Transistor Negative-Impedance Converters*

J. G. LINVILL†, ASSOCIATE, IRE

Summary—**Negative impedances having very stable characteristics are obtained with circuits using transistors. The physical characteristics of transistors, compactness, long life, simple power requirements, plus constancy of pertinent electrical parameters enhance their practical utility in the production of negative impedances.**

Introduction

NETWORK DESIGNERS have long been attracted to the potentialities of negative impedances. Their use in one form or another has been proposed for the reduction of parasitic losses, for the improvement of device characteristics, and for the design of new circuits impossible with purely passive elements. Vacuum tubes have been employed in the construction of negative impedances, and circuit configurations which exhibit negative impedances have long been known.[1–5]

In the telephone plant negative-impedance repeaters[1] have been employed to reduce the loss on telephone lines. The inherent limitations of vacuum tubes, their bulkiness, relatively short life, requirement of a heated filament, deterioration with age, and limited ruggedness have deterred a broader application of negative impedances. A wider usefulness of these circuits is made possible by the appearance of the transistor which, apparently, possesses none of these practical limitations and, in addition, has desirable electrical characteristics.

Junction transistors are particularly adapted for application in a circuit called a negative-impedance converter. Ideal negative-impedance converters are active four-poles which are completely characterized by two properties: the input current equals the output current and the input voltage equals the negative of the output voltage. Thus negative-impedance converters act impedance-wise as a sort of ideal transformer in which the input impedance is the negative of the load impedance.

Transistor converters are good approximations to ideal converters; the physical converter can be represented as an ideal converter with an impedance-conversion factor in the range -0.90 to -0.98, with a parasitic-series resistance at the input of a few tens of ohms. The conversion properties of the transistor converter are stable; the conversion factor depends, principally, upon the alpha of the transistors and this parameter varies insignificantly with bias conditions. The useful frequency range of transistor converters extends into the hundreds of kilocycles.

Negative-Impedance Converters—Balanced Open-Circuit Stable Type

A simple qualitative explanation of the negative-impedance converter circuit shown in Fig. 1(b) is as follows: Practically all of the current I_1, going into the transistor, flows out of the collector into the load, developing a voltage across it. This voltage is cross-coupled by the condensers back to the bases of the transistors, and thence to the input terminals practically undiminished because of the low impedance of the operating transistor between emitter and base. Hence input voltage is practically the negative of voltage across the load, and one sees at input the negative of load impedance.

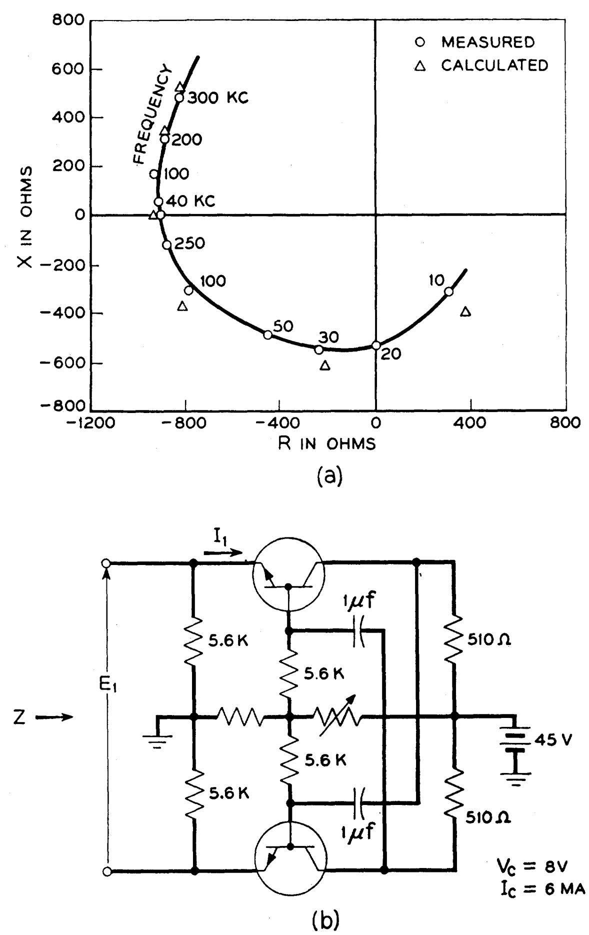

Fig. 1(a) and (b)—Open circuit stable negative-impedance converter and locus of its input impedance.

