Pathogen-Macrophage Model

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1 Introduction

This document presents the current Pathogen Macrophage model, which describes how macrophages and pathogens interact with each other.

2 Model Assumptions

- **Logistic Growth for Pathogens:** The pathogen population grows logistically, meaning that its growth rate slows as the population approaches a maximum carrying capacity.

- **Abiotic Growth for Macrophages:** Macrophages are generated at a constant rate, regardless of the pathogen population.

- **One-to-One Killing:** It is assumed that each interaction results in one macrophage killing one pathogen, and thus one macrophage dying in the process.

- **Interaction Term:** The interaction between macrophages and pathogens is quantified by the term $\mu\,M\,P$, where μ is a constant representing the hunting rate at which macrophages hunt and kill pathogens, and $\mu\,M\,P$ represents that the macrophages hunt pathogens at a rate proportional to each others population.

3 Mathematical Model

$$\frac{dM}{dt} = \delta - \sigma\,M - \mu\,M\,P, \tag{1}$$

$$\frac{dP}{dt} = \alpha\,P\left(1 - \frac{P}{\beta}\right) - \mu\,M\,P, \tag{2}$$

4 Parameter Definitions

The model includes five parameters, defined as follows:

δ The constant rate at which macrophages are generated

σ The natural death rate of macrophages.

α The growth rate of pathogens.

β The carrying capacity of pathogens.

μ The rate at which macrophages hunt pathogens