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Recent European fertility patterns: Fitting curves to 'distorted' distributions

T. CHANDOLA, D. A. COLEMAN, AND R. W. HIORNS

Abstract. Recent patterns of fertility in Europe show marked differences between countries. Recent United Kingdom and Irish fertility curves show 'distortions' in terms of a 'bulge' in early age fertility, distinct from the smoother curves of other European countries. These patterns may not be adequately described by mathematical functions used by previous studies to model fertility curves. A mixture model with two component distributions may be more appropriate. The suitability of the simple and mixture Hadwiger functions is examined in relation to the fertility curves of a number of European countries. While the simple Hadwiger model fits recent period age-specific fertility distributions for some countries, others which display a 'bulge' in early age fertility require a mixture Hadwiger model. Some of the parameters of the Hadwiger models appear to be related to familiar demographic indices. The simple and mixture Hadwiger models appear useful in describing and comparing fertility patterns across European countries.

INTRODUCTION

This paper reports on investigation of some 'distorted' fertility rate distributions revealed in the course of a broader research project to compare and analyse demographic trends in the industrial world since the second world war. A preliminary account of that project is given elsewhere (Coleman and Chandola, in press). One component of the research has been to analyse the differences and similarities between the United Kingdom population on the one hand and the population of other European countries on the other, to see to what extent British demographic behaviour is typical of the rest of Europe. One of several ways of doing that would be to see if a standard or average European distribution could be determined for each variable being studied, and then to establish the British position with respect to that variable. We are already familiar, for example, with the 'standard European population' used for making comparisons of mortality through direct standardisation.

This notion could be extended to other variables, for example to period fertility measures. Fertility can already be compared, of course, using a wide variety of existing measures; summary indices or averages such as total fertility, the mean age at childbearing or distributional measures based on birth order or parity. Few comparisons, however, are made based upon the detailed distribution of the age-specific fertility curve (by single year of age). Not all the information in that curve is conveyed by total fertility, the mean age at childbearing, the proportion of births or of total fertility accounted for by mothers over age 30 or other summary measures. Much remains to be described in respect of the variance, skew, kurtosis and symmetry of such curves. In general, recent trends in European fertility might lead us to expect a convergence upon an age-specific pattern with a relatively high mode, median and mean and with an increasing symmetry around the central measure.

The work behind this paper began by asking whether a standard European age-specific fertility curve could be defined, capable of being fitted by a simple mathematical model with a small number of parameters. Deviations from that 'standard' in different countries, regions or periods of time could then be accounted for statistically by variation in those parameters. The search for a common European age-specific fertility pattern which might serve as a standard was encouraged by the similarity of the distributions for a number of countries (Figure 1). These are countries which represent the better off, more developed, end of European economy and society, with fertility under effective control, such as Denmark. Could a curve be based upon an average of those countries, producing a standard to which others might be expected to conform as they developed further? In fact, while a number of fertility curves seemed suitable for this role, others showed marked deviation from the regularity expected in these distributions. The investigation of these irregular or 'distorted' distributions is the aim of this paper.

PREVIOUS WORK

Although modelling fertility curves has attracted the attention of demographers for many years,

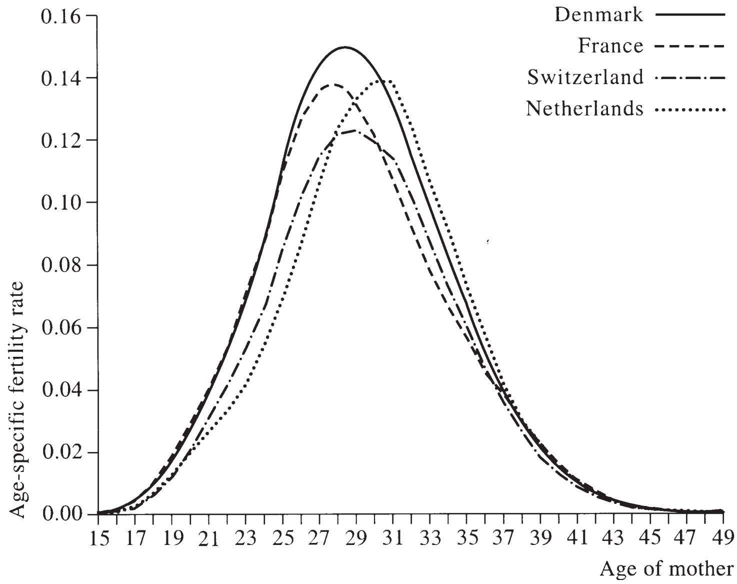

Figure 1. Fertility in Switzerland, Netherlands, Denmark, and France, 1994.

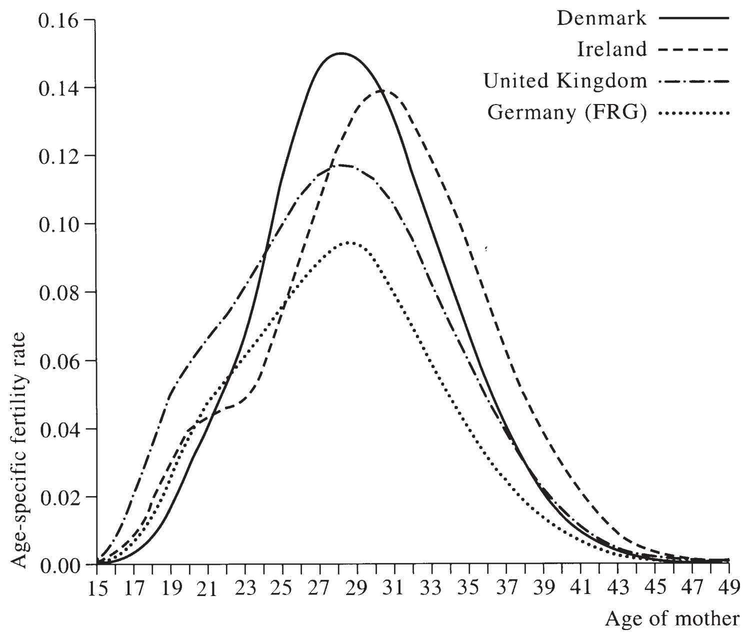

Figure 2. Fertility in the United Kingdom, Germany (FRG), Denmark, and Ireland, 1994.

