Decomposition- Class 4

José Manuel Aburto

Review

Kitagawa (1955)

$$
\begin{aligned}
\Delta CDR = & \underbrace{\sum_x \left(\frac{M_x(t_2) + M_x(t_1)}{2} \right) \left(\frac{N_x(t_2)}{N(t_2)} - \frac{N_x(t_1)}{N(t_1)} \right)}_{\text{Changes in x-composition}} + \\
& \underbrace{\sum_x \left(\frac{\frac{N_x(t_2)}{N(t_2)} + \frac{N_x(t_1)}{N(t_1)}}{2} \right) (M_x(t_2) - M_x(t_1))}_{\text{Changes in rates}}
\end{aligned} \tag{1}
$$

Review

LYL $\longrightarrow$ Andersen et al (2013), Erlangsen et al (2016)

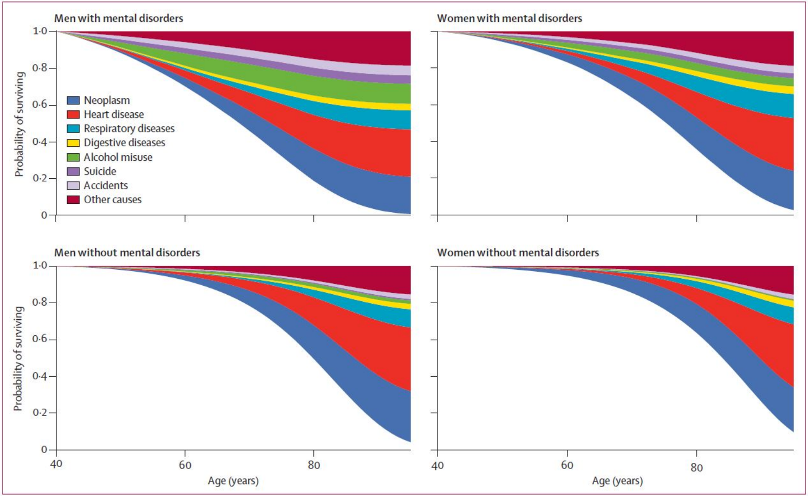


Figure 1: **Probabilities of survival and deaths from different causes for people with and without mental disorders aged between 40 and 94 years living in Denmark between 1995 and 2014**

Review

Life expectancy at age x is defined as

$$e(x)_{=}\frac{\int_x^\infty \ell(a)da}{\ell(x)}$$

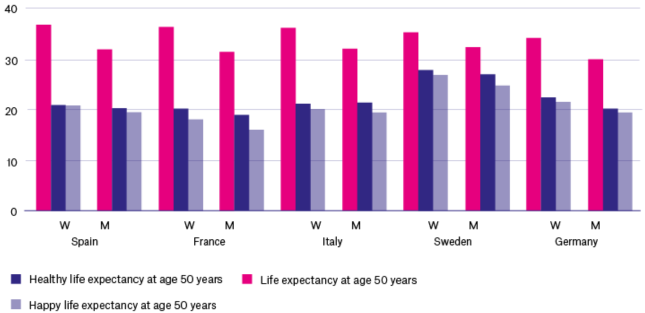
Then we can define disability-free life expectancy as

$$e^{DF}(x) = \frac{\int_x^\infty [1-\pi(a)]\ell(a)da}{\ell(x)}$$

Review

Happy life expectancy, Solé-Auró et al (2018)

Graph 3. **Life expectancy, healthy life expectancy and happy life expectancy.**



Note 1: data on life expectancy and healthy life expectancy were obtained from Eurostat. Data on happy life expectancy were obtained from the SHARE database.
Note 2: W: women; M: men.

Source: compiled by the author adapted from Solé-Auró *et al.* (2018) based on data from SHARE (2011) and Eurostat (2018).

Recap

Arriaga (1984) Effects of mortality change by age groups on life expectancies ($\sum_n \Delta_x =$ Total change):

$$
{}_n\Delta_x = \underbrace{\frac{\ell_x^1}{\ell_0^1}\left(\frac{{}_nL_x^2}{\ell_x^2} - \frac{{}_nL_x^1}{\ell_x^1}\right)}_{\text{Direct effect}} + \underbrace{\frac{T_{x+n}^2}{\ell_0^1}\left(\frac{\ell_x^1}{\ell_x^2} - \frac{\ell_{x+n}^1}{\ell_{x+n}^2}\right)}_{\text{Indirect and interaction effects}}
$$

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The impact of violence on Venezuelan life expectancy and lifespan inequality

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Abstract

Background

Venezuela is one of the most violent countries in the world. According to the United Nations, homicide rates in the country increased from 32.9 to 61.9 per 100 000 people between 2000 and 2014. This upsurge coincided with a slowdown in life expectancy improvements. We estimate mortality trends and quantify the impact of violence-related deaths and other causes of death on life expectancy and lifespan inequality in Venezuela.

