Population Studies
A Journal of Demography

ISSN: 0032-4728 (Print) 1477-4747 (Online) Journal homepage: https://www.tandfonline.com/loi/rpst20

Forecasting cohort incomplete fertility: A method and an application

Nan Li & Zheng Wu

Population Studies, Vol. 57, No. 3, 2003, pp. 303–320

Routledge
Taylor & Francis Group

Forecasting cohort incomplete fertility: A method and an application

Nan Li[1] and Zheng Wu[2]
[1]Max Planck Institute for Demographic Research, [2]University of Victoria

Drawing on insights from previous work on fertility forecasts, we develop a method for forecasting incomplete cohort fertility. Our approach involves two basic steps. First, we use a singular-value-decomposition (SVD) model to establish a relationship between the level and the age pattern of fertility for completed cohorts. This relationship is then applied to incomplete cohorts to obtain forecast fertility. We propose techniques to evaluate model assumptions and illustrate our method using cohort data from Canada, the USA, Norway, and Japan. With the exception of Japan, our results show that the model fits the data well, and that the youngest cohort whose total fertility can be reliably forecast is age 25 for Canada, the USA, and Norway. Our method is less applicable to Japan, where the youngest cohort whose total fertility could be forecast was age 35 or older. We discuss the limitations of our method in the context of model assumptions.

Keywords: fertility forecast; cohort fertility; SVD model

[Submitted September 2002; Final version accepted June 2003]

In the cohort-component population projection (Whelpton 1936), officially used by most countries that conduct population forecasts, a value of period total fertility is typically determined for a given future time, usually 3–5 decades from the time of the projection. Period total fertility is then projected to reach this predetermined value, normally considered the most plausible. Although assumptions about the long-term value often represent the most problematic component of this approach (Ryder 1990; Ahlburg and Lutz 1998), some of them are based on analyses of cohort total fertility (e.g., De Beer and De Jong 1996). Demographers generally agree that cohort fertility measures are better than period measures at reflecting how well a society is replacing itself. Indeed, Ryder (1983) has gone so far as to suggest that fundamental changes in fertility behaviour (the timing or level of fertility, or both) occur primarily through new cohorts being exposed to important historical events in childhood and early adulthood that influence their fertility aspirations. Sharing similar life experiences and a particular historical era may lead to a sharing of preferred fertility goals. Thus, the cohort measure is believed to be more reasonable and reliable than the period measure for projecting future fertility (see, e.g., Frejka and Calot 2001a; Lesthaeghe 2001). More importantly perhaps, it is well known that changes in the timing of childbearing influence period total fertility but not cohort total fertility, in which changes are usually steadier and more predictable.

Assuming the future level of period total fertility by analysing cohort total fertility is particularly appropriate for many developed countries, where a decrease in the former and an increase in the mean age at childbearing have occurred (Kohler et al. 2001; United Nations 2001). The motives for delayed childbearing among individuals from recent cohorts may be transient. Hence, these individuals could increase their fertility in later childbearing years, meaning that in the long term, period total fertility may end up higher than current levels. This catching-up hypothesis has influenced long-term fertility assumptions used in population forecasts in several low-fertility countries (e.g., De Beer and De Jong 1996). However, the long-term period total fertility will be lower than current levels if the difference in fertility between younger and older cohorts persists. Unfortunately, determining the most probable outcome is difficult, because at any given forecast launch time we know only the completed fertility of cohorts born a long time ago. We can never be certain about younger cohorts who have not passed through their childbearing years. Suppose, for instance, we are

ISSN 0032-4728 print/ISSN 1477-4747 online/03/030303-18 © 2003 Population Investigation Committee
DOI: 10.1080/0032472032000137826

forecasting fertility with period data from the year 2000 and that childbearing occurs from age 15 to 44. In this case, the youngest cohort with completed fertility was born in 1956, while the cohort born in 1985 is about to begin childbearing. How do we fill a gap between cohorts that can range up to 30 years?

In this paper, we propose a method that helps close this gap. Our approach involves two basic steps. First, we use a singular-value-decomposition (SVD) model to establish the relationship between the level and the age pattern of fertility for completed cohorts. Second, we apply this relationship to incomplete cohorts to obtain forecast fertility. We establish mechanisms to evaluate model assumptions and illustrate our method using cohort data from selected Western countries. In the following sections, we review recent attempts at fertility projection and forecasts, describe our method, and apply this method to cohort fertility data from Canada, the USA, Norway, and Japan. Our procedure applies better to some countries than others, because it depends on how well our model assumptions are satisfied. Our method is limited to cohorts that have begun but not yet finished childbearing, and is not intended to be a tool for forecasting period fertility or cohort fertility in general. However, we believe the method represents a step towards developing a reliable tool for cohort fertility forecasts.

Previous research

The relationship between cohort and period total fertility can be determined when specific changes in the timing of childbearing are known or can be correctly estimated (Ryder 1956). Assuming an identical rate of timing change in fertility at each age for a certain birth order in one year, Bongaarts and Feeney (1998) proposed a method of calculating an adjusted period total fertility level that would be its value if there were no such timing change. Underlying this adjustment is the assumption that the shape of the fertility schedule is invariant; the timing change only advances or postpones the schedule. A sensitivity analysis by Zeng and Land (2001) confirmed that the Bongaarts–Feeney method is generally robust even when the shape of the fertility schedule changes at a constant annual rate. Following this line of research, Kohler and colleagues developed several tempo-adjusted measures to extend the Bongaarts–Feeney method, e.g., allowing variant changes in the shape of the fertility schedule (Kohler and Philipov 2001), and taking into account the fecundity decline in delayed childbearing, particularly for higher-order births (Kohler and Ortega 2001a, b). Based on the most recent period fertility pattern, Kohler and Ortega (2001a, b) developed a method for projecting the overall level and the parity-specific fertility of cohorts that are beginning or have not yet completed childbearing under scenarios where observed postponement will either stop or continue.

These tempo-adjusted period measures improve conventional period measures by adjusting short-term change in the timing of fertility. However, they cannot predict whether changes in timing will be transient or persistent. Moreover, these measures are not adequate for inferring a cohort's complete fertility, partly because, from a behavioural perspective, it is difficult to justify why the age pattern of period fertility must represent that of cohort fertility (van Imhoff and Keilman 2000; van Imhoff 2001).

Estimating the total fertility of a cohort whose childbearing is incomplete is forecasting. In such cohorts, the average number of children women expect to have is a relevant fertility indicator, but such information, usually collected from social surveys, is unreliable (Ryder 1990). To be sure, some evidence supports using expectations data for fertility forecasts (e.g., Hendershot and Placek 1981; Bongaarts 1992). But most research questions the utility of expectations data for these purposes. For example, Westoff and Ryder (1977) show that estimates based on expectations data are typically much higher than the actual fertility realized, particularly for younger cohorts. Studies on childlessness also indicate that many eventually childless women become so through a series of postponements, until finally it is too late to have a child (e.g., Rindfuss et al. 1988).

Parameter models of age-specific fertility also have been used in cohort fertility forecasts (e.g., Bloom 1982; Chandola et al. 1999). In this approach, parameters in a fertility model are estimated using a cohort's completed fertility at younger ages, such as 15–30, and the incomplete fertility at ages older than 30 is extrapolated using the estimated parameters. Bloom (1982) and Chen and Morgan (1991) use this strategy to forecast first births; both use a modified version of the Coale and McNeil (1972) marriage model. Although their studies show that parameter models fit well with first birth (childlessness) data, they are problematic, as the parameters for a given cohort are estimated using solely that cohort's completed fertility at younger ages, normally ages under 25. In other words, the information on completed fertility observed from earlier cohorts is not taken into account. Arguably, the behaviour of earlier cohorts could indirectly reflect that of later cohorts.

Ryder (1990) argues that historical trends (*patterns*) *in fertility* should not be ignored in fertility forecasts. For example, to estimate unobserved fertility for a 30-year-old cohort at age 31, his approach suggests using the experience at age 31 observed in earlier cohorts. Acting on this principle, Ryder (1990) uses the observed age-parity-fertility data (trends) of earlier cohorts (e.g., for the past 15 years) to estimate the incomplete age-parity-specific fertility of later cohorts. Implicit in Ryder's approach is the assumption that fertility changes observed in these earlier cohorts will continue to follow the same pattern. Although this strategy extends historical fertility trends to forecast incomplete cohort fertility, it cannot use the information on completed portions of fertility observed in later cohorts (e.g., observed age-specific birth rates for ages younger than 30 in a 30-year-old cohort), a strategy used in parameter models.

Recognizing the foregoing limitations, some demographers have attempted to combine historical fertility trends observed in earlier cohorts with the completed fertility of later cohorts in their fertility forecasts. For example, in several recent studies of cohort fertility in low-fertility countries, including former socialist countries, Frejka and Calot (2001a–c) observe that, except in the USA, women born in the 1960s and 1970s are experiencing substantially lower fertility than earlier born women did at comparable ages. Their data suggest that only a fraction of the shortfall is likely to be made up when the women are older, which implies that the completed cohort fertility of women born in the 1960s and the 1970s will be lower than of those born earlier. Their analysis shows that to reach replacement level, these women would have to raise their fertility to an unusually high level to compensate for fertility deficits they experienced at younger ages. Frejka and Calot predict further declines in the cohort total fertility of these low-fertility societies, although they do not stipulate a model to quantify future declines.

Evans (1986) developed an innovative model to incorporate both historical trends and completed cohort fertility at younger ages in forecasting the birth rates of women aged 25–39. For each age between 25 and 39, she uses historical data (completed fertility in earlier cohorts) to estimate a linear model of the birth rate at that age. Evans considers two predictors: (a) the cumulated cohort fertility from age 24, and (b) the ratio of fertility at ages 15–19 to that at ages 20–24. The former measures the level of completed cohort fertility up to age 25 and the latter captures the pace of cohort fertility. Evans then applies this linear model to cohorts aged 25–39. For each cohort, the birth rate at each age is estimated using the parameters estimated from earlier completed cohorts and the information from its own completed fertility below age 25. This method improves on earlier models by using both historical trends of fertility and completed cohort fertility at younger ages, but has several shortcomings. First, using age 25 as a cut-off is neither well explained nor empirically based. Second, by using this cut-off age, birth rates of cohorts younger than age 25 cannot be forecast. Third, for a cohort older than 24, its completed fertility at ages over 24, which would otherwise be useful information, makes no contribution to the forecast.