there seems to have been relatively little recent interest in the fitting of curves to data from modern low-fertility countries or in comparing those countries. This is probably because all important problems in this connection have been resolved by earlier work. Previous work, it was assumed, would readily provide a quick solution to the choice of statistical model to fit the data and their variability, ideally with a small number of parameters which could be interpreted in demographic terms. Certainly a variety of options is available to curve-fitters for fertility. A number of curves have been shown to provide excellent fits to the one-year age-specific fertility rate distributions of populations with natural fertility, with transitional fertility, and with controlled fertility. These include for example the Coale–Trussell function (Coale and Trussell 1974, 1978; Hoem *et al.* 1981), the Pearson Type 1 curve (Mitra 1967; Romaniuk 1973) and Type III curves (Nurul-Islam and Ali Mallick

1987), the Beta and Gamma distributions equivalent to the Pearson Type I and III curves respectively (Hoem *et al.* 1981), the Hadwiger distribution (Gilje 1969; Hoem *et al.* 1981) and polynomials.

Other important classes of fertility models can also be considered. For example the important class of Gompertz relational models (Brass 1977; Pollard and Valkovics 1992) is intended to describe, project (Murphy 1982), and adjust data sets and the parameters defining the observed fertility distribution relative to the shape and dispersion of the standard. The Brass model (Brass 1977) is not an obvious choice for defining a standard. In the first place, its purpose is to measure deviations from a standard chosen on other or a priori grounds. The three parameters of the Gompertz relational fertility model (total fertility defining the level, β defining the location on the age axis relative to the standard, and β defining the spread relative to the standard) cannot readily be interpreted in terms of an underlying demographic meaning (Zaba 1989). That might not matter for our descriptive and comparative purposes if a suitable standard could be defined and if deviations from it were of a simple nature; indeed for those purposes it might serve very well. However the Brass model did not produce very good fits in the comparative tests undertaken by Hoem *et al.* (1981).

On the face of it, the Coale–Trussell model is unsuitable because of its orientation to the analysis of fertility control. It focuses on marital fertility, the parameters determining the shape of the curve being age of marriage, proportion marrying, and degree of fertility control. Nonetheless, very good results have been achieved by fitting the Coale–Trussell function to the overall fertility rates of modern post-transitional populations such as those of Denmark 1962–71 (Hoem *et al.* 1981) . The computation involved is rather daunting however. To determine the age distribution of fertility fitted to the Pearson Type 1 curve (equivalent to the Beta distribution), values for average age, variance, asymmetry, and kurtosis are needed although some simplifications enable estimates to be made from mean and modal ages of fertility only. Most work on this distribution has, however, been devoted to the projection of fertility and future births (Romaniuk 1973), as is the case with the Gompertz models.

Unmodified gamma and beta distributions may not generate curves of appropriate shape at the younger ages of the fertility distribution. However a modified gamma presented by Hoem *et al.* (1981) provided excellent fits to Danish data. Polynomials and splines elevated to a suitable order can be made to fit fertility curves but the lack of meaningful parameters makes these models ill-suited for comparative purposes and their use has not been taken further.

The problem addressed in this paper arose because there is considerable variation in the pattern of recent fertility among European countries which was not apparent in the age-specific distributions of earlier periods. Figure 2 shows the age-specific fertility rates by single year of the mother's age in the United Kingdom, Netherlands, Denmark, and Irish Republic in 1994. The graph shows considerable variation both in the timing and in the volume of fertility in these countries. The Danish curve is the smoothest, as noted above, while the United Kingdom and Irish curves display a marked hump in early fertility. Our attention was first drawn to this feature by M Gérard Calot (personal communication), using the detailed data available from his Observatorie Démographique Européen. An accurate representation of United Kingdom and Irish fertility by the models used in previous papers, and a parameterisation of their peculiarities, may not be expected because of the bulge at youthful ages in the fertility distributions of these two countries. This paper is an attempt at modelling these apparently 'distorted' European fertility curves, paying special attention to these particular features of United Kingdom and Irish fertility by using a 'mixture' model. A mixture of the Hadwiger functions is chosen to demonstrate a plausible approach.

METHOD

The distributions used for previous attempts at modelling fertility curves would be inadequate in describing the early bulge in the fertility curves of the United Kingdom, the Irish Republic, and a number of other countries. Another parsimonious mathematical model is needed which is able to describe the United Kingdom and Irish fertility curves as well as those with more usual distributions. Such a model could reveal other important parameters which need to be taken into account when comparing fertility between countries and across time and would thus increase our understanding of fertility patterns, especially the particular features of enhanced early fertility. It would be helpful if any additional parameters could not only measure the departure from symmetry in some populations but also throw light on the demographic processes involved.

