Lifespan dispersion in stagnant and decreasing periods of life expectancy in Eastern Europe

Jose Manuel Aburto*[1] and Alyson van Raalte†[2]

[1]Max Planck Institute for Demographic Research, European Doctoral School of Demography, Sapienza University of Rome
[2]Max Planck Institute for Demographic Research

January 14, 2016

Abstract

Life expectancy at birth has had an atypical pattern in Eastern European countries since the 1950s. Periods of rapid increase in life expectancy followed by stagnation and decreases have been documented. We analyze patterns of dispersion in age at death for these countries and its relationship with the average length of life. Data includes 12 countries since 1947 from the Human Mortality Database by sex. Two measures of dispersion were used. Life disparity $e^{\dagger}$ and Keyfitz's entropy $\mathcal{H}$. Life expectancy is made up of a combination of age-specific mortality rates so its change might be driven by an offsetting effect of improvements in old mortality and a reversal in middle-age mortality. In such way that the relationship between life expectancy and dispersion at death is not clear. We found that the negative relationship between life expectancy and lifespan variation holds even in atypical periods of stagnation. Furthermore, the relationship between both measures seems to be consistent over time simultaneously. Although Eastern European countries have experienced improvements in life expectancy during the last decade, high levels of life disparity remain. Pointing to a high prevalence of premature deaths, potentially avoidable.

*aburtoflores@demogr.mpg.de
†vanRaalte@demogr.mpg.de

Introduction

The 20th century was marked by sizable improvements in mortality and health in most countries in the world (World Health Organization 2000). However, these improvements were shattered in the second half of the past century, as Eastern European countries experienced an unprecedented period of stagnation and, in some countries, decreases in life expectancy at birth after 1979 (Chenet et al. 1996). Although improvements in infant mortality were documented during this period, an offsetting effect driven by increases in young and middle-aged mortality (from circulatory diseases, lung cancer, cirrhosis and accidents) led to a substantial deterioration in the health status of the populations in Czechoslovakia, Hungary and Poland (Chenet et al. 1996). Three of the wealthiest former socialist countries. Similarly, Russia experienced a brief rise in life expectancy in the 1980s, followed by a pronounced decline in males' life expectancy in the 1990s (over 5 years) (Cockerham 1997) mainly caused by premature adult mortality associated with alcohol consumption (Rehm et al. 2007).

Importantly, this period of increase in Russian longevity coincided with the implementation of Gorbachev's anti-alcohol campaign in the mid-1980s that lasted until 1987. Empirical evidence suggests that the rise in Russian life expectancy has been attributed to the success of this campaign (Bobadilla et al. 1997). Nevertheless, after the break down of the former Soviet Union, Russian's health status continued worsening and by 1994, life expectancy was even lower than their Soviet counterparts in 1960, for both men and women (Cockerham 1997). Although women also experienced deterioration in life expectancy in these nations, men were specially susceptible to dying prematurely, leading to large sex-differences in Poland, Hungary, Russia, and the European post-Soviet Republics (McKee and Shkolnikov 2001).

National trends in life expectancy are important and informative. Nonetheless, they conceal heterogeneity and variation at age at death. Several studies have found a negative correlation between life expectancy and the variation in the ages at death across time and countries in the context of improvements in averting premature deaths (e.g. Vaupel et al. (2011)) and the rectangularization of the survivorship (Wilmoth and Horiuchi 1999). These improvements have been due, in part, to the reduction of infectious diseases and the diffusion of medical services that have had major impact in averting infant and child mortality in most of the countries. However, this high negative correlation between life expectancy and variation at death could be spurious (for the data we use the correlation is 0-.89 and -0.94 with years lost due to death and Keyfitz's entropy respectively, see Appendix's figure 7). In some cases because life expectancy is defined in terms of some variation measure (e.g. Keyfitz's entropy, Gini coefficient). Another explanation is that since life is limited, the distribution of death is bounded at the right side. This means that people reaching higher ages increases regularly. However, in practice averting death at high age is increasingly difficult, therefore the proportion of individuals getting alive to high ages is small (Smits and Monden 2009). In spite of this fact, some countries, like the U.S., showed an unanticipated high variation in age at death due to mortality at very young and older ages compared with the average lifespan suggesting that countries with similar life expectancy could have different levels of lifespan inequality (Shkolnikov et al. 2003). In this sense, periods of mortality upheavals, like the ones observed in Eastern Europe, might be driven drastically by deterioration on the overall survival (Keyfitz and Caswell 2005), but also increases in variability of age at death, particularly when life expectancy declines or stagnates.

Variation at age at death depends on the interaction of saving lives at younger ages and at old ages simultaneously (Zhang and Vaupel 2009). The balance between both drives the compression (or expansion) of ages at death. Therefore, if lifespan variation is increasing alongside life expectancy decreases means that not only are people dying younger but they are also facing more uncertainty about the eventual timing of death. Consequently, addressing inequalities and unsureness in mortality during periods of continuous deterioration, rather than improvements, is a step forward to better understand the dynamics of mortality. Moreover, understanding mortality trajectories is important to shed light to create new strategies to improve populations' health and lessen the burden of premature mortality observed in Eastern European countries in order to rise life expectancy but also minimize the uncertainty of age at death simultaneously.

We study the average remaining life expectancy at time of death measured by $e^{\dagger}$ as a dispersion indicator. We do so by analyzing closely the relationship between $e^{\dagger}$ with Keyfitz's entropy $\mathcal{H}$ and life expectancy. Given the clear association between life expectancy and lifespan variability, the study of their trends over time is not particularly informative, as these will only reflect opposite trends of life expectancy and variability of age at death (Smits and Monden 2009) (See Appendix's figure 6). Hence, rather than focusing in observed levels, we explore the linkage in successive changes and look for clarification driving the changes over time implementing an innovative series of formulas. We argue that the study of variation at death with years lost to death ($e^{\dagger}$) should be approached from a dynamic perspective along with Keyfitz's entropy and life expectancy because the changes of the three of them complement with the changes of each other.

Theoretical background

Lifespan dispersion measures

Life disparity ($e^{\dagger}$) is defined as the average remaining life expectancy when death occurs; or life years lost due to death (Vaupel 1986). Its application is justified because of its easy interpretation and additive properties (Zhang and Vaupel 2009). For example, when death is very variable, some people will die before their expected age at death, contributing many lost years to life disparity. When people survive to older ages, the difference between the age at death and the expected remaining years decreases, and life disparity gets smaller. If all deaths occur at the same age the survival curve is rectangular and life disparity is zero. It can be expressed as

$$e_x^{\dagger} = \frac{1}{\ell_x} \int_x^{\infty} \ell(a)\mu(a)e(a)da \tag{1}$$

where $\ell(a)$, $\mu(a)$ and $e(a)$ are the survival function, the force of mortality, and life expectancy, respectively.

Keyfitz's entropy $\mathcal{H}$ gives the proportional response in life expectancy to a proportional change in the overall survival (Keyfitz and Caswell 2005). It is defined as

$$\mathcal{H} = \frac{\int_0^\infty \ell(a)\mu(a)e(a)ds}{e^o(0)}. \tag{2}$$

These measures hold a close relationship with life expectancy and both are sensitive to changes in the force of mortality. When $\ell(0) = 1$, life disparity can be expressed as $e^\dagger = \mathcal{H} * e_0^o$ (Vaupel and Canudas-Romo 2003).

A proportional increase in μ at all ages and its relationship with changes in life expectancy

We first analyze the effect of a change on the force of mortality in life expectancy as Keyfitz and Caswell (2005, p. 78) proposed to illustrate the meaning of the life table entropy. Then, we follow a similar procedure to show how a variation on the force of mortality can alter life disparity.

