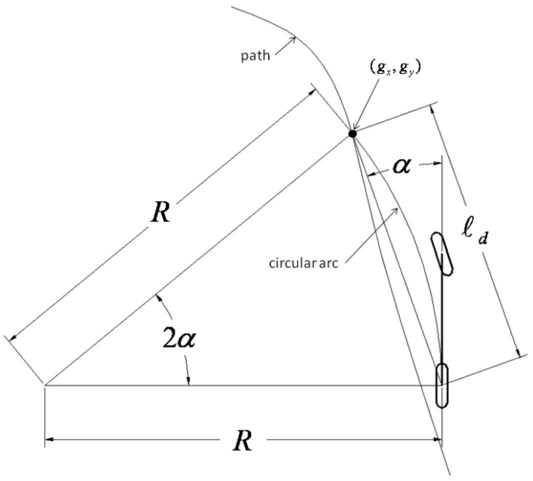
Geometric Bicycle Model

This model approximates the motion of a car reasonably well *at low speeds and moderate steering angles*.

Pure Pursuit

The pure pursuit method:

- The curvature of a circular arc that connects the rear axle location to a goal point on the path ahead of the vehicle.
- The goal point is determined from a look-ahead distance ℓ_d from the current rear axle position to the desired path.



Pure Pursuit geometry

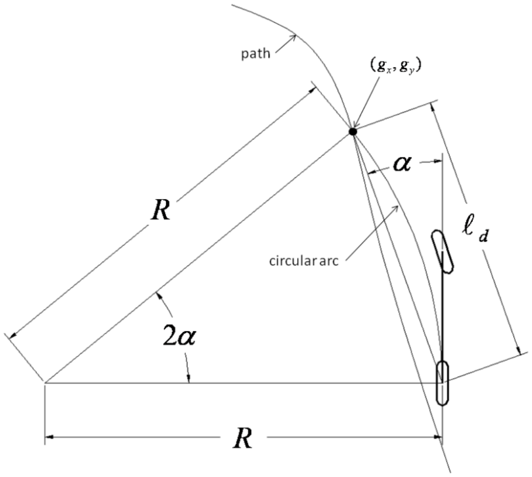
Pure Pursuit

The vehicle's steering angle δ can be determined using only the goal point location and the angle α between the vehicle's heading vector and the look-ahead vector.

$$\frac{\ell_d}{sin(2\alpha)} = \frac{R}{sin\left(\frac{\pi}{2} - \alpha\right)} \Rightarrow \frac{\ell_d}{2sin(\alpha)cos(\alpha)} = \frac{R}{cos(\alpha)} \Rightarrow \frac{\ell_d}{sin(\alpha)} = 2R$$

$$\left.\begin{aligned} \frac{1}{R} &= \frac{2sin(\alpha)}{\ell_d} \\ \tan\delta = \frac{L}{R} &\Rightarrow \delta = tan^{-1}\left(\frac{L}{R}\right) \end{aligned}\right\} \Rightarrow \delta(t) = tan^{-1}\left(\frac{2Lsin\big(\alpha(t)\big)}{\ell_d}\right)$$

$$sin(\alpha) = \frac{e_{\ell_d}}{\ell_d} \Rightarrow \delta(t) = tan^{-1}\left(\frac{2L}{\ell_d^2} e_{\ell_d}(t)\right)$$



Pure Pursuit geometry

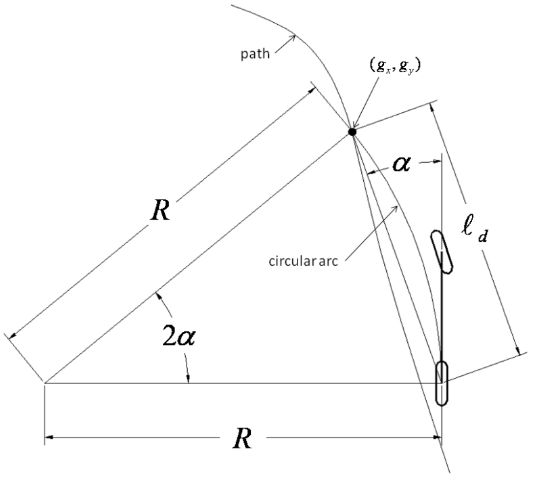
Pure pursuit is a **proportional** controller of the steering angle operating on a cross track error.

some look-ahead distance in front of the vehicle and having a gain of $\frac{2L}{\ell_d^2}$.