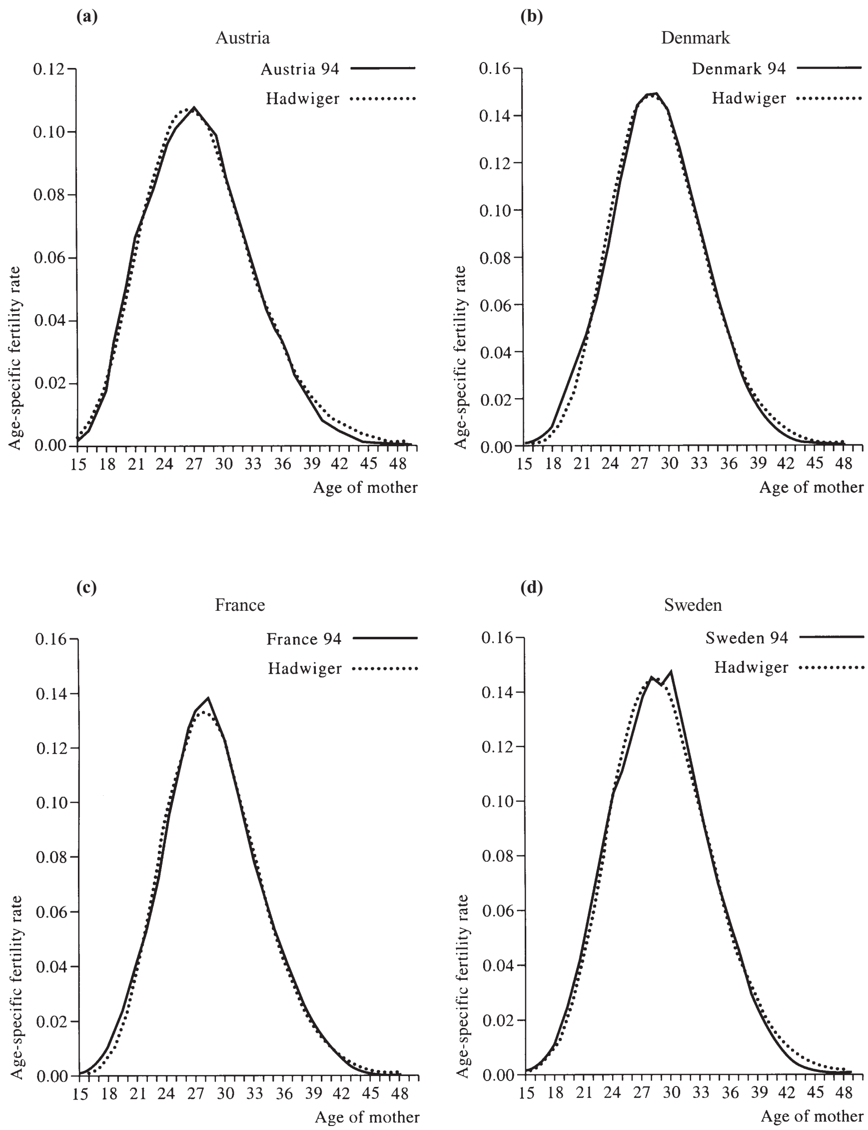

Figure 3. (a) Simple Hadwiger model estimates and fertility rates in Austria, Denmark, France and Sweden, 1994.

The Hadwiger function

Hoem *et al.* (1981) and Gilje (1969) found that the Hadwiger function provided a good fit to the fertility curves of modern populations. Hoem *et al.* (1981) also found that other mathematical functions could be used to fit the fertility curves in Denmark from 1962 to 1971 such as the Coale-Trussell and gamma density functions. These functions could also have been explored in relation to fitting the fertility curves of more recent European countries.

The Hadwiger function is expressed as

$$f(x)=\frac{ab}{c}\left(\frac{c}{x}\right)^{3/2}\exp\left\{-b^2\left(\frac{c}{x}+\frac{x}{c}-2\right)\right\}$$

where x is the age of mother at birth and a,b,c are the three parameters.

This model may fit well the smooth fertility curves such as the Danish fertility curve (Figure 2). However, the model may not fit the recent United Kingdom and Irish and other fertility curves well because of the excesses of fertility at younger ages (Figure 2). The possibility arises that distributions of this kind may represent not an homogeneous population but two somewhat different populations. If that is so, a combination of a pair of curves, rather than just one, may be more appropriate as a model. Accordingly a ‘mixture’ term was introduced in the Hadwiger function to separate two distributions that could, taken together, fit the humped fertility distributions of the

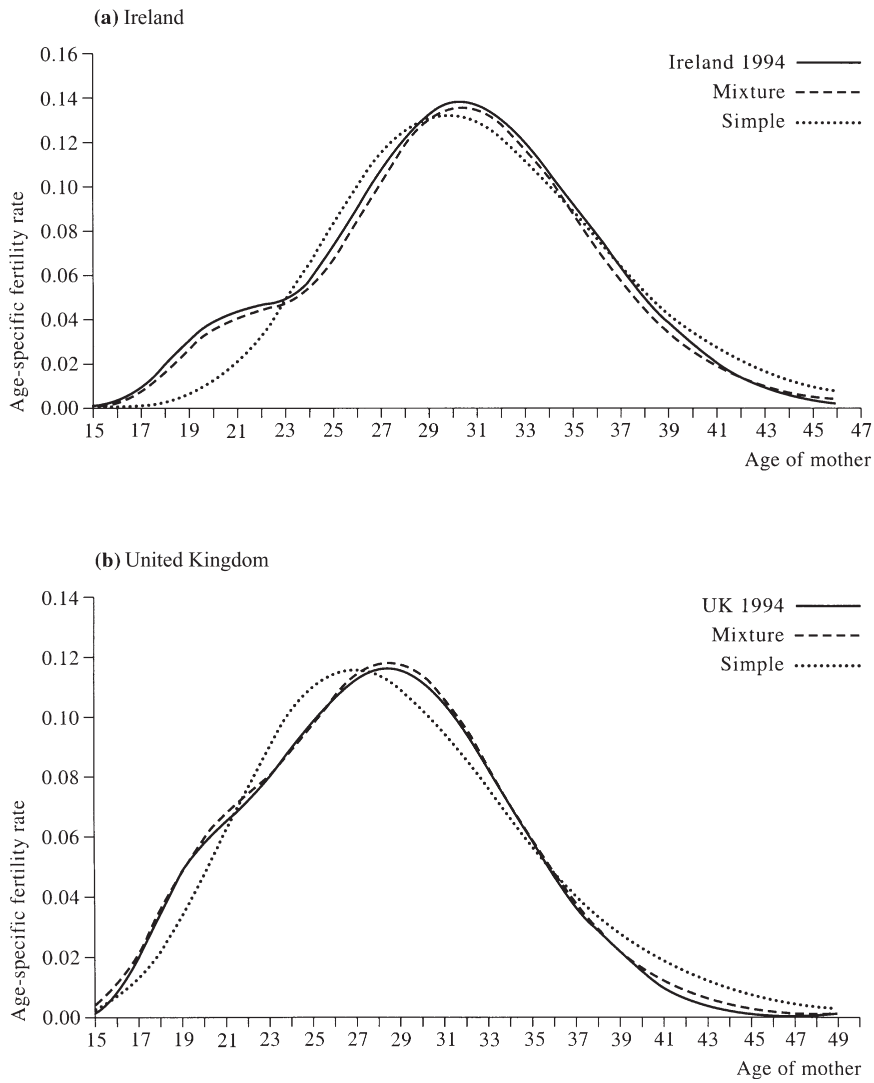

Figure 4. Simple and mixture Hadwiger models fitted to Irish and United Kingdom fertility curves, 1994.

