Decomposition techniques in population health research

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Recap

To avoid the interaction term, Kitagawa suggests

$$\Delta CDR = \underbrace{\sum_x \left(\frac{M_x(t_2)+M_x(t_1)}{2}\right)\left(\frac{N_x(t_2)}{N(t_2)} - \frac{N_x(t_1)}{N(t_1)}\right)}_{\text{Changes in x-composition}} + \underbrace{\sum_x \left(\frac{\frac{N_x(t_2)}{N(t_2)} + \frac{N_x(t_1)}{N(t_1)}}{2}\right)\left(M_x(t_2) - M_x(t_1)\right)}_{\text{Changes in rates}} \tag{1}$$

Recap

$$\dot{\bar{v}} = \underbrace{\bar{\dot{v}}}_{\text{Direct component}} + \underbrace{Cov(v, \acute{w})}_{\text{Structural or compositional component}} \tag{2}$$

Main result in Vaupel & Canudas-Romo (2002)

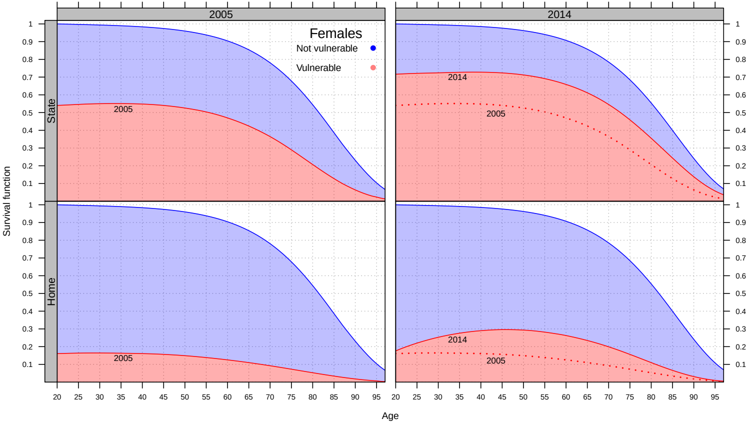
Recap

Life expectancy at age x is defined as

$$e(x)_{=}\frac{\int_x^\infty \ell(a)da}{\ell(x)}$$

Then we can define disability-free life expectancy as

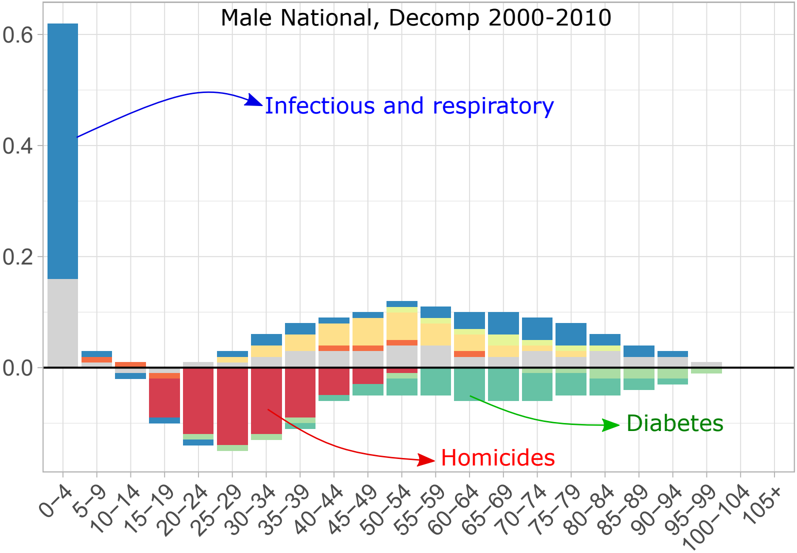
$$e^{DF}(x) = \frac{\int_x^\infty [1-\pi(a)]\ell(a)da}{\ell(x)}$$


2005
2014
Females
Not vulnerable
Vulnerable
State
Home
Survival function
Age
2005
2014
2005
2005
2014
2005
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95

Recap

Arriaga (1984) Effects of mortality change by age groups on life expectancies ($\sum_n \Delta_x =$ Total change):

$$
{}_n\Delta_x = \underbrace{\frac{\ell_x^1}{\ell_0^1}\left(\frac{{}_nL_x^2}{\ell_x^2} - \frac{{}_nL_x^1}{\ell_x^1}\right)}_{\text{Direct effect}} + \underbrace{\frac{T_{x+n}^2}{\ell_0^1}\left(\frac{\ell_x^1}{\ell_x^2} - \frac{\ell_{x+n}^1}{\ell_{x+n}^2}\right)}_{\text{Indirect and interaction effects}}
$$


Male National, Decomp 2000-2010
Infectious and respiratory
Diabetes
Homicides
0.6
0.4
0.2
0.0
0–4
5–9
10–14
15–19
20–24
25–29
30–34
35–39
40–44
45–49
50–54
55–59
60–64
65–69
70–74
75–79
80–84
85–89
90–94
95–99
100–104
105+

Outline

- Cause-deleted life tables
- Measures of variation in ages at death
- Linear integral decomposition

Cause-deleted life tables (based on Chapter 4, Preston et al 2001)

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The activity of a particular decrement *i* will be observed as it works itself out within a multiple decrement process.