Pure Pursuit

Basic procedures:

- Determine the current location of the vehicle;
- Find the path point closest to the vehicle;
- Find the goal point G;
- Transform the goal point to the vehicle coordinate;
- Calculate the curvature and request the vehicle to set the steering to that curvature $\delta(t) = tan^{-1}\left(\frac{2L}{\ell_d^2} e_{\ell_d}(t)\right)$;
- Update the vehicle's position.



Pure Pursuit geometry

Pure Pursuit Tuning

Most common way:

- Scaling the look-ahead distance (l_d) with speed;
- With set up a range: [min, max];

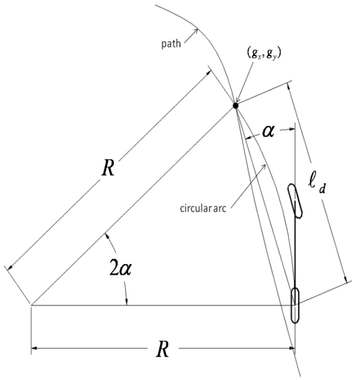
$$\delta(t) = \tan^{-1}\left(\frac{2L\sin\big(\alpha(t)\big)}{l_d}\right)$$

$$= \tan^{-1}\left(\frac{2L\sin\big(\alpha(t)\big)}{kv_x(t)}\right)$$

Characteristic:

1. A short look-ahead distance provides more accurate tracking while a longer distance provides smoother tracking.
2. k value that is too small will cause instability and a k value that is too large will cause poor tracking.
3. High level of robustness: e.g. good handling on the discontinuity in the path.

Challenges of Pure Pursuit



Pure Pursuit geometry

- How to pick the best look-ahead distance. Varying the look-ahead distance with speed is a common approach, but it makes sense that the look-ahead distance could be a function of path curvature and maybe even cross track error in addition to longitudinal velocity.
- Care should be taken to prevent over tuning Pure Pursuit to a specific course since changing the look-ahead distance simply changes the radius of curvature that the vehicle will travel and therefore can compensate for the increased (compared to the kinematic bicycle model prediction) radius that results from the understeer gradient of the vehicle. If tuned to insure stability, performance can be greatly reduced by cutting corners on the path due to a longer look-ahead distance.
- Steady state error in curves also becomes a problem as speed increases.

Stanley Controller

Stanley Method

The Stanley method is the path tracking approach used by Stanford University's autonomous vehicle entry in the DARPA Grand Challenge, Stanley.

- Uses the center of the front axle as a reference point
- Look at both the error in heading and the error in position relative to the closet point on the path
- Define an intuitive steering law to
    - Correct heading error
    - Correct position error
    - Obey max steering angle bounds

Stanley Method

Essentially, it is combined three requirements:

- Steer to align heading with desired heading (proportional to heading error): θ_e