United Kingdom and Irish Republic and other appropriate examples. The mixture function is expressed by

$$f(x)=m\left(\frac{b_1}{c_1}\right)\left(\frac{c_1}{x}\right)^{3/2}\exp\left\{-b_1^2\left(\frac{c_1}{x}+\frac{x}{c_1}-2\right)\right\}$$

$$+(1-m)\left(\frac{b_2}{c_2}\right)\left(\frac{c_2}{x}\right)^{3/2}\exp\left\{-b_2^2\left(\frac{c_2}{x}+\frac{x}{c_2}-2\right)\right\}$$

where m is the mixture parameter that determines the relative size of the two component distributions and b_1, c_1 and b_2, c_2 are the other parameters.

The fit of these two models (the simple Hadwiger and the Hadwiger mixture model) is tested using period data by single year of age from different European countries. The data were obtained from Eurostat. Furthermore, some potential explanations of the Hadwiger parameters are explored by examining their associations with more traditional demographic measures of fertility such as total fertility and the mean age and modal age of fertility. The fitted Hadwiger curves were also compared with the curves of marital and non-marital fertility (calculated as the number of marital or non-marital births to a mother by single year of age divided by the number of women in the population of that age).

RESULTS

Figure 3 shows that, for the 1994 data, the simple Hadwiger model fits the Danish, French, Austrian and Swedish fertility curves very well but the model does not accurately fit the United Kingdom and

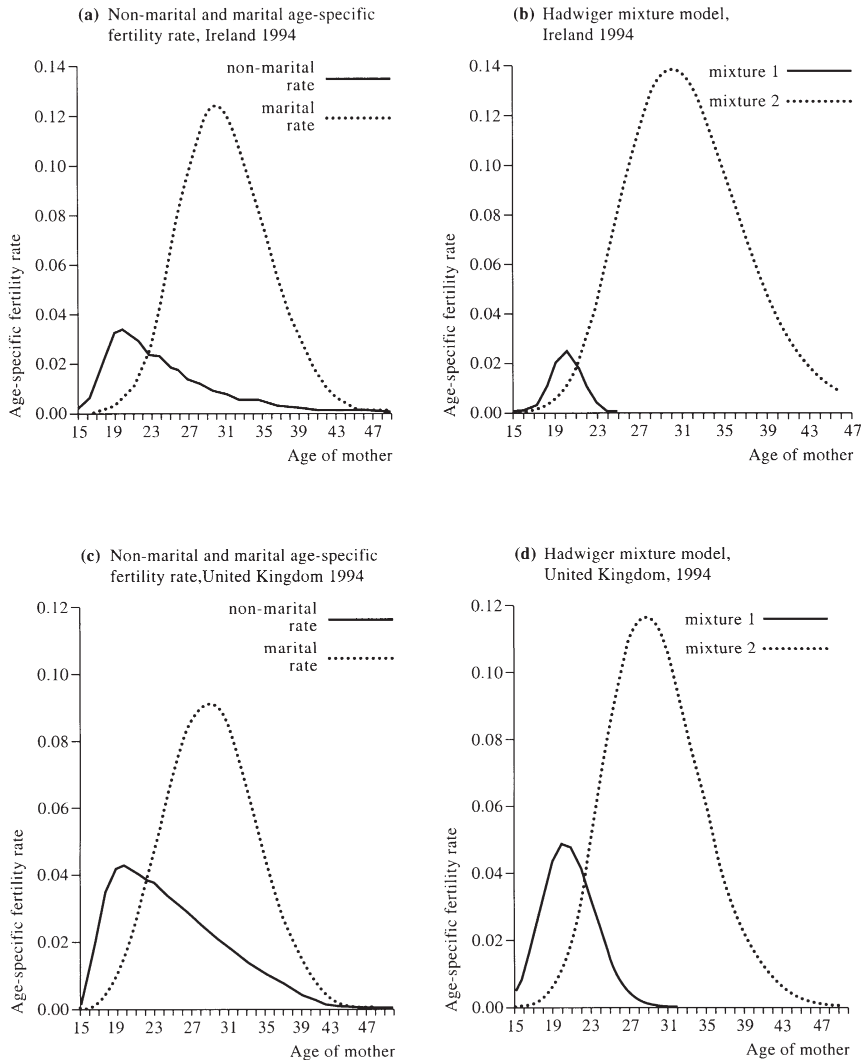

Figure 5. Comparison of relative sizes of Hadwiger mixtures 1 and 2, and marital and non-marital fertility, United Kingdom and Irish Republic, 1994.

Irish fertility curves (Figure 4). The hump in early fertility in the United Kingdom and Ireland cannot be accommodated by the simple Hadwiger model. On the other hand, the Hadwiger mixture model fits the United Kingdom and Irish fertility curves well. The separation of the simple Hadwiger function into two distributions appears to have resulted in a well fitting model for the fertility of the United Kingdom and Ireland in 1994. The mixture term 'm' in the Hadwiger mixture model determines the relative size of the two component distributions. When the Hadwiger mixture function is separated into two distributions, the resulting two curves show that in the United Kingdom and Ireland the humps in fertility are being caused by the early-age fertility in the group 'mixture1 ' (Figure 5). The mixture 1 group is about a quarter the size of the mixture 2 group in the United Kingdom (that is to say, the area under the curve is about 1/4 the size). In the Irish Republic, the mixture 1 group is much smaller – about a tenth the size of the mixture 2 group .