Following Evans' approach, this paper proposes a method of forecasting cohort fertility for women aged 15 and older who have started but not completed childbearing. We make two important departures from Evans' method. First, we choose the youngest cohort based on empirical data used in the forecast, i.e., where data permit, we can forecast the fertility of cohorts younger than age 25. Second, our method uses *all* completed fertility in the form of births by single year of age, rather than cumulated fertility or limiting completed fertility to age 25. We can, for instance, forecast cohort fertility for women aged 30 using information on their completed fertility up to age 29.

Method

Model assumptions

Our strategy requires three basic assumptions. First, we assume that fertility is a life-cycle arrangement and that women make conscious efforts to reach their fertility goal. Economic theories of fertility suggest that parents weigh economic and non-economic opportunity costs of childbearing before deciding how many children to have (e.g., Becker 1988; Hechter and Kanazawa 1997). There is a close association between the level of fertility at early childbearing ages and completed fertility because of the biological constraints of reproduction. Studies consistently find that the timing and level of fertility in young adulthood strongly influence completed fertility (e.g., Bumpass et al. 1978; Millman and Hendershot 1980; Morgan 1996; Morgan and Rindfuss 1999). Hence, a cohort's age-specific fertility at older ages is related to that at younger ages. This assumption also underlies the parameter models noted above (e.g., Bloom 1982; Chen and Morgan 1991).

Second, we assume that any difference in fertility between cohorts is smaller at older than at younger ages. This premise derives from observations that both age-specific cohort fertility rates are normally higher, and changes in fertility across cohorts are more pronounced at younger than at older ages. Parameter models of fertility forecasting suggest that changes in parameters usually affect fertility more at middle reproductive ages than at older reproductive ages (e.g., Bloom 1982). Post-war fertility trends in industrial societies have demonstrated that the overall decline in fertility is attributable largely to the reduction in fertility in women's early and middle reproductive careers (Foster 1990).

Third, we assume that the relationship between fertility at younger and older ages, established for the first assumption, varies little across cohorts born in a (historical) period when structural conditions for fertility changes are similar. This assumption stems from the heart of fertility theories that emphasize structural and cultural factors that condition the level of fertility and influence fertility changes. For example, demographic transition theory cites social and economic forces of industrialization as fundamental causes of fertility decline (Notestein 1945). Within this broad framework, Coale (1973) speaks of the cultural roots of high fertility in historical societies, while Caldwell (1982) points to the diffusion of Western ideas and mass education in the transformation of traditional fertility regimes in contemporary societies. There is evidence suggesting that period influences are more powerful than cohort influences in explaining fertility changes in the last century (Foster 1990; Ni Bhrolchain 1992), and that age patterns of fertility appear to conform to a family of curves with relatively little variation across populations (Coale and McNeil 1972; Coale and Trussell 1974).

To summarize, our first assumption means that our method will work well when there is a relationship between young and old age-specific fertility for a given cohort. Because this relationship may change across cohorts, our second assumption stipulates that these changes are more evident at younger than older ages. Our third assumption requires that these changes be relatively small and smooth across cohorts in a given historical period.

These three model assumptions are important for our forecasts. The first constitutes the basis for forecasting a cohort's incomplete fertility using its completed fertility at younger ages. The more years of completed fertility used in the forecast, the more reliable the forecast will be. Our second assumption enables us to extend fertility forecasts to a large age range because the cohort total fertility has most of its weight in the first half of the reproductive window in virtually all known human populations. The last assumption allows the use of data from more than one cohort, thereby creating a more robust forecast than would data from merely one cohort, as in, e.g., parameter models of forecasting. In practice, the success of our method will depend on how well these assumptions materialize: the more realistic our premises, the more reliable our forecast.

Although evidence supports their validity, we must emphasize that one or more of these assumptions may not be realistic for all countries at all times. They determine the applicability of our method but, as discussed in more detail below, they are not intended to apply to any fertility regime during any historical period. Thus, it would be a mistake to presume that our procedure can be used indiscriminately. As shown below, our method is more applicable in Canada, the USA, and Norway, but less applicable in Japan. To indicate the limits of our method, we have developed techniques to examine how well these assumptions are satisfied, and to quantify the degree of variations (stipulated in our assumptions) that the model can tolerate. In the following section, we introduce our method and then turn to these assessment techniques.

The SVD model for completed cohort fertility

Let the birth rate of women aged x in cohort t, born at time t and aged x at time $(t + x)$, be $f(x, t)$. Suppose the values of $f(x, t)$ in $\alpha \leq x \leq \beta$ (hereafter $\alpha = 15$ and $\beta = 44$) and $t_1 \leq t \leq t_2$ are known to us: in other words, the latest completed cohort, whose birth rate at age β is known, is born at time t_2. Let $a(x)$ be the over-cohort mean value of $f(x, t)$, we decompose $f(x, t)$ as

$$\begin{bmatrix} f(15, t_1) & \dots & f(15, t_1) \\ \dots & \dots & \dots \\ f(44, t_2) & \dots & f(44, t_2) \end{bmatrix} = \begin{bmatrix} a(15) & \dots & a(15) \\ \dots & \dots & \dots \\ a(44) & \dots & a(44) \end{bmatrix} +$$

$$\begin{bmatrix} b(15) \\ \dots \\ b(44) \end{bmatrix} [k(t_1) \dots k(t_2)] + \varepsilon(x,t)$$

or

$$f(x,t) = a(x) + b(x)k(t) + \varepsilon(x,t) \qquad (1)$$

In (1), $b(x)$ and $k(t)$ can be estimated by minimizing $\sum_{x=\alpha}^{\beta}\sum_{t=t_1}^{t_1}\varepsilon^2(x,t)$. Although the ordinary least-squares (OLS) method can be used to solve for $k(t)$ when $b(x)$ is given or vice versa, it cannot solve for both $k(t)$ and $b(x)$ simultaneously because there are no explanatory (right-hand side) variables in (1) to be regressed on as required in the OLS method. One strategy to estimate $b(x)$ and $k(t)$ (as $\hat{b}(x)$ and $\hat{k}(t)$, respectively) simultaneously is to use a SVD model (e.g., Lee and Carter 1992).

Using the SVD method, the fertility matrix of x by t, $f(x, t)$, is decomposed into an age vector $\hat{b}(x)$ and a cohort vector $\hat{k}(t)$. We may normalize $\hat{b}(x)$ such that its elements are summed up to unity. Thus, $\hat{k}(t)$ describes the change in the total fertility across cohorts and $\hat{b}(x)$ distributes the change into age groups. In other words, $\hat{b}(x)$ describes the (common) age pattern of the relationships between fertility changes at older and younger ages for completed cohorts. How well $\hat{b}(x)$ describes this pattern depends on how similar these relationships are, i.e., how well our third assumption holds. The more realistic this assumption, the better the $\hat{b}(x)$ works.

One important property of (1) is that using $\hat{b}(x)$ obtained from the SVD, $k(t)$ can *also* be estimated by solving the linear least-squares model

$$\min \sum_{x=\alpha}^{\beta} [f(x,t) - \hat{a}(x) - \hat{a}(x)k(t)]^2, \qquad (2)$$

as

$$\hat{k}(t) = \frac{\sum_{x=\alpha}^{\beta} [f(x,t) - \hat{a}(x)]\hat{b}(x)}{\sum_{x=\alpha}^{\beta} \hat{b}^2(x)}. \qquad (3)$$

As will be shown below, this property provides a foundation for our forecast.

The forecast of a cohort's incomplete fertility

As noted, using data from completed cohorts, we estimate $b(x)$ and $k(t)$ as $\hat{b}(x)$ and $\hat{k}(t)$. We now apply $\hat{b}(x)$ to an incomplete cohort and forecast its $K(t, n)$, where n is the number of years for which this cohort has completed its childbearing (i.e., the non-censored part of the incomplete cohorts).

We denote $f(y, t)$ for incomplete birth rates of women aged y in cohort t, who are born at time t and aged y at time $(t + y)$. For an incomplete cohort born at t, t is in the range $[t_2 + 1, t_2 + \beta - \alpha]$. Its completed birth rates, $f(x, t)$, are at ages $\alpha \leq x \leq \beta - t + t_2$, and its incomplete birth rates, $f(y, t)$, are at ages $\beta - t + t_2 < y \leq \beta$ (see Figure 1). The number of its completed birth rates, n, is $\beta - \alpha - t + t_2 + 1$. For example, if $t_2 = 1956$, then the incomplete cohorts were born between 1957 and 1985. For the cohort born in 1975, its complete birth rates are at ages 15–25, its n is 11, and so forth.