Recap

$$ {}^{*}_{n}p_x^i = \left[{}_n p_x\right]^{R_i} $$

$$ = \left[{}_n p_x\right]^{\frac{{}_nD_x^i}{{}_nD_x}} $$

Now with this simple relation we can create hypothetical scenarios.
`https://demographs.shinyapps.io/saudiarabia_wb2018/`

Recap

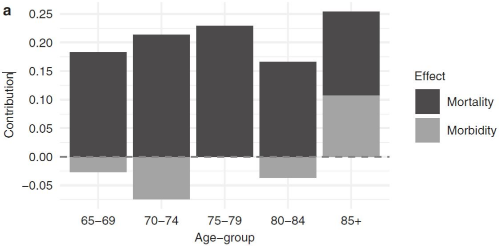
$$y_2 - y_1 = \sum_{i=1}^{n} \int_{x_{i1}}^{x_{i2}} \frac{\partial y}{\partial x_{i1}} dx_i = \sum_{i=1}^{n} c_i \tag{2}$$

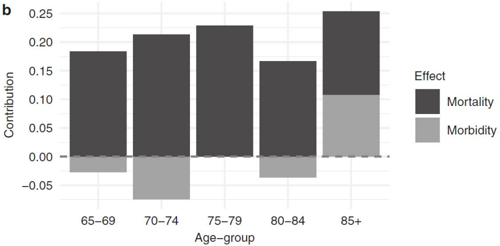
$$y_2 - y_1 = \sum_{i=1}^{n} c_i \tag{3}$$

c_i is the total change in y produced by changes in the i-th covariate, x_i.

Decomposing Gaps in Healthy Life Expectancy 7

Alyson A. van Raalte and Marília R. Nepomuceno


a
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
Contribution
65−69
70−74
75−79
80−84
85+
Age−group
Effect
Mortality
Morbidity


b
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
Contribution
65−69
70−74
75−79
80−84
85+
Age−group
Effect
Mortality
Morbidity

Advanced topics in formal demography

The change over time in e_0 is given by

$$\frac{\partial e_o}{\partial t} = \int_0^\infty \rho(x)\mu(x)\ell(x)e(x)dx \tag{4}$$

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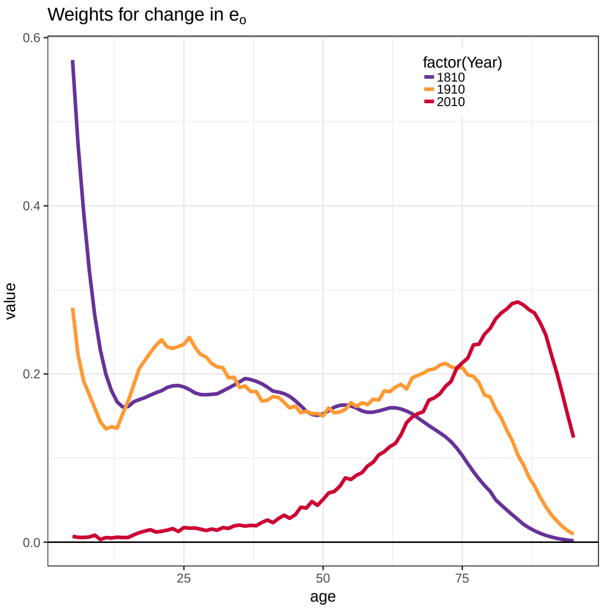
$$\frac{\partial e_o}{\partial t} = \int_0^\infty \rho(x)\mu(x)\ell(x)e(x)dx \tag{4}$$

where

$$\rho(x) = -\frac{\partial\mu(x)/dt}{\mu(x)}$$

$\dot{e_o} =$ **is a weighted total of rates of mortality improvement** ρ (Vaupel & Canudas Romo, 2003)


Weights for change in e$_o$
0.6
0.4
0.2
0.0
value
factor(Year)
1810
1910
2010
25
50
75
age

The change over time in lifespan equality $\eta = -\ln\left(\frac{e^\dagger}{e_o}\right)$ is given by

$$\frac{\partial \eta}{\partial t} = \dot{\eta} = \frac{\dot{e_o}}{e_o} - \frac{\dot{e}^\dagger}{e^\dagger} \tag{5}$$

$$\dot{e}^\dagger = -\dot{e}_o - \int_0^\infty \frac{\partial \ell(x)}{\partial t} \ln \ell(x) dx$$

