Decomposition techniques in population health research

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Kitagawa (1955)

To avoid the interaction term, Kitagawa suggests

$$\Delta CDR = \underbrace{\sum_x \left(\frac{M_x(t_2)+M_x(t_1)}{2}\right)\left(\frac{N_x(t_2)}{N(t_2)} - \frac{N_x(t_1)}{N(t_1)}\right)}_{\text{Changes in x-composition}} + \underbrace{\sum_x \left(\frac{\frac{N_x(t_2)}{N(t_2)} + \frac{N_x(t_1)}{N(t_1)}}{2}\right)(M_x(t_2) - M_x(t_1))}_{\text{Changes in rates}} \tag{1}$$

Direct vs Compositional

Let $\bar{v}(y)$ denote the mean value of $v(x, y)$ over x as

$$E(v) = \bar{v}(y) = \frac{\int_0^\infty v(x,y)w(x,y)dx}{\int_0^\infty w(x,y)dx}, x \text{ continuous}$$
$$= \frac{\sum_x v(x,y)w(x,y)dx}{\sum_x w(x,y)dx}, x \text{ discrete} \qquad (2)$$

where $v(x, y)$ is some demographic function and $w(x, y)$ is some weighting function.

$$\dot{\bar{v}} = \underbrace{\bar{\dot{v}}}_{\text{Direct component}} + \underbrace{Cov(v, \acute{w})}_{\text{Structural or compositional component}} \tag{3}$$

Main result in Vaupel & Canudas-Romo (2002)

$$\dot{e}_o(t) = \int_0^\infty \rho(x)e(x)f(x)dx \tag{4}$$

can be written as:

$$\dot{e}_o(t) = \bar{\rho}(t)e^\dagger(t) + Cov(\rho, e_x) \tag{5}$$

where $e^\dagger = \int_0^\infty e(x)f(x)da$ is the average life lost at time of death (Vaupel & Canudas-Romo, 2003).

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Lifespan Dispersion in Times of Life Expectancy Fluctuation: The Case of Central and Eastern Europe

José Manuel Aburto[1,2] · Alyson van Raalte[2]

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Abstract
Central and Eastern Europe (CEE) have experienced considerable instability in mortality since the 1960s. Long periods of stagnating life expectancy were followed by rapid increases in life expectancy and, in some cases, even more rapid declines, before more recent periods of improvement. These trends have been well documented, but to date, no study has comprehensively explored trends in lifespan variation. We improved such analyses by incorporating life disparity as a health indicator alongside life expectancy, examining trends since the 1960s for 12 countries from the region. Generally, life disparity was high and fluctuated strongly over the period. For nearly 30 of these years, life expectancy and life disparity varied independently of each other, largely because mortality trends ran in opposite directions over different ages. Furthermore, we quantified the impact of large classes of diseases on life disparity trends since 1994 using a newly harmonized cause-of-death time series for eight countries in the region. Mortality patterns in CEE countries were heterogeneous and ran counter to the common patterns observed in most developed countries. They contribute to the discussion about life expectancy disparity by showing that expansion/compression levels do not necessarily mean lower/higher life expectancy or mortality deterioration/improvements.

Source: Aburto & van Raalte

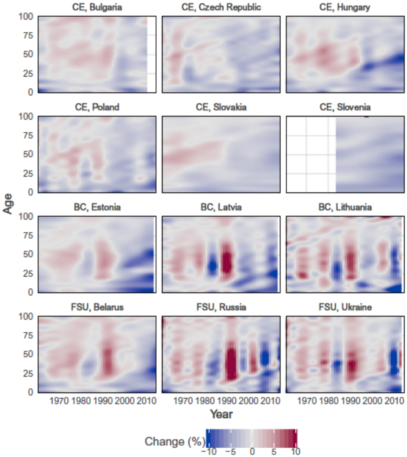

Fig. 1 Male mortality surface showing rates of mortality improvements. The regular white areas indicate no data available. *Source:* Own calculations based on Human Mortality Database (2016) data

Source: Aburto & van Raalte

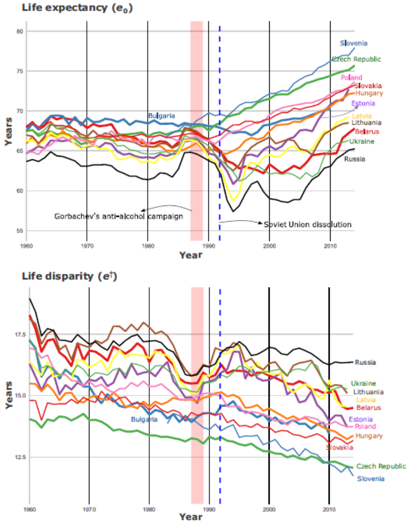

Fig. 2 Trends in male life expectancy (e_0) and lifespan disparity ($e^\dagger$) for 12 Eastern European countries, 1960–2014. *Source:* Own calculations based on Human Mortality Database (2016) data