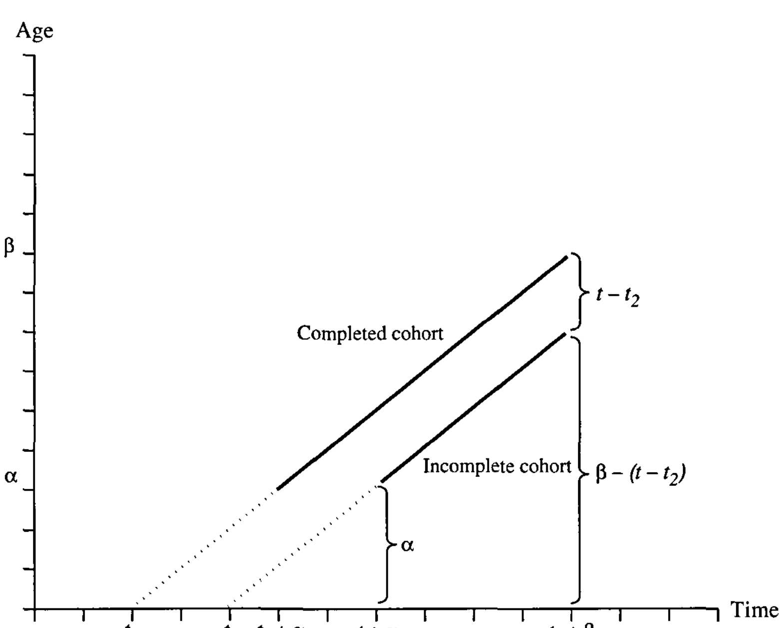

Figure 1 Childbearing ages for completed and incomplete cohorts

As noted, $\hat{b}(x)$ distributes the change in cohort total fertility into age groups and describes the age pattern of relationships between fertility changes at younger and older reproductive ages for completed cohorts. Under our third assumption, such relationships are similar among cohorts born in the same historical period. This assumption enables us to apply $\hat{b}(x)$ obtained from the completed cohorts to the incomplete cohorts.

Under our first assumption, a cohort's total fertility is related to its fertility at younger ages. Thus, we choose $K(t,n)$ to minimize the error in describing completed fertility at younger ages

$$\min \sum_{x=\alpha}^{\alpha+n-1} [f(x,t) - \hat{a}(x) - \hat{b}(x)K(t,n)]^2, \tag{4}$$

and obtain

$$K(t,n) = \frac{\sum_{x=\alpha}^{\alpha+n-1} [f(x,t) - \hat{a}(x)]\hat{b}(x)}{\sum_{x=\alpha}^{\alpha+n-1} \hat{b}^2(x)}, \tag{5}$$

$$n = \beta - \alpha - t + t_2 + 1.$$

Using (1) and the estimated $K(t, n)$ in (5), $f(y, t)$ is forecasted as

$$F(y,t) = \hat{a}(y) + \hat{b}(y)K(t,n), \alpha + n \leq y \leq \beta. \tag{6}$$

Using (5) and (6), $f(y, t)$ can be forecast for any cohort that has begun childbearing, regardless of n. Logically, of course, the more completed years of childbearing used (an increase in n), the more reliable the forecasts. But how many completed years of childbearing are required to provide a reliable forecast? We address this issue in the following section.

Assessing model assumptions

How reliable is our method? When do we apply it? We developed two criteria to assess our model assumptions and to determine whether our approach applies to a given population for a specific historical period.

Our first assessment involves examining $\hat{b}(x)$ obtained from completed cohorts. If our second assumption holds well, the absolute values of $\hat{b}(x)$ at older reproductive ages, which allocate the change in the cohort total fertility into older reproductive ages, should be noticeably smaller than those at middle reproductive ages. The smaller these values, the smaller the n required. Under such circumstances, $k(t)$ may be reliably estimated using cohort fertility at younger ages because ignoring the $\hat{b}(x)$ at older ages that are close to zero in (3) will not significantly affect the estimated $k(t)$. However, when the absolute values of $\hat{b}(x)$ at older ages are relatively large, a larger n would be needed. In general, when our second assumption holds well, the smallest-acceptable n would be considerably less than 30, the entire length of the reproductive career.

Our second assessment evaluates our first and third assumptions; to do so, we focus our attention on completed cohorts. We begin by assuming that the cohort birth rates at age 44, $f(44,t)$, are unknown. We forecast them as $F(44,t)$, using (5) and (6). Comparing the forecast rates with the observed rates, we compute forecast errors, $\delta(44,t,n) = f(44,t) - F(44,t)$, where $n = 29$ is the number of completed birth rates used in the forecast. Similarly, suppose that cohort birth rates at age 43 are unknown, forecast errors for $\delta(43,t,28)$ can also be computed. Repeating this process, we obtain forecast errors for $n = 1$–30. In general, we write

$$\delta(x,t,n) = f(x,t) - F(x,t), n = 1 - 30. \tag{7}$$

It is clear that when $n = 30$, $F(x, t)$ is the SVD model value of $f(x, t)$, and $\delta(x, t, 30)$ is the SVD description error.

A value of $\delta(x, t, n)$ for a single cohort and a single age may not tell us much about how well our model fits. However, as in an OLS model, values can be used collectively to evaluate the overall fit of our model. Analogously to the use of R^2 in the OLS model, we evaluate our model using a simple index, and call the explanation ratio

$$R(n) = 1 \frac{\sum_{x=\alpha}^{\beta} \sum_{t=t_1}^{t_2} \delta(x, t, n)^2}{\sum_{x=\alpha}^{\beta} \sum_{t=t_1}^{t_2} [f(x, t) - \hat{a}(x)]^2}. \tag{8}$$

Clearly, $R(n)$ is a function of n, the number of completed cohort birth rates used in the forecast. In general, $R(n)$ increases with n, implying that more information creates better forecasts. In other words, it indicates how the fit of the model changes with n.

Returning to our three model assumptions, our third should hold well if $R(30)$ is close to 1 when modelling errors are minimum. As noted, $R(n)$ declines with a decrease in n. When n declines and $R(n)$ falls below a threshold level, say 0.9, assumption 1 may become suspect. In other words, our first assumption is sound so long as $R(n)$ stays above a threshold level for a given n. Moreover, smooth trends in cohort fertility and mean age at childbearing should also indicate that the third assumption may be satisfied.

In sum, determining whether our method is applicable requires evaluation of all three assumptions. We do not recommend our approach when one or more of these assumptions are seriously violated, particularly when the absolute values of $\hat{b}(x)$ at older ages are excessively large and (or) $R(n)$ is significantly below the threshold value. Further, the smallest-acceptable n should be determined jointly by $\hat{b}(x)$ and $R(n)$.

Application

We apply our method to fertility data from Canada, the USA, Norway, and Japan only. We wanted some regional representation in our selection, and of course our choices depended upon data availability. Further, our intent is to demonstrate our method, not to address any substantive questions regarding current or future trends in fertility. Of course, researchers may consider applying our method to a wider range of populations in the future.

Our data came from several sources. Age-specific birth rates for Canada from 1956 to 1983 came from Foster (1990) and, from 1984 to 1995, from the *Demographic Yearbook* (United Nations 1987–97); the most recent data from Statistics Canada (2002a, b). American fertility data from 1955 to 1980 are available from the Office of Population Research at Princeton University (Office of Population Research 2001), and, from 1981 to 2000, from the National Centre for Health Statistics (National Centre for Health Statistics 2000, 2001). Norwegian data came from Brunborg and Mamelund (1994) from 1961 to 1993 and from Statistics Norway (2002) from 1994 to 2000. Finally, Japanese data came from Matsukura (2002). In Table A1, we present the values of $\hat{a}(x)$ and its standard error.

We begin our exercise by examining the model assumptions. As noted, smooth trends of cohort total fertility and its mean age at childbearing should indicate that our third assumption holds well. In low-fertility countries, 'smooth' trends typically reflect declining cohort total fertility and a rising mean age at childbearing, although some countries may experience different patterns. Of course, our method will fail if there are no clear trends in these events, as do most, if not all, forecasting methods.

Evaluating our third assumption, Figure 2 shows the trends in cohort total fertility and mean age at childbearing for selected cohorts in our four chosen countries. Although we have data before the earliest dates shown in the figure, we limited the intervals to reveal the trends. The reason that we truncated these data is because the mean age at childbearing in the earlier years exhibited falling trends in these countries and we needed to obtain 'smooth' (monotonic) trends. Inclusion of these earlier data would have seriously violated our third assumption.

Figure 2 shows that, except for Japan, a clear decline in cohort total fertility and a rise in mean age at childbearing are evident. In the Japanese case, although mean age at childbearing rises steadily for the period in question, we observe distinct changes in cohort total fertility in the cohorts of 1945–48, suggesting that our method may not be as applicable here as in the other three countries.

Further examining the third assumption, we apply equation (1) to the fertility data shown in Figure 2. We find that the explanation ratio, $R(30)$, is as high as 0.97 for Canada, 0.94 for the USA, 0.91 for Norway, but 0.68 for Japan (see Figure 4). With the exception of Japan, these values indicate that our modelling errors are very small. Recall that equation (8) shows that $R(30)$ compares errors when using the latest cohort's fertility with those when using our model in describing earlier completed cohort fertility. Thus, for example, $R(30) = 0.97$ for Canada indicates that our model improves the description of the earlier completed cohort fertility by 97 per cent. Overall, with the exception of Japan, these values indicate that assumption 3 is acceptable. In the case of Japan, the low value of $R(30)$ reflects the erratic trend of cohort fertility in Japan (see Figure 2). Focusing on the trend in more recent years, when it is more consistent, should improve $R(30)$. In an unreported analysis, we replaced $a(x)$ by $f(x, t_2)$, the latest completed cohort in equation (1). As expected, $R(30)$ rose to 0.89.