Actually, the emitter current can vary from the collector current by not more than a few per cent and the emitter-base resistance of typical units typically biased is about 25 ohms; hence this analysis is quite accurate.

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† Bell Telephone Laboratories, Inc., Murray Hill, N. J.

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2 G. Crisson, "Negative impedances and the twin 21-type repeater," *Bell Sys. Tech. Jour.*, vol. 10, pp. 485–513; July, 1931.

3 L. C. Verman, "Negative circuit constants," Proc. I.R.E., vol. 19, pp. 676–681; April, 1931.

4 E. W. Herold, "Negative resistance and devices for obtaining it," Proc. I.R.E., vol. 23, pp. 1201–23; October, 1935.

5 E. L. Ginzton, "Stabilized negative impedances," *Electronics*, vol. 18, pp. 140–144; September, 1945.

A direct analysis of the equivalent circuit of Fig. 2(a) indicates that the input impedance of the converter is

$$\frac{E_1}{I_1} = -\alpha Z_N \frac{Z_g}{Z_g + Z_d} + 2r_\epsilon + (1-\alpha)\left[2r_b + \frac{(2Z_d + Z_A)2Z_g}{2Z_d + Z_A + 2Z_g}\right], \quad (1)$$

or at the operating frequencies, where $Z_d \to 0$,

$$\frac{E_1}{I_1} = (1 - 2\alpha)Z_N + 2r_\epsilon + (1-\alpha)2r_b. \quad (2)$$

The only approximation made in arriving at the above relations is that Z_A/r_c is negligible, a very good approximation for practical circuits with a load in the thousands of ohms as r_c is of the order of megohms.

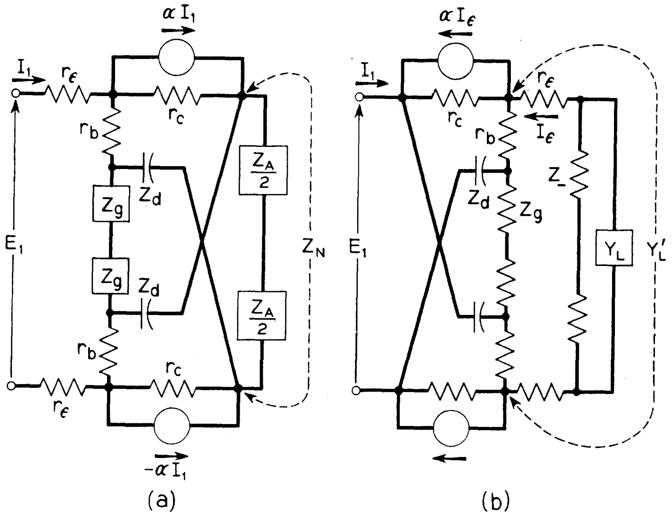


Fig. 2—Equivalent circuits of open- and short-circuit stable converters. (a) Z_n is the parallel combination of Z_A and $2Z_d + 2Z_g$.

(b) $Y_L' = \cfrac{1}{2r_\epsilon + \cfrac{1}{Y_L + \cfrac{1}{2Z}}}$.

Since the alpha's of good transistors are between 0.95 and 1.0, the multiplier of Z_N in (2) is close to the ideal value of -1.0. The deviation of the input impedance from $-Z_N$ associated with the second and third terms of (2) is small also: r_ϵ is inversely proportional to emitter current being 13 ohms at 2 ma.; the third term is a few per cent of r_b, which is a few hundred ohms.

