Project 2: PINNs for the Lotka-Volterra Model

Brin Summer School on SciML

Consider the following (scaled) Lotka-Volterra systems that models the dynamics of prey population x and predator population y:

$$x'(t) = \alpha x(t)(1 - Kx(t)) - \beta x(t)y(t) \tag{1}$$
$$y'(t) = -\gamma y(t) + \delta x(t)y(t) \tag{2}$$
$$x(0) = x_0,\ y(0) = y_0 \tag{3}$$

where:

- $\alpha > 0$ is the prey birth/growth rate
- $\beta > 0$ is the predation rate and controls the effect of presence of predators on the prey death rate
- $\gamma > 0$ is the predator death rate
- $\delta > 0$ controls the effect of presence of prey on the predator growth rate
- $K \geq 0$ with $1/K$ denoting the carrying capacity for prey. If $K = 0$, the prey growth rate is exponential in the absence of predators, while with $K > 0$ the growth rate is logistic with the total prey population capped.

Design a PINN that takes t as input to find the solution of the above system with $\alpha = 80, \beta = 15, \gamma = 15, \delta = 4, x_0 = 10, y_0 = 5$. Reference solutions are provided for $K = 0, 0.001, 0.01$. Investigate the performance of the PINN if

- The collocation points are chosen to cover the whole interval $[0, 2]$
- The collocation points are chosen only from the $[0, 1.5]$. Is the PINN solution accurate when tested on $t \in [1.5, 2]$? Here you are testing the PINN's extrapolation capabilities.
- Does the PINN performance differ depending on the value of K?

Bonus task: You can also train a PINN that takes K as input, so that a single network can be used to approximate the solution for all K.