It could be hypothesised that, in respect of these countries, the mixture 1 group represents, to some extent, non-marital fertility. Fewer women are married at such a relatively young age, and mean age at maternity for births outside marriage tends to be lower than for births within marriage. The hump in the United Kingdom and Irish fertility curves could be caused by a relatively large proportion of teenage fertility, most of it non-marital. Figure 5 shows that this hypothesis is partly supported by the non-marital and marital fertility curves for the two countries. The curves resemble the Hadwiger mixture distributions. The smaller mixture 1 group resembles the curve for non-marital fertility,

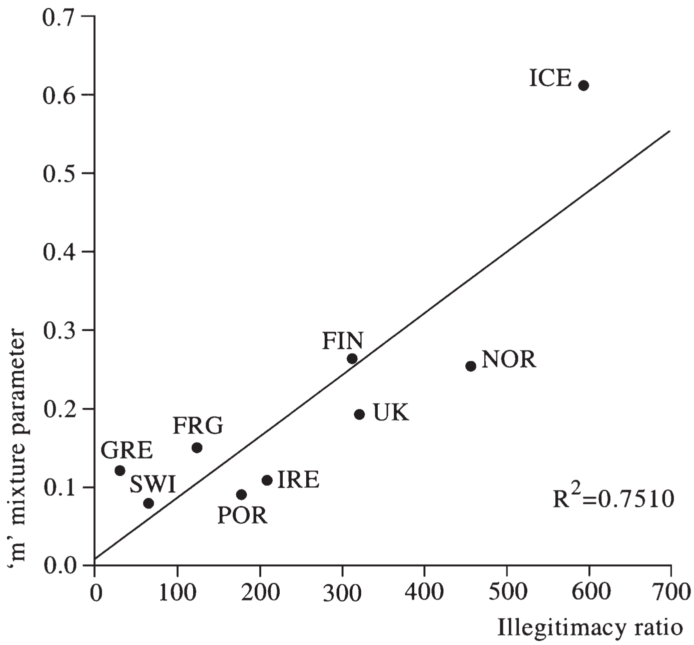

Figure 6. Assocation between illegitimacy ratio and Hadwiger mixture parameter for selected European countries, 1994.

although the size of the right-end tail of the non-marital fertility distribution is under-estimated. This under-estimation of the size of non-marital fertility among later age groups results in a comparatively larger mixture 2 group. While the Hadwiger mixture function does not accurately reflect the size of the non-marital and marital fertility curves, it does manage to reflect the difference in age structure between the marital and non-marital curves in these two countries. Of course, other factors apart from marital and non-marital fertility are likely to be reflected in the mixture model.

However, the United Kingdom and the Irish Republic have by no means the highest levels of non-marital fertility (illegitimacy ratio) in Western Europe. How far do other countries support the hypothesis that the mixture 1 group represents non-marital fertility? Figure 6 examines the association between the Hadwiger mixture parameter and non-marital fertility in other selected European countries in 1994. There is some evidence of a linear association between the illegitimacy ratio and the Hadwiger mixture parameter. Increasing illegitimacy is associated with a larger mixture parameter; the relative size of the two fertility distributions in the Hadwiger mixture model thus appears to be related to the ratio of non-marital fertility to marital fertility in the small number of countries represented in this sample.

This conclusion is, however, subject to an important qualification. There are a number of European countries with high proportions of births outside marriage where the simple Hadwiger distribution fits the fertility distribution quite well without recourse to a mixture model. For example, despite having Europe's highest levels of births outside marriage, the fertility curves for Sweden and Denmark in 1994 appear to be fitted well by the simple Hadwiger model (Figure 3). It is evident that these populations do not show the heterogeneity seen in the United Kingdom distribution with respect to the timing and volume of marital and non-marital fertility; nor do they have the exceptionally high teenage fertility rates (almost all extra-marital) characteristic of this country. In Sweden and Denmark and other countries it may be hypothesised that individuals or cohabiting couples producing births outside marriage are behaving, demographically, much more like married couples than is the case in the United Kingdom or the Irish Republic. Are the age distributions of marital and non-marital fertility in the former cases more similar to each other than in the United Kingdom and the Irish Republic? Some evidence for this hypothesis is shown in Figure 7. This shows that, in Denmark and Sweden in 1994, the distributions of marital and non-marital fertility were similar in timing and volume. The similarity in the distribution of marital and non-marital fertility may mean there is no need for a Hadwiger mixture function to describe the pattern of fertility in these and other countries with regular fertility curves.