Let $\mu(x)$ be the force of mortality at age x. The probability of surviving from birth to age x is, with a radix of $\ell(0) = 1$, is

$$\ell(x) = e^{-\int_0^x \mu(a)da}$$

so that life expectancy at age x is given by

$$e^o(x) = \int_x^\infty e^{-\int_0^s \mu(a)da} ds \tag{3}$$

Since we are interested in life expectancy at birth, we fixed x at zero and make $\mu(a)$ vary in the age-dimension. Therefore, life expectancy at birth can be re-expressed as

$$e_0^o(\mu(a)) = \int_0^\infty e^{-\int_0^s \mu(a)da} ds. \tag{4}$$

Consider a constant increase in μ of $\delta > 0$. Then, the new mortality function is $(1+\delta)\mu(a)$ and the new probability of surviving from birth to age x is

$$\ell_x((1+\delta)\mu(a)) = e^{-\int_0^x (1+\delta)\mu(a)da} = \left[e^{-\int_0^x \mu(a)da}\right]^{1+\delta} = [\ell_x(\mu(a))]^{1+\delta}.$$

Then, the new life expectancy at birth is

$$e_0^{o*}((1+\delta)\mu(a)) = \int_0^\infty \ell(s)^{1+\delta} ds. \tag{5}$$

A relative increase in mortality should cause a relative decrease in life expectancy. The derivative of (5) with respect to δ will show the effect of a small change δ on life expectancy. That is

$$\frac{\partial e_0^{o*}}{\partial \delta} = \int_0^\infty \frac{\partial}{\partial \delta} e^{\ln(\ell(s)^{1+\delta})} ds = \int_0^\infty \frac{\partial}{\partial \delta} e^{(a+\delta)\ln(\ell(s))} ds = \int_0^\infty [\ln \ell(s)] \ell(s)^{1+\delta} ds \tag{6}$$

If δ is small, Keyfitz and Caswell (2005) approximated a relative change in life expectancy at birth as

$$\frac{\Delta e_0^o}{e_0^o} = \left[\frac{\int_0^\infty \ell(s)\ln(\ell(s))ds}{\int_0^\infty \ell(s)ds}\right]\delta = \left[\frac{-\int_0^\infty \ell(s)\mu(s)e(s)ds}{\int_0^\infty \ell(s)ds}\right]\delta = -\mathcal{H}\delta = \left[\frac{-e^\dagger}{e_0^o}\right]\delta \tag{7}$$

Since $0 < \ell(x) < 1$, the ratio in (7) is negative, causing a decrease in life expectancy. The negative of this ratio is the equivalent of Keyfitz's entropy $\mathcal{H}$ in (2). This means that a proportional increase in mortality rates at all ages result in a proportional decrease in life expectancy of δ times $\mathcal{H}$.

A proportional increase in μ at all ages and its relationship with changes in life disparity

From (1) and (7), we have that

$$e_0^\dagger = e^\dagger = \int_0^\infty \ell(s)\mu(s)e(s)ds = -\int_0^\infty \ell(s)\ln(\ell(s))ds. \tag{8}$$

Evaluating (8) with the new force of mortality $(1+\delta)\mu(a)$ yields

$$e^{\dagger *} = -\int_0^\infty \ell_s((1+\delta)\mu(a))\ln(\ell_s((1+\delta)\mu(a)))ds. = -\int_0^\infty \ell(s)^{1+\delta}\ln[\ell(s)^{1+\delta}]ds \tag{9}$$

Similarly, as in (6), the derivative of $e^{\dagger *}$ with respect to δ will show the effect of a small change δ on life disparity.

$$\begin{aligned}
\frac{\partial}{\partial\delta}e^{\dagger *} &= \frac{-\partial}{\partial\delta}\int_0^\infty \ell(s)^{1+\delta}\ln[\ell(s)^{1+\delta}]ds \\
&= \frac{-\partial}{\partial\delta}(1+\delta)\int_0^\infty \ell(s)^{1+\delta}\ln\ell(s)ds \\
&= \frac{-\partial}{\partial\delta}\left[(1+\delta)\frac{\partial e_0^{o*}}{\partial\delta}\right] \\
&= -\frac{\partial e_0^{o*}}{\partial\delta} - (1+\delta)\frac{\partial^2 e_0^{o*}}{\partial\delta^2}
\end{aligned} \tag{10}$$

Two results are important from (10). First, that an increase in the force of mortality results in an increase in life disparity. The first term in (10) cannot be negative since the first derivative of e_0^{o*} is always negative (Keyfitz and Caswell 2005). We know from (7) that $\frac{\partial e_0^{o*}}{\partial\delta}$ can be approximated (for small δ) as

$$\frac{\partial e_0^{o*}}{\partial\delta} \approx \delta\int_0^\infty \ell(s)\ln(\ell(s))ds.$$

Thus, the second derivative in (10) can be approximated as

$$\frac{\partial^2 e_0^{o*}}{\partial\delta^2} \approx \frac{\partial}{\partial\delta}\delta\int_0^\infty \ell(s)\ln(\ell(s))ds \approx \int_0^\infty \ell(s)\ln(\ell(s))ds, \tag{11}$$

which is always negative since $0 < \ell(x) < 1$. Therefore, the second term in (10) is always positive for small δ.

The second main result from equation (10) is that it shows that life expectancy and life disparity change in opposite directions when a variation in the survival function happens. Recently, similar results have been found using calculus of variations (Fernandez and Beltrán-Sánchez 2015).

The relationship between relative changes in life expectancy and disparity over time

Consider the equality (Vaupel and Canudas-Romo 2003, p. 205, eq. 18):

$$e^\dagger(t) = \mathcal{H}(t)e_0^o(t) \tag{12}$$

Since we are interested in relative changes over time we calculate the derivative (indicated by a dot over the functions) of (12) respect to t divided by $e^\dagger(t)$ applying Leibniz law, that is

$$\begin{aligned}\frac{\dot{e^\dagger(t)}}{e^\dagger(t)} &= \frac{[\mathcal{H}(t)\dot{e}_0^o(t)]}{\mathcal{H}(t)e_0^o(t)} \\ &= \frac{\dot{\mathcal{H}(t)}e_0^o(t) + \mathcal{H}(t)\dot{e_0^o(t)}}{\mathcal{H}(t)e_0^o(t)} \\ &= \frac{\dot{\mathcal{H}(t)}}{\mathcal{H}(t)} + \frac{\dot{e_0^o(t)}}{e_0^o(t)}.\end{aligned} \tag{13}$$

Which implies that relative changes in life disparity are driven by relative changes in life expectancy and in the life table entropy.

Moreover, from equation (13) follows that

$$\dot{e^\dagger(t)} = e^\dagger(t)\left[\frac{\dot{\mathcal{H}(t)}}{\mathcal{H}(t)} + \frac{\dot{e_0^o(t)}}{e_0^o(t)}\right] \tag{14}$$

which implies that $e^\dagger$ and e_0^o do not have a linear link. For example, if these to measures would have a linear link, then they could be expressed as $e^\dagger = \alpha e_0^o + \beta$, and the changes over time then could be expressed as $\dot{e^\dagger} = \alpha\dot{e_0^o}$. However, although life expectancy and disparity react in opposite ways to changes, equation (14) states that variation on $e^\dagger$ depends on the dynamics of both entropy and life expectancy.

Data & Methods

We used death counts, exposures and life tables from the Human Mortality Database (2015) for 12 countries from 1947 to the most recent year available. The countries included in the study are Belarus, Bulgaria, Czech Republic, Hungary, Poland, Russia, Slovakia, Ukraine, Slovenia, Estonia, Latvia and Lithuania. These data contain information on life table's measures (e.g. d_x, ℓ_x, e_x,q_x) by single age, sex and country. To examine the changes in age-specific mortality we calculated rates of mortality improvement as $\rho(x,t+1) = -\ln\frac{m(x,t+1)}{m(x,t)}$ with smoothed mortality surfaces (Rau et al. 2013). For each population, we investigated life expectancy and life disparity since birth. We decided not to analyze variability at death conditioned on survival to any older age, as previous studies have done, because of the high proportion of deaths concentrated in younger ages (see Appendix's figure 9) . Furthermore, our decision is also founded by the upsurge in young age mortality in the 1990s (see Appendix's figure 9) and the low levels of life expectancy that European countries experienced in the last century.

Using data for males and females, we calculated life expectancy at birth or mean age at death ($e^o_0(t)$), life disparity from birth or years lost due to death ($e^\dagger$) and Keyfitz's entropy $\mathcal{H}$ as described in the theoretical background for all countries included in the study and the years available. The life table functions needed to calculate these measures are taken from the period life tables from the Human Mortality Database (2015) with the last age, $\omega = 110$, including all deaths above it. Although this right-truncation might underestimate variation in mortality if these deaths are shifting toward upper ages, we believe their impact is negligible since almost all deaths are concentrated in younger ages (see Appendix's figure 9).

Changes over time of lifespan dispersion measures and life expectancy and their magnitude vary depending on the dynamics of age-specific deaths rates. In order to analyze such patterns we estimate relative changes calculating first differences conditioned on the initial year. The resulting ratios are equal to zero if no changes were observed, positive values indicate increases while negative indicate decreases in the indicators. In addition, to analyze the dynamics that equation (12) suggests, we estimated regression surfaces. We fitted four regression models which predict changes in life disparity using changes in life expectancy and changes in Keyfitz's entropy during the period. The first model is a standard linear regression model. The remaining three are non-parametric; two are variants of LOESS (Cleveland and Devlin 1988), and the third one uses local regressions (Loader 2006).

To capture trends in lifespan disparity and expectancy we broke the period of study in four. A period with sizable increases in life expectancy before 1960; then from 1960 to 1970 which was characterized by life expectancy stagnation and little improvements; 1988-1995, which includes the years just after Gorbachev's anti-alcohol campaign was stopped and the Soviet Union's break up distinguished by decreases in the mean age at death; and from 1996 to the maximum year available in the data set marked by significant improvements in life expectancy. Everything was carried out using `R`.

