Single Photon Interference

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Abstract

In this experiment, the particle-wave duality of the photon is observed. To see this phenomenon, the diffraction and interference experiment was conducted by using a single slit, a double slit. To be specific, the laser is used to examine the multiphoton limit, and the bulb is used to catch out the single photon limit. As a result, one can find that the theory of light is physically acceptable.

1 Introduction

1.1 Purpose of Experiment

This experiment aims the examination the behavior of light. Especially, by handling sensitive optical equipment, the intensity of light in distinct slits is monitored. This implies that diffraction and interference which are the phenomenon of general waves occur when light traverses.[1]

1.2 Fraunhofer Diffraction

According to the Huygnece principle and Kirchihoff integration, one can derive the Fraunhofer formula which states the effect of incident wave far from the wave source. The wave function ψ_p is

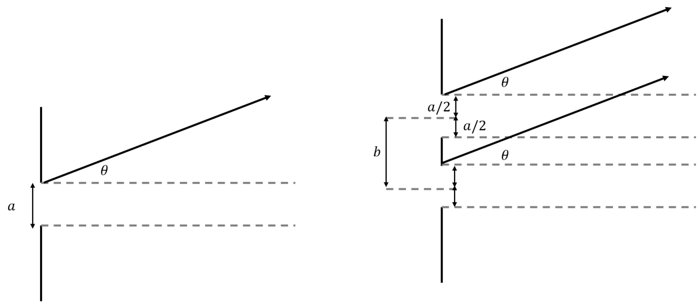
$$\phi_p(\mathbf{r}) \propto \iint\limits_{Aperture} \phi(\mathbf{r}')e^{-i\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}\, d^2r' \tag{1}$$

where $\phi(\mathbf{r}')$ is the complex amplitude at the aperture, and $\mathbf{k}$ is the wave vector of the wave. Since $I \propto |\phi_p|^2$, the intensity at P, observation point, is

$$I(\theta) = I_0 \left(\frac{\sin\beta(\theta)}{\beta(\theta)}\right)^2 \tag{2}$$

where $\beta(\theta) = \frac{1}{2}ka\sin(\theta)$ with slit width a.

[1]In this experiment, a tremendous of data is obtained. This means that there are a lot of figures so it is hard to show every figure in this document. Moreover, the figure cannot be reduced due to the great number of data. Therefore, supplementary material is provided. Please see the figure which is not in the article or when the figure is too small to determine.



(a) (b)

Figure 1: (a) : single slit aperture. (b) : double slit aperture

1.3 Diffraction by Non-Monochromatic

Equation (2) holds only for monochromatic waves whose wave number is fixed. However, the laser which is used in this experiment has a small line width, which implies that it is not monochromatic. Therefore, the fringe pattern by ψ_p is affected by the line width of the light source.

$$I(\lambda, \theta) = I_0(\lambda) \left(\frac{\sin \beta(\lambda, \theta)}{\beta(\lambda, \theta)} \right)^2 \cos^2 \gamma(\lambda, \theta)$$

(3)

where $\beta = \frac{1}{2} ka \sin \theta$, $\gamma = \frac{1}{2} kb \sin \theta$ with slit width a and the distance b from the single slit to the other(See Figure 1). If the line width is sufficiently small, considering driving damped oscillation system, the spectral length can be approximated by Lorentzian form.

$$g(\lambda) = C_0 \frac{\sigma}{(\lambda - \lambda_0)^2 + \sigma^2} \tag{4}$$

The equation (4) is the Lorentzian form whose Full Width at Half Maximum(FWHM) is 2σ where

$$\int_0^\infty g(\lambda) d\lambda = 1 \tag{5}$$

By introducing Lorentzian form to the existing formula, one can get the model considering the line width of the incident wave.

$$I(\theta) = \int_0^\infty I_0(\lambda) \left(\frac{\sin \beta(\lambda, \theta)}{\beta(\lambda, \theta)} \right)^2 \cos^2 \gamma(\lambda, \theta) g(\lambda) d\lambda \tag{6}$$

1.4 Wave-Particle Duality

Historically, the fundamentals of light had been a long debate. Isaac Newton argued that light is a particle. This is strongly believed for a long time until electrodynamics was established. Maxwell contended that light is an electromagnetic wave and Thomas Young observed the fringe pattern of light. This caused the belief of physicists that light is a wave. However, this explanation faced trouble, the photoelectric effect. Wave optics cannot explain the behavior of the light. This problem was settled by Albert Einstein, who is one of the most famous physicists. He assumed that light is a particle and successfully explain the phenomenon with his light quantum theory. This triggered the concept of wave-particle duality. De-Broglie suggested the idea of matter wave and Davisson & Germer demonstrated the duality of matter by showing the diffraction of the electron. This experiment is devised to observe the duality of the wave, especially the fringe pattern of light.

1.5 Asymmetry Slit

If the slit becomes asymmetric, equation (3) does not work. To explain the behavior of light passing through the asymmetric slit, some assumptions are necessary. First of all, every ray progress in parallel and the distance from the slit to the detector is sufficiently large. Also, any diffraction does not affect the progressing light. This means that the laser isn't blocked. The following is the schematization of an asymmetric double slit.

