Module 1 Supplementary Materials

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1 Distribution of R_{DUT} and C_{DUT}

In this section, the figures of distribution of R_{DUT} and C_{DUT} are listed for each reference resistor. We plotted every distribution of following three figures with the Gaussian curve(in bold line) whose mean value and standard deviation corresponds to that of DUT components.

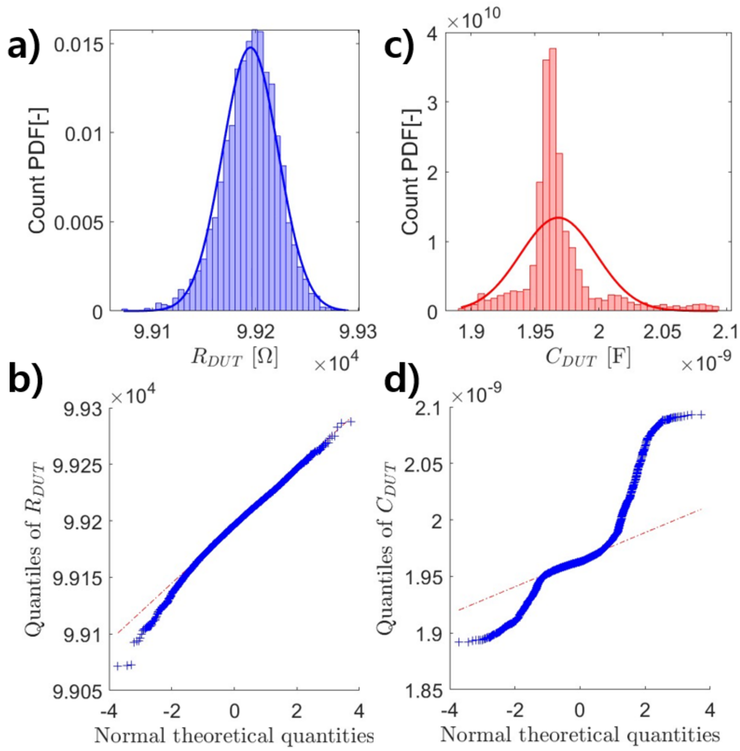

Figure S1: The distribution of DUT components with reference resistor $4.7k\Omega$ (a) The distribution of R_{DUT}. (b) QQ-plot of R_{DUT} (c) The distribution of C_{DUT} (d) QQ-plot of C_{DUT}

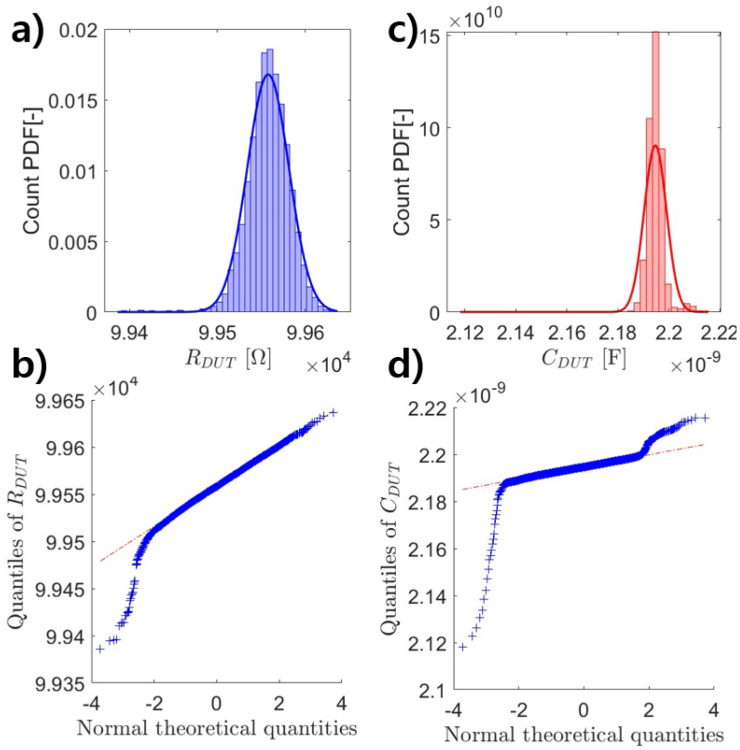

Figure S2: The distribution of DUT components with reference resistor $7.5k\Omega$ (a) The distribution of R_{DUT} (b) QQ-plot of R_{DUT} (c) The distribution of C_{DUT} (d) QQ-plot of C_{DUT}

$\overline{R}_{DUT}$	std(R_{DUT})	Kurt(R_{DUT})	$\overline{C}_{DUT}$	std(C_{DUT})	Kurt(C_{DUT})
99194.99	26.9949	3.593031	1.96827240642512e-09	2.97967163394303e-11	7.2915
99557.79	23.76962	7.3828	2.19471234581578e-09	4.41869622129199e-12	81.89342
99778.73	22.62912	3.387266	2.01237970907741e-09	4.52329094205898e-11	4.467948

Table S1: The statistical value of R_{DUT} and C_{DUT}. For the value $*$, $\overline{*}$ is the mean, std($*$) is the standard deviation, and Kurt($*$) is the kurtosis.