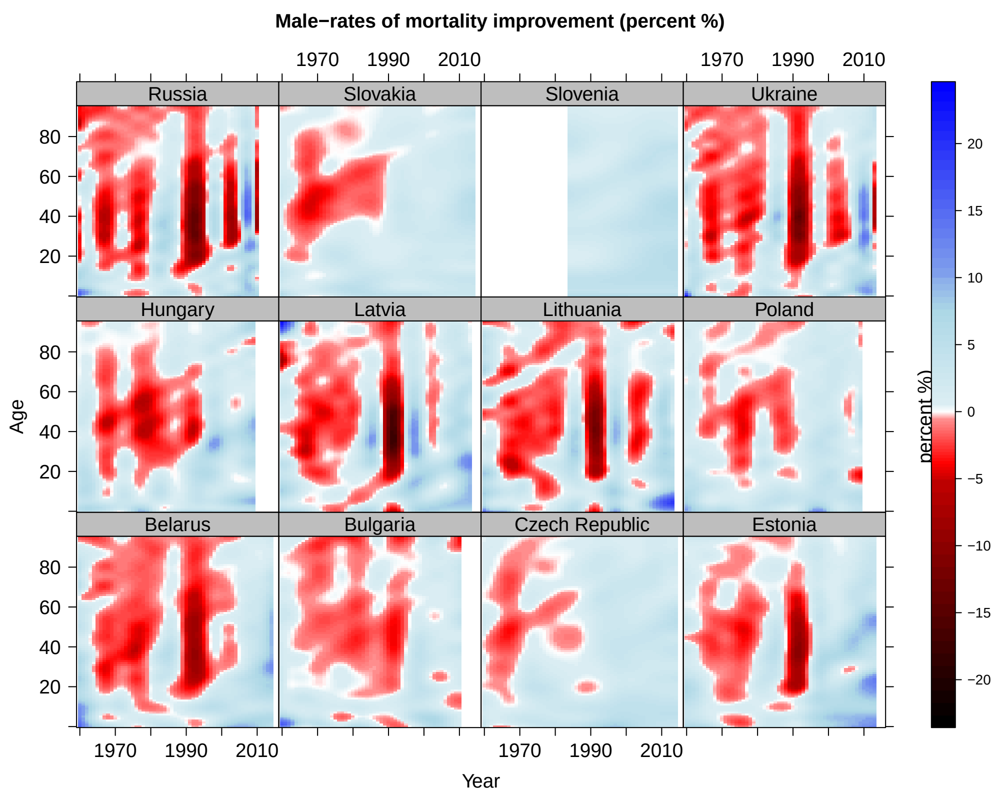
Results

Figure 1 shows the age-specific rates of mortality improvements for males in 12 Eastern European countries (results for females are shown in Appendix's figure 8). The respective values are expressed in percent. Little changes or no improvements (-0.5% to 0.5%) are depicted in white. Improvements in mortality are shown in blue and mortality deterioration in red. Darker tones mean major changes in mortality rates.

Almost every country experienced a continuous 20-year period of mortality deterioration, from the mid-1960s to the mid-1980s. Mortality rates exhibited increases mainly concentrated in the ages between 20 and 80 years. After 1985, 8 countries experienced sizable improvements in mortality (Russia, Ukraine, Latvia, Lithuania, Poland, Belarus, Czech Republic and Estonia) for a period of 5-10 years. Opposing this trend, in the 1990s almost every country presented an intense worsening in mortality rates after 20 years, particularly in the countries part of former Soviet Union. Nevertheless, since the last decade of the past century and the beginning of the 2000s Slovakia, Slovenia, Hungary, Latvia, Poland, Belarus, Estonia and Czech Republic have reduced age-specific mortality rates in almost every age. The biggest improvements are related to the most recent years. However, Russia, Ukraine and Poland's recent data point towards a downturn in population's health in middle and adult ages.

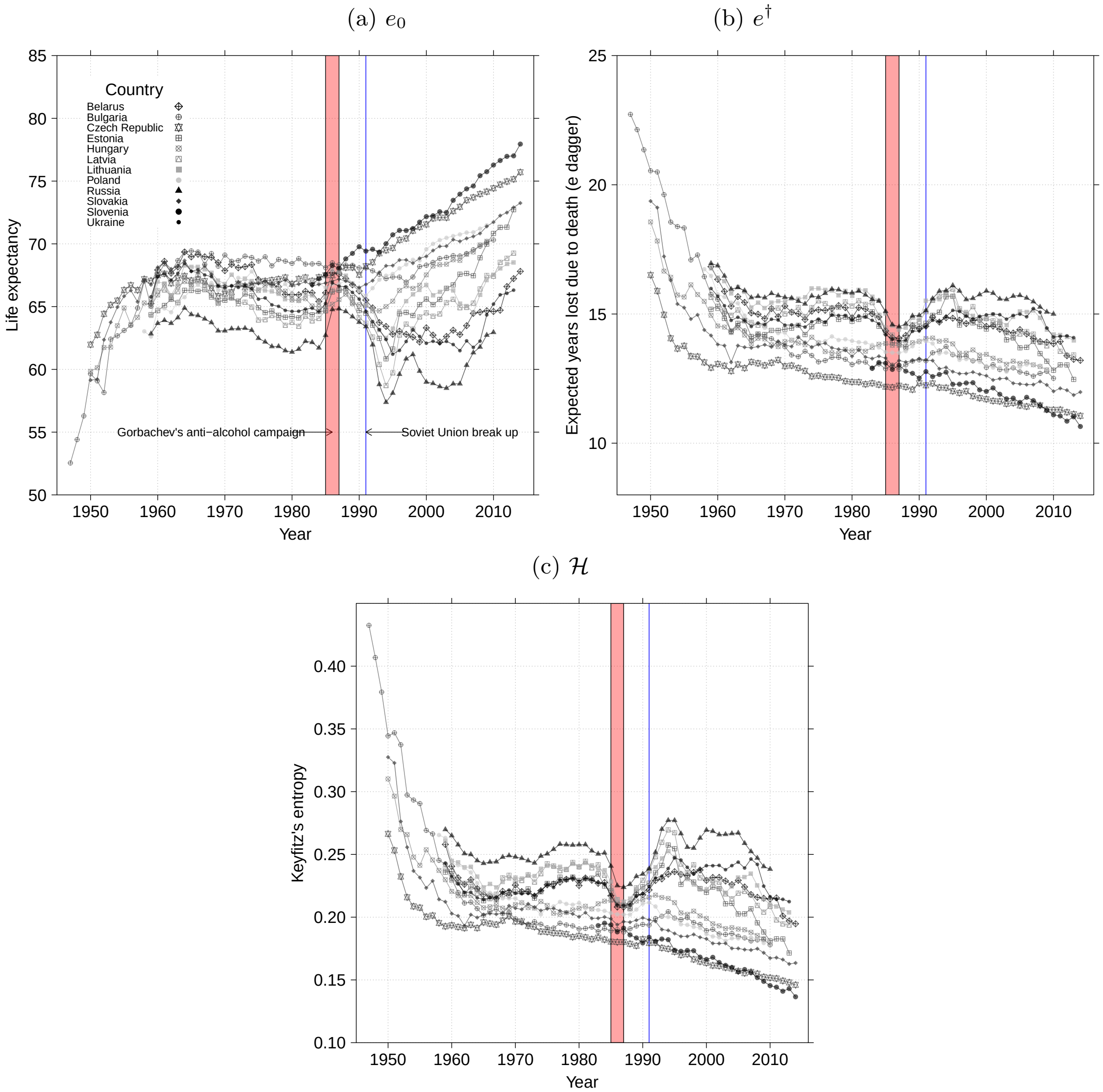
Results for women are similar than men's in Russia, Ukraine, Latvia, Lithuania, Belarus and Bulgaria but with significantly lower magnitude. (Appendix's figure 8). Importantly, some countries such as Slovakia, Slovenia, Poland, Czech Republic and Estonia experienced a continuous trend of mortality improvements in almost the entire period.

Figure 1: Male mortality surface showing rates of mortality improvements



Source: own calculations based on Human Mortality Database (2015) data. Note: The regular white areas indicate no data available. Russia, Hungary, Bulgaria and Poland after 2010. Slovenia before 1983.

