1. A function $f : R^n \to R^m$ is called affine if for any $x, y \in R^n$ and any $\alpha, \beta \in R$ with $\alpha + \beta = 1$, we have

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(x)$$

   (a) Let $x, y \in R^m, \alpha, \beta \in R$. Take $f(\alpha x + \beta y) = A(\alpha x + \beta y) + b$. Without loss of generality, take an entry z_i from $z \in R^m$, z the result of $f(\alpha x + \beta y)$. Let $c_{i1}, c_{i2}, ..., c_{im}$ be the coefficients in the ith row of matrix A (A must be $m \times m$ for Ax to make sense). Then $z_i = c_{i1}(\alpha x_1 + \beta y_1) + ... + c_{im}(\alpha x_m + \beta y_m) + b_i = c_{i1}(\alpha x_1)c_{i1}(\beta y_1) + ... + c_{im}(\alpha x_m) + c_{im}(\beta y_m) + \alpha + \beta b_i$ (By distribution of scalar multiplication and the fact that $\alpha + \beta = 1$) $= A(\alpha x) + A(\beta y) + (\alpha + \beta)b = \alpha A(x) + \beta A(y) + \alpha b + \beta b = \alpha A(x) + \alpha b + \beta A(y) + \beta b$ (By commutativity of vector addition, homogenity of matrix multiplication) $= f(\alpha x) + f(\beta y)$.$\blacksquare$

   (b) Let $g(x) = f(x) - f(0)$. Then $g(x) = A(x) + b - (A(0) + b) = A(x) + b - b = A(x)$. Then $g(x)$ is linear by the fact that every matrix product is a linear operation. Therefore for any $f(x)$, the matrix A is unique. Then b is unique, since it is uniquely determined by $A_i + b_i = 1$.