2 Background Circuit Inductance

On the other hand of parasitic element of devices, background circuit conductor can have parasite inductance. In fact, comparison group which was introduced to compensate for external circuit effects showed anomalous behavior on phase difference. The phase of reference voltage is slightly delayed about total voltage, even if there is no devices between detecting points of reference voltage and total voltage. The delay linearly increased as system frequency increased. We linearly regressed phase difference about frequency to consider parasite inductance of external circuit. However, cal-

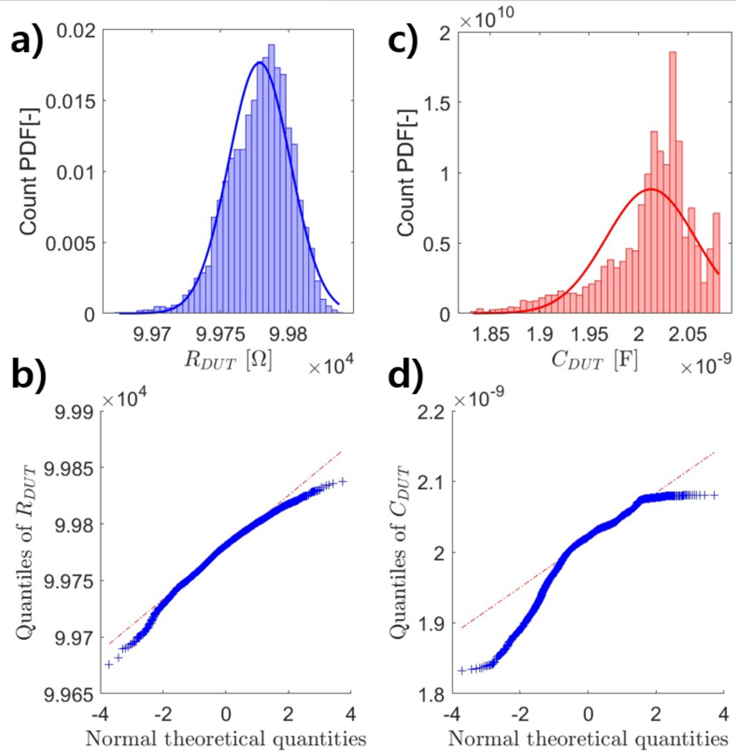

Figure S3: The distribution of DUT components with reference resistor $12.2k\Omega$ (a) The distribution of R_{DUT} (b) QQ-plot of R_{DUT} (c) The distribution of C_{DUT} (d) QQ-plot of C_{DUT}

culated inductance of circuit was not constant about reference resistance, so that it is improper to introduce circuit inductance to calibrate experimental result. In conclusion, we directly subtracted phase difference by phase delay of comparison group, and divided voltage ratio by voltage ratio of comparison group.

3 Parasitic Inductance

To confirm the effect of parasitic inductance(L_p) on DUT resistance(R_{DUT}), we evaluated R_{DUT} in function of L_p. We considered two cases: L_p in series with R_{ref}, L_p in series with R_{DUT}. Since we performed our experiment in low-frequency limit, L_p should be small enough so we can approximate R_{DUT} to first order. We made plots of coefficient of first order($\partial R_{\mathrm{DUT}}/\partial L_p|_{L_p=0}$), and determined whether L_p is significant cause of the experimental error.

Case 1: Assume parasitic inductance in series with reference resistor.