As noted, our second model assumption requires that the difference in fertility between cohorts be small at older reproductive ages. To evaluate this premise, we compute $\hat{b}(x)$ for the four countries. Recall that $\hat{b}(x)$ distributes the change in cohort total fertility across cohorts into age groups. Numerically, a greater absolute value of $\hat{b}(x)$ corresponds to a larger change in the birth rate at age x, and the converse is true for a smaller value of $\hat{b}(x)$. If

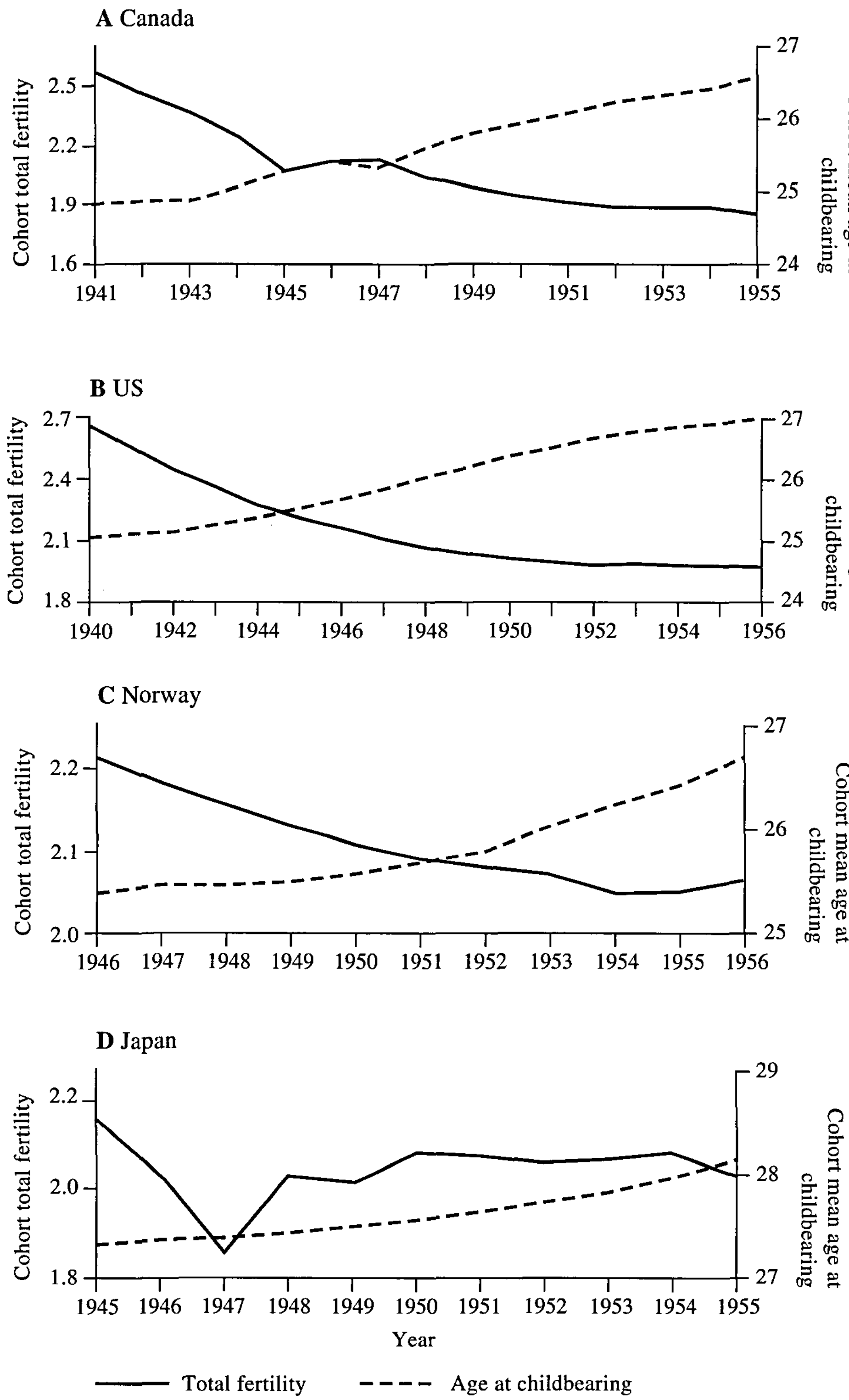

Figure 2 Cohort total fertility and cohort mean age at childbearing, Canada, USA, Norway, and Japan
Sources: For Canada: Foster (1990), *Demographic Yearbook* (United Nations 1987–97), Statistics Canada (2002a, b). For the USA: Office of Population Research (2001), National Center for Health Statistics (2000, 2001). For Norway: Brunborg and Mamelund (1994), Statistics Norway (2002). For Japan: Matsukura (2002)

assumption 2 holds well, absolute values of $\hat{b}(x)$ at older reproductive ages should be relatively small. Figure 3 shows that the $\hat{b}(x)$ for Canada, the USA, and, to a lesser extent, Norway are acceptable. However, the $\hat{b}(x)$ for Japan suggests that assumption 2 may not stand well. In other words, a reasonable forecast may require a larger n in Japan than in the other countries.

As noted, the explanation ratio $R(n)$ is designed to assess our first model assumption. When $R(n)$ falls

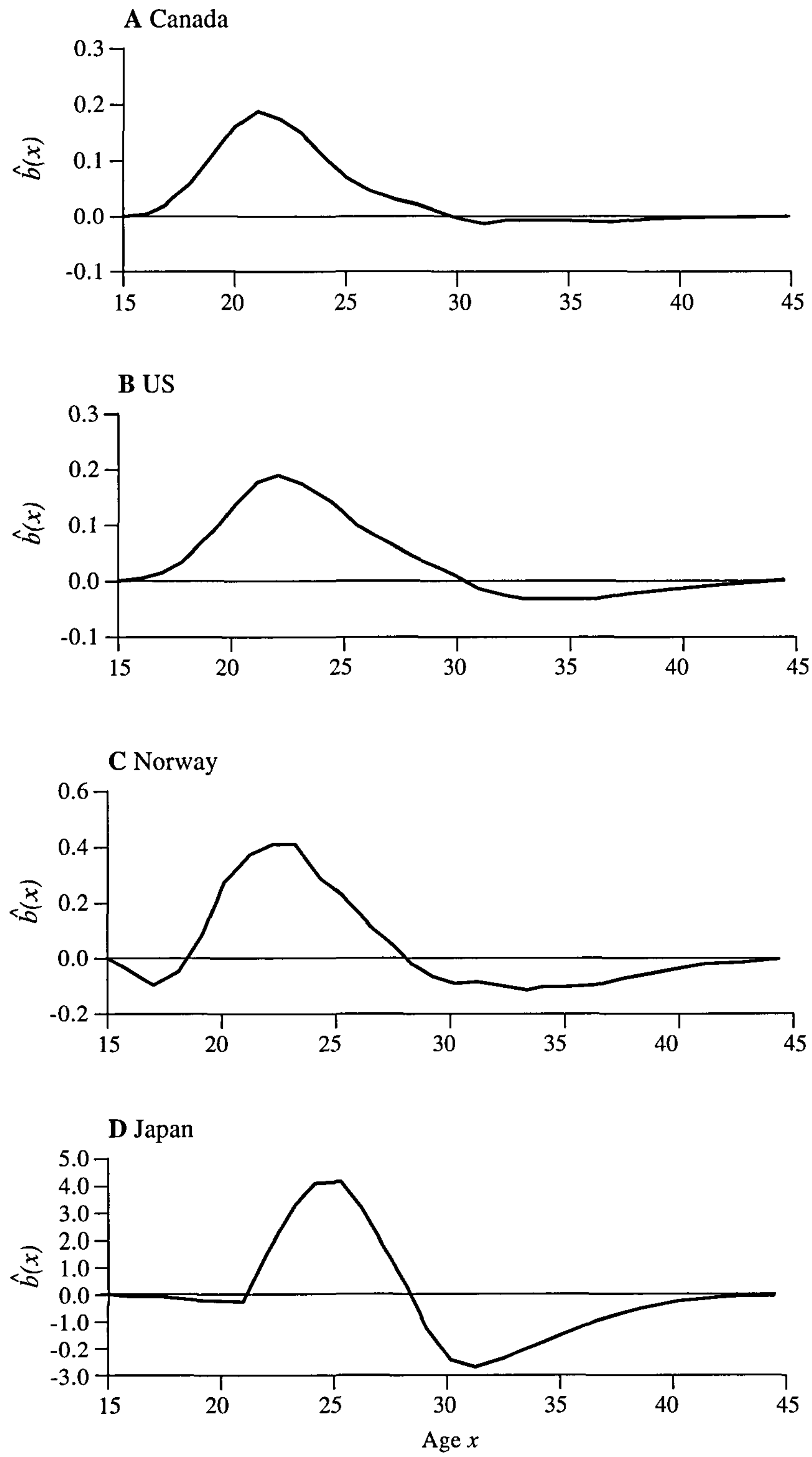

Figure 3 $\hat{b}(x)$, Canada, USA, Norway, and Japan
Source: As for Figure 2

below a threshold level, assumption 1 may become questionable. Using the birth rates of the initial n reproductive ages of completed cohorts, the values of $K(t,n)$ can be estimated by (5), and the 'incomplete' birth rates forecast by (6). Using (7) and (8), the values of $R(n)$ were computed and are presented in Figure 4.

