Decomposition- Class 2

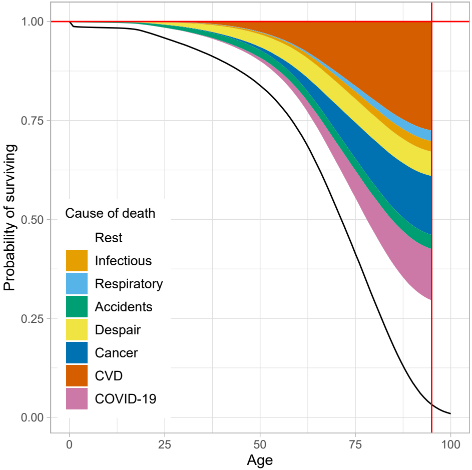
José Manuel Aburto

Kitagawa (1955)

To avoid the interaction term, Kitagawa suggests

$$\Delta CDR = \underbrace{\sum_x \left(\frac{M_x(t_2)+M_x(t_1)}{2}\right)\left(\frac{N_x(t_2)}{N(t_2)}-\frac{N_x(t_1)}{N(t_1)}\right)}_{\text{Changes in x-composition}} + \underbrace{\sum_x \left(\frac{\frac{N_x(t_2)}{N(t_2)}+\frac{N_x(t_1)}{N(t_1)}}{2}\right)(M_x(t_2)-M_x(t_1))}_{\text{Changes in rates}} \quad (1)$$

Life years lost: Andersen et al (2013)



Source: Aburto et al 2022

Outline

- Prevalence-based lifetables
- Difference in life expectancy

Sullivan's (1971) method

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Most widely method to calculate Disability-free life expectancy

Data used in Sullivan's method:

- Mortality data from a period lifetable
- Disability (or any) prevalence from cross-sectional survey

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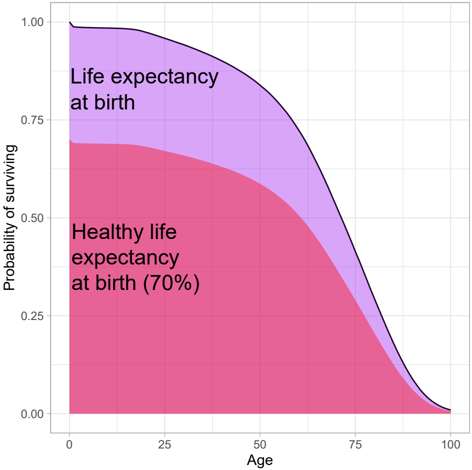
It represents the conditional probability that an individual of this cohort is disabled at age a given he/she survived up to age a.


Life expectancy
at birth
Healthy life
expectancy
at birth (70%)
Probability of surviving
1.00
0.75
0.50
0.25
0.00
0
25
50
75
100
Age

Additional info

- For a statistical foundation of the method see Imai & Soneji (2007) (including confidence intervals)
- For a guide see: Health Expectancy Calculation by the Sullivan Method: A Practical Guide
- See Decomposing Gaps in Healthy Life Expectancy. Van Raalte and Nepomuceno 2020

Challenge 3: Get (any source) age-specific prevalences and apply the Sullivan method in R. Follow the practical guide if you want to and be aware of the method's limitations (e.g. same mortality schedule).

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Long and happy living: Trends and patterns of happy life expectancy in the U.S., 1970–2000

Yang Yang

Department of Sociology, Population Research Center and Center on Aging at NORC, The University of Chicago, 1126 E. 59th Street, Chicago, IL 60637, USA

Available online 27 August 2007

Table 2
Life expectancy in happiness by age and sex: U.S. 1970–2000

Expectation of life	Men					Women				
	1970	1980	1990	2000	1970–2000	1970	1980	1990	2000	1970–2000
At age 30 years										
Total (TLE)	40.5	42.8	44.0	45.9	5.4	46.9	49.3	50.1	50.5	3.6
Happy (HapLE)	34.8	37.5	39.3	41.6	6.8	40.8	43.0	43.9	44.4	3.6
Very happy (VHapLE)	13.9	14.7	15.2	15.8	1.9	17.3	17.3	16.7	16.0	−1.3
Pretty happy (PHapLE)	20.9	22.8	24.2	25.8	4.9	23.5	25.7	27.2	28.4	4.9
Unhappy (UHapLE)	5.7	5.2	4.7	4.3	−1.4	6.1	6.3	6.2	6.1	0.0
% of life expectancy										
Total	100.0	100.0	100.0	100.0		100.0	100.0	100.0	100.0	
Happy	86.0	87.8	89.3	90.7	4.7	86.9	87.2	87.6	87.9	1.0
Very happy	34.4	34.5	34.4	34.4	0.0	36.9	35.1	33.3	31.6	−5.3
Pretty happy	51.6	53.3	54.9	56.3	4.7	50.1	52.2	54.3	56.3	6.2
Unhappy	14.0	12.2	10.7	9.3	−4.7	13.1	12.8	12.4	12.1	−1.0

Sullivan method

Exercise 1

Life expectancy decomposition

In the 1980s, several authors developed similar approaches: Pollard (1982), Andreev (1982), Arriaga (1984), Pressat (1985) ...