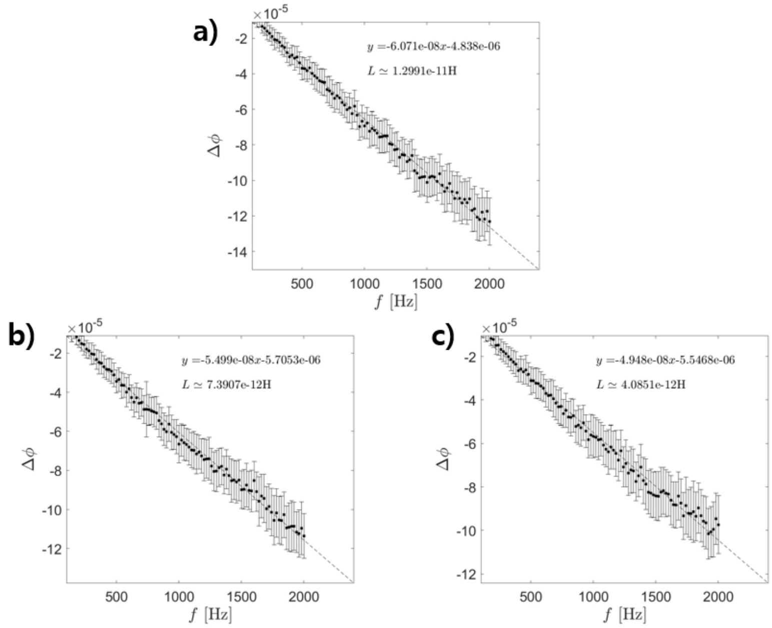

Figure S4: The linear regression of phase delay about frequency for (a) $4.7k\Omega$ (b) $7.5k\Omega$ (c) $12.2k\Omega$

$$
\begin{aligned}
Z &= R_{\mathrm{ref}} + i\omega L_p + \frac{1}{\frac{1}{R_{\mathrm{DUT}}} + i\omega C_{\mathrm{DUT}}} \\
&= R_{\mathrm{ref}} + i\omega L_p + \frac{R_{\mathrm{DUT}}}{1 + i\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}}} \\
&= \frac{(R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{DUT}}) + i(\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}} R_{\mathrm{ref}} + \omega L_p)}{1 + i\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}}}
\end{aligned}
$$

$$
V = \frac{R_{\mathrm{ref}}}{|Z|} = R_{\mathrm{ref}} \sqrt{\frac{1 + \omega^2 C_{\mathrm{DUT}}^2 R_{\mathrm{DUT}}^2}{(R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{DUT}})^2 + \omega^2 (C_{\mathrm{DUT}} R_{\mathrm{DUT}} R_{\mathrm{ref}} + L_p)^2}}
$$

$$
\begin{aligned}
\tan\theta &= \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} \\
&= \frac{(\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}} R_{\mathrm{ref}} + \omega L_p) - \omega C_{\mathrm{DUT}} R_{\mathrm{DUT}} (R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{DUT}})}{(R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{DUT}}) + (\omega^2 C_{\mathrm{DUT}}^2 R_{\mathrm{DUT}}^2 R_{\mathrm{ref}} + \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{DUT}})} \\
&= \frac{-\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}}^2 + \omega L_p + \omega^3 L_p C_{\mathrm{DUT}}^2 R_{\mathrm{DUT}}^2}{R_{\mathrm{ref}} + R_{\mathrm{DUT}} + \omega^2 C_{\mathrm{DUT}}^2 R_{\mathrm{DUT}}^2 R_{\mathrm{ref}}}
\end{aligned}
$$

We solved V and θ for R_{ref} through MATLAB, and approximated to first order with respect to L_p The following is a plot of coefficient of first order against varying frequency.

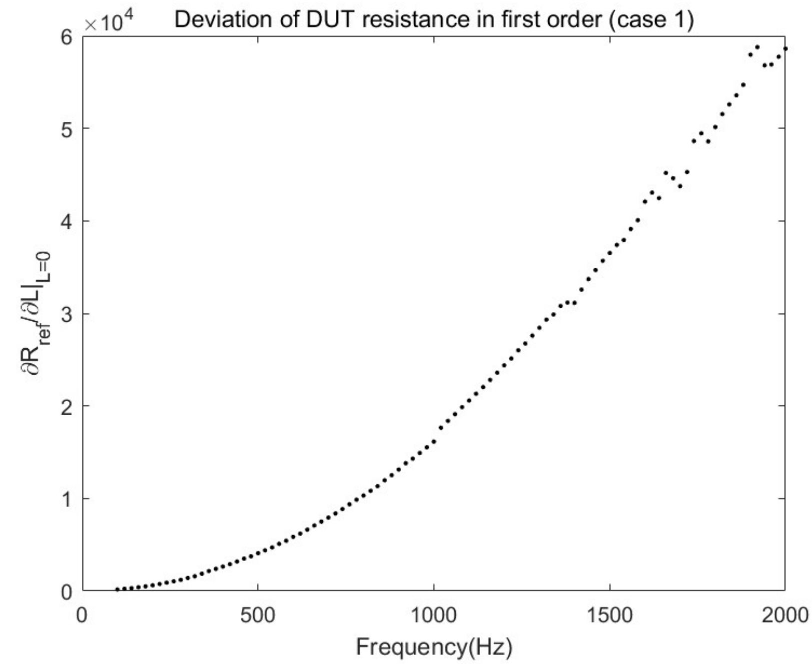

Figure S5: Deviation of R_{DUT} in first order. It corresponds to the case 1

Case 2: Assume parasitic inductance in series with DUT resistor.