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where

$$\begin{aligned} \dot{e}^\dagger &= -\dot{e}_o - \int_0^\infty \frac{\partial \ell(x)}{\partial t} \ln \ell(x) dx \\ &= -\int_0^\infty \rho(x)\mu(x)\ell(x)[K(x) + e(x)]dx \end{aligned} \tag{6}$$

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where

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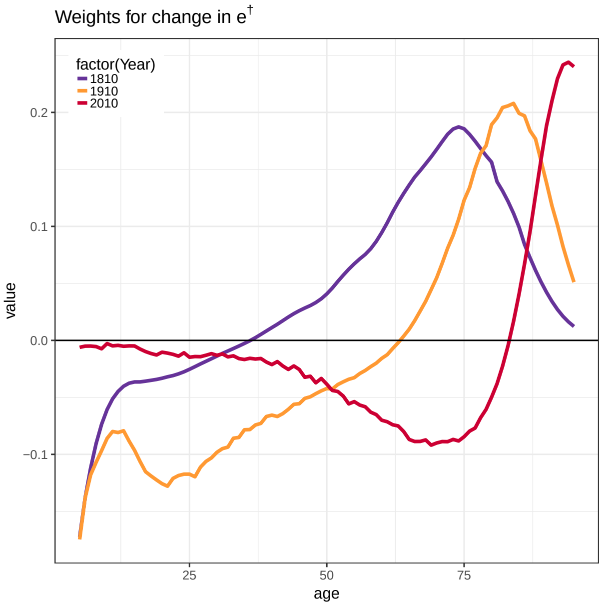
$\dot{e}^\dagger =$ **weighted total of rates of mortality improvement** ρ


Weights for change in e†
factor(Year)
1810
1910
2010
0.2
0.1
0.0
−0.1
value
25
50
75
age

We derive these expressions considering

$$e^{\dagger} = \int_0^{\infty} \mu(x) \int_x^{\infty} \ell(a) da dx$$

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$$e^{\dagger} = \int_0^{\infty} \mu(x) \int_x^{\infty} \ell(a) da dx$$

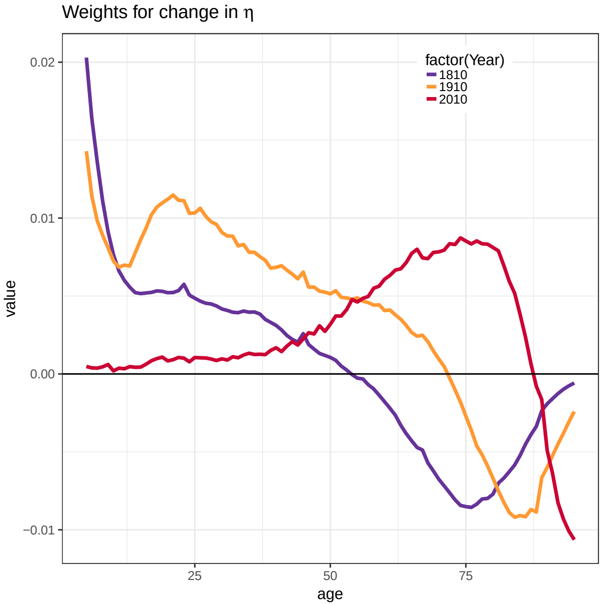
which leads to

$$\dot{e}^{\dagger} = \int_0^{\infty} \frac{\partial \mu(x)/\partial t}{\mu(x)} \ell(x) e(x) dx + \int_0^{\infty} \mu(x) \int_x^{\infty} \frac{\partial \ell(a)}{\partial t} da dx$$

$$\dot{\eta} = \int_0^\infty \rho(x)\left[\frac{e(x)}{e_o} + \frac{e(x)}{e^\dagger} + \frac{K(x)}{e^\dagger}\right]dx$$
$$= \int_0^\infty \rho(x) W(x) dx$$

$$\dot{\eta} = \int_0^\infty \rho(x) \left[\frac{e(x)}{e_o} + \frac{e(x)}{e^\dagger} + \frac{K(x)}{e^\dagger} \right] dx$$
$$= \int_0^\infty \rho(x) W(x) dx$$

Key point: change in η over time is a weighted total of ρ


Weights for change in η
factor(Year)
1810
1910
2010
0.02
0.01
0.00
−0.01
value
25
50
75
age

Separating contributions over time in early and late components

Given the threshold age a^η, changes over tome for η can be expressed as

$$\dot{\eta} = \underbrace{\int_0^{a^\eta} \rho(x)W(x)dx}_{\textit{early component} < a^\eta} + \underbrace{\int_{a^\eta}^{\infty} \rho(x)W(x)dx}_{\textit{late component} > a^\eta}$$