Figure 2: Trends in males' e_0 , $e^\dagger$ and $\mathcal{H}$ for 12 Eastern European countries, 1960-2014



(a) e_0 (b) $e^\dagger$ (c) $\mathcal{H}$

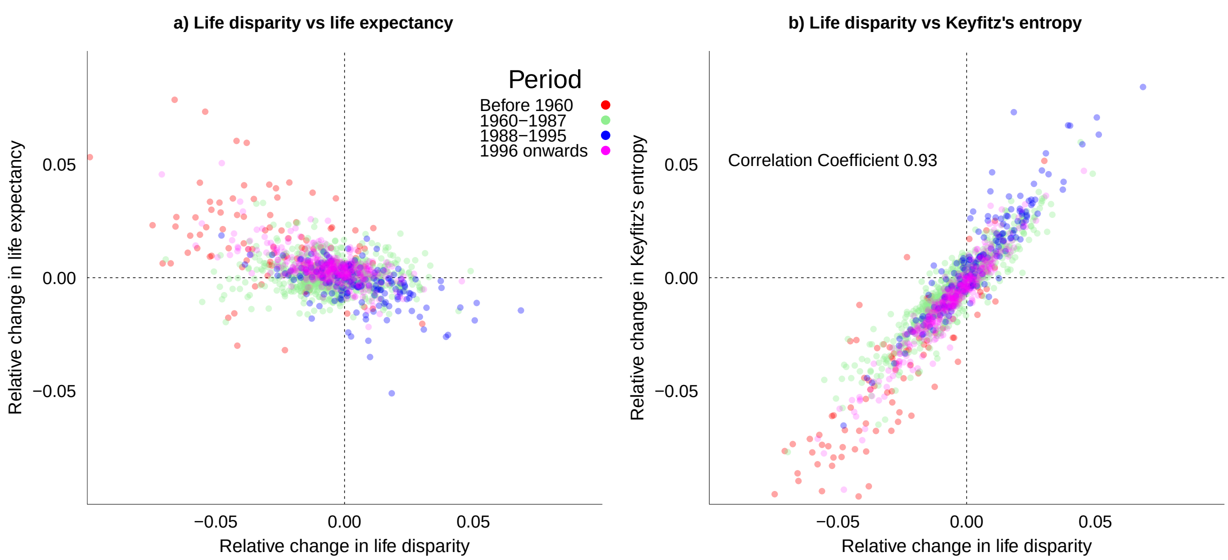
Source: own calculations based on Human Mortality Database (2015) data.

Figure 2 shows male's life expectancy at birth e_0 , lifespan disparity $e^\dagger$ and Keyfitz's entropy $\mathcal{H}$ for Eastern European countries from 1947 to 2014 (panels a), b) and c) respectively). All countries experienced marked increases in life expectancy before 1960. In contrast, from 1960 to 1984 life expectancy stagnated for most of the countries, some of them even experienced decreases (e.g. Russia, Latvia, Estonia, Ukraine). This period was followed by a notable increase in life expectancy in the mid-1980s. However, in 1987 life expectancy among these countries started to diverge. Slovenia and the Czech Republic exhibited a continuous increase from that point up to now. Hungary, Poland, Bulgaria stagnated for a short period and then continued an upward trend until 2014. The rest of the countries (Russia, Latvia, Estonia, Ukraine, Belarus and Lithuania) experienced a marked decrease in life expectancy from 1988 to 1993. From that point on, all of them have improved life expectancy. The trends for both male and females are similar (figure with females' results are shown in the Appendix). Yet, the magnitude of the changes is shorter for women and the level of life expectancy is significantly higher than men.

Opposing the trends in life expectancy, life disparity $e^{\dagger}$ (panel b) and Keyfitz's entropy $\mathcal{H}$ (panel c) , exhibited similar tendencies for all countries but in an inverted manner. Bulgaria, Slovakia, Hungary and the Czech Republic experienced a remarkable decrease in life disparity before 1960, from 22 to 13 years. Followed by a period of stagnation until the mid 1980's. From that point on, changes are similar to the ones observed in life expectancy with a negative scale. Russia, Lithuania, and Latvia presented the highest levels of lifespan disparity during the whole period. In contrast, the Czech Republic and Slovenia showed the lowest levels since 1950. It is worth noting that although Slovenia is the record holder in the lowest life disparity throughout the entire period, it was just the life expectancy record holder in the 1950's.

The relationship between relative changes in life disparity with relative changes in life expectancy (panel a) and Keyfitz's entropy (panel b) are shown in figure 3 for Eastern European Countries. Red dots show data from before 1960; green dots, correspond to the period of stagnation between 1960 and 1987; blue dots to 1988-1995. Finally, magenta dots account for the most present period.

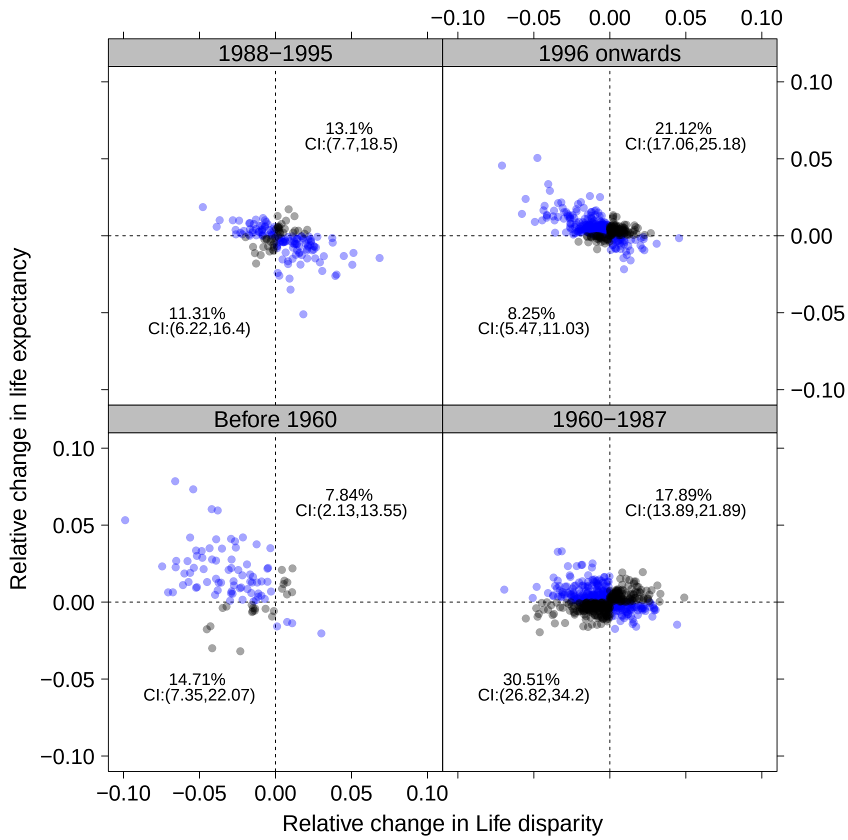
Figure 3: Relationship between differences in $e^{\dagger}$ and $\mathcal{H}$ with e_0 for Eastern European countries, 1960-2014



Source: own calculations based on Human Mortality Database (2015) data.

Results in panel a) reveal that the high correlation between $e^{\dagger}$ and life expectancy do not holds when looking to relative changes between them. For instance, if life disparity and life expectancy are negatively related, then a positive relative change in life expectancy (y-axis) should correspond to a negative relative change in life disparity (x-axis). Therefore, the differences should be concentrated in the upper-left and lower-right axes. Opposing this, relative changes in life disparity are highly correlated with changes in Keyfitz's entropy with a correlation coefficient of 0.93. This is consistent with the response of both measures to alterations on the overall survival, as the theoretical background suggested. Negative changes in life disparity correspond to negative changes in Keyfitz's entropy along the entire period and vice versa.

Figure 4: Proportion of observations corresponding to decreases in life expectancy and disparity for Eastern European countries, 1960-2014

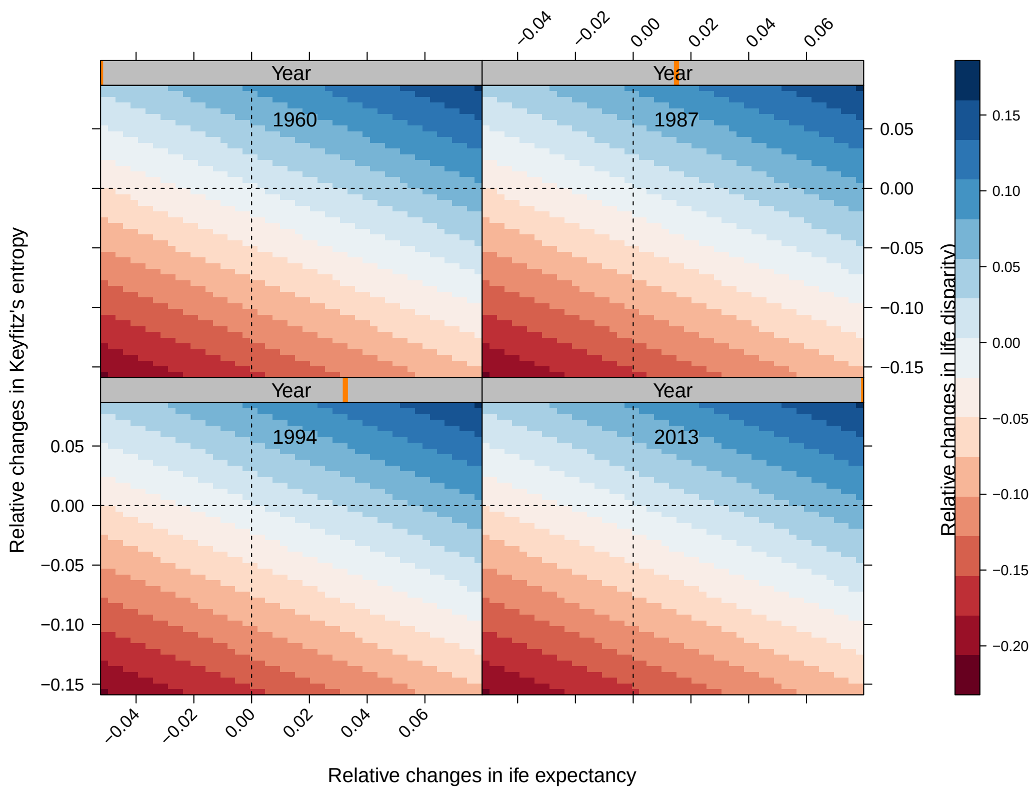


Source: own calculations based on Human Mortality Database (2015) data.