Source: Aburto & van Raalte

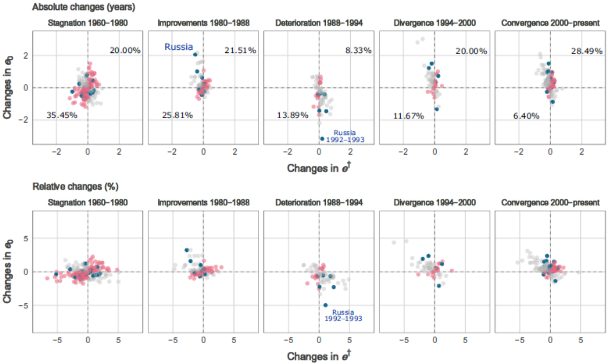

Fig. 3 Absolute and relative yearly changes in life expectancy and lifespan disparity, 1960–2010. Data for Slovenia begin in 1983. The black dots are related to changes experienced in Russia. The percentages correspond to the total changes occurred during each period. *Source:* Own calculations based on Human Mortality Database (2016) data

Source: Aburto & van Raalte

Challenge 1: Using data from the HMD, give a descriptive answer to the questions: Is there a female advantage in life disparity at shared levels of life expectancy? Does it matter if you measure life disparity from birth, age 50, or age 65? Use data for 5 countries over long time series.

Outline

- Prevalence-based lifetables
- Difference in life expectancy

Sullivan's (1971) method

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Most widely method to calculate Disability-free life expectancy

Data used in Sullivan's method:

- Mortality data from a period lifetable
- Disability (or any) prevalence from cross-sectional survey

Theoretical definitions

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$$e(x) = \frac{\int_x^\infty \ell(a)da}{\ell(x)}$$

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It represents the conditional probability that an individual of this cohort is disabled at age a given he/she survived up to age a.

Additional info

- For a statistical foundation of the method see Imai & Soneji (2007) (including confidence intervals)
- For a guide see: Health Expectancy Calculation by the Sullivan Method: A Practical Guide
- See Decomposing Gaps in Healthy Life Expectancy. Van Raalte and Nepomuceno 2020

Challenge 2: Get (any source) age-specific prevalences and apply the Sullivan method in R. Follow the practical guide if you want to and be aware of the method's limitations (e.g. same mortality schedule).

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Long and happy living: Trends and patterns of happy life expectancy in the U.S., 1970–2000

Yang Yang

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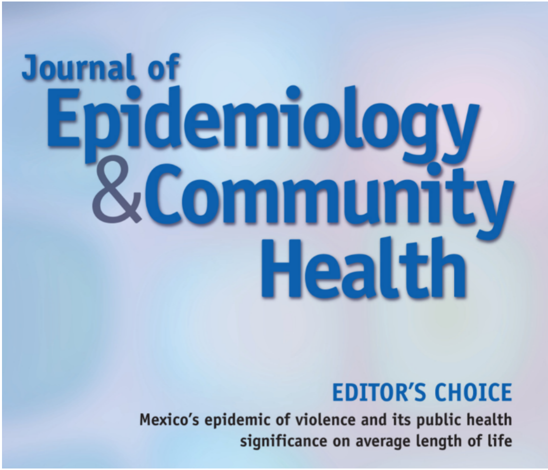
Available online 27 August 2007

Table 2
Life expectancy in happiness by age and sex: U.S. 1970–2000

Expectation of life	Men					Women				
	1970	1980	1990	2000	1970–2000	1970	1980	1990	2000	1970–2000
At age 30 years										
Total (TLE)	40.5	42.8	44.0	45.9	5.4	46.9	49.3	50.1	50.5	3.6
Happy (HapLE)	34.8	37.5	39.3	41.6	6.8	40.8	43.0	43.9	44.4	3.6
Very happy (VHapLE)	13.9	14.7	15.2	15.8	1.9	17.3	17.3	16.7	16.0	−1.3
Pretty happy (PHapLE)	20.9	22.8	24.2	25.8	4.9	23.5	25.7	27.2	28.4	4.9
Unhappy (UHapLE)	5.7	5.2	4.7	4.3	−1.4	6.1	6.3	6.2	6.1	0.0
% of life expectancy										
Total	100.0	100.0	100.0	100.0		100.0	100.0	100.0	100.0	
Happy	86.0	87.8	89.3	90.7	4.7	86.9	87.2	87.6	87.9	1.0
Very happy	34.4	34.5	34.4	34.4	0.0	36.9	35.1	33.3	31.6	−5.3
Pretty happy	51.6	53.3	54.9	56.3	4.7	50.1	52.2	54.3	56.3	6.2
Unhappy	14.0	12.2	10.7	9.3	−4.7	13.1	12.8	12.4	12.1	−1.0