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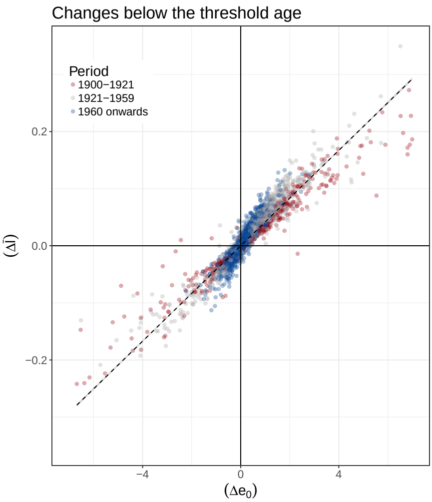
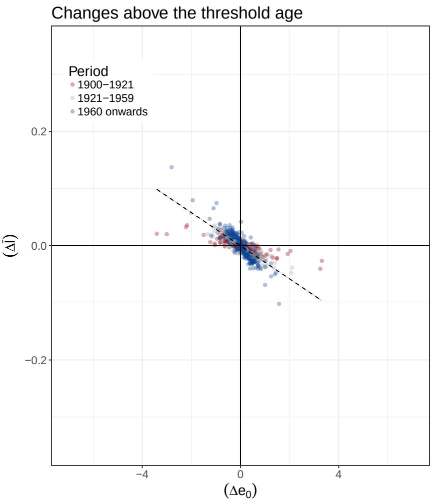
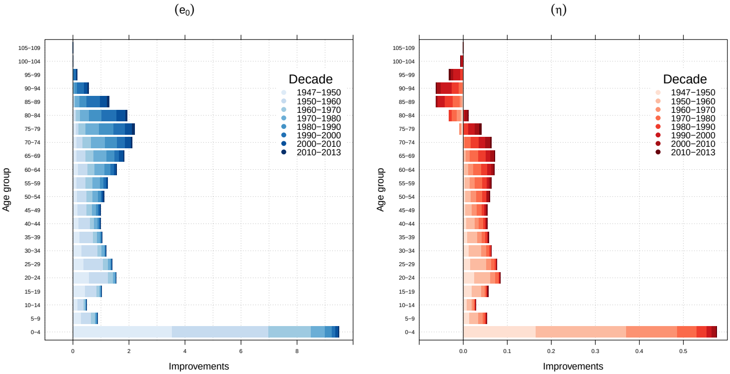
or

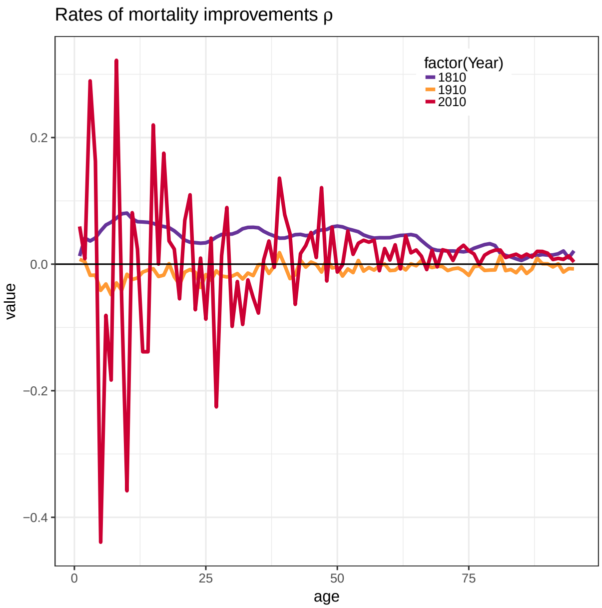
$$\dot{\eta} = \underbrace{\left[\frac{\dot{e}_o[x|x < a^\eta]}{e_o} - \frac{\dot{e}^\dagger[x|x < a^\eta]}{e_o}\right]}_{\textit{early component}} + \underbrace{\left[\frac{\dot{e}_o[x|x > a^\eta]}{e_o} - \frac{\dot{e}^\dagger[x|x > a^\eta]}{e_o}\right]}_{\textit{early component}}$$


Changes below the threshold age
Period
1900−1921
1921−1959
1960 onwards
0.2
0.0
−0.2
−4
0
4
$(\Delta \bar{l})$
(Δe_0)


Changes above the threshold age
Period
1900−1921
1921−1959
1960 onwards
0.2
0.0
−0.2
−4
0
4
$(\Delta \bar{l})$
(Δe_0)

Changes over time of lifespan equality




Rates of mortality improvements ρ
factor(Year)
1810
1910
2010
0.2
0.0
−0.2
−0.4
value
0
25
50
75
age