Following Vaupel & Canudas-Romo (2002)

$$\dot{e}_o(t) = \int_0^\infty \rho(x)e(x)f(x)dx \tag{4}$$

can be written as:

$$\dot{e}_o(t) = \bar{\rho}(t)e^\dagger(t) + Cov(\rho, e_x) \tag{5}$$

where $e^\dagger = \int_0^\infty e(x)f(x)da$ is the average life lost at time of death (Vaupel & Canudas-Romo, 2003).

Reversing the order of integration

$$\frac{\partial}{\partial t} e_o(t) = \dot{e}_o(t) = \int_0^\infty \frac{\partial}{\partial t} \ell(x,t)\, dx$$
$$= -\int_0^\infty \ell(x,t) \int_0^x \frac{\partial}{\partial t} \mu(a,t)\, da\, dx$$
$$= -\int_0^\infty f(x) \int_0^x g(y)\, dy\, dx$$

$$\int_0^\infty f(x) \int_0^x g(y)\,dy\,dx$$
$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

$$\int_0^\infty f(x) \int_0^x g(y)\, dy\, dx$$

$$\int_0^\infty \int_{0=y}^{x=y} f(x)g(y)\, d_{\underset{\uparrow}{\mathrm{y}}}\, dx$$

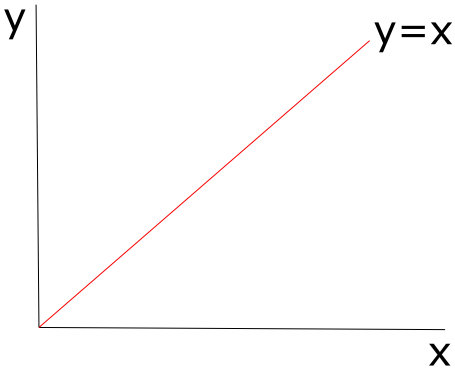
$$\int_0^\infty f(x) \int_0^x \; g(y)\, dy\, dx$$

$$\int_{0=x}^{\infty=x} \int_{0=y}^{x=y} f(x)g(y)\, d\underset{\uparrow}{\mathrm{y}}\, d\overset{\downarrow}{\mathrm{x}}$$

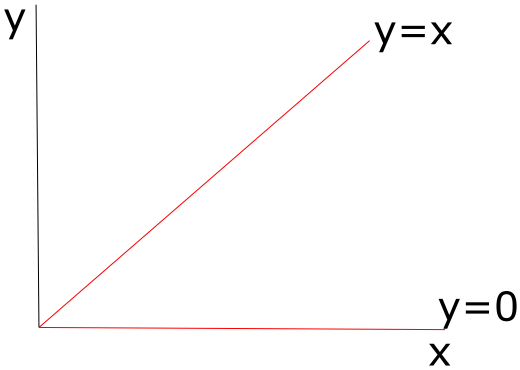
$$\int_0^\infty f(x) \int_0^x g(y)\, dy\, dx$$

$$\int_{0=x}^{\infty=x} \int_{0=y}^{x=y} f(x)g(y)\, d\underset{\uparrow}{\mathrm{y}}\, d\overset{\downarrow}{\mathrm{x}}$$

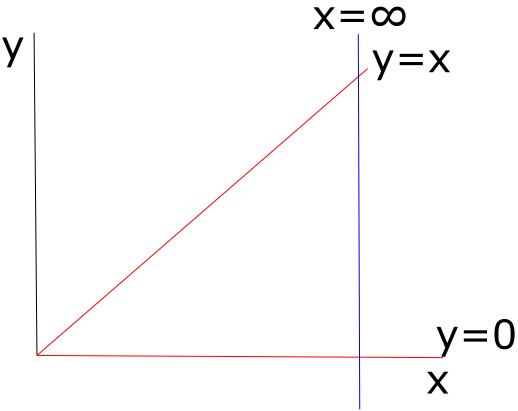
Graph those 4 lines


y
y=x
x

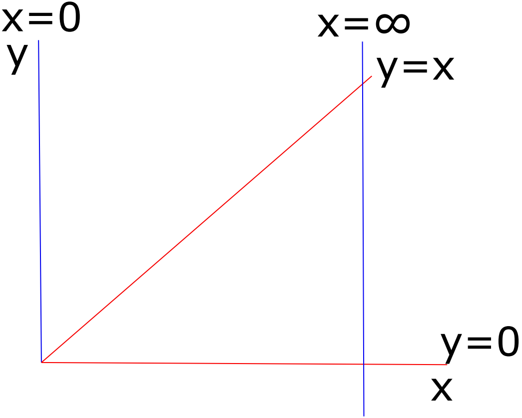
$$\int_{0=x}^{\infty=x} \int_{0=y}^{x=y} f(x)g(y)\, d\underset{\uparrow}{\mathrm{y}}\, d\overset{\downarrow}{\mathrm{x}}$$