We have noted that $R(n)$ shows how the fit of the model varies with n. However, the values of $R(n)$, as computed using equations (7) and (8), may not fully validate the method. This is because, although the birth rates at the first n ages of completed cohorts are assumed to be known and $K(t,n)$ is computed accordingly, the values of $\hat{a}(x)$ and $\hat{b}(x)$ are calculated from

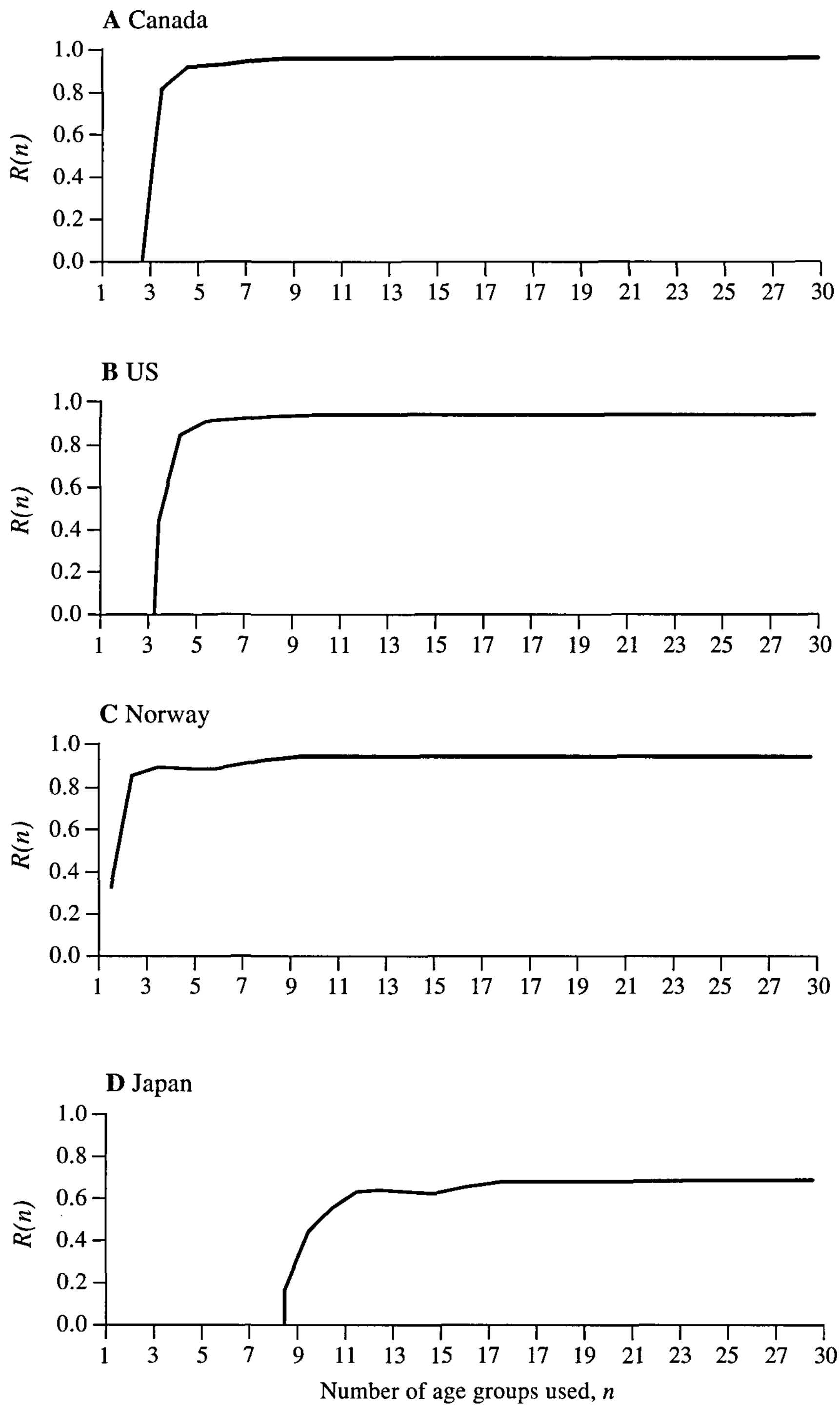

Figure 4 Explanation ratios, Canada, USA, Norway, and Japan
Source: As for Figure 2

the birth rates of completed cohorts. A modified approach would assume that the birth rates of a fraction of completed cohorts were known. Then, $\hat{a}(x)$ and $\hat{b}(x)$ would be calculated from the known segment of cohort fertility, and $K(t,n)$ computed from the 'unknown' segment. Clearly, one weakness of this approach is that the value of the fraction is arbitrary. As an exercise, we set the fraction as the first two-thirds of the completed cohorts and recalculated $R(n)$. We found the new values of $R(n)$ were marginally higher than those reported in Figure 4. Because the fraction 2/3 was chosen arbitrarily, we decided not to report these values. We acknowledge an anonymous reviewer for suggesting this modified strategy.

Figure 4 shows that, with the exception of Japan, levels of $R(n)$ are high and remain virtually unchanged when $n \geq 10$. High levels of $R(n \geq 10)$

suggest that reliable forecasts of incomplete fertility can be made for cohorts born as late as 1975–76, who have completed birth rates for age 15–24. In fact, rather high levels of $R(5–9)$ for Canada, the USA, and Norway suggest that we might even press for a few more years of forecasts. However, for Japan, $R(n < 10)$ would not be acceptable.

As noted, the smallest acceptable n must be determined by examining both $\hat{b}(x)$ and $R(n)$. Our evaluations of $\hat{b}(x)$ and $R(n)$ indicate that for Canada, the USA, and Norway, the minimum n should be around 10, while the corresponding figure for Japan should be at least 20. In other words, for the former countries, reliable forecasts can be made for cohorts that have completed a minimum of 10 years of their reproductive career (age 25 or older), whereas for Japan, a minimum of 20 years (age 35 or older) are required.

Using these criteria, we now turn to the results of our forecasts. We first computed $K(t, n)$, using (5) and the birth rates of the initial n reproductive age groups of incomplete cohorts. Figure 5 presents observed $\hat{k}(t)$ and estimated $K(t, n)$, shown in solid and broken lines, respectively. The values of $\hat{k}(t)$ and its standard error are given in Table A2.

Except for the USA, $K(t, n)$ declines over the cohorts until the 1975 cohort (t = 1975, n = 10) for Canada, the 1976 cohort for Norway (n = 10), and the 1965 cohort (n = 20) for Japan. For the USA, while the decline is evident for observed $\hat{k}(t)$, $K(t, n)$ shows a trend toward rising somewhat until the 1976 cohort (n = 10).

From (1), a decline in $\hat{k}(t)$ corresponds to a decline in the cohort total fertility, which has been observed in Figure 2, although it is less evident in Japan. However, a decline in $\hat{k}(t)$ or $K(t, n)$ does not necessarily mean an increase in the cohort mean age at childbearing. To describe the decline *and* the delay in cohort fertility adequately, we require the knowledge of $\hat{b}(x)$. In Figure 3, we saw that the values of $\hat{b}(x)$ are positive at early reproductive ages and become negative after age 28 or so. Accordingly, from (1), a decline in $\hat{k}(t)$ or $K(t, n)$ leads to a decline in fertility at younger ages at which the values of $\hat{b}(x)$ are positive, but to an increase in fertility at older ages at which the values of $\hat{b}(x)$ are negative, which in turn results in an increase in the cohort mean age at childbearing.

Our forecasts in Figure 5 suggest also that the decline in fertility may well continue in Canada, Norway, and Japan, while this trend may have levelled off in the USA. Clearly, these data show that forecast $K(t, n)$ is not a simple extrapolation of $\hat{k}(t)$. As we noted, $K(t, n)$ is determined by the $\hat{b}(x)$ of earlier completed cohorts as well as the completed fertility of later incomplete cohorts.

Using (6), we forecast incomplete birth rates of cohorts for Canada, the USA, Norway, and Japan. Figure 6 presents observed and forecast cohort total fertility; Figure 7 shows the corresponding mean age at childbearing. Our earlier analyses of $R(n)$ and $\hat{b}(x)$ suggest that forecasts for the cohorts of 1956–75 for Canada, 1957–76 for the USA and Norway are most probably reliable. For Japan, caution must be exercised and forecasts should be limited to the cohorts of 1956–65. We extend forecasts to more recent cohorts for the purpose of illustration. It is apparent that for the more recent cohorts (later than 1965 for Japan and 1975 for the other countries), forecast values become unstable and highly improbable.

Figure 6 shows that, overall, forecast cohort total fertility levels parallel the estimated $K(t, n)$ shown in Figure 5. We observe that, in Canada, the decline in cohort fertility may continue: specifically, forecast cohort total fertility may decline from 1.85 for the cohort of 1955 to 1.57 for the cohort of 1975. The figures for the USA are less dramatic: from 1.97 to 1.95 for the cohorts of 1956 and 1976, respectively. For Norway, forecast cohort total fertility rises from 2.06 for the cohort of 1956 to 2.13 for the cohort of 1961, and then declines to 1.69 for the cohort of 1976. For Japan, although observed cohort fertility shows no visible sign of decline, forecast levels suggest a clear downward trend, from 1.98 for the cohort of 1955 to 1.60 for the cohort of 1965. These results further demonstrate that forecast cohort total fertility is by no means a simple extrapolation of observed trends.

A somewhat reversed pattern is observed for the cohort mean age at childbearing. Figure 7 shows that the observed mean age rose steadily for all four countries. Our forecasts indicate that, except for the USA, the rising trend may continue, although the rate of increase is likely to decline, particularly in Canada and Norway. In the USA, the increase in the mean age may level off in the decades to come: in fact, we forecast a slight decline for more recent American cohorts (1963–76).

Discussion and conclusions

In this study, we have established a method to forecast incomplete cohort fertility, drawing on prior studies in this field, particularly those that employ information on historical trends of fertility (e.g., Ryder 1990) and a cohort's completed fertility (e.g., Bloom 1982; Chen and Morgan 1991). Our method