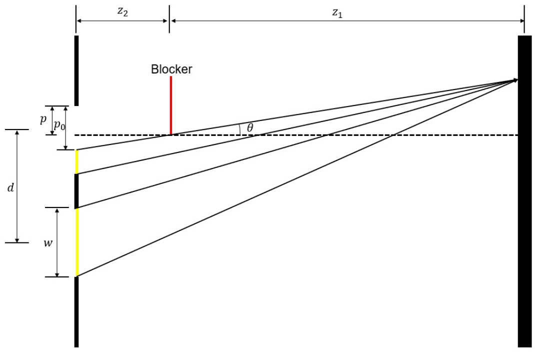


Figure 2: A diagram of asymmetric double slit diffraction

Since the blocker doesn't adjoin with the slit, one can notice that there is a hidden width which depends on the blocker position, the distance from the blocker to double slit, and the position of detector. The following is the intensity of light in this system.

$$I = I_0 \frac{\sin^2(\beta - \phi) + \sin^2(\beta) + 2\cos(2\gamma - \phi)\sin(\beta - \phi)\sin(\beta)}{\beta} \tag{7}$$

Here, $p_0 = p + z_1 \tan(\theta), \beta = \pi w \sin(\theta)/\lambda, \gamma = \pi d \sin(\theta)/\lambda, \phi = \pi p_0 \sin(\theta)/\lambda.$

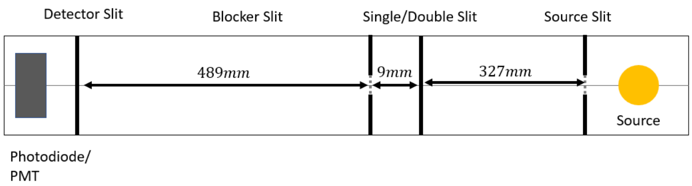
2 Body

In this experiment, U-channel, Photodiode, Photomultiplier(PMT), PCIT, Manual multimeter, several slits, and the oscilloscope are used. Laser and the light bulb are used as the light source. While using a laser, the photodiode detects the signal. However, since the intensity of the light bulb is too weak to be detected by the photodiode, the photomultiplier is employed to amplify the photon emitted by the bulb. This magnified signal is counted by PCIT. Oscilloscope is used to detect the voltage of the photodiode. Three double slits(From now on, they are called slit 14, slit 15, and slit 16.) are used in this task and the single-slit experiment is conducted by using one double slit(slit 14) blocking one slit.

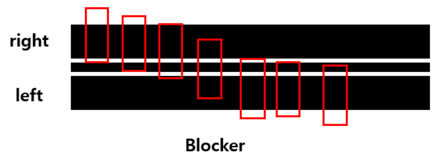
3 Experiment & Result

3.1 Align

First of all, some tasks to align must be conducted to get precise results. The light ray must pass the center of every slit exactly. Otherwise, the results are distorted extremely since. The aligned information is in the following table. The data for alignment is on the supplementary materials. On top of that, since the blocker slit is used to block the light emitted to slits(it does not block but it plays a such role), one should know the exact position of the detector. Therefore, after alignment is carried out, the positions of the blocker is observed. .



(a)



(b)

Figure 3: (a) : Experiment Set. (b) : principal location of detector.

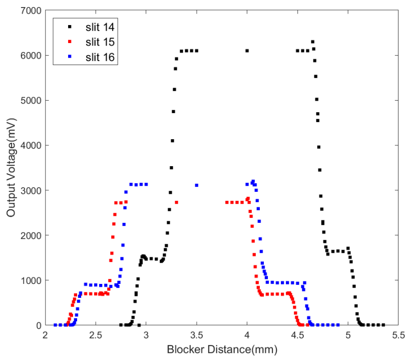


Figure 4: The output voltage varying to the position of blocker

Slit	Peak Difference(%)	Visibility	Blocker Location(mm)
14	2.9252 ± 0.47986	35.08 ± 1.2289	4.90 4.80 4.69 4 3.35 3.16 3.1
15	2.07 ± 1.28	39.09 ± 4.137	4.2 4.13 4.01 3.3 2.7 2.62 2.4
16	1.536 ± 0.4316	36.914 ± 2.3523	4.25 4.2 4.08 3.5 2.85 2.72 2.4

Table 1: Alignment Information and the observed location of detector as order of Figure 3(b).