Importantly, results from panel a) show different patterns on the dynamics between life expectancy and life disparity by period. On periods of stagnation and mortality deterioration negative changes in life expectancy are more likely to coincide with reductions on life disparity than periods with improvements in mortality. Figure 4 shows such proportions by period. On the period 1960-1987, 30.5% of the observations corresponded to negative changes in life disparity and reductions in life expectancy at the same time. After Gorbachev's alcohol policy, 11.3 percent. In addition, this figure exhibits that it is also possible to increase life disparity simultaneously with life expectancy. Both life expectancy and life disparity are made up of a combination of age-specific mortality rates. Therefore, the atypical pattern observed in Figure 4 (black dots) could be explained by the contribution of improvements (deterioration) in specific ages, captured by changes in life expectancy and changes on the overall survival explained by Keyfitz's entropy. As (13) shows, the change in life disparity is made up of relative changes in both life expectancy and Keyfitz's entropy, depending on the dynamics of the mortality rates (See Appendix's figure 10).

Figure 5 shows the regression surface estimated by a linear model following equation (13) with relative changes in life disparity $e^\dagger$ as response variable adding the year as co-variate. We estimated three additional models, parametric and non-parametric, that show similar results, see Appendix's figure 11 for all the outcomes. Blue colors indicate positive changes in $e^\dagger$, while red colors correspond to negative changes. White or very light colors refer to no changes. On the y-axis are the relative changes of Keyfitz's entropy and in the x-axis the corresponding values for changes in life expectancy.

Figure 5: Dynamics of relative changes in life disparity, Keyfitz's entropy and life expectancy.



Source: own calculations based on Human Mortality Database (2015) data.

The results displayed in figure 5 show similar dynamics regardless of the year. They also show that changes in one measure depend on the changes of the remaining two. The magnitude of such changes give the value on each measure. For instance, we expect that improvements on the overall survival, captured by a decline in Keyfitz's entropy, should coincide with an increase in life expectancy and reduce life disparity (low-right quadrant). However, if a little change on the entropy results in a large change on life expectancy, then life disparity increases.

Now we focus on the unexpected patterns that refer to the proportions exhibited in figure 4. These surfaces show that when life expectancy and life disparity decrease is mainly due to a decline on Keyfitz's entropy as can be seen in the low-left quadrant without blue color. Similarly, these surfaces show that if life expectancy increases along with life disparity, is because Keyfitz's entropy is increasing. It is worth emphasizing that there is a threshold in which two of these measures may change while the other one remains the same, which underscore the importance of studying the three measures simultaneously.

Discussion

Life expectancy and disparity trends

Mortality trends experienced an atypical pattern in Eastern Europe since 1950 relative to other European regions. The marked deterioration on mortality at all ages, and in particular at younger ones, resulted in the life expectancy decreases and stagnation in these countries coinciding with high levels of expected remaining years of life when death occurred ($e^{\dagger}$). Relative to the trend observed in other European regions and in the record life expectancy (Oeppen and Vaupel 2002), European Countries lagged behind. Nevertheless, these countries experienced considerable improvements on the average age at death before 1960, followed by a fairly large period of stagnation (1960-1990) with a mean life expectancy around 66 years. After Gorbachev's anti-alcohol campaign was implemented and the Soviet Union broke up, some of these countries exhibited and unprecedented decrease in life expectancy (figure 2). Russia and Latvia's male life expectancy declined from 64 in 1991 to 57 and 58 respectively. To put this in perspective, Russia and Latvia were having the same level of life expectancy as Slovakia used to have in 1959 and contradicting the best practice upward tendency of 2.5 years every decade (Oeppen and Vaupel 2002), underscoring the large disparities among them that still remain. This reversal in life expectancy was mainly driven by mortality at younger ages (15-75) caused by hazardous alcohol consumption (Shkolnikov et al. 2001, Leon 2011). However, in the last decade life expectancy has showed significantly improvements in all the Eastern European countries, yet high levels of inequality between and inside the countries remains (Leon 2011).

In parallel, results show an astounding decline in life disparity before 1960, from 23 to less than 15 years, almost a 10-year decrease in a 13 year period. This improvements were reversed as Eastern European countries experienced a slowdown in life disparity reduction until the mid-1980's. Shkolnikov et al. (2003) attribute the decrease in lifespan variation in Russia after 1987 (figure 2, panel b) to the success of Gorbachev's anti-alcohol campaign. After 1991, life disparity increased and then started to decrease in the early 2000s. Although, with a lower rate, not comparable with the period prior to 1960. Nutritional patterns are likely to have contributed to the changes in life disparity after 1991. The standard diet in the region is characterized by a high consumption of cholesterol-rich foods, sugar, salt and bread. Which, may account for the high mortality patterns caused by circulatory diseases and cancer in premature ages (McKee and Shkolnikov 2001).

Life expectancy and its relationship with life disparity

Trends in life disparity suggest a relationship with those observed in life expectancy since they coincide over the period in an inverse way. This relationship has been previously studied in diverse contexts and a bouquet of measures have been proposed (Shkolnikov et al. 2003, Van Raalte and Caswell 2013). For instance, Edwards and Tuljapurkar (2005) found that achieving the best practice means reducing inequalities as opposed to pushing the aging boundary among industrialized countries. Similarly, Vaupel et al. (2011) performed a big scale study to determine the contribution of progress in avoiding premature deaths to the improvements in life expectancy and life disparity. They found that the countries that have successfully averted premature deaths have the higher life expectancy and the lower life disparity levels. Recently, other authors have studied the variability withing age groups and by socioeconomic status (Engelman et al. 2010, van Raalte et al. 2014). Albeit the strong relationship, our results show heterogeneity when looking by period. This can be explained by the pattern observed in life expectancy relative to life disparity. The variation among the age at death and the mortality distribution between Eastern European countries can account for this inconsistency in the results compared with previous research (Vaupel et al. 2011). The high proportion of deaths at younger ages and the rickety pattern of mortality over the time can justify the variation by period observed. In addition to alcohol-related mortality and circulatory diseases, this might be due to the lack of determination of policymakers to avoid deaths due to injuries and violence in the central and Eastern European region (McKee and Shkolnikov 2001). Although this findings strengthen the relationship between life disparity and life expectancy they are not useful when measuring lifespan variability with life disparity $e^{\dagger}$ and Keyfitz's entropy $\mathcal{H}$ separately, since in both formulas for $e^{\dagger}$ and $\mathcal{H}$ remaining life expectancy is weighted by the density of deaths (equations (1), (2)). In the case of $\mathcal{H}$, this relationship is even stronger because of the division by life expectancy itself. Therefore the high correlation between these measures is spurious.

Changes in life expectancy and lifespan variability

The fact that the record holder in low life disparity is not always the record holder in life expectancy over the period suggests an underlying process that drives some part of both measures independently. Looking into the changes over time in both measures and their relationship is a step forward to understand clearer life expectancy and disparity. Our results show that the missing part to fully understand the puzzle is Keyfitz's entropy and we argue that the analysis of these three measures should focus on changes, as equation (13) states. This make sense because of the dynamics of the rates of mortality improvements (figure 1) in these particular countries. From them, it is clear that periods of mortality deterioration, which led to declines in life expectancy, occurred almost at every age, affecting the overall survival which can be measured by Keyfitz's entropy. More worrisome, however, is that most of these countries were not able to improve populations' health by reducing deaths at young ages until the early 2000's, and in come cases (e.g. Russia, Ukraine, Latvia, Lithuania) they have not and the scenario is not improving, as recent data suggests.