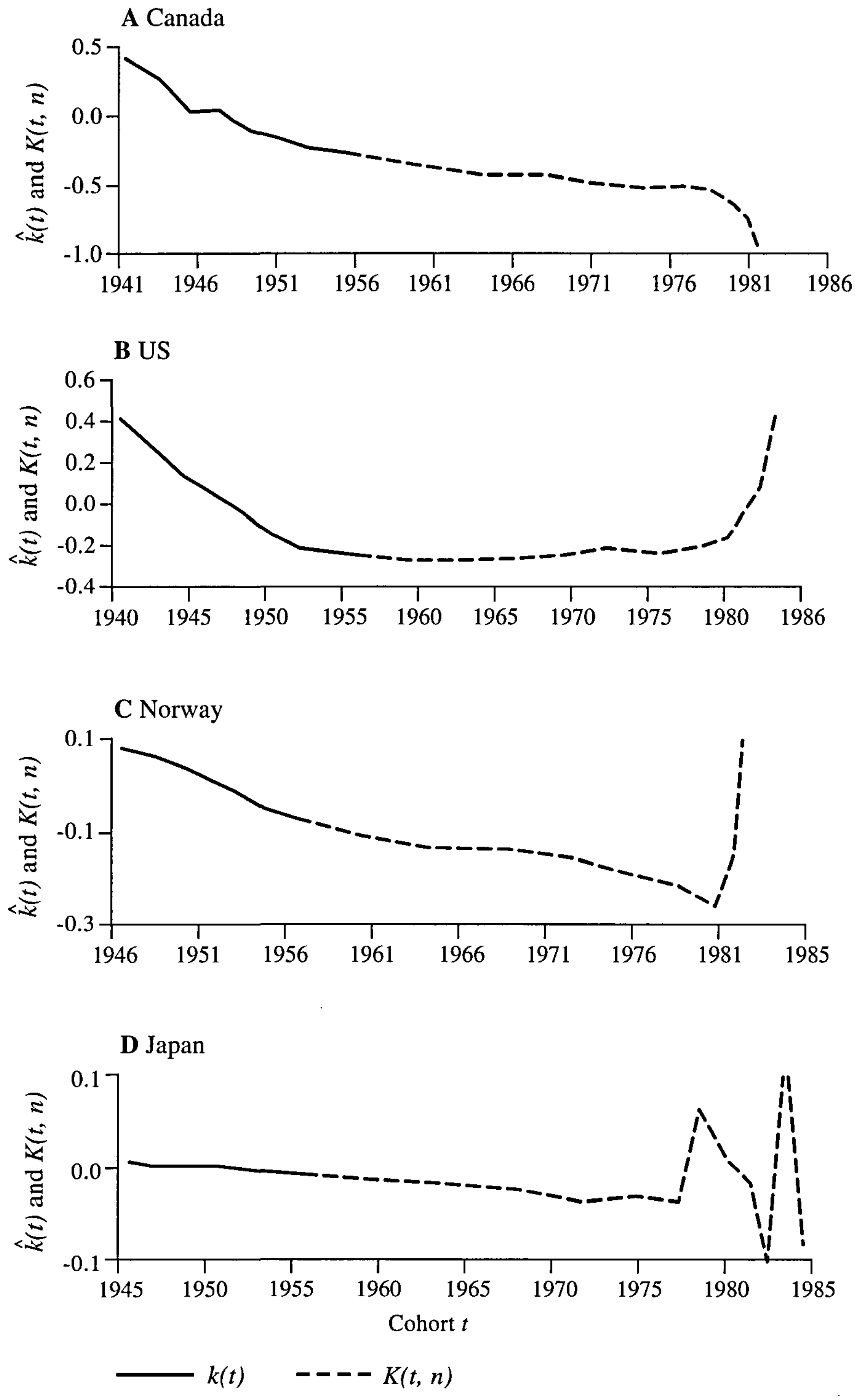

Figure 5 $\hat{k}(t)$ and $K(t, n)$, Canada, USA, Norway, and Japan
Source: As for Figure 2

improves on earlier models, particularly that of Evans (1986), in several respects. We determine the latest cohort for forecasts using an empirically based method (i.e., $R(n)$ and $\hat{b}(x)$), and also make full use of fertility data for both completed and incomplete cohorts. As a result, we can make more reliable forecasts of incomplete age-specific birth rates as young as age 25.

Based on the SVD model, which has been successfully applied in mortality (Lee and Carter 1992) and period fertility forecasts (Lee 1993), our model decomposes a fertility matrix of age by cohort into an age vector and a cohort vector. Our method calculates these vectors using completed fertility of earlier cohorts so that we establish a relationship between the level and the age pattern of fertility. As the age pattern of cohort fertility tends to be similar (our third model assumption), we apply the age vector

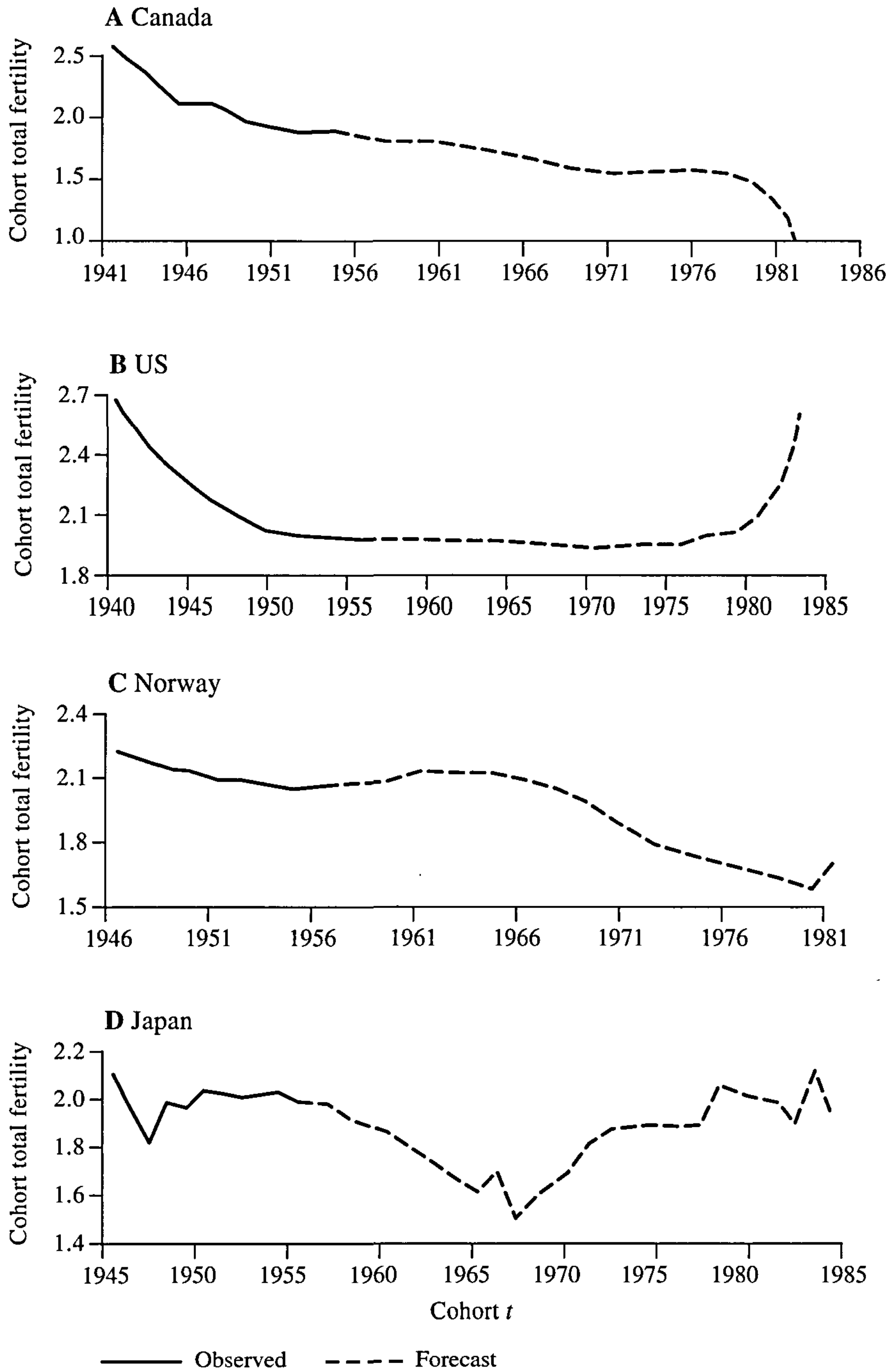

Figure 6 Observed and forecast cohort total fertility, Canada, USA, Norway, and Japan
Source: As for Figure 2

($\hat{b}(x)$) from earlier cohorts that have completed childbearing to later cohorts that have not yet completed. Because a cohort's total fertility tends to relate to its fertility at younger ages (our first model assumption), we estimate a cohort-vector value ($K(t, n)$) for a later cohort that minimizes error in describing its fertility at younger ages. With the age vector and the estimated cohort-vector value, we are able to forecast incomplete fertility for a later cohort. Further, because fertility varies little at older reproductive ages (our second model assumption), we can extend our forecasts to cohorts considerably younger than 44 years of age.

From a behavioural perspective, our model is a simple one which assumes that fertility is determined entirely by age. Clearly, estimated fertility rates are more reliable for some age groups than for others, and a smaller variance of modelling error generally indicates a better estimate. To adjust for the variance of modelling error, we re-estimated $k(t)$ using the weighted least-squares method (we acknowledge an anonymous reviewer for suggesting this strategy).

