# Three-Equation Model

James Lynch

March 2025

## 1 Coefficients Explained

$\phi$ The growth rate of specialist macrophages

$\eta$ The pathogen level at which the specialist macrophage production rate is half of its maximum

$\psi$ The death rate of specialist macrophages

$\delta$ The growth rate of generalist macrophages

$\sigma$ The death rate of generalist macrophages

$\mu$ The rate at which macrophages hunt and destroy pathogens (and themselves in the process)

$\alpha$ The intrinsic growth rate of the pathogen

$\beta$ The carrying capacity of the pathogen population

$\kappa$ The rate at which specialist macrophages hunt and destroy pathogens

## 2 Equations

**Specialist Equation**

$$\frac{dS}{dT} = \frac{\phi P}{\eta + P} - \psi S$$

**Generalist Equation**

$$\frac{dG}{dT} = \delta - \sigma G - \mu G P$$

**Pathogen Equation**

$$\frac{dP}{dT} = \alpha P\Big(1 - \frac{P}{\beta}\Big) \; - \; \mu G P \; - \; \kappa S P$$

# 3 Scaling Choices

**1** $\frac{t}{a} = \mathbf{T}$

**2** $\mathbf{P} = \mathbf{p} \cdot \beta$

**3** $\mathbf{G} = \mathbf{g}\ \frac{\delta}{\sigma}$

**4** $\mathbf{S} = \mathbf{s}\ \frac{\phi}{\psi}$

# 4 Scaled Equations Equations

**Specialist Equation**

$$\frac{ds}{dt} = \frac{\psi}{\alpha}(\frac{p}{\frac{\eta}{\beta} + p} - s)$$

**Generalist Equation**

$$\frac{dg}{dt} = \frac{\sigma}{\alpha} - \frac{\sigma}{\alpha}g - \frac{\mu\beta}{\alpha}pg$$

**Pathogen Equation**

$$\frac{dp}{dt} = p(1-p) - \frac{\mu\delta}{\sigma\alpha}pg - \frac{\kappa\phi}{\psi\alpha}ps$$

# 5 variable choices

**a** $\mathrm{a} = \frac{\psi}{\alpha}$

**e** $\mathrm{h} = \frac{\eta}{\beta}$

**b** $\mathrm{b} = \frac{\sigma}{\alpha}$

**m** $\mathrm{m} = \frac{\mu\beta}{\alpha}$

**r** $\mathrm{r} = \frac{\mu\delta}{\alpha\sigma}$

**k** $\mathrm{k} = \frac{\kappa\phi}{\alpha\psi}$

# 6 Final Equations

**Specialist Equation**

$$\frac{ds}{dt} = a(\frac{p}{h+p} - s)$$

**Generalist Equation**

$$\frac{dg}{dt} = b(1 - g - mpg)$$

**Pathogen Equation**

$$\frac{dp}{dt} = p(1 - p) - rpg - kps$$