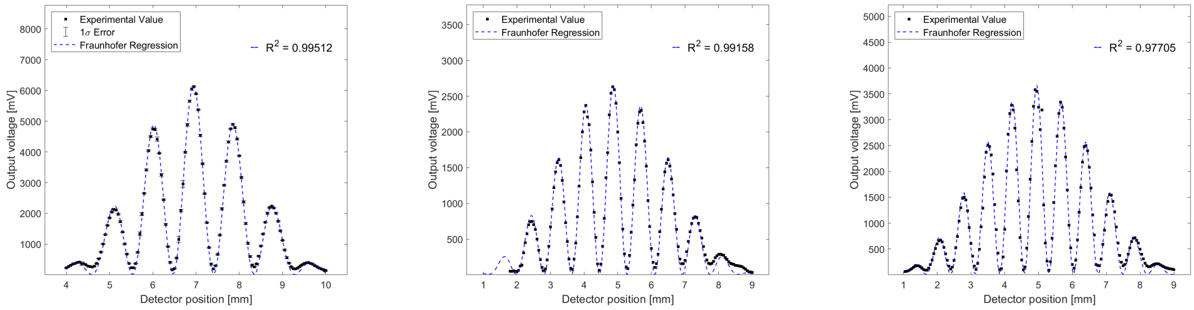
3.2 Multi-photon limit

3.2.1 Interference of single & double slit

For this section, every slit is used respectively. The proper position of the blocker is selected to make a double slit and single slit referring to Figure 4 and Table 1. Equation (2) is used to fitting both double and single slits. The regression coefficient of monochromatic fittings which is achieved by equation (2) and their uncertainty are listed in the supplementary materials.

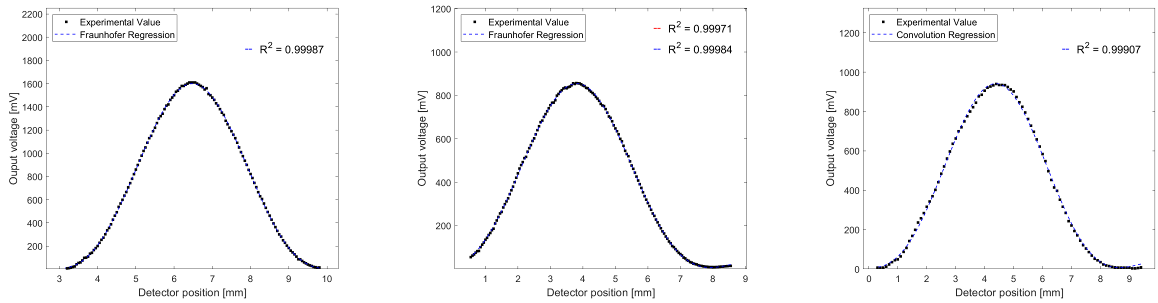
From the fitted data, the t-tests are conducted with 5% significance level to check the equality of the left and right width of the double slit. The results of the test are in Table 2. The null hypothesis, that is, the width of slits is the same, is rejected for each slit except slit 16.

However, the regression can be improved by considering the line width of the laser. One can see that the monochromatic fitting discord with the actual data at the local minima. To introduce equation (6) in regression, a special technique which is called convolution fitting is



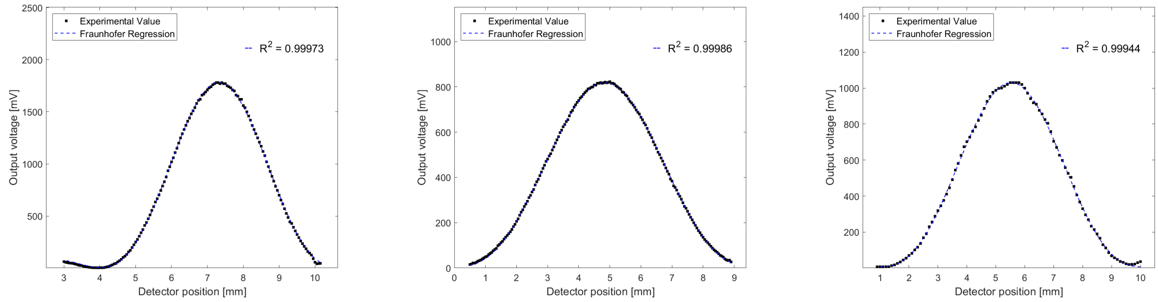
(a) (b) (c)

Figure 5: The result of double slits for a laser. (a) : slit 14, (b) : slit 15, (c) : slit 16.



(a) (b) (c)

Figure 6: The diffraction of light in left side single slits. (a) : 14, (b) : 15, (c) : 16.



(a) (b) (c)

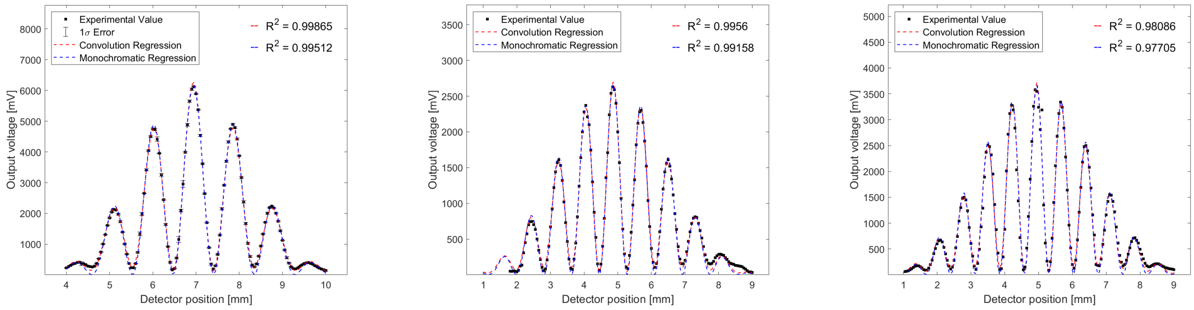
Figure 7: The diffraction of light in right side single slits. (a) : 14, (b) : 15, (c) : 16.

