

CS 11 Quiz #4

W15 $x \equiv_m y, y \equiv_m z \Rightarrow x \equiv_m z$

proof. Suppose $x \equiv_m y$ and $y \equiv_m z$. This implies $m|(y-x)$ and $m|(z-y)$
 then $y-x=ma$ for some integer a
 and $z-y=mb$ for some integer b

But then, $z-x = z-y + y-x$

$$= ma + mb$$

$$= m(a+b)$$

Since integers are closed under addition, $a+b \in \mathbb{Z}$

So $m|(z-x)$, which implies $x \equiv_m z$. Therefore

\equiv_m is transitive. ~~■~~

2 Write down every equivalence relation on set $\{a, b, c\}$

Relation R on set S is equivalence relation if R is reflexive, symmetric and transitive for set $S = \{a, b, c\}$

$\{a\} \{b\} \{c\}$ aRa, bRb, cRc

$\{a, b\} \{c\}$ aRb, aRa, cRc, bRb, bRa

$\{b, c\} \{a\}$ bRb, bRc, aRa, cRc, cRb

$\{a, c\} \{b\}$ aRa, aRc, cRa, bRb, cRc

$\{a, b, c\}$ aRa, aRb, aRc
 bRa, bRb, bRc
 cRa, cRb, cRc