Figure 7 shows that, in Austria in 1994, the non-

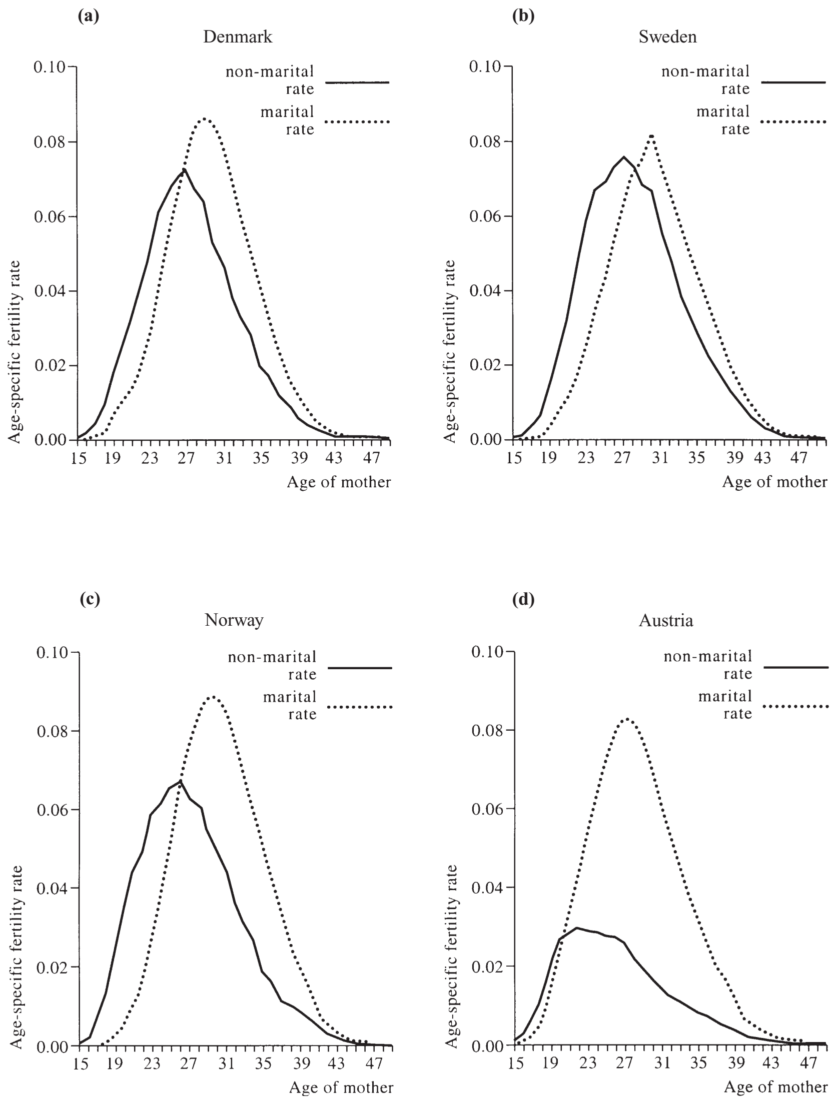

Figure 7. (a) Non-marital and marital age-specific fertility rates in Denmark, Sweden, Norway, and Austria, 1994.

marital fertility curve is subsumed by the marital fertility curve. Non-marital fertility in Austria in 1994 was lower relative to the Scandinavian countries. Moreover, the distributions of marital and non-marital fertility in Austria were not notably heterogeneous in terms of the timing of fertility. This may explain why the simple Hadwiger model is sufficient to describe the Austrian fertility curve (Figure 3). On the other hand, the Norwegian marital and non-marital fertility curves for 1994 (Figure 7) show that the timing and volume of non-marital fertility is distinct from the marital fertility curve. If these two curves are combined, the resulting fertility curve shows a bulge in early-age fertility, similar to that of the United Kingdom and Irish fertility curves, as a result of the volume of fertility, mostly non-marital, among relatively younger women. This suggests that the Hadwiger mixture model may fit the Norwegian fertility curve well because of the difference in the volume and timing of non-marital fertility compared to marital fertility, as is the case with the United Kingdom and Irish fertility curves.

As far as the data available to the authors can reveal, the ‘distorted’ fertility distribution in the United Kingdom has arisen only since the 1970s and in the Irish Republic only since the 1980s . In 1970 the fertility rate distribution of the former was much more symmetrical and could easily be fitted by a simple (unmixed) Hadwiger distribution. The emergence of the distortion is charted in Figure 8. Fertility rates for the age group 15–19 have remained relatively constant since 1980, and have been ‘left behind’ by the reduction and retardation

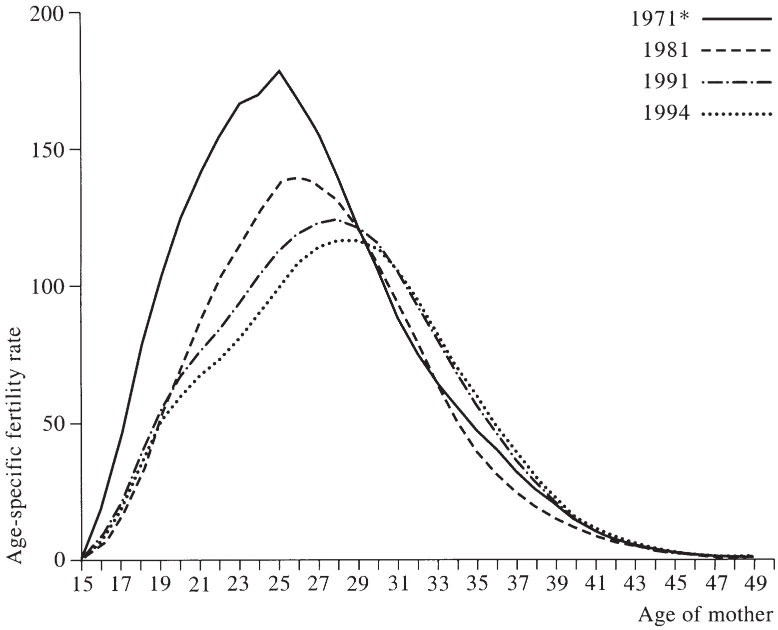

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Figure 8. Age-specific fertility rate in the United Kingdom 1971–1994.

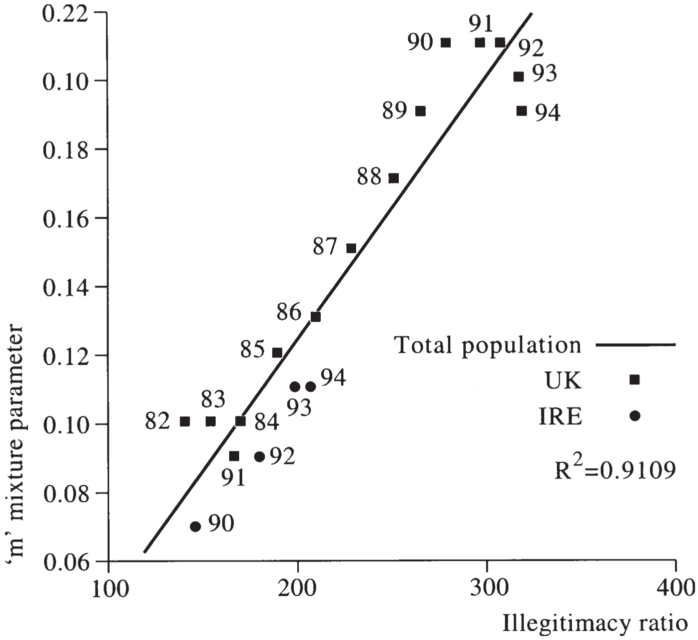

Figure 9. Association between illegitimacy ratio and Hadwiger mixture parameter estimates, United Kingdom and Ireland 1982–94.

of the rest of the fertility curve in the process of the move to older mean ages at maternity as in the rest of Europe.