Slit	left width(m)	right width(m)	p-value
14	$9.60 \times 10^{-5} \pm 9.97 \times 10^{-8}$	$1.01 \times 10^{-4} \pm 1.54 \times 10^{-7}$	0.02257
15	$8.14 \times 10^{-5} \pm 9.04 \times 10^{-8}$	$7.15 \times 10^{-5} \pm 3.65 \times 10^{-8}$	0.00624
16	$7.83 \times 10^{-5} \pm 2.49 \times 10^{-7}$	$7.51^{-5} \pm 1.64 \times 10^{-7}$	0.05932

Table 2: t-test result that confirms the width of slits are different

conducted. Figure 8 is the refined fitting of the double slit. The regression of single slit is on the supplementary materials.

The following is about the spectral strength of the laser. In this analysis, only the results of double slits are considered with the same weights. This will be discussed later. See the supplementary materials to check the estimated spectral strength of each slit.



(a) (b) (c)

Figure 8: The fringe pattern of double slits with refined fitting method. (a) : slit 14, (b) : slit 15, (c) : slit 16.

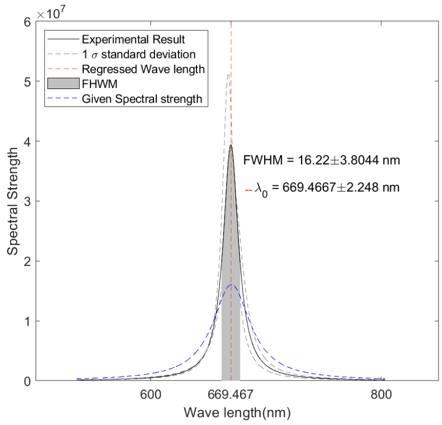


Figure 9: The spectral strength considering every result of slits.

3.2.2 Interference : asymmetric double slit

In this section, the behavior of light passing various asymmetric slits is observed. The fitting is conducted by using equation (7). The following is measured data and the result of the fitting.

3.3 Single Photon Limit

3.3.1 PMT Preparation

For every experiment using a light bulb, the photo-multiplier must be employed to grab the signal from the bulb. Therefore, the role of PMT is pivotal. To minimize the error from PMT, some preparation is necessary. First of all, one has to set the driving voltage of PMT. PMT amplifies a photon by the fixed voltage. However, for high voltage, PMT magnifies the signal even though there is no input signal. Moreover, for low voltage, PMT does not work properly despite of present input photon. Therefore, it is quite important to find the proper voltage for PMT. In this section, PCIT is used to count the incoming photon.

As one can see in Figure 10, the criterion of Pulse count is selected to 10, not 1. This is because we tried to get photons transmitted from the light bulb as much as possible even though the noise is more likely to affect the result. The upper bound of PMT voltage is 690.2844±172V and the lower bound is 601.9726±212V. As a result, the driving voltage is set by 660±2V. Then, one has to select the threshold voltage of PCIT. The signal of PMT and PCIT is compared by oscilloscope. The followings are the results.

As one can see that 0.04V is proper to the threshold voltage. The word "proper" means that

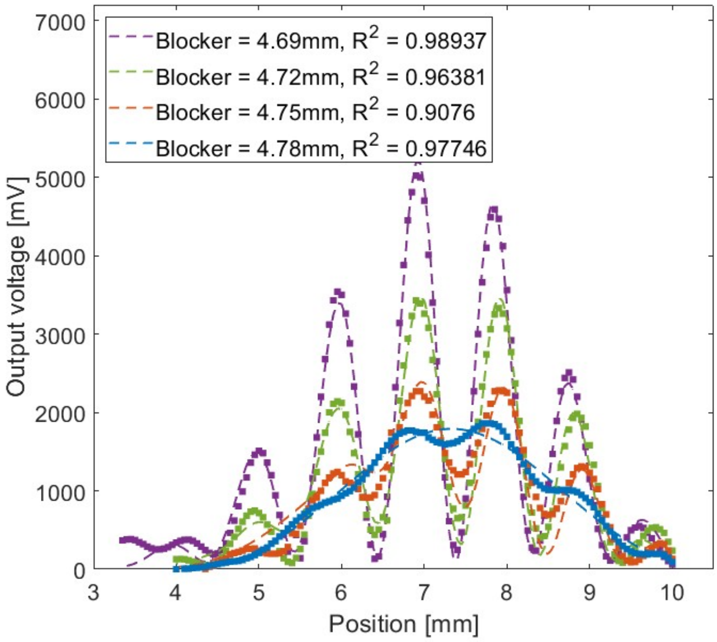
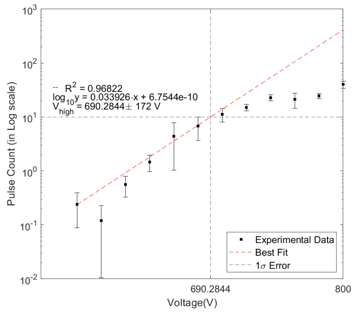
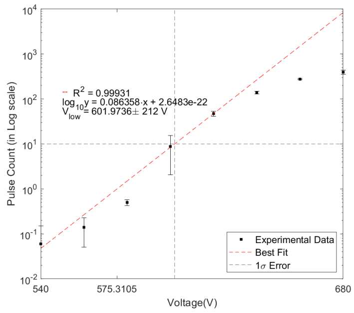


Figure 10: Asymmetric Fraunhofer fitting.