'What would happen if...'

- The rate of decrement from $\mu^i(x)$ if i were the only decrement $({}^*m^i(x))$ differs from what it would be if i were working in the presence of other decrements $(m^i(x))$.

- Assume the force of decrement of cause i is proportional all-cause force of decrement:

$$\mu^{i}(a) = R_i \cdot \mu(a) \quad \text{for} x \leq a \leq x + n$$

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$$ {}_n^*p_x^i = e^{-\int_x^{x+n} \mu^i(a)da} = e^{-\int_x^{x+n} R_i \cdot \mu(a)da}$$

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$$ {}^{*}_{n}p^{i}_{x} = [{}_{n}p_{x}]^{R_i}$$

$$ {}_n^{*}p_x^{i} = \left[{}_np_x\right]^{R_i} $$

$$ = \left[{}_np_x\right]^{\frac{{}_nD_x^i}{{}_nD_x}} $$

Now with this simple relation we can create hypothetical scenarios.

`https://life-expectancy.org`

Lifespan variation indicators

Indicators	•	Conventional life table notation
Life disparity	$e^{\dagger}$	$\sum_{y=0}^{\omega} d_y e(y)$
Gini coefficient	G	$1 - \frac{1}{e_0} \sum_{y=0}^{\omega} \ell_{y+1}^2$
Theil's index	T	$\sum_{y=0}^{\omega} d_y \left[\frac{\bar{x}_y}{e_0} \ln \frac{\bar{x}_y}{e_0} \right]$
Mean logarithmic deviation	MLD	$\sum_{y=0}^{\omega} d_y (\ln(e_0/\bar{x}_y)$
Variance	V	$\sum_{y=0}^{\omega} d_y (\bar{x}_y - e_0)^2$
Standard deviation	SD	$\sqrt{V}$
Interquartile range	IQR	

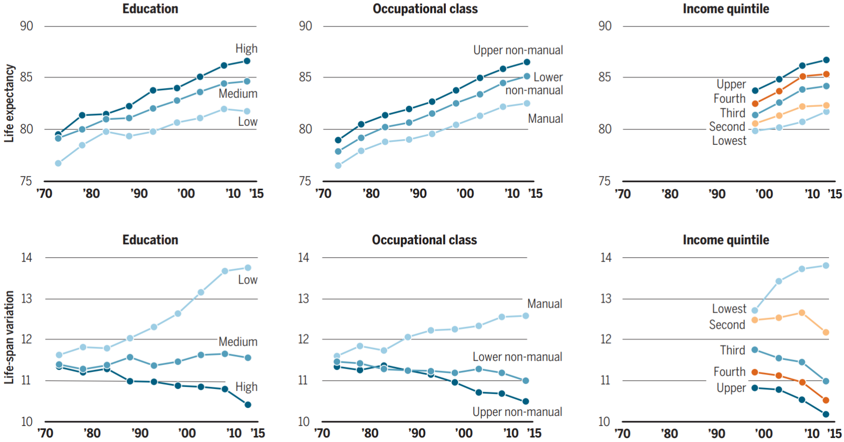
Table 1 Pearson correlation coefficients between pairs of indices, calculated from birth (ages 0–110+) in the top panel and calculated conditional on survival to age 10 (ages 10–110+) in the bottom panel, for all female and male life tables in the Human Mortality Database (7,516 in total)

	$e^\dagger$	G	T	MLD	S	V	IQR
$e^\dagger$	1.000						
G	.978	1.000					
T	.947	.991	1.000				
MLD	.965	.991	.992	1.000			
S	.981	.933	.893	.930	1.000		
V	.987	.945	.911	.944	.996	1.000	
IQR	.967	.966	.948	.956	.920	.944	1.000
	$e^\dagger_{10}$	G_{10}	T_{10}	MLD_{10}	S_{10}	V_{10}	IQR_{10}
$e^\dagger_{10}$	1.000						
G_{10}	.986	1.000					
T_{10}	.978	.995	1.000				
MLD_{10}	.979	.990	.995	1.000			
S_{10}	.986	.962	.961	.973	1.000		
V_{10}	.984	.964	.971	.980	.998	1.000	
IQR_{10}	.981	.978	.977	.976	.958	.966	1.000

Source: van Raalte & Caswell (2013)

Trends in life expectancy and life-span variation for Finnish females, 1971–1975 to 2011–2014

Life expectancy is the average age at death, and life-span variation is the standard deviation, conditional upon survival to age 30, with age-specific death rates frozen at those observed in the given year. See supplementary materials for data and methods, including trends for males (which are qualitatively similar), and robustness checks using alternative measures of life-span variation.



How can we decompose by age and cause of death these (any) function?

Linear integral decomposition (Horiuchi et al 2008)

- Relies on the assumption that values of the covariates change **continuously**, or gradually, along an actual or hypothetical dimension

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- Relies on the assumption that values of the covariates change **continuously**, or gradually, along an actual or hypothetical dimension
- Applied when difference in a dependent variable is expressed as a sum of the effects of differences in its covariates. (TFR,mean completed parity)

- Let y be a demographic function (e.g. $e^{\dagger}$), which is differentiable, of n covariates (e.g. rates) denoted by $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]$.