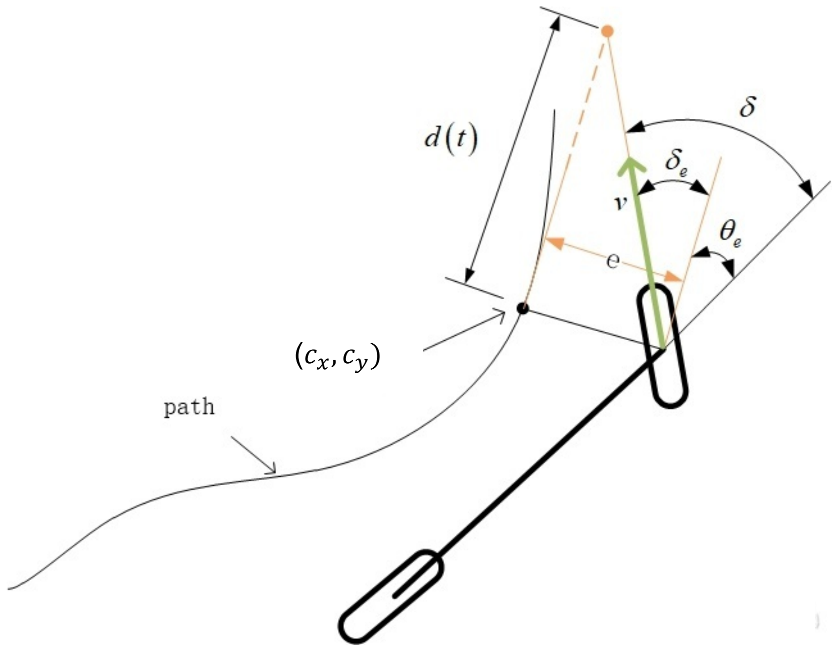
$$\delta(t) = \theta_e(t)$$

- Steer to eliminate cross track error: e
  - Essentially proportional to error
  - Inversely proportional to speed
  - Limit effect for large error with the inverse tan
  - Gain k determined experimentally

$$\delta(t) = \delta_e(t) = tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right)$$

- Max and min steering angles:

$$\delta(t) \in [\delta_{min}, \delta_{max}]$$



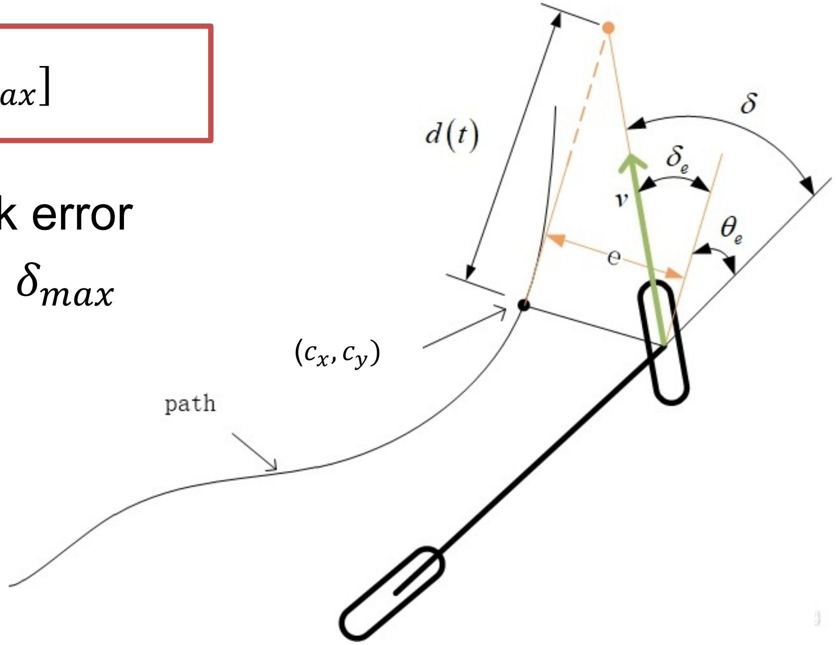
Stanley method geometry

Stanley Method

- Stanley control law

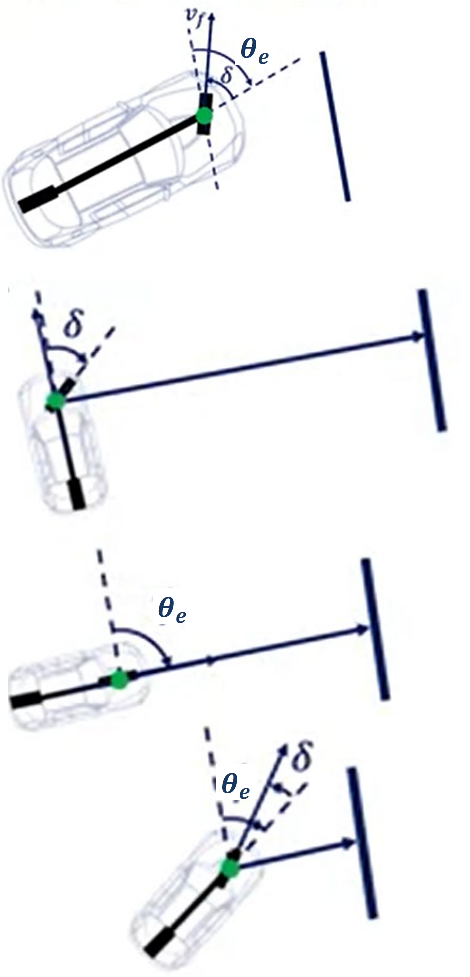
$$\boxed{\delta(t) = \theta_e(t) + tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right), \delta(t) \in [\delta_{min}, \delta_{max}]}$$