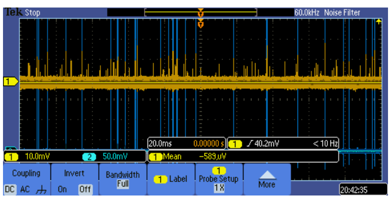


(a)

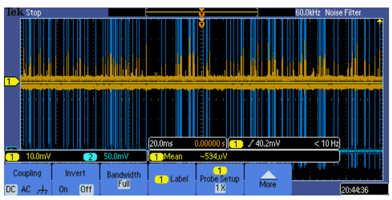


(b)

Figure 11: (a) : upper bound of voltage. (b) : lower bound of voltage.



(a)



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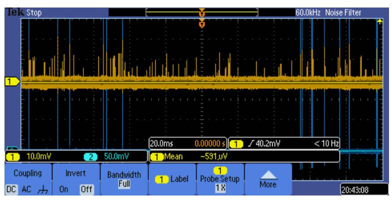


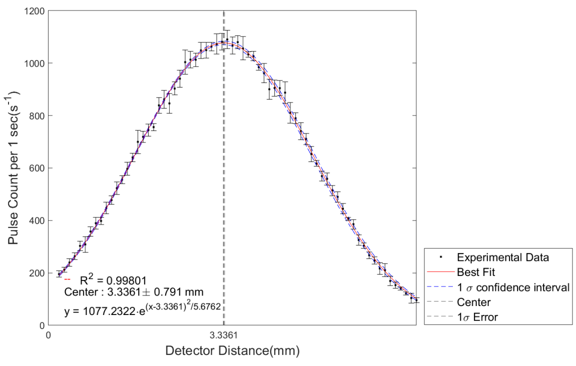
(c)

Figure 12: (a) : PCIT voltage = 0.04. (b) : PCIT voltage = 0.03. (c) : PCIT voltage = 0.06. The yellow one is the signal from PMT and the blue picks are the one from PCIT. As the voltage decreases for same input, PCIT counts more signal to pulse, that is photon.

the PCIT counts the exact photon, not the noised signal. As PCIT counts the photon precisely in some threshold voltage, that voltage is appropriate. For the case of 0.03V, PCIT counts too many signals as the photon. However, in the case of 0.06V, PCIT does not count even though the signal is big enough. Therefore the PCIT threshold voltage is 0.04 ± 0.01V. For simplicity, the PCIT threshold voltage is set to 0.04V.[2]

3.3.2 Transfer Function of Detector Blocker Measurement

There are some errors in U-Channel due to the irregular light and inconsistent incident position where the photon collides with PMT. To refine the limitation of these apparatuses, the transfer function of the detector and blocker is observed. Note that the Gaussian fit is conducted. As a result, the central position of detector is chosen to 3.3161 mm. Moreover, one can calculate the power of bulb according to its intensity.

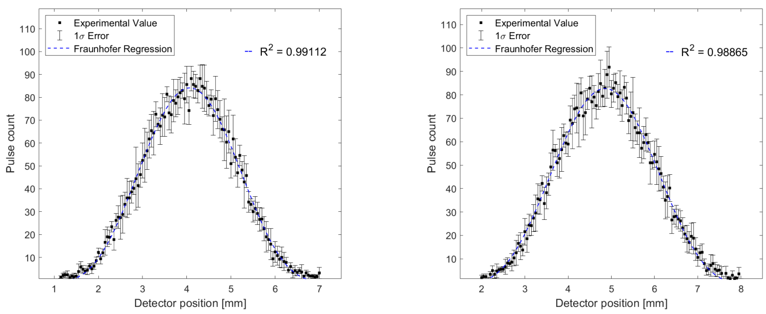


(a) (b)

Figure 13: (a) : Transfer function of detector and blocker. (b) : the pulse count in one second as changing the bulb power.

3.3.3 Diffraction & Interference: Bulb

The experiment is carried out in the bulb power 3. The single slit experiment of bulb is conducted only for slit 14.



(a) (b)

Figure 14: The single slit result of bulb. (a) : left single slit (b) : right single slit

The table below is the result of t - test whether the intensities of each slit are same. The following test is conducted by using the result of fitting above. Here the number of photon what PCIT counts is the test statistics.

