Appendix A1. Formulation

April 4, 2021

0.1 Summary

Data Preparation Below is a representation of graduate timeslots to facilitate understanding.

0.1.1 Input data:

- I: the set of classes.
  - a: the set of double-unit class (i.e. 3 units for graduate and 4 units for undergraduate classes)
  - b: the set of single-unit class (i.e. 1.5 units for graduate and 2 units for undergraduate classes) taken in full semester
  - c: the set of single-unit class taken in the first half of semester
  - d: the set of single-unit class taken in the second half of semester
- J: the set of classrooms
- T: the set of timeslots. Below are 7 kinds of timeslots:
  - A: full semester for 1.5/2 hours twice a week
  - B: full semester for 3/4 hours once a week
  - C: full semester for 1.5/2 hours once a week
  - D: first half for 1.5/2 hours twice a week
  - E: second half for 1.5/2 hours twice a week
  - F: first half for 3/4 hours once a week
  - G: second half for 3/4 hours once a week
- S: the set of days of week, including Monday, Tuesday, Wednesday, Thursday, Friday
- V_s: the set of timeslots on day of week s. eg. V_M indicates the timeslots on Monday (includ. M and MW).
- K: the set of professors
- L_k: the set of classes that professor k teaches
- M: the set of back-to-back/consecutive timeslots. Timeslots are defined as back-to-back if the interval between two timeslots is less than 30 minutes
- N: the number of total classes to be allocated
- U_{ij}: the space utilization rate for allocating class i to classroom j, calculated by seats offered over capacity
- z_{ij}: whether the seats offered for class i is less than or equal to the capcaity of classroom j
- O: the set of conficting timeslot pairs
- W_U, W_Q, W_R: the weights assigned on U,Q,R respectively (by default, they are 1,1,0.2 respectively)

Decision variables: Let X_{ijt} denote whether class i is allocated to classroom j at timeslot t(binary) ($i \in I$, $j \in J$, $t \in T$)

Auxillary Variable: - y_{jt}: the total number of classes allocated to classroom j at time t (integer) - w_{kt}: the total number of classes allocated to professor k at time t (integer) - U: the average classroom utilization rate (in %) of the schedule (contineous) - R: the number of professors who are allocated at least one back-to-back class (integer) - Q: the number of professors who have to work more than two days a week (integer) - $H_{kt_1t_2}$: whether professor k teaches back to back classes at consecutive timeslots $(t_1, t_2) \in M$ (binary) - r_k: whether professor k is allocated at least one back-to-back class (binary) - Z_{ks}: whether professor k is allocated any class on day of week s (binary) - q_k: whether professor k has to work more than two days a week (binary)

Unless otherwise noted, every summation of i is over I, of j is over J and of t is over T.

Objective Maximize the schduling score

$$\text{Maximize} \qquad W_U * U - W_Q * Q + W_R * R$$

$$U = \frac{1}{N} \sum_{i,j,t} U_{ij} X_{ijt} \qquad \text{average utilization rate}$$

- U_{ij}: the space utilization rate for allocating class i to classroom j, calculated by seats offered over capacity - N: the number of total classes to be allocated - R and Q will be defined in constraint 5 and 6

Constraint 1

English description: Each class eventually only occupies one classroom and one timeslot. More specifically, if it's a double-unit class, then it takes one of the timeslot in A/B. If it's a single-unit class taught throughtout the full semester, then it takes one of the timeslots in C. If it's a single-unit taught in the first half, then it takes one of the timeslots in D/F. If it's a single-unit class taught in the second half, then it takes one of the timeslots in E/G

$$\begin{aligned}
\sum_{t,j} X_{ijt} &= 1 \quad \text{For every class } i \in I \\
\sum_{t \in A \cup B} \sum_{j} X_{ijt} &= 1 \quad \text{For every double-unit class } i \in a. \\
\sum_{t \in C} \sum_{j} X_{ijt} &= 1 \quad \text{For every single-unit class } i \in b \text{ taught throughtout the full semester.} \\
\sum_{t \in D \cup F} \sum_{j} X_{ijt} &= 1 \quad \text{For every single-unit class } i \in c \text{ taught in the first half.} \\
\sum_{t \in E \cup G} \sum_{j} X_{ijt} &= 1 \quad \text{For every single-unit class } i \in d \text{ taught in the second half .}
\end{aligned}$$

Constraint 2

English description: Each classroom can not be occupied by more than one class at the same time.

$$
\begin{aligned}
\sum_i X_{ijt} &\leq 1 && \text{for each room } j\text{, timeslot } t. \\
y_{jt_1} + y_{jt_2} &\leq 1 && \text{For each classroom } j \text{ and conflicting time slots } (t_1, t_2) \in O. \\
y_{jt} &= \sum_i X_{ijt} && \text{For each classroom } j \text{ and time slot } t.
\end{aligned}
$$

- y_{jt}: the total number of classes allocated to classroom j at time t (integer)

Constraint 3

English description: Each class can not be allocated to any classrooms that fail to accommodate the number of seats offered by the class.

$$\sum_t X_{ijt} \leq z_{ij} \qquad \text{for each class } i\text{, classroom } j$$

- z_{ij}: whether the seats offered for class i is less than or equal to the capcaity of classroom j

Constraint 4

English description: Each professor can not be allocated to more than one classroom or class at the same time.