The association between the Hadwiger mixture parameter and the illegitimacy ratio for the United kingdom and the Irish Republic is further explored in Figure 9. There appears to be a linear association for both countries: the mixture parameter increases over time and also with increasing non-marital fertility. However, the dip in the mixture parameter in the 1990s for the United Kingdom suggests that the association between the Hadwiger mixture parameter and non-marital fertility may not be straightforward; other aspects of fertility are likely to influence the parameter.

The other parameters in the Hadwiger simple and

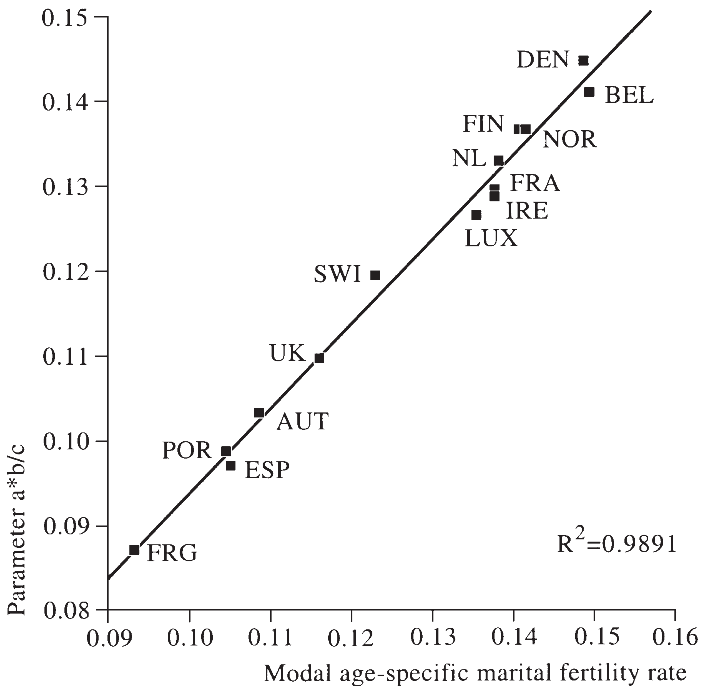

Figure 10. Scatter of modal age-specific fertility rate and parameter *a*b/c* selected countries, 1994.

mixture models may also have demographic interpretations. Some possible interpretations of the parameters for the simple Hadwiger model are explored in Figures 10 and 11. The parameter a is strongly associated with total fertility and is about 0.56 times the total fertility of these countries in 1994. The *c* parameter is strongly correlated with the mean age of motherhood in them. The *b* parameter helps to determine the height of the curve in the formula *ab/c* which is highly associated with the maximum age-specific fertility rate (mode) in these countries.

It is possible that the 'c_1' and 'c_2' parameters in the Hadwiger mixture model reflect the mean age of motherhood in the non-marital and marital fertility groups. Although there is limited evidence for this hypothesis, Figure 12 shows that for both the United Kingdom and Ireland, the 'c_2' parameter has become increasingly large over time. This could be a reflection of the increase in the mean age of marital motherhood in both countries over the same time period. The figure also shows that the 'c_1' parameter decreases for these two countries in the 1990s. This is contrary to what might be expected from trends in the mean age of non-marital motherhood which has increased over the same time period in both countries. Thus the Hadwiger mixture parameters cannot be interpreted exclusively in terms of marital and non-marital fertility.

DISCUSSION

The attempt to fit Hadwiger models to recent fertility curves in Europe has met with some success. Firstly, the simple Hadwiger model appears to fit the smooth fertility curves of some European countries relatively well. These curves, when averaged, may provide a candidate for the European standard fertility distribution proposed at the beginning of the paper. Secondly, the Hadwiger mixture model fits the 'humped' fertility curves of certain other European countries which cannot be fitted by the simple model. These results could allow the comparison of fertility and some of its components in European countries in terms of the parameters of the Hadwiger model.

Hoem *et al.* (1981) contended that the Hadwiger parameters do not have any demographic interpretation when fitted to fertility curves. This paper offers evidence that the parameters may in fact have some demographic interpretations. Firstly, the strong correlations found between the parameters for the Hadwiger simple model ('*a*', '*b*' and '*c*') and total fertility, maximum age-specific fertility rate, and the mean age of motherhood suggest that the Hadwiger parameters may be interpreted in more traditional demographic terms. Furthermore, the two distributions in the Hadwiger mixture model appear to be related to marital and non-marital fertility, at least to some degree in some

