Why PageRank Converges ?

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The PageRank equation is:

$$\vec{P}^{t+1} = \lambda \vec{A} \vec{P}^{t} + \mu \vec{Q} \tag{1}$$

where $0 \leq \lambda, \mu \leq 1, \vec{A}$ is a transition matrix with each column summing to 1 and $\vec{Q}$ can be any vector with the same dimension of $\vec{P}$.

Why PageRank (1) converges?

Prove([1]):

Suppose $\vec{P}^0 = \pi$, we have $\vec{P}^n = (\lambda\vec{A})^n\pi + \sum_{k=0}^{n-1}(\lambda\vec{A})^k\mu\vec{Q}$. Since $0 \leq \lambda, \mu \leq 1$ and the eigenvalues of the transition matrix $\vec{A}$ are in $[-1, 1]$, we have $\lim\limits_{n\to\infty}(\lambda\vec{A})^n = \vec{0}$ and $\lim\limits_{n\to\infty}\sum_{k=0}^{n-1}(\lambda\vec{A})^k = (\vec{I} - \lambda\vec{A})^{-1}$. So $\vec{P}^n$ finally converges to $\vec{P}^* = (\vec{I} - \lambda\vec{A})^{-1}\mu\vec{Q}$. So the value of $\vec{P}^*$ is only relavant to $\vec{A}$, $\vec{Q}$, λ and μ and the convergence is irrelevant to $\vec{Q}$ •

References

[1] Wei Feng, Jianyong Wang. *Incorporating Heterogeneous Information for Personalized Tag Recommendation in Social Tagging Systems.* KDD, 2012