The independence of the input impedance with respect to the point of operation is important in the negative-impedance converter. In this connection, the transistor circuit is very good indeed. The significant quantity, α, varies insignificantly with the point of operation. The emitter resistance, r_ϵ, is inversely proportional to emitter current, but its value is so small that this variation is of no consequence for load impedances above a few hundred ohms.

The locus of measured-input impedance of a converter which was constructed is shown in Fig. 1(a) along with a number of calculated points which were evaluated using (1). In connection with the calculation, it should be pointed out that the α of transistors is frequency-dependent, varying roughly according to the relationship

$$\alpha = \frac{\alpha_0}{1 + jf/f_{c\alpha}}, \quad (3)$$

where $f_{c\alpha}$ is called the alpha cut-off frequency. The cut-off frequency used in the calculation of high-frequency points shown on Fig. 1(a) is 1.75 mc.

The fact that the locus of input impedance of the circuit of Fig. 1(a) encloses the origin in the clockwise sense indicates that the circuit is open-circuit stable. Open-circuit stability is a characteristic of converters in which the emitters are at the input terminal pair, independent of the nature of the passive load or of the cross-coupling networks, as is proved in the Appendix.

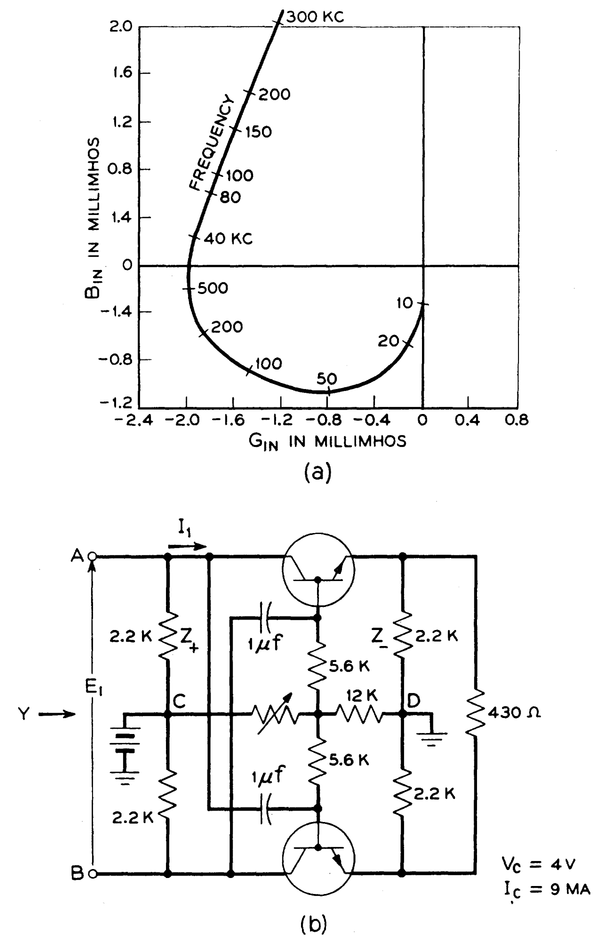


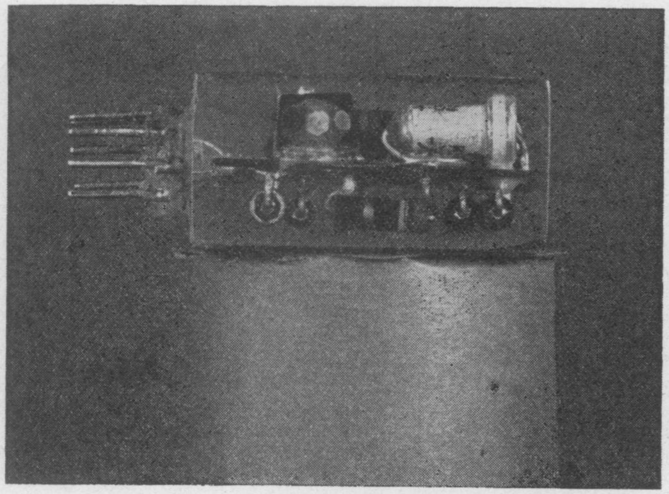
Fig. 3(a) and (b)—Short circuit stable negative-impedance converter and locus of its input impedance.