- If there is no heading error, large cross track error case. Steering angle will be capped by both δ_{max} and $tan^{-1}(t) \in (-\frac{\pi}{2}, \frac{\pi}{2})$



Stanley Method

- For large heading error, steer in opposite direction
  - The larger the heading error, the larger the steering correction
  - Fixed at limit beyond maximum steering angle, assuming no cross track error
- For large positive cross track error
  - $tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right) \approx \frac{\pi}{2} \rightarrow \delta(t) \approx \theta_e(t) + \frac{\pi}{2}$
  - As heading changes due to the steering angle, the heading correction counteracts the cross track correction, and drives the steering angle back to zero
  - The vehicle approaches the path, cross track error drops, and steering command stars to correct heading alignment.



Stanley controller analysis

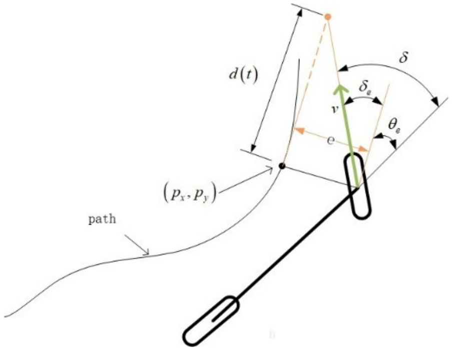
The error dynamics when not at max steering angle are:

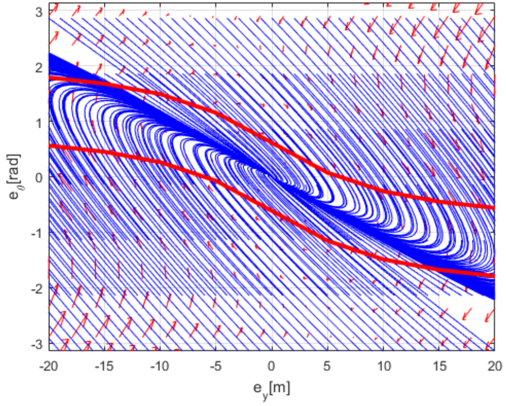
$$\dot{e}(t) = -v_f(t)\sin(\delta_e) = -v_f(t)\sin(\tan^{-1}\frac{ke(t)}{v_f(t)})$$

$$= \frac{-ke(t)}{\sqrt{(1+\left(\frac{ke(t)}{v_f}\right)^2)}}$$

For the small cross track error, leads to exponential decay characteristics

$$\dot{e}(t) \approx -ke(t)$$





Phase portrait of the system

Stanley Method Adjustment

1. Low speed operation
    - Inverse speed can cause numerical instability
    - Add positive softening constant to controller

$$\delta(t) = \theta_e(t) + tan^{-1}\left(\frac{ke(t)}{k_s + v_{\mathrm{f}}(t)}\right)$$

2. Extra damping on heading
    - Becomes an issue at higher speed in real vehicle
3. Steer into constant radius curves
    - Improves tracking on curves by adding a feedforward term on heading

Stanley Method Summary

- the Stanley method is more intuitive to tune when compared to Pure Pursuit, but it suffers from similar pitfalls when tuning.
- The Stanley tracker can be over-tuned to a specific course in a similar manner because the only way it can overcome dynamic effects is with a high gain that may lead to instability on other paths.
- In contrast to Pure Pursuit, a well-tuned Stanley tracker will not ”cut corners” but rather overshoot turns. This effect can be attributed to not having a look-ahead.
- Similar to the Pure Pursuit method, steady state errors in curves at moderate speeds become significant.

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感谢聆听！

Thanks for Listening