Journal of
Epidemiology
&Community
Health
EDITOR'S CHOICE
Mexico's epidemic of violence and its public health
significance on average length of life

An example

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An example

- Numerous studies have documented an increase in drug-related violence in Mexico after 2005.
- This is particularly notable in Mexico, which had one of the lowest homicide rates in Latin America
- Few studies have used the perception of vulnerability to assess the impact of violence on the quality of life.
- Aim: calculate the number of years that Mexicans would spend living with vulnerability.

Table 1 Prevalence of self-reported vulnerability among Mexicans (%), 2005, 2010 and 2014

	State			Home		
Age-group	Female	Male	p Value*	Female	Male	p Value*
2005						
20–29	54	49	0.000	16	13	0.000
30–39	56	51	0.000	17	13	0.000
40–49	56	49	0.000	16	13	0.000
50–59	54	46	0.000	15	12	0.000
60–69	51	43	0.000	14	9	0.000
70–79	44	41	0.125	10	10	0.723
80–89	37	35	0.478	9	7	0.387
90+	27	40	0.160	11	12	0.876
2010						
20–29	67	63	0.000	17	13	0.000
30–39	71	65	0.000	21	17	0.000
40–49	72	67	0.000	23	18	0.000
50–59	71	67	0.000	23	19	0.000
60–69	67	65	0.085	21	19	0.025
70–79	63	59	0.016	17	15	0.115
80–89	58	54	0.200	14	10	0.036
90+	54	47	0.357	19	15	0.495
2014						
20–29	72	63	0.000	22	16	0.000
30–39	74	63	0.000	28	21	0.000
40–49	74	65	0.000	30	24	0.000
50–59	74	67	0.000	30	25	0.000
60–69	72	66	0.000	27	24	0.007
70–79	68	63	0.001	23	18	0.000
80–89	62	59	0.166	17	17	0.871
90+	63	55	0.250	13	14	0.715

Data from the ENSI-2005, ENSI-2010[15] and ENVIPE-2014[16] surveys.
*p Values of differences in the prevalence of vulnerability between females and males were estimated with a normal z-test.

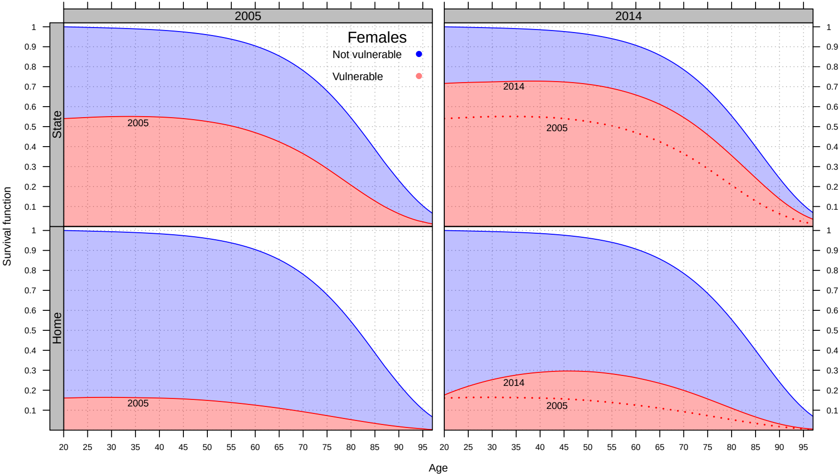
2005
2014
Females
Not vulnerable
Vulnerable
State
Home
Survival function
Age
2005
2014
2005
2005
2014
2005

Table 2 Mexican life expectancy with and without vulnerability at selected ages, 2005, 2010 and 2014