Negative-Impedance Converters—Balanced Short-Circuit Stable Type

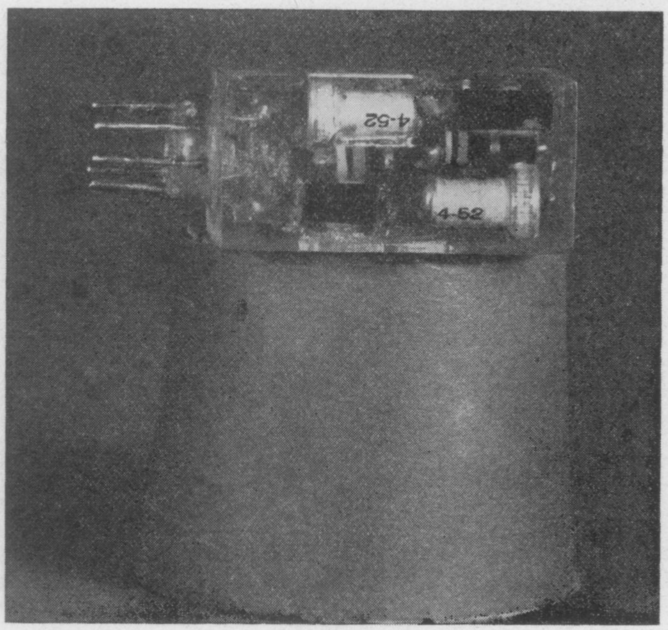
The circuit diagram of the short-circuit stable converter is shown in Fig. 3(b), and the equivalent circuit of the significant part of it is shown in Fig. 2(b). A straightforward analysis of the equivalent circuit indicates the input admittance to be

$$\frac{I_1}{E_1} = \frac{-\alpha Y_L' + \frac{1}{2Z_g} + (1-\alpha)\left[1+\frac{r_b}{Z_g}\right]Y_L'}{1+\frac{Z_d}{Z_g}+(1-\alpha)Y_L'\left[2r_b+\frac{Z_d}{Z_g}(2r_b+2Z_g)\right]} \cdot \quad (4)$$

From (4) it is apparent that in the short-circuit stable type, deviations from ideal character, i.e., $I_1/E_1 = -Y_L'$, are small but of different nature from those in the open-circuit stable type.



(a)



(b)

Fig. 4(a) and (b)—Two views of a short-circuit stable converter in a plastic plug-in unit ($\frac{3}{4}\times\frac{3}{4}\times 2$ in.).

Fig. 3 shows element values of a circuit which was constructed, and the locus of measured-input admittance over a wide range of frequencies. The fact that the locus (only a part of which is shown in Fig. 3) encloses the origin in the clockwise sense indicates the short-circuit stability of the device. It is shown in the Appendix that any converter of the type shown in Fig. 3 (the input terminals are at the collectors) will be short-circuit stable with a resistive termination independent of the nature of the passive cross-coupling networks provided that the magnitude of alpha remains less than one. Also the converter must be stable (short-circuit) for any passive load provided that alpha is real and less than one.

A plastic plug-in unit including transistors, cross-coupling condensers, and biasing circuits of a short-circuit stable converter is shown in Fig. 4.

In connection with the overload characteristics of the converters, it is important that as the open-circuit stable type is overloaded, the amount of negative input resistance decreases. As the short-circuit stable type of converter is overloaded, the amount of negative input conductance decreases. These statements indicate that the circuits are less susceptible to oscillation when overloaded than when they are under normal load levels.

Unbalanced Negative-Impedance Converters

In the telephone repeater application, the balanced configurations of converters described in the foregoing are ideal. However, for other applications, it is frequently desirable to have a common-ground connection. It is desirable, also, in an active circuit to have the power supply grounded. A small change in the circuits already described adapts them to the unbalanced configuration.