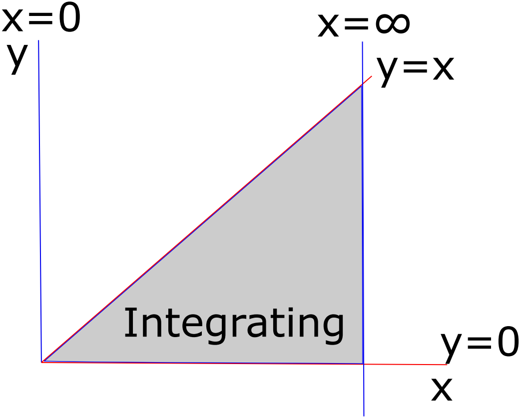

y
y=x
y=0
x

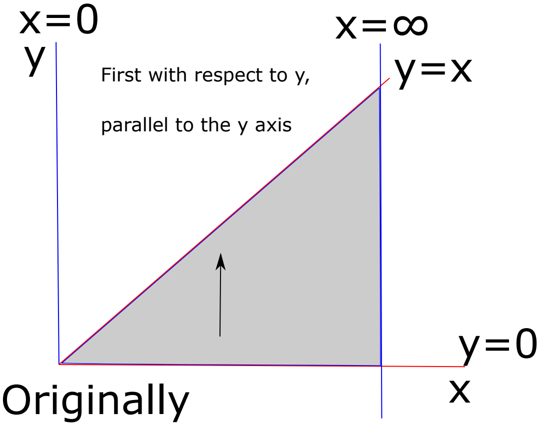
$$\int_{0=x}^{\infty=x} \int_{0=y}^{x=y} f(x)g(y)\, d_{\underset{\uparrow}{\mathrm{y}}}\, d\overset{\downarrow}{\mathrm{x}}$$

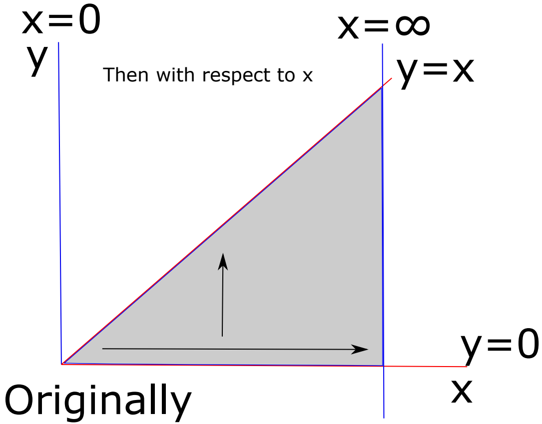

x=∞
y
y=x
y=0
x

$$\int_{0=x}^{\infty=x} \int_{0=y}^{x=y} f(x)g(y)\, d\underset{\uparrow}{\mathrm{y}}\, d\overset{\downarrow}{\mathrm{x}}$$


x=0
y
x=∞
y=x
y=0
x


x=0
y
x=∞
y=x
Integrating
y=0
x


x=0
y
x=∞
y=x
First with respect to y,
parallel to the y axis
y=0
x
Originally


x=0
y
x=∞
y=x
Then with respect to x
y=0
x
Originally

$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

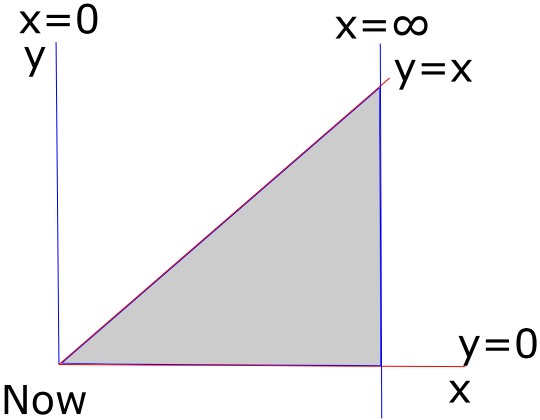
$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

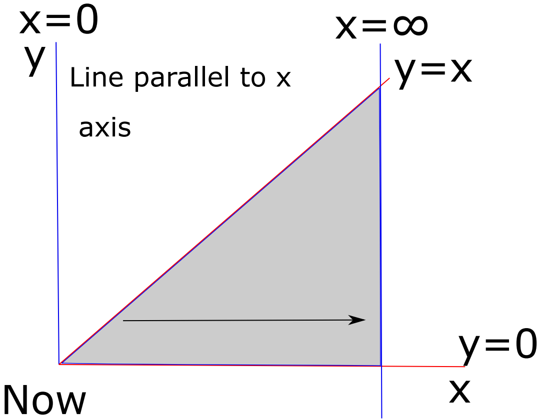
But we want to reverse the order of integration

