Introduction to Computer Vision S24 Assignment #1

Due April 1, '24/(Mon)

## 1. Camera Calibration

- A $3 \times 4$ camera projection matric $\mathbf{P}$ maps a world point $\mathbf{X}_i = [X_i \quad Y_i \quad Z_i \quad 1]^T$ to its corresponding image point $\mathbf{x}_i = \omega[u_i \quad v_i \quad 1]^T$ such that $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$, which gives two equations w.r.t. the unknown parameters of $\mathbf{P} = [m_{ij}]$:

$$m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14} - m_{31}u_iX_i - m_{32}u_iY_i - m_{33}u_iZ_i - m_{34}u_i = 0$$
$$m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24} - m_{31}v_iX_i - m_{32}v_iY_i - m_{33}v_iZ_i - m_{34}v_i = 0$$

- Given $n \geq 6$ pairs of corresponding points, we have an over-determined system of equations, $\mathbf{Ap} = \mathbf{0}$, where $\mathbf{p}$ is the vectorized form of $\mathbf{P}$.
- Consider the SVD of $\mathbf{A}$ as $\mathbf{A} = \mathbf{UDV}^T$. Then the least square solution $\mathbf{p} = argmin_{\mathbf{p}}||\mathbf{Ap}||$, $||\mathbf{p}||$=1 is the last column of $\mathbf{V}$, which is the singular vector corresponding to the smallest singular value. Then, you can construct the camera projection matrix $\mathbf{P}$ using $\mathbf{p}$ up to scale.

a) Show that the solution **p** is the last column of **V** correspoding to the smallest singular value of **A**.

b) Given the following corresponding points, determine the camera projection matrix **P** using the SVD method.

| $(u$ | $v)$ | $(X$ | $Y$ | $Z)$ |
|---|---|---|---|---|
| 880 | 214 | 312.747 | 309.140 | 30.086 |
| 43 | 203 | 305.796 | 311.649 | 30.356 |
| 270 | 197 | 307.694 | 312.358 | 30.418 |
| 886 | 347 | 310.149 | 307.186 | 29.298 |
| 745 | 302 | 311.937 | 310.105 | 29.216 |
| 943 | 128 | 311.202 | 307.572 | 30.682 |
| 476 | 590 | 307.106 | 306.876 | 28.660 |
| 419 | 214 | 309.317 | 312.490 | 30.230 |
| 317 | 335 | 307.435 | 310.151 | 29.318 |
| 783 | 521 | 308.253 | 306.300 | 28.881 |
| 235 | 427 | 306.650 | 309.301 | 28.905 |
| 665 | 429 | 308.069 | 306.831 | 29.189 |
| 655 | 362 | 309.671 | 308.834 | 29.029 |
| 427 | 333 | 308.255 | 309.955 | 29.267 |
| 412 | 415 | 307.546 | 308.613 | 28.963 |
| 746 | 351 | 311.036 | 309.206 | 28.913 |
| 434 | 415 | 307.518 | 308.175 | 29.069 |
| 525 | 234 | 309.950 | 311.262 | 29.990 |
| 716 | 308 | 312.160 | 310.772 | 29.080 |
| 602 | 187 | 311.988 | 312.709 | 30.514 |

c) Another method to determine **P** is using the pseudo inverse. By setting $m_{34}$=1, we have $\mathbf{Ap} = \mathbf{b}$, and the least square solution $\mathbf{p} = argmin_{\mathbf{p}}||\mathbf{Ap} - \mathbf{b}||$, become $\mathbf{p} = \mathbf{A}^{+}\mathbf{b}$, where $\mathbf{A}^{+} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ is the pseudo inverse of **A**. Determine **P** for the data in b) using the pseudo inverse method.

**Implementation & Submission instructions:**

- Implementation instruction: Use Python for your implementation.
- Submission instructions: Upload the electronic file with the report, source code, and data in a single zip format with the name "**ICV_assignment#1_yourname.zip**" on the ETL class

homepage.

- The report should include the brief description of the problems, code, results, and discussions
- Reference
  - Quick tutorial page on how to use Matlab
  - Quick tutorial page on how to use Image Processing Toolbox
  - OpenCV
  - OpenCV-Python Tutorials

Note: All works should be individual-based. NO copy is allowed.