

## CIS 600-Assignment 3

Show your work for all answers.

1. 50 pts. From Tan et al. text Exercise 5.2. Association rules  
Consider the table below.

	Customer ID	Transaction ID	Items Bought
1	1	0001	{a, d, c}
2	1	0024	{a, b, c, e}
3	2	0012	{a, b, d, e} ✓ ✓
4	2	0031	{a, c, d, e}
5	3	0015	{b, c, e}
6	3	0022	{b, d, e} ✓ ✓
7	4	0029	{c, d}
8	4	0040	{a, b, c}
9	5	0033	{a, d, e}
10	5	0038	{a, b, e}

$$\text{support}(\text{itemset}) = \frac{\# \text{ of transactions containing set}}{\text{total \# of transactions}}$$

- (a) Compute the support for items {c}, {b, d}, and {b,d,e} by treating each transaction ID as a market basket.

$$\begin{aligned} \{c\} &\rightarrow 6 \rightarrow 6/10 = .6 \rightarrow 60\% \\ \{b, d\} &\rightarrow 2 \rightarrow 2/10 = .2 \rightarrow 20\% \\ \{b, d, e\} &\rightarrow 2 \rightarrow 2/10 = .2 \rightarrow 20\% \end{aligned}$$

- (b) Use the results in (a) to compute the confidence for the association rules {b, d} → {e} and {c} → {b,d}. Is confidence a symmetric measure?

$$\begin{aligned} \text{Confidence}(\{A\} \rightarrow \{B\}) &= \text{support}(\{A, B\}) / \text{support}(\{A\}) \\ \text{support}(\{b, d, e\}) &= .2, \text{ support}(\{b, d\}) = .2 \rightarrow \text{Confidence}(\{b, d\} \rightarrow \{e\}) = .2 / .2 = 1 \\ \text{support}(\{b, c, d\}) &= 0, \text{ support}(\{c\}) = .6 \rightarrow \text{Confidence}(\{c\} \rightarrow \{b, d\}) = 0 / .6 = 0 \end{aligned}$$

- (c) Repeat (a) by treating each customer ID as a market basket. Each item should be treated as a binary variable (1 if an item appears in at least one transaction bought by the customer, and 0 otherwise).

$$\text{Market Basket: } \text{support}(\text{itemset}) = \frac{\# \text{ of Customers buying all items in itemset}}{\text{Total \# of customers}}$$

$$\begin{aligned} \text{support}(\{c\}) &= \frac{4}{5} = .8 \rightarrow 80\% \\ \text{support}(\{b, d\}) &= \frac{2}{5} = .4 \rightarrow 40\% \\ \text{support}(\{b, d, e\}) &= \frac{2}{5} = .4 \rightarrow 40\% \end{aligned}$$

- (d) Use the results of (c) to compute the confidence of the association rules  $\{b, d\} \rightarrow \{e\}$  and  $\{e\} \rightarrow \{b, d\}$

$$\{b, d\} \rightarrow \{e\} : .4 / .4 \rightarrow 1$$

$$\{e\} \rightarrow \{b, d\} : .4 / .8 = .5$$

- (e) Suppose  $s_1$  and  $c_1$  are the support and confidence values of an association rule  $r$  when treating each transaction ID as a market basket. Also,  $s_2$  and  $c_2$  are the support and confidence values of an association rule  $r$  when treating each customer ID as a market basket. Discuss whether there are any relationships between  $s_1$  and  $s_2$  or  $c_1$  and  $c_2$ .

There is generally no relationship or correlation between  $s_1$  and  $s_2$  or  $c_1$  and  $c_2$ , the relationship depends on distribution of items among transactions and customers. Comparison of these values may yield insight into customer buying patterns. High  $s_1$  compared to  $s_2$  suggests customers buy items in an itemset across multiple transactions.

2. From Tan et al. text Exercise 5.6. Association rules

Consider the table below.

Transaction ID	Items Bought
1	{Milk, Beer, Diapers}
2	{Bread, Butter, Milk}
3	{Milk, Diapers, Cookies}
4	{Bread, Butter, Cookies}
5	{Beer, Cookies, Diapers}
6	{Milk, Diapers, Bread, Butter}
7	{Bread, Butter, Diapers}
8	{Beer, Diapers}
9	{Milk, Diapers, Bread, Butter}
10	{Beer, Cookies}

- (a) What is the maximum number of associations rules that can be extracted from this data (including rules that have zero support)?

$n = 6$  unique items: Milk, beer, diapers, bread, butter, cookies

$$3^n - 2^n \rightarrow 3^6 - 2^6 = 665$$

- (b) What is the maximum size of frequent itemsets that can be extracted (assuming minsup > 0)?

• size of longest transaction  
 $\rightarrow 4$

- (c) Write an expression for the maximum number of size itemsets that be derived from this data set.

$n$  choose  $k \rightarrow n C k$

$n = \text{unique items}$   
 $k = \text{size of itemset}$

$$6 C k$$

(d) Find an itemset (of size 2 or larger) that has the largest support.

- Bread and Butter appear in more transactions together than any other size 2 or larger item set.

(e) Find a pair of items, a and b, such that the rules  $\{a\} \rightarrow \{b\}$  and  $\{b\} \rightarrow \{a\}$  have the same confidence.

$\{Milk, Diapers\}, \{Bread, Butter\}, \{Beer, Diapers\} \rightarrow 4$

$\{Milk, Bread\}, \{Milk, Butter\}, \{Diapers, Bread\}, \{Diapers, Butter\} \rightarrow 3$

$Diapers \rightarrow 7$

$Milk \rightarrow 5$

$Bread \rightarrow 5$

$Butter \rightarrow 4$

$Beer \rightarrow 3$

$Cookies \rightarrow 3$

$\Rightarrow$  None have the same confidence

$\{A\} \rightarrow \{B\}$  vs  $\{B\} \rightarrow \{A\}$

2. 50 pts. From tan et al text Exercise 3.12. Learning objective is to show understanding of classifier performance analysis.

Consider a labeled data set containing 100 data instances, which is randomly partitioned into two sets A and B, each containing 50 instances. We use A as the training set to learn two decision trees,  $T_{10}$  with 10 leaf nodes and  $T_{100}$  with 100 leaf nodes. The accuracies of the two decision trees on data sets A and B are shown in the table below.

Data Sets	Accuracy	
	$T_{10}$	$T_{100}$
A	0.86	0.97
B	0.84	0.77

Based on the accuracies shown above, which classification model you expect to have better performance on unseen instances?

$T_{10}$  will do better against unseen instances, given it performs approximately the same with the B data set as with A, the training data.  $T_{100}$ 's accuracy drops too much with the testing data, suggesting overfitting to the training set.

Now, you have tested  $T_{10}$  and  $T_{100}$  on the entire data set (A + B) and found the classification accuracy of  $T_{10}$  on the entire set (A + B) is 0.85, whereas the classification accuracy of  $T_{100}$  on the data set (A + B) is 0.87. Based on this new information and your observations from the table, which classification model would you finally choose for classification?

$T_{10}$  is still the better choice; with the combined data it is clear that the accuracies are comparable. It is important to note that A+B is held seen by both models,  $T_{10}$  still performs much better with unseen data.