In periods of mortality improvements it is more likely to increase life expectancy and decrease years lost due to death (figure 4). However, if a little change on the entropy results in a large change on life expectancy, then life disparity increases, pointing to improvements in reducing older deaths (Zhang and Vaupel 2009) (figure 5). This is consistent with previous research that found a threshold age ($a^\dagger$), which is important for overall survival since improvements in reducing early (before $a^\dagger$) deaths reduces variation in lifespans depending on the value of Keyfitz's entropy, while improvements in late (after $a^\dagger$) deaths increases variation in lifespans. Nevertheless, no study has addressed the relationship between life disparity and life expectancy in the context of life expectancy stagnation/decrease focusing in the Eastern European case.

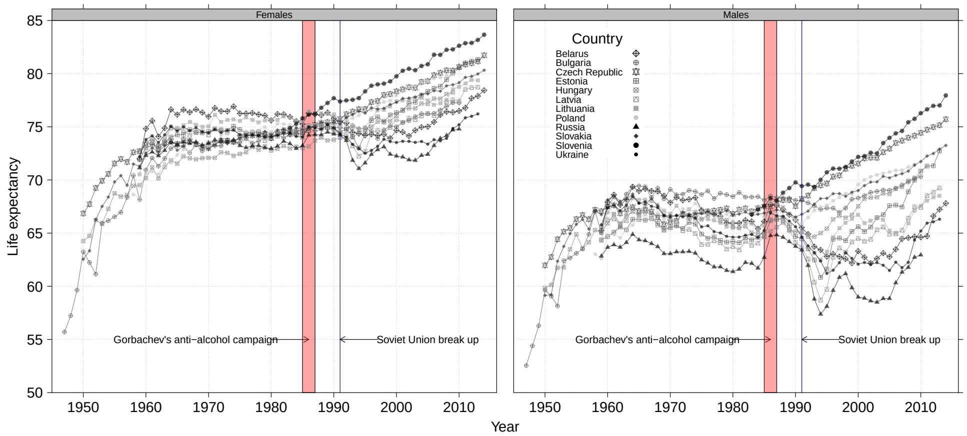
Concluding remarks regarding the Eastern European case

Eastern European countries face larger inequalities in mortality, partly due to socioeconomic inequalities in accessibility, utilization and quality of health care services (Kunst 2009). Although the most recent data confirms an upward trend in life expectancy, there still exist life disparity at age at death, which means that these populations are facing high uncertainty on when they are likely to die. In order to achieve higher life expectancy and lower life disparity, efforts should focus on averting deaths at all ages. Premature deaths can be avoided if public health interventions are successful and well performed. Therefore, efforts to reduce inequalities at age at death is a step towards achieving the best practice life expectancy. The public health framework should focus its endeavors in minimizing injuries and violence in premature deaths to lessen mortality at young ages. Regulations on building, equipment and health care access should be improved as they still remain in a poor quality level (McKee and Shkolnikov 2001). Eastern European countries have a characteristic lifestyle that contributes to the risk of premature mortality. Thus, particular policies should be implemented to reduce this risk. Our results show that Russia, Ukraine, Poland, and Bulgaria still face mortality deterioration in young ages. Recent evidence suggests that the frequency of overweight and obese children in Eastern European countries increased between the 1980's and the early 2000's (Bodzsar and Zsakai 2014), which can lead to increasing life disparity and decreasing life expectancy in the future. Since lifestyle is influenced by social circumstances, the lifespan disparity reduction and the increase in life expectancy should be a goal approached from different perspectives. Given the magnitude of the challenges will be different from country to country.

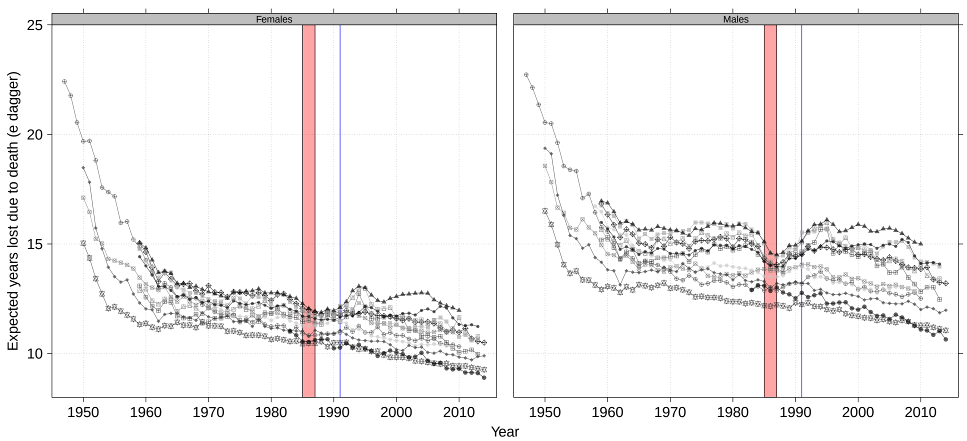
A Supplemental material

Figure 6: Trends in e_0, $e^\dagger$ and $\mathcal{H}$ for 12 Eastern European countries by sex, 1946-2014