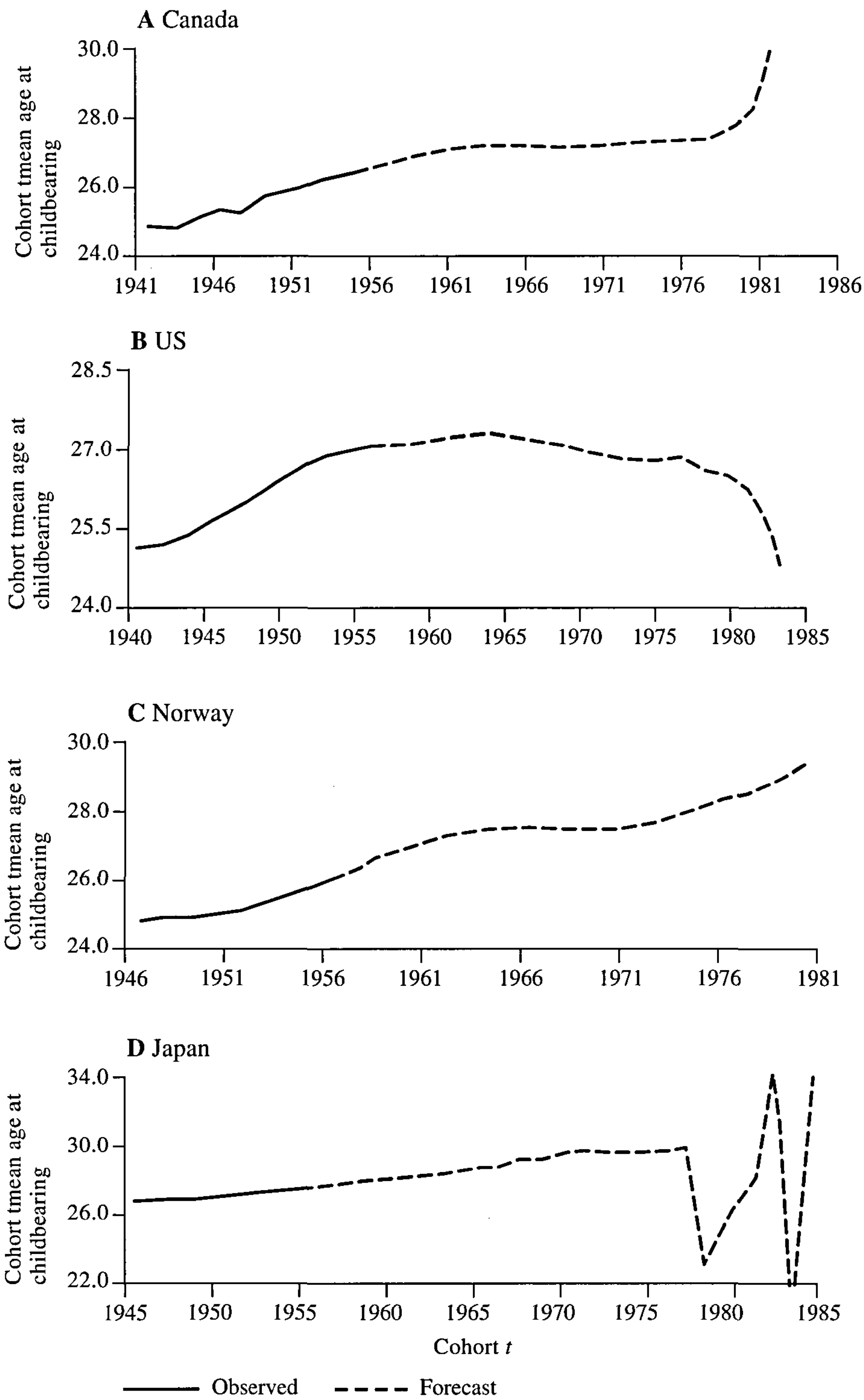

Figure 7 Observed and forecast cohort mean age at childbearing, Canada, USA, Norway, and Japan
Source: As for Figure 2

The results are virtually identical to those reported in Figure 5 (results not shown).

We have noted that the success of our method depends on how well its model assumptions hold true. For example, if there is a significant variation in the age pattern of cohort fertility between cohorts (i.e., our third assumption is violated), our SVD model may not effectively use the historical trends of fertility for forecasts. This violation will result in a low value of the explanation ratio at $n = 30$, as in the case of Japan. When a cohort's total fertility is not closely related to its fertility at younger ages (i.e., our first assumption is violated), we are unable to estimate reliably a cohort's incomplete fertility at older ages using its completed fertility at younger ages. This violation will result in a rapid decline in the explanation ratio as n declines (from 30). The fact that, with the exception of Japan, our explanation ratios remain high and virtually constant from $n = 30$ to some low number of n suggests that our first assumption is generally sound. Finally, a significant variation in cohort fertility at older ages violates our second assumption, and this again has occurred to some extent when applying our method to Japan. Overall, our forecasts provide reasonable expectations of what may happen to future fertility in Canada, the USA, Norway, and, to a lesser extent, Japan.

Our forecasts suggest that in Canada, the decline in cohort fertility is likely to continue. Cohort total fertility will decline from 2.57 for the 1941 cohort to possibly 1.57 for the 1975 cohort. The corresponding cohort mean age at childbearing may rise, however, from 24.8 to 27.4. Our forecasts paint a somewhat different picture for future American fertility, showing that the long-term decline in cohort total fertility may halt and the increase in mean age at childbearing may level off for more recent cohorts. The future trend of Norwegian cohort fertility is more dramatic: forecast total fertility will first increase slightly and then decline, to the surprise of the many forecasters who rely on extrapolation methods. Our prediction for Japan is even more interesting: although historical (observed) cohort fertility shows no clear pattern of change, we forecast a declining trend for the cohorts born in 1955–65. While prior studies also predict future declines in cohort fertility in countries like the ones reported here (e.g., Frejka and Calot 2001a, b), our method enables us to quantify these declines.

Aside from meeting model assumptions, we noted that our method is not intended as a general tool for fertility forecasts, nor is it assumed to work well in all countries at all times. We have shown, for instance, that our method did not work as well for Japan as for Canada, the USA, and Norway. The model assumptions must be carefully examined before our method can be applied. Further, even when our model assumptions are perfectly satisfied, we cannot provide reliable forecasts for young cohorts that have just begun their reproductive careers. For example, our application restricts our forecasts to cohorts older than age 25 for Canada, the USA, and Norway, and age 35 for Japan. In other words, our method requires information on a cohort's first ten or more years of complete reproductive history in order to forecast its incomplete fertility reliably. Although a few more years of forecasts might be attempted depending upon how well our model assumptions are met, caution must be taken in extending the horizon of the forecast. Finally, while our method may have improved short-term cohort fertility forecasts, forecast cohort fertility does not automatically equal future period fertility. However, a reliable forecast of future cohort total fertility will provide a sound basis for estimating future period total fertility.

Appendix

Table A1 Values of $\hat{a}(x)$, Canada, USA, Norway, and Japan

	Canada		USA		Norway		Japan	
Age	$\hat{a}(x)$	SE	$\hat{a}(x)$	SE	$\hat{a}(x)$	SE	$\hat{a}(x)$	SE
15	0.0049	0.0001	0.0058	0.0001	0.0035	0.0003	0	0
16	0.0173	0.0004	0.0183	0.0004	0.0179	0.0010	0.0003	0
17	0.0408	0.0012	0.0395	0.0009	0.0496	0.0017	0.0016	0
18	0.0724	0.0034	0.0697	0.0021	0.0882	0.0012	0.0053	0.0001
19	0.1031	0.0065	0.1038	0.0044	0.1200	0.0027	0.0135	0.0004
20	0.1268	0.0094	0.1310	0.0073	0.1412	0.0053	0.0293	0.0008
21	0.1420	0.0109	0.1485	0.0096	0.1550	0.0070	0.0555	0.0012
22	0.1512	0.0104	0.1538	0.0103	0.1585	0.0074	0.0963	0.0025
23	0.1535	0.0087	0.1537	0.0096	0.1586	0.0068	0.1445	0.0039
24	0.1525	0.0061	0.1502	0.0082	0.1537	0.0050	0.1880	0.0049
25	0.1501	0.0041	0.1440	0.0064	0.1462	0.0042	0.2111	0.0048
26	0.1438	0.0028	0.1364	0.0049	0.1344	0.0027	0.2155	0.0041
27	0.1342	0.0019	0.1273	0.0037	0.1213	0.0018	0.2029	0.0035
28	0.1229	0.0015	0.1173	0.0027	0.1092	0.0013	0.1800	0.0034
29	0.1093	0.0010	0.1066	0.0020	0.0968	0.0017	0.1520	0.0033

Table A1 Continued

Age	Canada $\hat{a}(x)$	Canada SE	USA $\hat{a}(x)$	USA SE	Norway $\hat{a}(x)$	Norway SE	Japan $\hat{a}(x)$	Japan SE
30	0.0954	0.0013	0.0952	0.0016	0.0828	0.0018	0.1224	0.0033
31	0.0793	0.0013	0.0831	0.0015	0.0728	0.0020	0.0950	0.0032
32	0.0655	0.0010	0.0720	0.0018	0.0609	0.0023	0.0740	0.0030
33	0.0531	0.0009	0.0618	0.0020	0.0527	0.0025	0.0569	0.0026
34	0.0425	0.0007	0.0531	0.0021	0.0443	0.0022	0.0426	0.0021
35	0.0337	0.0008	0.0451	0.0020	0.0363	0.0022	0.0316	0.0016
36	0.0265	0.0008	0.0378	0.0018	0.0285	0.0017	0.0234	0.0012
37	0.0201	0.0008	0.0310	0.0016	0.0228	0.0014	0.0165	0.0008
38	0.0152	0.0007	0.0247	0.0013	0.0179	0.0012	0.0116	0.0006
39	0.0117	0.0006	0.0192	0.0010	0.0124	0.0008	0.0081	0.0004
40	0.0069	0.0003	0.0142	0.0008	0.0091	0.0008	0.0052	0.0003
41	0.0048	0.0002	0.0102	0.0006	0.0056	0.0005	0.0033	0.0002
42	0.0033	0.0002	0.0069	0.0003	0.0038	0.0004	0.0019	0.0001
43	0.0022	0.0001	0.0044	0.0002	0.0022	0.0002	0.0011	0.0001
44	0.0013	0.0001	0.0025	0.0001	0.0012	0.0001	0.0005	0

Source: As for Figure 2.

Table A2 Values of $\hat{k}(t)$, Canada, USA, Norway, and Japan

Cohort, t	Canada $\hat{k}(t)$	Canada SE	USA $\hat{k}(t)$	USA SE	Norway $\hat{k}(t)$	Norway SE	Japan $\hat{k}(t)$	Japan SE
1940	–	–	0.4197	0.0156	–	–	–	–
1941	0.4352	0.0209	0.3522	0.0110	–	–	–	–
1942	0.3535	0.0089	0.2827	0.0117	–	–	–	–
1943	0.2872	0.0118	0.2191	0.0146	–	–	–	–
1944	0.1700	0.0155	0.1554	0.0161	–	–	–	–
1945	0.0511	0.0136	0.1042	0.0161	–	–	0.0682	0.0057
1946	0.0320	0.0108	0.0562	0.0156	0.0056	0.0011	0.0540	0.0043
1947	0.0574	0.0144	0.0106	0.0156	0.0035	0.0004	0.0481	0.0038
1948	–0.0305	0.0050	–0.0404	0.0168	0.0012	0.0010	0.0359	0.0036
1949	–0.0920	0.0050	–0.0924	0.0170	0.0029	0.0004	0.0257	0.0038
1950	–0.1341	0.0048	–0.1412	0.0148	0.0018	0.0005	0.0074	0.0054
1951	–0.1773	0.0069	–0.1806	0.0109	0.0014	0.0005	–0.0120	0.0058
1952	–0.2103	0.0071	–0.2064	0.0078	–0.0002	0.0005	–0.0307	0.0044
1953	–0.2310	0.0081	–0.2226	0.0072	–0.0019	0.0004	–0.0525	0.0026
1954	–0.2397	0.0100	–0.2319	0.0088	–0.0032	0.0002	–0.0669	0.0029
1955	–0.2714	0.0111	–0.2391	0.0113	–0.0046	0.0004	–0.0772	0.0067
1956	–	–	–0.2455	0.0140	–0.0064	0.0006	–	–

Source: As for Figure 2.