	Age	Life expectancy (95% CI)	Vulnerable expectancy (95% CI)		Vulnerable expectancy (% of total)	
			State	Home	State	Home
2005						
Females	20	59.2 (59.2 to 59.3)	30.1 (29.7 to 30.5)	8.4 (8.2 to 8.7)	51	14
	40	40.0 (40.0 to 40.1)	19.4 (19.1 to 19.9)	5.2 (5.1 to 5.6)	49	13
	60	22.4 (22.4 to 22.5)	9.6 (9.3 to 10.0)	2.4 (2.3 to 2.7)	43	11
	80	9.2 (9.2 to 9.3)	2.9 (2.5 to 3.4)	0.7 (0.5 to 1.0)	31	8
Males	20	54.4 (54.4 to 54.5)	24.9 (24.5 to 25.4)	6.5 (6.3 to 6.9)	46	12
	40	36.5 (36.5 to 36.6)	15.9 (15.6 to 16.4)	4.1 (3.9 to 4.4)	44	11
	60	20.2 (20.2 to 20.3)	8.0 (7.7 to 8.4)	2.1 (1.9 to 2.4)	40	11
	80	8.6 (8.6 to 8.7)	2.6 (2.2 to 3.0)	1.0 (0.7 to 1.4)	30	12
2010						
Females	20	59.3 (59.3 to 59.4)	39.8 (39.4 to 40.2)	11.7 (11.5 to 12.1)	67	20
	40	40.1 (40.1 to 40.2)	26.6 (26.3 to 27.0)	8.1 (7.9 to 8.4)	66	20
	60	22.5 (22.5 to 22.5)	14.0 (13.7 to 14.4)	4.1 (3.9 to 4.4)	62	18
	80	9.3 (9.3 to 9.4)	5.1 (4.8 to 5.5)	1.4 (1.2 to 1.7)	55	16
Males	20	53.8 (53.8 to 53.9)	34.0 (33.6 to 34.4)	8.6 (8.4 to 8.9)	63	16
	40	36.4 (36.4 to 36.5)	22.8 (22.5 to 23.3)	6.1 (5.9 to 6.4)	63	17
	60	20.2 (20.2 to 20.3)	12.0 (11.7 to 12.4)	3.1 (2.9 to 3.3)	59	15
	80	8.5 (8.5 to 8.6)	4.4 (4.1 to 4.8)	1.0 (0.8 to 1.3)	52	12
2014						
Females	20	59.5 (59.0 to 60.1)	42.3 (41.6 to 43.0)	15.3 (15.0 to 15.8)	71	26
	40	40.3 (39.8 to 40.8)	28.2 (27.7 to 28.9)	10.6 (10.3 to 11.0)	70	26
	60	22.6 (22.3 to 23.0)	15.1 (14.7 to 15.6)	5.2 (4.9 to 5.5)	67	23
	80	9.4 (9.3 to 9.6)	5.5 (5.2 to 5.9)	1.4 (1.2 to 1.7)	59	15
Males	20	54.4 (53.7 to 55.1)	34.6 (34.0 to 35.4)	11.1 (10.8 to 11.5)	64	20
	40	36.6 (36.1 to 37.1)	23.4 (22.9 to 24.1)	8.0 (7.7 to 8.4)	64	22
	60	20.4 (20.1 to 20.7)	12.7 (12.3 to 13.2)	4.1 (3.8 to 4.4)	63	20
	80	8.7 (8.6 to 8.9)	5.0 (4.6 to 5.4)	1.4 (1.2 to 1.7)	57	16

Authors' calculations, data from the ENSI-2005, ENSI-2010[15] and ENVIPE-2014[16] surveys, and period life tables.[17 18]

Life expectancy decomposition

In the 1980s, several authors developed similar approaches: Pollard (1982), Andreev (1982), Arriaga (1984), Pressat (1985) ...

We focus on Arriaga's method

Arriaga (1984)

Arriaga (1984) Effects of mortality change by age groups on life expectancies ($\sum_n \Delta_x =$ Total change):

$$
{}_n\Delta_x = \underbrace{\frac{\ell_x^1}{\ell_0^1}\left(\frac{{}_nL_x^2}{\ell_x^2} - \frac{{}_nL_x^1}{\ell_x^1}\right)}_{\text{Direct effect}} + \underbrace{\frac{T_{x+n}^2}{\ell_0^1}\left(\frac{\ell_x^1}{\ell_x^2} - \frac{\ell_{x+n}^1}{\ell_{x+n}^2}\right)}_{\text{Indirect and interaction effects}}
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- Change in mortality rates between ages x and $x+n \longrightarrow$ effect of change in number of years lived between x and $x+n$ on life expectancy.