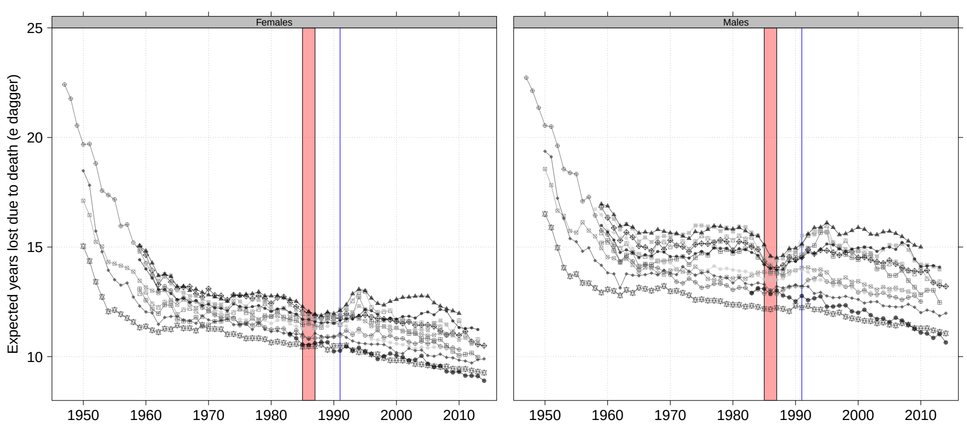
(a) e_0



(b) $e^\dagger$

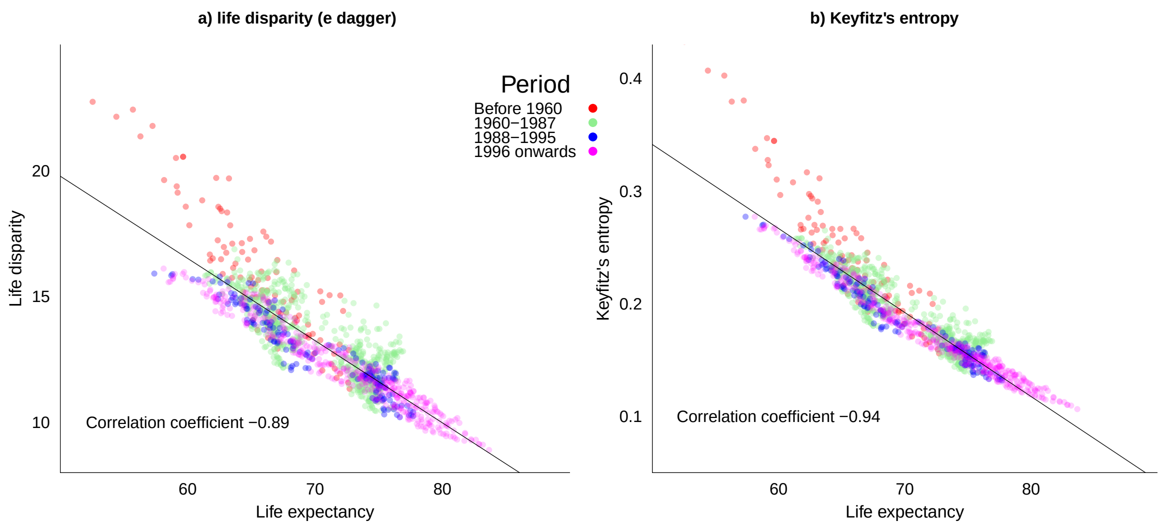


(c) $\mathcal{H}$



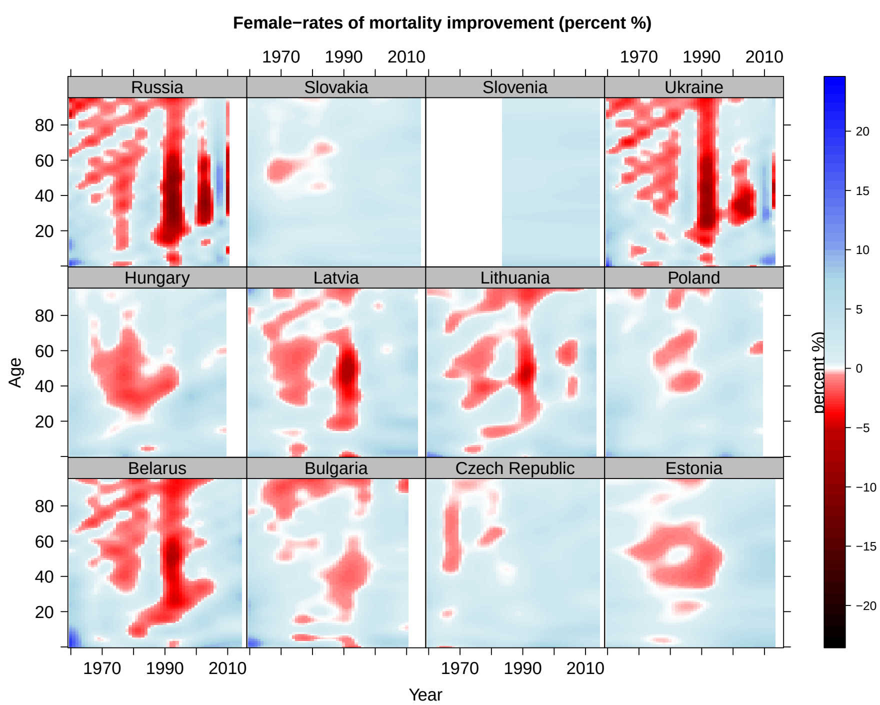
Source: own calculations based on Human Mortality Database (2015) data.

Figure 7: Relationship between $e^{\dagger}$ and $\mathcal{H}$ with e_0 for Eastern European countries, 1947-2014



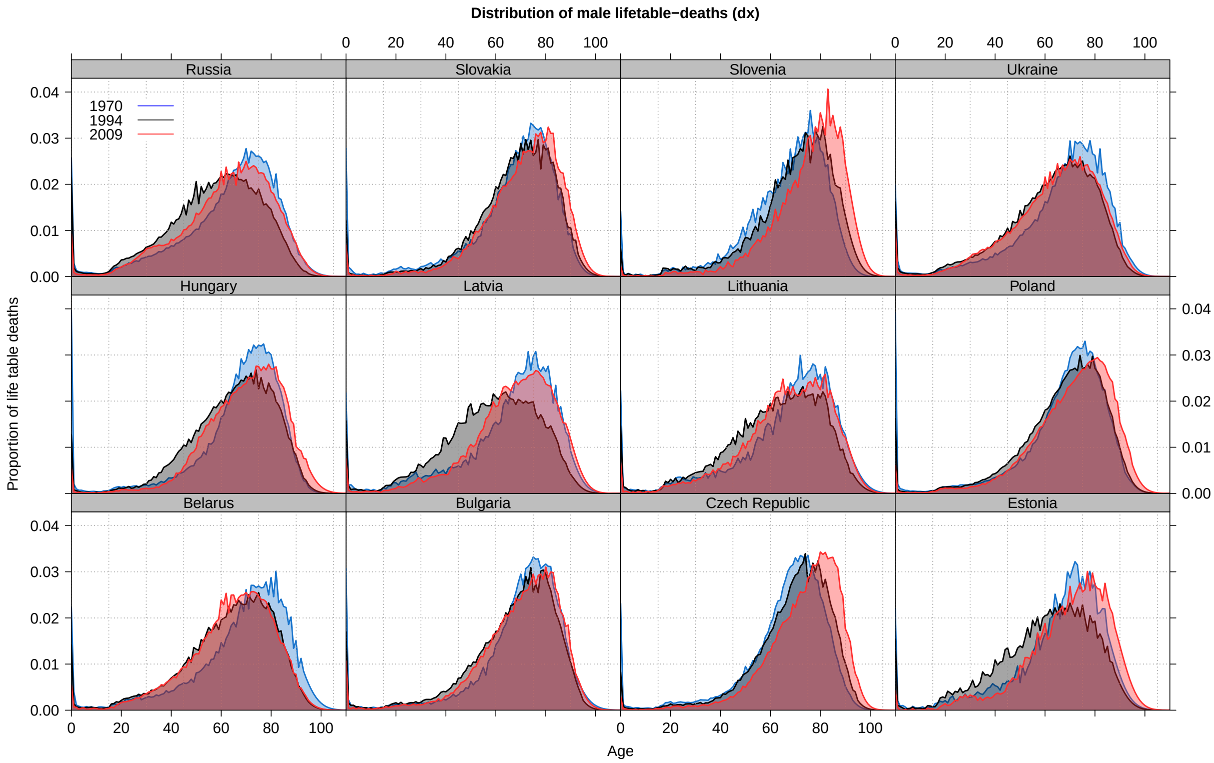
Source: own calculations based on Human Mortality Database (2015) data.

Figure 8: Female mortality surface showing rates of mortality improvements.



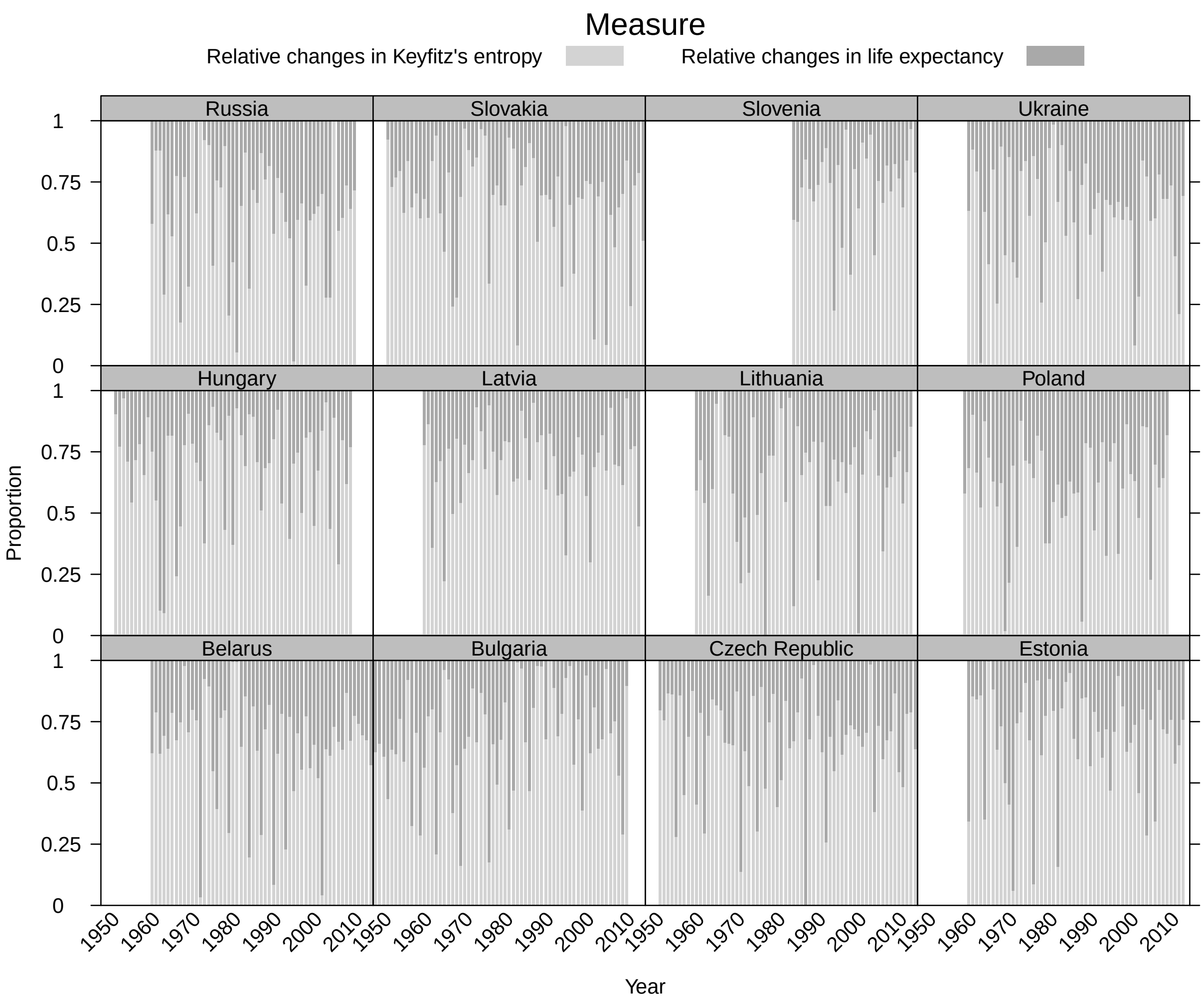
Source: own calculations based on Human Mortality Database (2015) data.