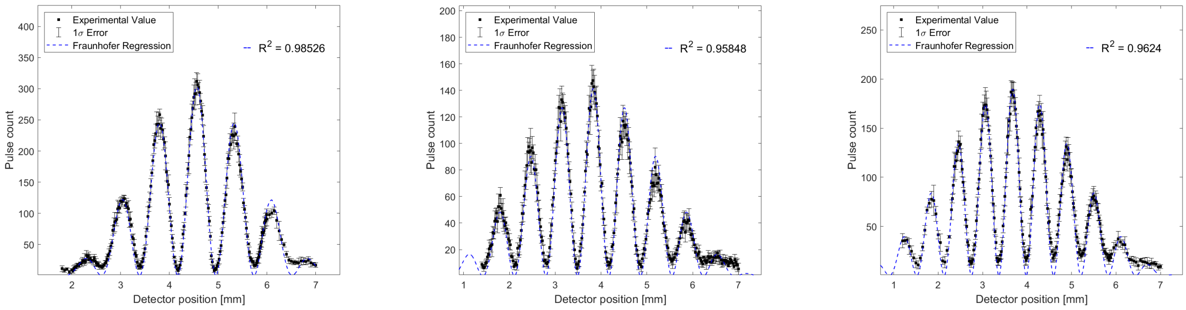
The null hypothesis is not rejected. This implies that the intensity from two apertures in double slit is same.

[2]This is much smaller than the one in manual. However, as a rule of thumbs, after the slits equiped, the signal outgoing from PCIT is too small to conduct further experiment.

Slit	Left $N_0(cnt)$	Right $N_0(cnt)$	p-value
14	$27.21680089 \pm 0.177087876$	$26.74928727 \pm 0.186806413$	0.069

Table 3: t-test to confirm the discrepancy of intensity of light between two slits

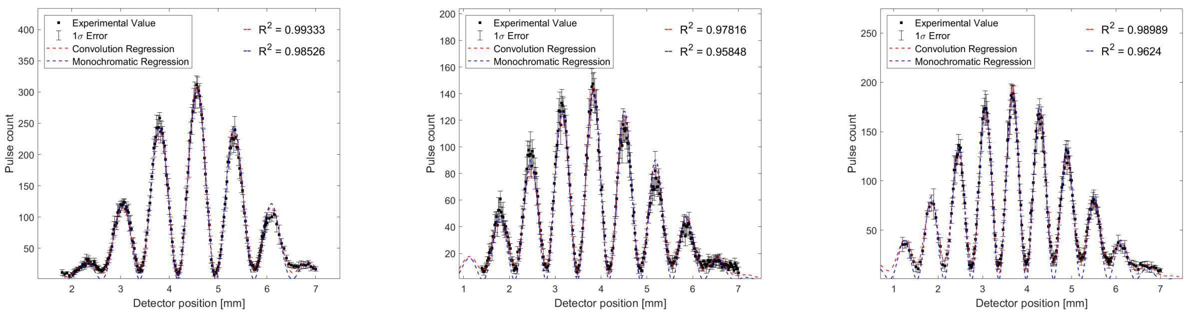
Then, like in the experiment in laser, the interference of light from the bulb appears. The followings are the result of the double slit for the light bulb. The fittings are conducted by the Fraunhofer formula.



(a) (b) (c)

Figure 15: The result of double slit for the light bulb : Fraunhofer model

However, similar to the case of laser, the fitting can be developed by introducing the model taken into account the line width.



(a) (b) (c)

Figure 16: The fringe pattern of double slit for the light bulb. (a) : slit 14, (b) : slit 15, (c) : slit 16

The following figures are the spectral length of light bulb. The wave length of light bulb is estimated by 560 ± 2.8583nm.

In this experiment,by chaining the bulb power, the intensity of double slit 14 is also detected. The result is below. One can see that regardless of the intensity of bulb, the interference regularly appear.

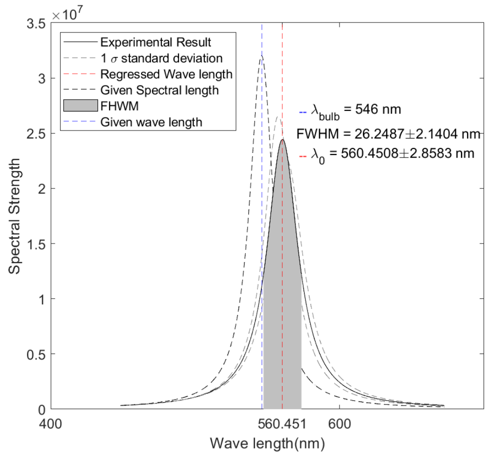


Figure 17: The total spectral strength for the case of bulb.

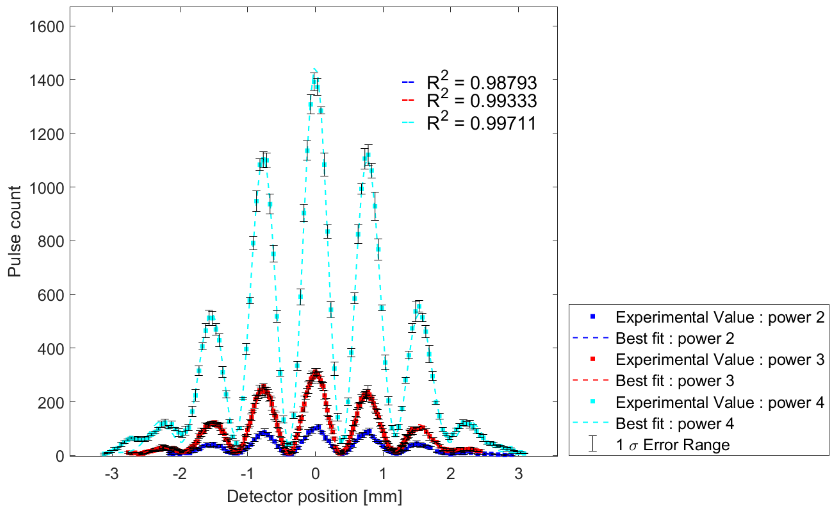


Figure 18: The intensity for the slit 14. The bulb power is changed from 2 to 4.