Breaking down the above:

- Each professor can't be assigned to a certain timeslot in more than one classroom or class

$$\sum_{i \in L_k} \sum_{j \in J} X_{ijt} \leq 1 \quad \text{for each professor } k\text{, timeslot } t$$

- As the timeslots are designed in a way that thay can overlap. Another constraint is added to make sure the classes taught by a certain professor won't have time conflicts. For example, M/W 8-9:30am first half semester and M 8-11am full semester overlaps. In this case, we say that slot t_1 and t_2 conflicts with each other. The set of conflicting timeslots is O.

$$
\begin{aligned}
w_{kt_1} + w_{kt_2} &\leq 1 && \text{For each professor } k \text{ and conflicting time slots } (t_1, t_2) \in O. \\
w_{kt} &= \sum_{i \in L_k} \sum_j X_{ijt} && \text{For each professor } k \text{ and time slot } t.
\end{aligned}
$$

- w_{kt}: the total number of classes allocated to professor k at time t (integer)
- L_k: the set of classes that professor k teaches

Constraint 5

English description: Make the number of professors who have to work more than 2 days a week as small as possible.

Break it down:

Timeslots like MW 8:00 - 9:30 AM and M 9:30 - 11:00 AM are consecutive, let's say the set of consecutive timeslots (t_1, t_2) is M.

1. For each professor, we check whether there are consecutive slots occupied by the same professor's class. The middle part of formula is the total number of classes professor k has to teach at two consecutive timeslots t_1 and t_2 plus 1.

For easy reference, let's call this number a_1. a_1 takes 3 different values: 1,2,3. - If $a_1 \in [1,2]$, i.e. the professor doesn't teach back-to-back classes at consecutive timeslots t_1 and t_2, then $H_{kt_1t_2}$ must be 0 to satisfy the LHS. - If $a_1 = 3$, i.e. the professor do teach consecutive classes at t_1 and t_2, then $H_{kt_1t_2}$ must be 1 to satisfy the RHS.

That is, $H_{kt_1t_2}$ is a binary auxillary variable indicating whether the professor teaches back-to-back classes at consecutive timeslots t_1 and t_2. N is the total number of classes to be allocated by the model, which is a sufficiently large upper bound.

$$3H_{kt_1t_2} \leq \sum_{i \in L_k} \sum_{j} X_{ijt_1} + \sum_{i \in L_k} \sum_{j} X_{ijt_2} + 1 \leq H_{kt_1t_2} N + 2 \quad \text{for each prof } k \text{ and consecutive time slots } (t_1, t_2) \in M$$

- M: the set of back-to-back/consecutive timeslots. Timeslots are defined as back-to-back if the interval between two timeslots is less than 30 minutes
- N: the number of total classes to be allocated
- $H_{kt_1t_2}$: whether professor k teaches back to back classes at consecutive timeslots $(t_1, t_2) \in M$ (binary)

2. Then, for each professor k, sum over all consecutive timeslots $(t_1, t_2) \in M$ to check whether he/she is allocated at least one back-to-back class. Let's say it is represented by a binary auxillary variable r_k

$$r_k \leq \sum_{(t_1,t_2) \in M} H_{kt_1t_2} \leq N r_k \quad \text{for each prof } k \in K$$

- r_k: whether professor k is allocated at least one back-to-back class (binary)

3. Sum over all professors to get the total number of professor who are allocated at least one back-to-back class

$$R = \sum_{k \in K} r_k$$

- R: the number of professors who are allocated at least one back-to-back class (integer)

Constraint 6

English description: Make the number of professors who have to work more than 2 days a week as small as possible.

Break it down:

1. For each professor k, check whether any class is allocated for him/her on day of week s. Let's say it is represesetned by a binary auxillary variable Z_{ks}.

The middle part of the fomula is the number of courses professor k teaches on day of week s. For easy reference, let's call this number a_2, if $a_2 = 0$, then Z_{ks} must be 0 to satisfy the LHS. If $a_2 > 0$, then Z_{ks} must be 1 to satisfy the RHS. N is the total number of classes to be allocated by the model, which is larger than any possible number of classes a professor would teach on a single day.

$$Z_{ks} \leq \sum_{i \in L_k} \sum_{j} \sum_{t \in V_s} X_{ijt} \leq Z_{ks} N \quad \text{for each prof } k \in K \text{ and each day of week } s \in S$$

- S: the set of days of week, including Monday, Tuesday, Wednesday, Thursday, Friday
- V_s: the set of timeslots on day of week s. eg. V_M indicates the timeslots on Monday (includ. M and MW).
- Z_{ks}: whether professor k is allocated any class on day of week s (binary)
- N: the number of total classes to be allocated

2. Then, for each professor k, check whether he/she has to work more than two days a week. Let's say it is represented by a binary auxillary variable q_k.

The middle part sums Z_{ks} over all days of week $s \in S$ to get how many days professor k has to work per week. For easy reference, let's call this number a_3. If $a_3 \in [0, 1, 2]$, then q_k must be 0 to satisfy the LHS. If $a_3 \geq 3$, then q_k must be 1 to satisfy the RHS.

$$3q_k \leq \sum_{s \in S} Z_{ks} \leq N q_k + 2 \quad \text{for each prof } k \in K$$

- q_k: whether professor k has to work more than two days a week (binary)

3. Finally, sum over all professors to get the number of professors who have to work more than 2 days a week,Q

$$Q = \sum_{k \in K} q_k$$

- Q: the number of professors who have to work more than two days a week (integer)