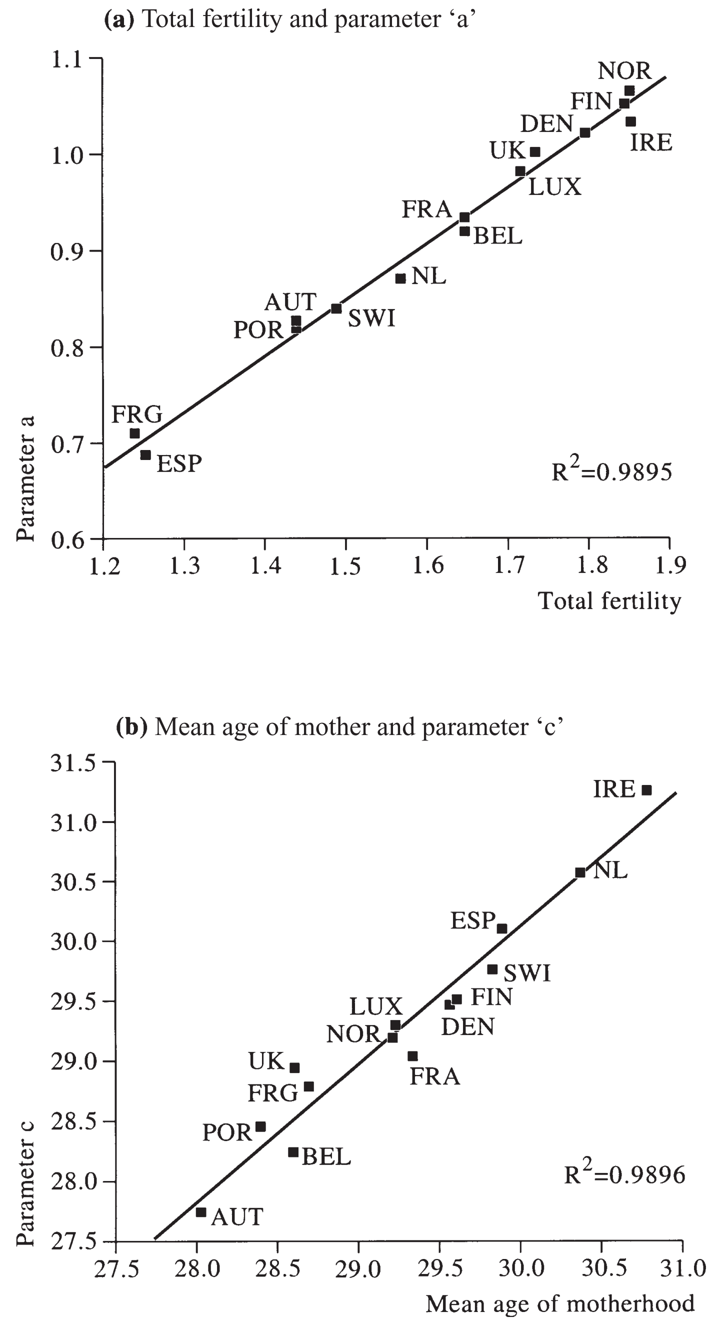

Figure 11. Regression of total fertility and Hadwiger parameter 'a', and regression of mean age of motherhood and parameter 'c', selected countries, 1994.

national populations. The mixture term, which partly determines the relative size of the two fertility distributions, is associated with the illegitimacy ratio both cross-sectionally across different European countries and longitudinally within the same country at least for the United Kingdom and the Irish Republic. Finally, the trends in the 'c_2' parameter in the mixture model for these two countries suggest that the parameter may represent the mean age of marital motherhood. However, some caution in interpreting the Hadwiger mixture parameters must be exercised owing to the lack of a corresponding association between the 'c_1' parameter and the mean age of non-marital motherhood.

There are, however, a number of examples where the proportion of births outside marriage is high and yet where the overall distribution can easily be fitted by a simple, not a mixture, model. This lack of heterogeneity in the pattern of fertility in some

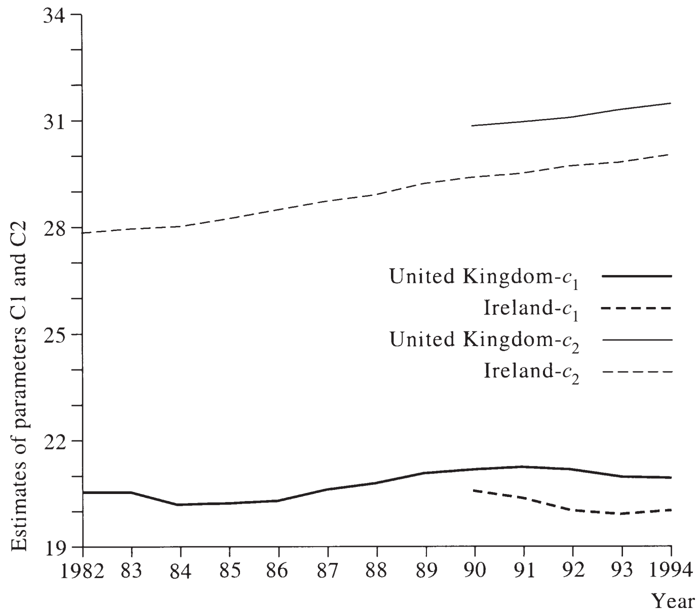

Figure 12. Trends in 'c' parameters of the Hadwiger mixture model for the United Kingdom and Ireland.

countries could be a result of a relatively small proportion of non-marital births (as in Austria) or a similarity in the timing and other demographic characteristics of births inside and outside marriage (such as in Denmark or Sweden). One might speculate that in those latter populations that are well fitted by a simple Hadwiger model, the behaviour characterised by the term 'Second Demographic Transition', notably births outside marriage, has been adopted by the mainstream of society in such a way that there is not much difference between the mother's characteristics for marital and non-marital births. In other words, for populations requiring a mixture model, the adoption of the 'Second Demographic Transition' has, at least in part, made the society more heterogeneous in respect of fertility characteristics, at least for the time being.

One of the original intentions of the paper was to determine whether a standard or average European fertility distribution could be found from which deviations, such as those of the fertility of Britain and Ireland, could be measured, analysed, and understood. Although such a standard distribution was not developed for this paper, the differentiation of fertility curves into some which can be fitted by simple Hadwiger models and others by Hadwiger mixture models is a useful step towards formulating a standard.

Furthermore, the increasing size of the mixture parameter in the Hadwiger mixture model for the United Kingdom and Ireland over time suggests that changes in the timing and volume of fertility in these countries are captured by this model; such changes in the patterns of fertility may not be adequately reflected by more conventional measures of fertility such as total fertility or the mean age of motherhood. The exploratory results of this paper indicate that further research into the use of the Hadwiger simple and mixture models in fitting fertility curves could be useful in comparing and explaining fertility patterns in countries over time.

NOTES

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