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- Change in mortality rates between ages x and $x+n \longrightarrow$ effect of change in number of years lived between x and $x+n$ on life expectancy.
- Contribution resulting from the person-years to be added because additional survivors at age $x+n$ are exposed to new mortality conditions.

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- Change in mortality rates between ages x and $x+n \longrightarrow$ effect of change in number of years lived between x and $x+n$ on life expectancy.
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Note: for the open-ended age group there is only direct effect:
${}_\infty\Delta_x = \frac{\ell_x^1}{\ell_0^1}\left(\frac{T_x^2}{\ell_x^2} - \frac{T_x^1}{\ell_x^1}\right)$

Age-specific decomposition Exercise

Extension of causes of death Suppose there are $i = 1, 2 \ldots, k$ causes of death. Following multiple decrements operations

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$$ {}_n\Delta_x^i =_n \Delta_x \frac{{}_n m_{x,i}^2 -_n m_{x,i}^1}{{}_n m_x^2 -_n m_x^1} $$

Extension of causes of death Suppose there are $i = 1, 2 \ldots, k$ causes of death. Following multiple decrements operations

$$
\begin{aligned}
{}_n\Delta_x^i &=_n \Delta_x \frac{{}_n m_{x,i}^2 -_n m_{x,i}^1}{{}_n m_x^2 -_n m_x^1} \\
&=_n \Delta_x \frac{{}_n R_{x,i}^2[{}_n m_x^2] -_n R_{x,i}^1[{}_n m_x^1]}{{}_n m_x^2 -_n m_x^1}
\end{aligned}
$$

where ${}_n R_{x,i}^t$ is the proportion of deaths from cause i in age groups x to $x + n$ in population t, and ${}_n\Delta_x$ is the contribution of all-cause mortality differences in the same age group. ${}_n m_x^t$ is the mortality rate in the same age groups of population t.

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Example

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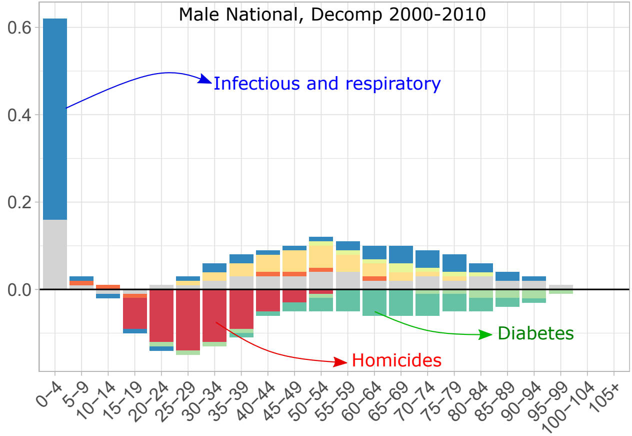
- This region has the **highest** homicide rate in the world (16.3 per 100,000).
- Central American countries $\longrightarrow$ **upsurge** in homicides in the new century.
- **In Mexico, rates doubled between 2007 and 2012** (9.3 $\longrightarrow$ 18.6).

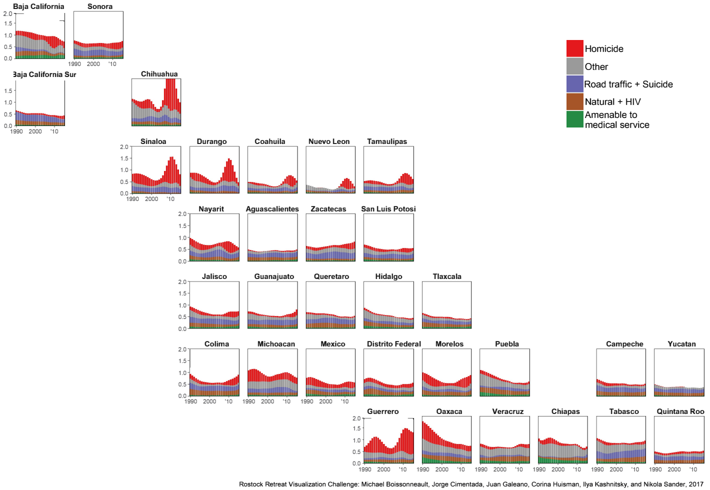
As a result

- Male life expectancy **stagnated** in the first decade of the 2000's (∽72y)



Application to temporary life expectancy

- Males between 15 and 50



Challenge 3: Extend the age-decomposition to age-cause-decomposition (5 causes) for the difference in life expectancy in a country of your choice, for example from the Human cause-of-death database, GBD, WHO, etc.)