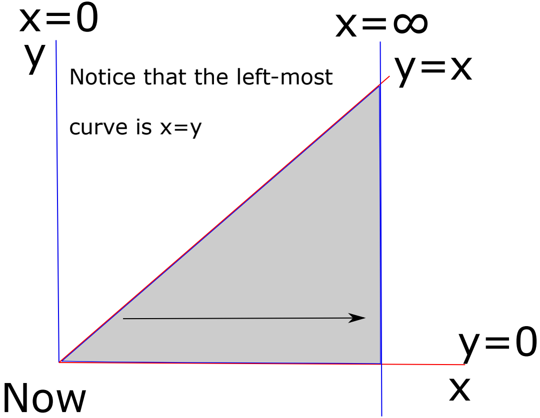
$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

But we want to reverse the order of integration

$$\int_?^? \int_?^? f(x)g(y)\,dx\,dy$$


x=0
y
x=∞
y=x
y=0
x
Now

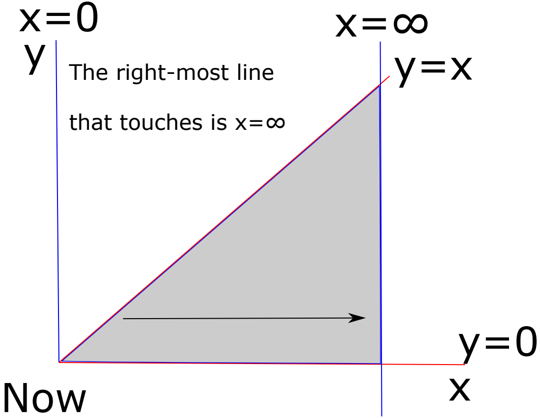

x=0
y
Line parallel to x
axis
x=∞
y=x
y=0
Now
x


x=0
y
x=∞
y=x
Notice that the left-most
curve is x=y
y=0
Now
x

$$\int_0^{\infty} \int_0^{x} f(x)g(y)\, dy\, dx$$

But we want to reverse the order of integration

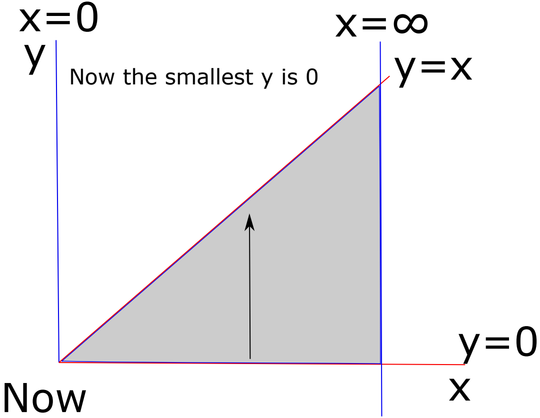
$$\int_?^{?} \int_y^{?} f(x)g(y)\, dx\, dy$$


x=0
y
x=∞
y=x
The right-most line
that touches is x=∞
y=0
Now
x

$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

But we want to reverse the order of integration

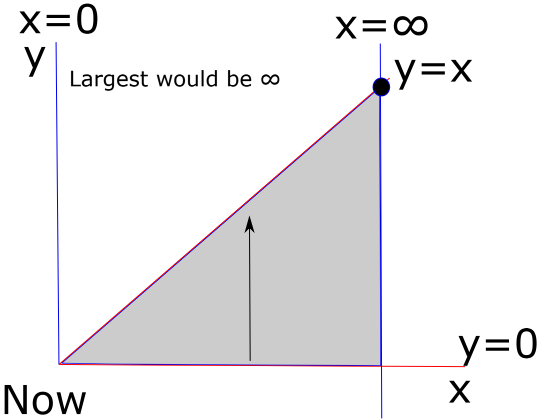
$$\int_?^? \int_y^\infty f(x)g(y)\,dx\,dy$$


x=0
y
Now the smallest y is 0
x=∞
y=x
y=0
Now
x

$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

But we want to reverse the order of integration

$$\int_0^? \int_y^\infty f(x)g(y)\,dx\,dy$$


x=0
y
Largest would be ∞
x=∞
y=x
y=0
Now
x

$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

But we want to reverse the order of integration

$$\int_0^\infty \int_y^\infty f(x)g(y)\,dx\,dy$$

$$\int_0^\infty \int_0^x f(x)g(y)\,dy\,dx$$

But we want to reverse the order of integration

$$\int_0^\infty \int_y^\infty f(x)g(y)\,dx\,dy$$

Now plug in the original functions

$$\int_0^\infty \int_y^\infty \ell(x)\frac{\partial}{\partial t}\mu(y)\,dx\,dy$$

$$\int_0^\infty \int_0^x f(x) g(y)\, dy\, dx$$

But we want to reverse the order of integration

$$\int_0^\infty \int_y^\infty f(x) g(y)\, dx\, dy$$

Now plug in the original functions

$$\int_0^\infty \int_y^\infty \ell(x) \frac{\partial}{\partial t} \mu(y)\, dx\, dy$$

$$\int_0^\infty \frac{\partial}{\partial t} \mu(y) \int_y^\infty \ell(x)\, dx\, dy$$