4 Discussion

4.1 Error Analysis : Slit width

Consider the slit width. According to Table 2, the left width and the right width of the 14 and 15 slits are quite different.[3] Also, the given width of slit(0.085mm) is different with the estimated width.[4] It is probably due to the defect of the slit or fault in the analysis.

The defect of the slit is suspicious due to how the slit is kept in the laboratory. The slit can be damaged by friction with the table or contaminated due to invisible materials such as human finger print or dust.

In addition, the regression could be wrong. The width of the slit is a result of the single-slit regression. However, one can notice that some of the single-slit data do not reach their local minima. This prevents one to get the width of the slit. The main reason for the failure to reach the minima is the micrometer which is attached outside of the U-tube. Besides the backlash which is possible to handle, the micrometer rotates irregularly, even though one rotates. Due

[3]It is suspicious that slit 16 also has asymmetric slit width since its p-value is slightly bigger than the significance level which means that it is almost success to reject the null hypothesis.

[4]K.Sol(2021), Experiment Manual Single Photon Interference *2.0*, Department of Physics and Astronomy, Seoul National University, p11.

to this reason, it is impossible to detect reasonable data from the boundary of the micrometer. Since the range of the detector does not contain the point where the local minimums are made, only one end can be observed. This is reason why only double slit results are considered when calculating the spectral strength. To settle this problem, a more broad slit should be used so that the diffraction pattern occurs in the range of a micrometer.

4.2 Asymmetric Double Slit fitting

In this section, the new method to explain the shape of asymmetric slit is introduced. The method employ the fast Fourier transform.

4.2.1 Forms of the Diffracted Waves

The Fresnel diffraction regime satisfies the following conditions :

$$a/z << 1 \,\&\, ka^2/z > 1 \tag{8}$$

where a is the size of the aperture, D is the propagation length from the aperture, k the size of the wave vector. $ka^2/z > 1$ implies that $ka^2/z << 1$ does not hold.

In these conditions and when the slit on the xy plane, the diffracted wave(ψ) for an incoming wave at the aperture(ψ_{inc}) can be approximated to[5]:

$$\psi(x, y, z) = -\frac{ik}{2\pi}\frac{e^{ikz}}{z} \times \int_A \psi_{inc}(x', y', 0)e^{\frac{-ik}{2z}((x-x')^2+(y-y')^2)}d^2r' \tag{9}$$

The Fraunhofer diffraction regime satisfies the following conditions :

$$a/z << 1 \,\&\, ka^2/z << 1 \tag{10}$$

where a is the size of the aperture, z is the propagation length from the aperture, k the size of the wave vector.

In these conditions, the diffraction pattern can be approximated to:

$$\psi(\mathbf{r}) \approx -\frac{ik}{2\pi}\frac{e^{-kr}}{r}\int_A \psi_{inc}(\mathbf{r'})e^{-i\mathbf{k}\cdot\mathbf{r'}}d^2r' \tag{11}$$

4.2.2 Fourier Optics

To simulate the Fresnel and Fraunhofer diffraction, one of the most convenient way is to use FFT(fast Fourier Transform). Equation (11) says that the Fresnel diffraction is a 2D convolution.

$$\tilde{\psi} = \tilde{\psi}_{inc} \times H_F(f_x, f_y; z) \tag{12}$$

where $\tilde{\psi}$ is Fourier transform of ψ and $H_F(f_x, f_y; z)$ is the Fourier transfer of the Fresnel impulse function(h_F)

$$h_F(x, y; z) = -\frac{ik}{2\pi}\frac{e^{ikz}}{z}e^{\frac{-ik}{2z}((x-x')^2+(y-y')^2)} \tag{13}$$

$$H_F(f_x, f_y; z) = e^{ikz}e^{-i\pi\lambda z(f_x^2+f_y^2)} \tag{14}$$

[5]Anupam Garg, Classical Electromagnetism in a Nutshell, Princeton University, pp.182-187.

Therefore, the inverse Fourier transform can be used to restore ψ. Also referring to equation (11), one can note that the Fraunhofer diffraction is a 2D Fourier transform, and the waves at the detector simply follow $\tilde{\psi}_{inc}$

A numerical simulation for the asymmetric slits were carried out by using MATLAB. For simulating the diffraction pattern, the Fresnel diffraction of the double slits at the blocker plane were truncated, only allowing the waves that pass the blocker to act as a new incoming wave for the Fraunhofer diffraction of the blocker itself(19). Details and the code are attached to Supplementary Information.