$$
\begin{aligned}
Z &= R_{\mathrm{ref}} + \frac{1}{\frac{1}{R_{\mathrm{DUT}}+i\omega L_p} + i\omega C_{\mathrm{DUT}}} \\
&= R_{\mathrm{ref}} + \frac{R_{\mathrm{DUT}} + i\omega L_p}{1 + i\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}}} \\
&= \frac{(R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{ref}}) + i(\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}} R_{\mathrm{ref}} + \omega L_p)}{1 - \omega^2 L_p C_{\mathrm{DUT}} + i\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}}}
\end{aligned}
$$

$$
V = \frac{R_{\mathrm{ref}}}{|Z|} = R_{\mathrm{ref}} \sqrt{\frac{(1 - \omega^2 L_p C_{\mathrm{DUT}})^2 + \omega^2 C_{\mathrm{DUT}}^2 R_{\mathrm{DUT}}^2}{(R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{ref}})^2 + \omega^2 (C_{\mathrm{DUT}} R_{\mathrm{DUT}} R_{\mathrm{ref}} + L_p)^2}}
$$

$$
\begin{aligned}
\tan\theta &= \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} \\
&= \frac{(\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}} R + \omega L_p) - \omega C_{\mathrm{DUT}} R_{\mathrm{DUT}} (R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R)}{(1 - \omega^2 L_p C_{\mathrm{DUT}})(R + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{ref}}) + (\omega^2 C_{\mathrm{DUT}}^2 R_{\mathrm{DUT}}^2 R_{\mathrm{ref}} + \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{DUT}})} \\
&= \frac{-\omega C_{\mathrm{DUT}} R_{\mathrm{DUT}}^2 + \omega L_p + \omega^3 L_p C_{\mathrm{DUT}}^2 R_{\mathrm{DUT}} R_{\mathrm{ref}}}{(1 - \omega^2 L_p C_{\mathrm{DUT}})(R_{\mathrm{ref}} + R_{\mathrm{DUT}} - \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{ref}}) + (\omega^2 C_{\mathrm{DUT}}^2 R_{dut}^2 R_{\mathrm{ref}} + \omega^2 L_p C_{\mathrm{DUT}} R_{\mathrm{DUT}})}
\end{aligned}
$$

Again, we solved V and θ for R_{ref} through MATLAB, and approximated to first order with respect to L_p The following is a plot of coefficient of first order against varying frequency.

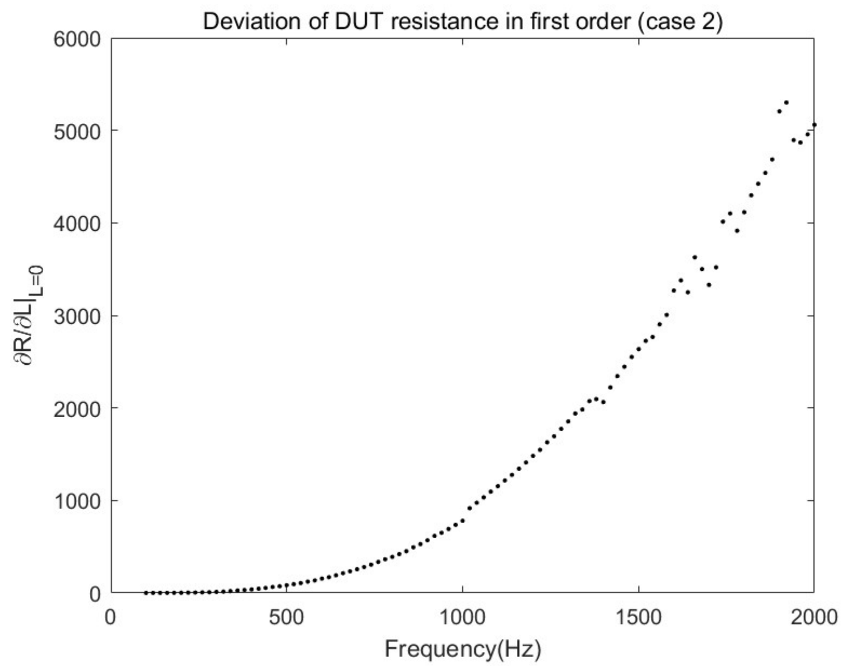

Figure S6: Deviation of R_{DUT} in first order. It corresponds to the case 2

In both cases, coefficient of first order turned out to be positive. Our experimenal result denoted that R_{DUT} becomes smaller as frequency gets higher. Since L_p is minuscule but still positive, we determined that L_p should not be the main cause of the error.