Figure 9: Distribution of deaths for males 1970,1994 and 2009



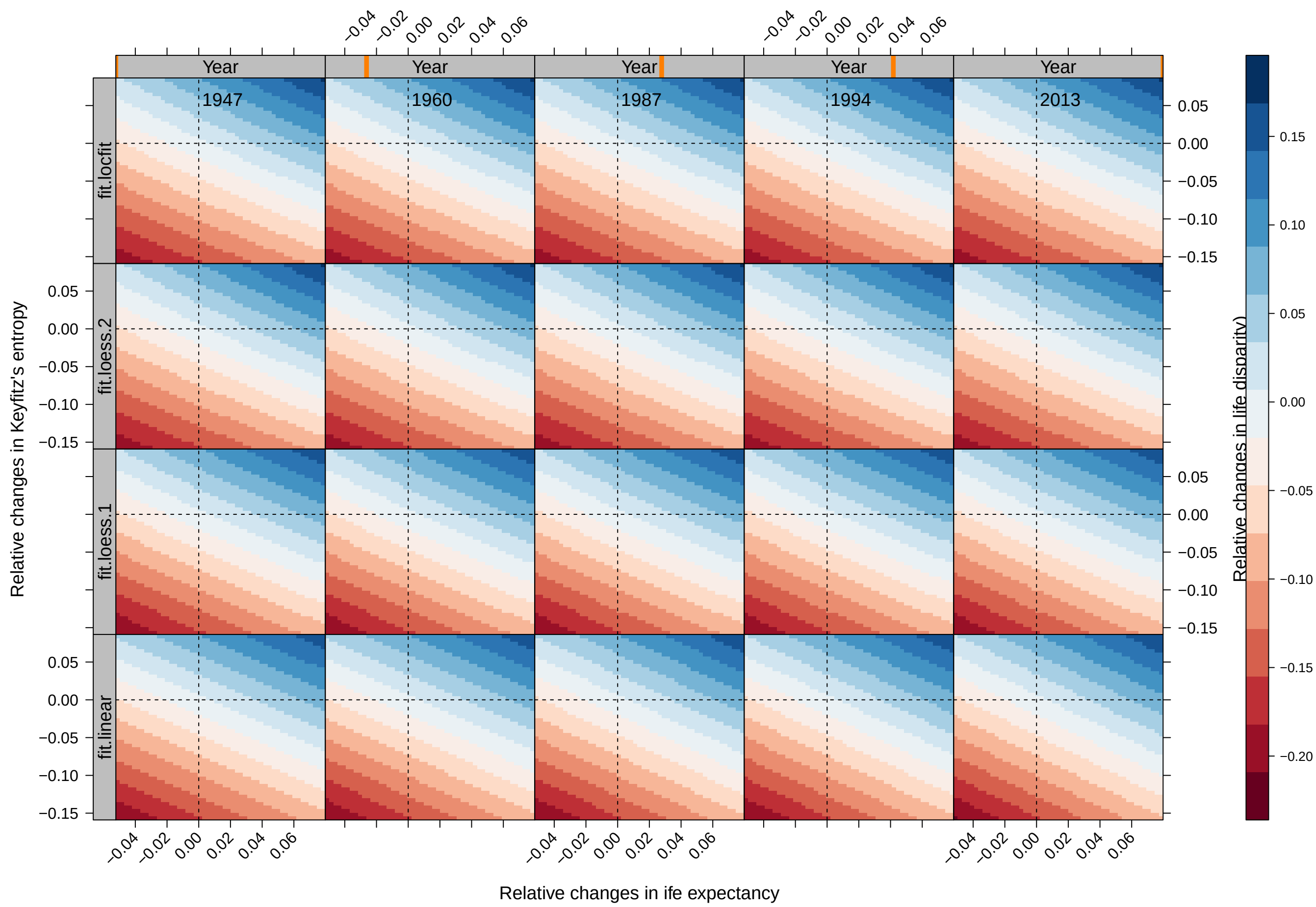
Source: own calculations based on Human Mortality Database (2015) data.

Figure 10: Contributions of changes in life expectancy and Keyfitz's entropy to relative changes in life disparity.



Source: own calculations based on Human Mortality Database (2015) data.

Figure 11: Dynamics of relative changes in life disparity, Keyfitz's entropy and life expectancy estimated by four models.



Source: own calculations based on Human Mortality Database (2015) data.

References

Bobadilla, J. L., Costello, C. A., Mitchell, F., on Population, N. R. C. U. C., et al. (1997). *The Anti-Alcohol Campaign and Variations in Russian Mortality*. National Academies Press (US).

Bodzsar, E. B. and Zsakai, A. (2014). Recent trends in childhood obesity and overweight in the transition countries of eastern and central europe. *Annals of human biology*, 41(3):263–270.

Chenet, L., McKee, M., Fulop, N., Bojan, F., Brand, H., Hort, A., and Kalbarczyk, P. (1996). Changing life expectancy in central europe: is there a single reason? *Journal of Public Health*, 18(3):329–336.

Cleveland, W. S. and Devlin, S. J. (1988). Locally weighted regression: an approach to regression analysis by local fitting. *Journal of the American Statistical Association*, 83(403):596–610.

Cockerham, W. C. (1997). The social determinants of the decline of life expectancy in russia and eastern europe: A lifestyle explanation. *Journal of Health and Social Behavior*, 38(2):117–130.

Edwards, R. D. and Tuljapurkar, S. (2005). Inequality in life spans and a new perspective on mortality convergence across industrialized countries. *Population and Development Review*, 31(4):645–674.

Engelman, M., Canudas-Romo, V., and Agree, E. M. (2010). The implications of increased survivorship for mortality variation in aging populations. *Population and Development Review*, 36(3):511–539.

Fernandez, O. E. and Beltrán-Sánchez, H. (2015). The entropy of the life table: A reappraisal. *Theoretical Population Biology*, 104(1):26–45.

Human Mortality Database (2015). University of california, berkeley (usa), and max planck institute for demographic research (germany).

Keyfitz, N. and Caswell, H. (2005). *Applied mathematical demography*, volume 47. Springer.

Kunst, A. E. (2009). Socioeconomic inequalities in health in central and eastern europe: synthesis of results of eight new studies. *International journal of public health*, 54(4):197–200.

Leon, D. A. (2011). Trends in european life expectancy: a salutary view. *International journal of epidemiology*, 40(2):271–277.

Loader, C. (2006). *Local regression and likelihood*. Springer Science & Business Media.

McKee, M. and Shkolnikov, V. (2001). Understanding the toll of premature death among men in eastern europe. *BMJ: British Medical Journal*, 323(7320):1051.

Oeppen, J. and Vaupel, J. W. (2002). Broken limits to life expectancy. *Science*, 296(5570):1029–1031.

Rau, R., Bohk, C., Muszyńska, M. M., and Vaupel, J. (2013). Rates of mortality improvement on the lexis surface. Technical report, Max Planck Institute for Demographic Research.

Rehm, J., Sulkowska, U., Mańczuk, M., Boffetta, P., Powles, J., Popova, S., and Zatoński, W. (2007). Alcohol accounts for a high proportion of premature mortality in central and eastern europe. *International journal of epidemiology*, 36(2):458–467.

Shkolnikov, V., McKee, M., and Leon, D. A. (2001). Changes in life expectancy in russia in the mid-1990s. *The Lancet*, 357(9260):917–921.

Shkolnikov, V. M., Andreev, E. E., and Begun, A. Z. (2003). Gini coefficient as a life table function: computation from discrete data, decomposition of differences and empirical examples. *Demographic Research*, 8(11):305–358.

Smits, J. and Monden, C. (2009). Length of life inequality around the globe. *Social Science & Medicine*, 68(6):1114–1123.

Van Raalte, A. A. and Caswell, H. (2013). Perturbation analysis of indices of lifespan variability. *Demography*, 50(5):1615–1640.

van Raalte, A. A., Martikainen, P., and Myrskylä, M. (2014). Lifespan variation by occupational class: compression or stagnation over time? *Demography*, 51(1):73–95.

Vaupel, J. W. (1986). How change in age-specific mortality affects life expectancy. *Population Studies*, 40(1):147–157.

Vaupel, J. W. and Canudas-Romo, V. (2003). Decomposing change in life expectancy: A bouquet of formulas in honor of nathan keyfitzs 90th birthday. *Demography*, 40(2):201–216.

Vaupel, J. W., Zhang, Z., and van Raalte, A. A. (2011). Life expectancy and disparity: an international comparison of life table data. *BMJ open*, 1(1):e000128.

Wilmoth, J. R. and Horiuchi, S. (1999). Rectangularization revisited: Variability of age at death within human populations*. *Demography*, 36(4):475–495.

World Health Organization (2000). The world health report 2000: health systems: improving performance.

Zhang, Z. and Vaupel, J. W. (2009). The age separating early deaths from late deaths. *Demographic Research*, 20(29):721–730.