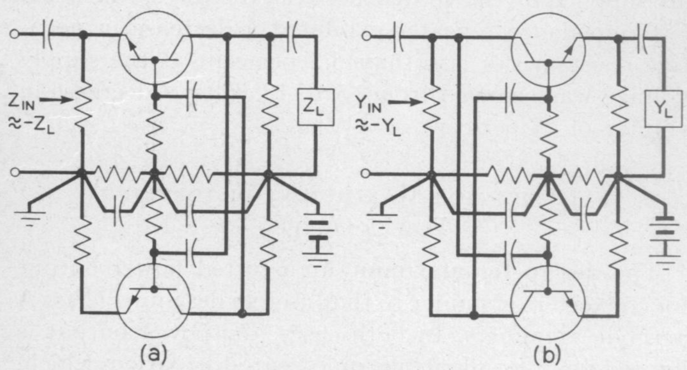


Fig. 5—Unbalanced negative impedance converters. (a) Open-circuit stable type. (b) Short-circuit stable type.

Fig. 5 shows unbalanced negative-impedance converters of both the open-circuit stable and the short-circuit stable types. It is interesting that in the unbalanced converters, the roles of the two transistors are quite different. In the figure, the upper transistor in each type plays the same role that the transistors of the balanced circuits played. The load current passes through it. However, the lower transistors in Fig. 5 act as phase inverters. They essentially sense the phase of voltage at the collector of the upper transistor and feed to its base a voltage of the opposite phase. The phase inverters could be replaced by suitable transformers or phase-shift networks if it were desirable to do so.

The Application of Transistors to Negative-Impedance Repeaters

Vacuum-tube converters have been employed successfully in the exchange plant as negative-impedance repeaters. Transistor converters are more nearly ideal converters than are the vacuum-tube version. Deviation from ideal character in the vacuum-tube model depends upon limited size of g_m and the inequality of $\mu-1/\mu+1$ to 1. Corresponding deviations from ideal character to that obtained in a transistor model with parameter values typical of junction transistors would be obtained with a triode having a g_m of 47,000 micromhos and a μ of 50.

In the application of a negative-impedance repeater, the input terminals of the negative-impedance converter are placed in series or in shunt with the line, the output being terminated in an appropriate passive network. Thus connected, the repeater increases the transmission of the line in both directions. If a repeater is to be connected in series with the line, an open-circuit stable converter is used. Open-circuit stable repeaters are also called the series type for this reason. If the repeater is to be connected across the line (called a shunt repeater), a short-circuit stable converter is used.

One very significant merit of the transistor repeater is the fact that changes in the supply voltage cause small changes in the negative resistance that is presented at the input terminals. For central office applications this point is not of great importance, but there is possibility of using transistor repeaters along the line where they are supplied by the voltage between the wires. One negative-impedance repeater exhibited a decrease in negative resistance of less than 5.5 per cent as the supply voltage was changed from 45 to 12 volts, a decrease in voltage of 73 per cent.

Design for Maximum Undistorted Power Output

The design for maximum-undistorted power output for converters is similar to that used in designing Class A pentode amplifiers. In both cases, the power output is limited by the allowable quiescent dissipation which, for the transistor, is V_cI_c, V_c and I_c being the quiescent-collector voltage and current. In the converter the amplitude of incremental voltages and currents for the input terminals, the output terminals, and the collectors of the transistors are practically the same. Hence for distortion-free operation, the maximum magnitudes of the ac components of voltage and current at all of the points are V_c and I_c. The quiescent operating point is determined by the allowable dissipation, V_cI_c, and the load impedance, V_c/I_c. The maximum power available at the input terminals, half of the allowable dissipation of a transistor, is 25 mw for the M1752 transistor. Actually, part of this power is lost in the biasing resistors at the input. Laboratory circuits were constructed which provided powers out as high as 17 mw.