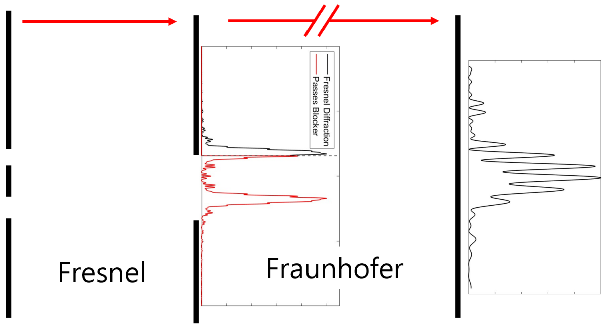


Figure 19: The simulation schematic, where the slit, blocker postitions and the Fresnel and Fraunhofer regions are noted. Example intensity patterns are plotted at the end of each region.

4.2.3 Result of Optics

The result of Fourier optics is below. One can notice that overall R^2 is improved than Figure 10.

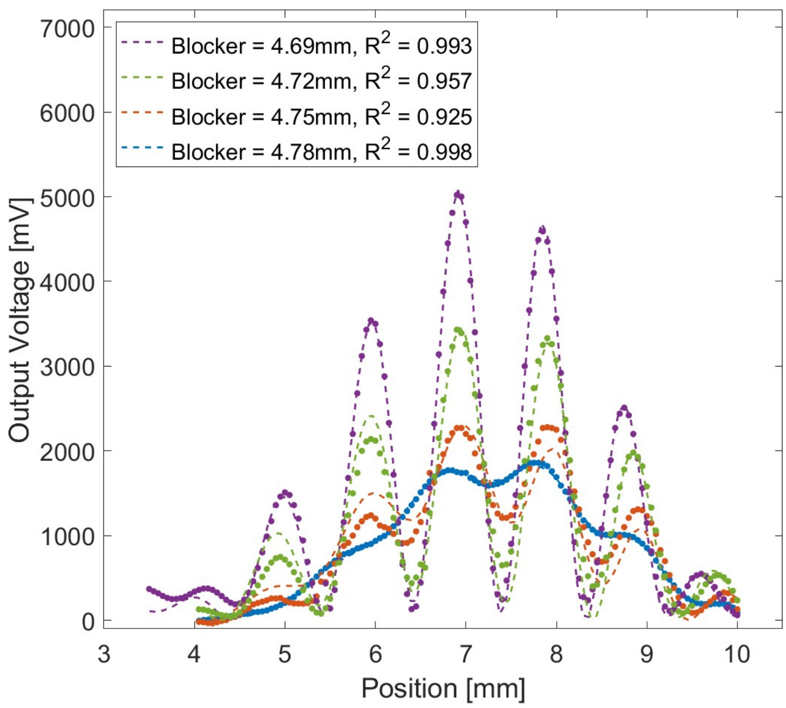


Figure 20: Result of Fourier Optics

4.3 Single Photon & Multi Photon Limit

Let $N(x)$ be the count profile with respect to the detector's location x and N_{det} be the detected photon count. Here, a denotes the slit width, w is the width of the U-channel, and Q_{eff} is the quantum efficiency of the PMT.

$$P = \left(N_{det} \frac{\int N(x)dx}{a \max(N(x))} \right) \frac{w}{aQ_{eff}} \frac{h\lambda}{c} \tag{15}$$

Let N be the total number of photons emanate from the source in a second is

$$N = \left(N_{det} \frac{\int N(x)dx}{a \max(N(x))} \right) \frac{w}{aQ_{eff}} \tag{16}$$

The resulting number of photons emitted per second and the power of the bulb is the following with each corresponding 2σ error.

$$N = (1.44 \pm 0.29) \times 10^8 \, [/s] \tag{17}$$

$$P = (5.11 \pm 1.0) \times 10^{-11} \, [J/s] \tag{18}$$

From the uncertainty principle, one can get the *effective wave packet size* of a photon due to the localized photon and assumption that $\Delta x \Delta p$ is the order of $\hbar/2$. Therefore, the effective wave packet size is

$$\Delta x \sim \frac{h}{4\pi\Delta p} \sim \frac{\lambda_0^2}{4\pi\Delta\lambda} \sim 1.96 \pm 0.15\mu\text{m (bulb) } 4.40 \pm 1.06\mu\text{m (laser)} \tag{19}$$

Moreover, since the photon takes approximately 3ns to pass the U-tube, one can calculate how many photon in there at the moment. The number of photon emitted from light bulb in one second was calculated above, and by the manual, the laser used the $1mW$ power. From this information, one can derive following expressions.

$$N_{bulb} = (1.44 \pm 0.29) \times 10^8 \, [/s] \tag{20}$$

$$N_{laser} = 3.37 \times 10^{15} \, [/s] \tag{21}$$

By using the equation 13,14, the number of photon in the tube at the moment is approximately 3.7^{-3} in average for the case of bulb and 8.6^4 for the laser. Therefore, it is evident that the experiment conducted by light bulb is single photon limit. In addition, the distance from a photon to the other one is sufficiently large, especially, larger than their effective wave packet size.

5 Conclusion

In this experiment, the fringe patterns of diverse slits are observed. To be specific, the diffraction and interference phenomenons are detected in multi-photon limit by using laser and single photon limit by using light bulb. To measure such matters, additional alignments and detection are conducted. As a result, the light behaves as one expected. This implies that the theory of light can explain the nature exactly.

References

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