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$$y(t) = f(\boldsymbol{x}(t)) = f(x_1(t), x_2(t), \ldots, x_n(t)) \quad (4)$$

By the fundamental theorem of calculus

$$y(t_2) - y(t_1) = \int_{t_1}^{t_2} \frac{d}{dt} y(t) dt$$

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Applying the chain rule for partial derivatives of a composite function

$$y(t_2) - y(t_1) = \int_{t_1}^{t_2} \left[\sum_{i=1}^{n} \frac{\partial}{\partial x_i(t)} y(t) \cdot \frac{d}{dt} x_i(t) \right] dt \tag{6}$$

Exchange of integration and summation, and applying the substitution rule for definite integrals

$$y(t_2) - y(t_1) = \sum_{i=1}^{n} \int_{x_i(t_1)}^{x_i(t_2)} \frac{\partial}{\partial x_i(t)} y(t) dx_i(t)$$

Exchange of integration and summation, and applying the substitution rule for definite integrals

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(Simplifying notation)

$$y_2 - y_1 = \sum_{i=1}^{n} \int_{x_{i1}}^{x_{i2}} \frac{\partial y}{\partial x_{i1}} dx_i = \sum_{i=1}^{n} c_i \quad (8)$$

$$y_2 - y_1 = \sum_{i=1}^{n} c_i \tag{9}$$

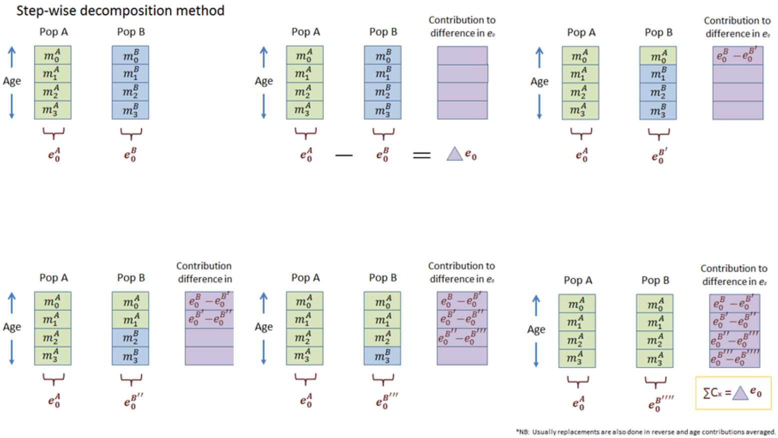
c_i is the total change in y produced by changes in the i-th covariate, x_i.

Important:Theoretical foundation for decomp analysis: implies that even if a dependent variable is not an additive function of its covariates, a change in the dependent variable can be expressed as a sum of effects of the covariates.

Stepwise decomposition (Andreev et al 2002)

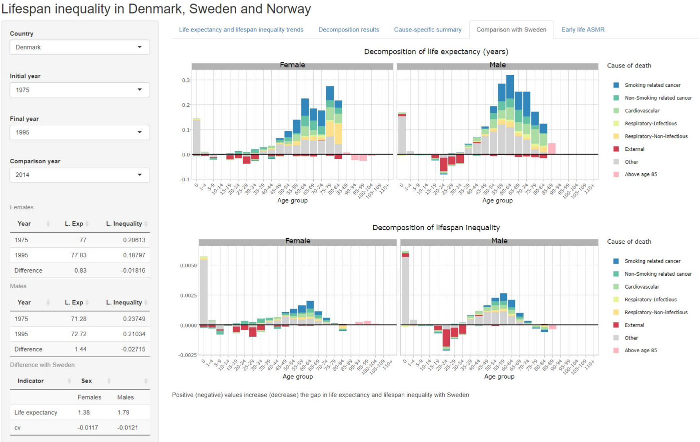
- To decompose a function for which there is no analytic solution (or for which the analytic solution is complicated).
- Step-wise decomposition alters the rates one element at a time (the element usually age), and recalculates the index function (i.e. life expectancy or any other summary measure).

Stepwise decomposition



Exercise 1: Mexico's example

Extending to causes of death



Females

Year	L. Exp	L. Inequality
1975	77	0.20613
1995	77.83	0.18797
Difference	0.83	-0.01816

Males

Year	L. Exp	L. Inequality
1975	71.28	0.23749
1995	72.72	0.21034
Difference	1.44	-0.02715

Difference with Sweden

Indicator	Sex	
	Females	Males
Life expectancy	1.38	1.79
cv	-0.0117	-0.0121

https://jmaburto.shinyapps.io/DK_App/

Exercise 2: by cause of death

Challenge 1 Use the linear integral model to decompose the change in the standard deviation of the age-at-death distribution and life expectancy by age and cause of death for 3 countries you might be interested in (over time or between them). Interpret the results of life expectancy alongside standard deviation. Make it interesting. You can use data from HCoD, HMD, WHO, GBD.