Design to Minimize Effect of Temperature Sensitivity of the Saturation Current I_{co}

Junction transistors are not greatly influenced by changes in temperature, except in one respect. The collector current which flows in the absence of emitter current, I_{co}, is extremely temperature sensitive. Proper design of converter circuits can make its effect on circuit performance small. At toom temperatures I_{co} ordinarily amounts to a few microamperes. For junction transistors of type M1752, the change in I_{co} from room temperature to 150F can be expected to be less than 100 microamperes. The effect of increased I_{co} is to decrease the maximum-power output by decreasing the amount of signal which leads to distortion. In some circuits the changes in I_{co} are amplified and reduce very greatly maximum-undistorted power output. Analysis of the equivalent circuit of Fig. 6 with the two pessimistic approximations, $r_c=\infty$, $\alpha=1$, gives the relationships

$$\Delta i_\epsilon \cong \Delta I_{co}\frac{R_g+\dfrac{R_1R_2}{R_1+R_2}}{R_-+r_\epsilon}, \tag{5}$$

$$\Delta I_c \cong \Delta I_{co}\left(1+\frac{R_g+\dfrac{R_1R_2}{R_1+R_2}}{R_-+r_\epsilon}\right), \tag{6}$$

$$\Delta V_c \cong -\Delta I_{co}\left[R_g+\frac{R_1R_2}{R_1+R_2}+R_+\left(1+\frac{R_g+\dfrac{R_1R_2}{R_1+R_2}}{R_-+r_\epsilon}\right)\right]. \tag{7}$$

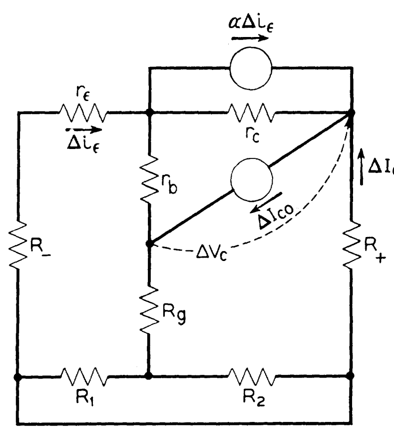


Fig. 6—Equivalent circuit to determine effect of ΔI_{co}.

On the basis of (6) and (7), to minimize variations in the point of operation, one should keep the ratio

$$\frac{R_g+\dfrac{R_1R_2}{R_1+R_2}}{R_--r_\epsilon}$$

to small values. The change in I_c of a few hundred microamperes, obtained when the above ratio is a few units, is still a small fraction of the usual quiescent-collector currents which are a few milliamperes.

Acknowledgment

The author is pleased to acknowledge the suggestions of R. L. Wallace, J. L. Merrill, J. A. Weller, and G. Raisbeck. Wallace observed that transistors should be ideal in the El type of repeater and suggested a first-circuit configuration to effect the substitution. Merrill and Weller have given suggestions on the negative-impedance repeater problem in general and on the application of transistors to it. Wallace and Raisbeck have made helpful comments on revision of the manuscript.

Appendix

Open and Short-Circuit Stability of Different Types of Converters

The property of open-circuit stability holds for transistor converters in which the input terminals are the emitter terminals, independent of the nature of the load or the cross-coupling networks, as long as these are passive. For converters in which the input terminals are the collectors of the transistors, one finds the property of short-circuit stability from the input terminals holds, independent of the nature of the passive cross-coupling network, for any resistive load as long as $|\alpha| \leqq 1$, or for any passive load as long as α is real and less than 1.

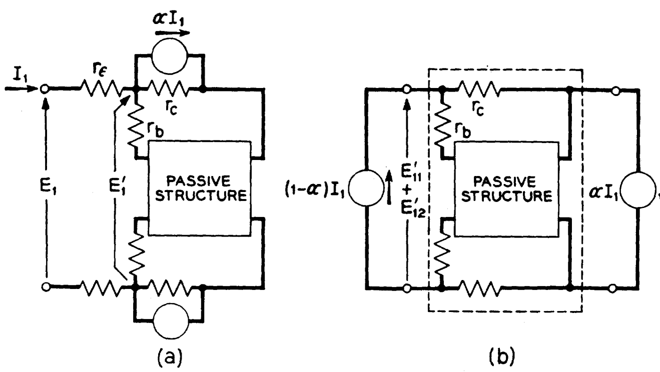


Fig. 7(a) and (b)—Open-circuit stable converter with arbitrary cross-coupling and equivalent circuit.