Notes

1 Nan Li is Research Scientist at the Max Planck Institute for Demographic Research, Konrad-Zuse-Strasse 1, 18057 Rostock, Germany. Zheng Wu is Professor of Sociology at the University of Victoria, PO Box 3050, Victoria, BC V8W 3P5, Canada. E-mail: zhengwu@uvic.ca

2 The authors thank Jiajian Chen, Andrew Foster, Nico Keilman, and Rikiya Matsukara for providing the data for analysis. For comments on an earlier draft of this paper, we thank Thomas K. Burch, Joshua Goldstein, Nico Keilman, Shripad Tuljapurkar, and the Editors and two anonymous referees. Support was provided by the Department of Sociology at the University of Victoria. An earlier version of this paper was presented at the

Annual Meeting of the Population Association of America, Washington, DC, March 2001.

References

Ahlburg, D. A. and W. Lutz. 1998. Introduction: the need to rethink approaches to population forecasts, *Population and Development Review* 24(Suppl.): 1–15.

Becker, G. S. 1988. Family formation and macro behavior, *American Economic Review* 78: 1–13.

Bloom, D. E. 1982. What's happening to the age at first birth in the United States? A study of recent cohorts, *Demography* 19: 351–370.

Bongaarts, J. 1992. Do reproductive intentions matter?, *International Family Planning Perspectives* 18: 102–108.

Bongaarts, J. and G. Feeney. 1998. On the quantum and tempo of fertility, *Population and Development Review* 24: 271–291.

Brunborg, H. and S.-E. Mamelund. 1994. Kohort- og periodefrukbarhet i Norge 1820–1993, Report No. 94/27. Oslo: Statistics Norway.

Bumpass, L. L., R. Rindfuss, and R. B. Janosik. 1978. Age and marital status at first birth and the pace of subsequent fertility, *Demography* 15: 75–86.

Caldwell, J. C. 1982. *Theory of Fertility Decline.* London: Academic Press.

Chandola, T., D. A. Coleman, and R. W. Hiorns. 1999. Recent European fertility patterns: fitting curves to 'distorted' distributions, *Population Studies* 53: 317–329.

Chen, R. and S. P. Morgan. 1991. Recent trends in the timing of first birth in the United States, *Demography* 28: 513–533.

Coale, A. J. 1973. The demographic transition reconsiderd, in *Proceedings: International Population Conference, 1973, Volume 1.* Liege: International Union for the Scientific Study of Population.

Coale, A. and D. McNeil. 1972. The distribution by age at first marriage in a female cohort, *Journal of the American Statistical Association* 67: 743–749.

Coale, A. J. and T. J. Trussell. 1974. Model fertility schedules: variations in the age structure of childbearing in human populations, *Population Index* 44: 203–213.

De Beer, J. and A. De Jong. 1996. National population scenarios for countries of the European economic area, *Maandstatistiek van de Bevolking* 44: 7–19.

Evans, M. D. R. 1986. American fertility patterns: a comparison of white and nonwhite cohorts born 1903–56, *Population and Development Review* 12: 267–293.

Foster, A. 1990. Cohort analysis and demographic translation: a comparative study of recent trends in age-specific fertility rates from Europe and North America, *Population Studies* 44: 287–315.

Frejka, T. and G. Calot. 2001a. Cohort reproductive patterns in low-fertility countries, *Population and Development Review* 27: 103–132.

Frejka, T. and G. Calot. 2001b. L'évolution du calendrier des naissances par génération dans les pays à basse fécondité à la fin du XX^e siécle, *Population* 56: 397–420.

Frejka, T. and G. Calot. 2001c. Cohort childbearing age patterns in low-fertility countries in the late 20th century: is the postponement of births an inherent element? Paper presented at the IUSSP Seminar on 'International Perspectives on Low Fertility: Trends, Theories and Policies', Tokyo, 21–23 March 2001.

Hechter, M. and K. Kanazawa. 1997. Sociological rational choice theory, *Annual Review of Sociology* 23: 191–214.

Hendershot, G. E. and P. J. Placek (eds.). 1981. *Predicting Fertility: Demographic Studies of Birth Expectations.* Toronto: Lexington Books.

Kohler, H.-P., F. C. Billari, and J. A. Ortega. 2001. Towards a theory of lowest-low fertility. The Max Planck Institute for Demographic Research Working Paper (WP 2001-032), Rostock, Germany.

Kohler, H.-P. and J. A. Ortega. 2001a. Period parity progression measures with continued fertility postponement: a new look at the implications of delayed childbearing for cohort fertility. The Max Planck Institute for Demographic Research Working Paper (WP 2001-001), Rostock, Germany.

Kohler, H.-P. and J. A. Ortega. 2001b. Parity progression ratios with continued fertility postponement: assessing the implications of delayed childbearing for fertility in Sweden, the Netherlands and Spain. Manuscript, The Max Planck Institute for Demographic Research, Rostock, Germany.

Kohler, H.-P. and D. Philipov. 2001. Variance effects in the Bongaarts–Feeney formula, *Demography* 38: 1–16.

Lee, R. and L. Carter. 1992. Modeling and forecasting the time series of U.S. mortality, *Journal of the American Statistical Association* 87: 659–671.

Lee, R. 1993. Modeling and forecasting the time series of U.S. fertility: age patterns, range, and ultimate level, *International Journal of Forecasting* 9: 187–202.

Lesthaeghe, R. 2001. Postponement and recuperation: recent fertility trends and forecasts in six Western European countries. Paper presented at the IUSSP Seminar on 'International Perspectives on Low Fertility: Trends, Theories and Policies', Tokyo, 21–23 March 2001.

Matsukura, R. 2002. Age-specific fertility rates. Special Tabulations. Tokyo: Population Research Institute, Nihon University.

Millman, S. R. and G. E. Hendershot. 1980. Early fertility and lifetime fertility, *Family Planning Perspectives* 12: 139–149.

Morgan, S. P. 1996. Characteristic features of modern American fertility, *Population Development Review* 22(Suppl.): 19–63.

Morgan, S. P. and R. R. Rindfuss. 1999. Reexamining the link of early childbearing to marriage and to subsequent fertility, *Demography* 36: 59–75.

National Centre for Health Statistics. 2000. *National Vital Statistics Report 48*. Available: http://www.cdc.gov/nchs/products/pubs/pubd/mvsr/mvsr.htm

National Centre for Health Statistics. 2001. *National Vital Statistics Report 49*. Available: http://www.cdc.gov/nchs/products/pubs/pubd/mvsr/mvsr.htm

Ni Bhrolchain, M. 1992. Period paramount? A critique of the cohort approach to fertility. *Population Development Review* 18: 599–629.

Notestein, F. 1945. Population—the long view, in T. W. Schultz (ed.), *Food for the World*. Chicago: University of Chicago Press, pp. 13–31.

Office of Population Research. 2001. *U.S. Cohort and Period Fertility Tables, 1917–1980*. Princeton, NJ: Princeton University Press. Available: http://opr.princeton.edu/archive/heuser

Rindfuss, R. R., S. P. Morgan, and G. Swicegood. 1988. *First Births in America: Changes in the Timing of Parenthood*. Berkeley: University of California Press.

Ryder, N. B. 1956. Problems of trend determination during a transition in fertility, *Milbank Memorial Fund Quarterly* 34: 5–21.

Ryder, N. B. 1983. Cohort and period measures of changing fertility, in R. A. Bulatao and R. Lee (eds.), *Determinants of Fertility in Developing Countries*. New York: Academic Press, pp. 737–756.

Ryder, N. B. 1990. What is going to happen to American fertility?, *Population and Development Review* 16: 433–454.

Statistics Canada. 2002a. Canadian Statistics: Age-specific Fertility Rate. Ottawa: Statistics Canada. Available: http://www.statcan.ca/english/Pgdb/People/Health/health08.htm

Statistics Canada. 2002b. Births, 1998–1999. Special Tabulations. Ottawa: Health Division, Statistics Canada.

Statistics Norway. 2002. Age-specific Fertility Rates, Total Fertility and Reproduction Rates. Available: http://www.ssb.no/english/yearbook/tab/t-020210-070.html

van Imhoff, E. 2001. On the impossibility of inferring cohort fertility measures from period fertility measures, *Demographic Research* 5: 21–64. Available: http://demographic-research.org

van Imhoff, E. and N. Keilman. 2000. On the quantum and tempo of fertility: comment, *Population Development Review* 26: 549–553.

United Nations. 1987–1997. *Demographic Yearbook*. New York: United Nations.

United Nations. 2001. *World Population Prospects: The 2000 Revision*. New York: United Nations.

Westoff, C. F. and N. B. Ryder. 1977. The predictive validity of reproductive intentions, *Demography* 14: 431–453.

Whelpton, P. K. 1936. An empirical method of calculating future population, *Journal of the American Statistical Association* 31: 457–473.

Zeng, Y. and K. C. Land. 2001. A sensitivity analysis of the Bongaarts–Feeney method for adjusting bias in observed period total fertility rates, *Demography* 38: 17–28.