We focus on Arriaga's method

Arriaga (1984)

Effects of mortality change by age groups on life expectancies ($\sum_n \Delta_x =$ Total change):

$$
{}_n\Delta_x = \underbrace{\frac{\ell_x^1}{\ell_0^1}\left(\frac{{}_nL_x^2}{\ell_x^2} - \frac{{}_nL_x^1}{\ell_x^1}\right)}_{\text{Direct effect}} + \underbrace{\frac{T_{x+n}^2}{\ell_0^1}\left(\frac{\ell_x^1}{\ell_x^2} - \frac{\ell_{x+n}^1}{\ell_{x+n}^2}\right)}_{\text{Indirect and interaction effects}}
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- Change in mortality rates between ages x and $x+n \longrightarrow$ effect of change in number of years lived between x and $x+n$ on life expectancy.

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Note: for the open-ended age group there is only direct effect: ${}_\infty\Delta_x = \frac{\ell_x^1}{\ell_0^1}\left(\frac{T_x^2}{\ell_x^2} - \frac{T_x^1}{\ell_x^1}\right)$

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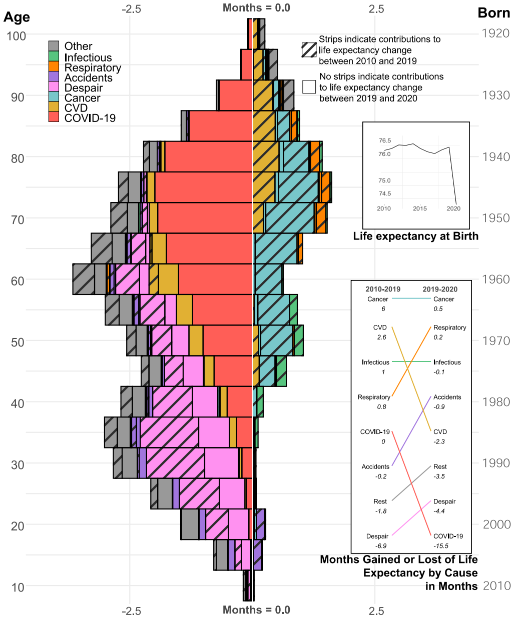
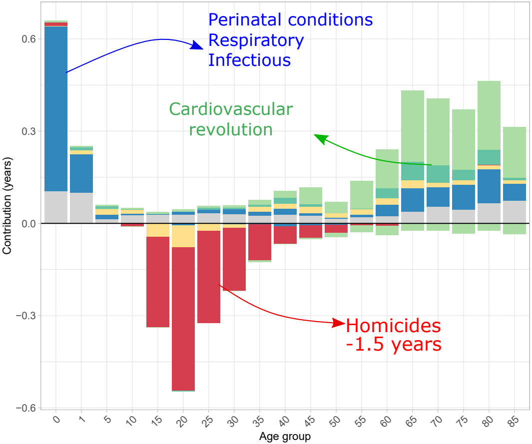
$$
\begin{aligned}
{}_n\Delta_x^i &=_n \Delta_x \frac{{}_n m_{x,i}^2 -_n m_{x,i}^1}{{}_n m_x^2 -_n m_x^1} \\
&=_n \Delta_x \frac{{}_n R_{x,i}^2[{}_n m_x^2] -_n R_{x,i}^1[{}_n m_x^1]}{{}_n m_x^2 -_n m_x^1}
\end{aligned}
$$

where ${}_n R_{x,i}^t$ is the proportion of deaths from cause i in age groups x to $x+n$ in population t, and ${}_n\Delta_x$ is the contribution of all-cause mortality differences in the same age group. ${}_n m_x^t$ is the mortality rate in the same age groups of population t.


Age
Months = 0.0
-2.5
2.5
Born
Other
Infectious
Respiratory
Accidents
Despair
Cancer
CVD
COVID-19
Strips indicate contributions to life expectancy change between 2010 and 2019
No strips indicate contributions to life expectancy change between 2019 and 2020
Life expectancy at Birth
2010-2019
2019-2020
Cancer 6
Cancer 0.5
CVD 2.6
Respiratory 0.2
Infectious 1
Infectious -0.1
Respiratory 0.8
Accidents -0.9
COVID-19 0
CVD -2.3
Accidents -0.2
Rest -3.5
Rest -1.8
Despair -4.4
Despair -6.9
COVID-19 -15.5
Months Gained or Lost of Life Expectancy by Cause in Months

Venezuela: Change in male life expectancy at birth by cause of death 1996-2013, from 68.6 to 70.6



García & Aburto 2019. International Journal of Epidemiology

Arriaga's method

Exercise 2