The open-circuit stability of the converter shown in Fig. 7 is identified with the fact that the impedance,

$$Z = \frac{E_1}{I_1},$$

has no poles in the right half of the complex-frequency plane.[1] The impedance Z is simply

$$2r_\epsilon + \frac{E_1'}{I_1}.$$

One can evaluate

$$\frac{E_1'}{I_1}$$

as being

$$\frac{E_{11}'}{I_1} + \frac{E_{12}}{I_1}$$

for the structures shown in Fig. 7(b), where E_{11}' is associated with the left source and E_{12}' is associated with the right source. Clearly,

$$Z = \frac{E_1}{I_1} + 2r_\epsilon + Z_{11}(1 - \alpha) + Z_{12}(\alpha), \tag{8}$$

where Z_{11} and Z_{12} are the driving-point and transfer impedances of the passive four-pole shown in the dotted lines of Fig. 7(b). The poles of Z are the poles of Z_{11}, which are the same as those of Z_{12} and the pole of α which is in the left half-plane. The poles of Z_{11} are in the left half-plane since the structure is a passive one. Hence this type of converter must be open-circuit stable.

A converter in which the input terminals are at the collectors is shown in Fig. 8. The short-circuit stability is to be verified by the fact that the input admittance $Y = I_1/E_1$ has no poles in the right half of the complex-frequency plane. To evaluate Y one considers Fig. 8(b) in which I_1 is represented as the sum of the current through the current generator, plus that into the other structure, I_{11}. The emitter and load currents are split into two paralleled parts. The current I_{12} is simply $-\alpha I_\epsilon$, and $(1-\alpha)I_\epsilon$ is determined as the product of E_1 and the transfer admittance Y_{21} of the structure shown

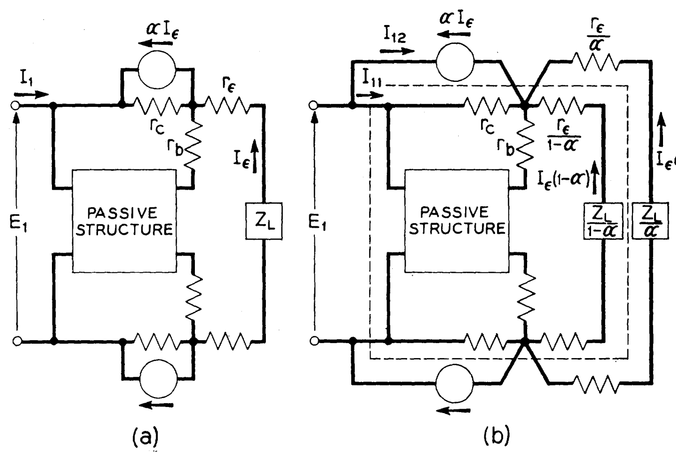


Fig. 8(a) and (b)—Short-circuit stable converter with arbitrary cross-coupling and equivalent circuit.

inside the dotted lines in Fig. 8(b). By consideration of Fig. 8(b), one sees that

$$Y = \frac{I_1}{E_1} = \frac{I_{11}}{E_1} + \frac{I_{12}}{E_1} = Y_{11} + \left(\frac{-\alpha}{1-\alpha}\right)Y_{21}, \tag{9}$$

where Y_{11} and Y_{21} are the driving-point and transfer (from input to loop containing $Z_L/1-\alpha$) admittances of the network shown in Fig. 8(b) inside the dotted lines. This network is certainly a passive structure if α is real and less than one, or if $|\alpha| \leqq 1$ and Z_L is resistive. The poles of Y_{11} and Y_{21} are those of Y; and under the above conditions, the passivity of the network inside the dotted lines of Fig. 8(b) assures that these poles lie in the left half-plane. Hence the type of converter illustrated is short-circuit stable.