Lifespan dispersion in stagnant and decreasing periods of life expectancy in Eastern Europe

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Abstract

Life expectancy at birth has had an atypical pattern in Eastern European countries since the 1950s. Periods of rapid increase in life expectancy followed by stagnation and decreases have been documented. I analyze patterns of dispersion in age at death for these countries and its relationship with the average length of life. Data includes 12 countries since 1947 from the Human Mortality Database by sex. Two measures of dispersion were used. Life disparity $e^{\dagger}$ and Keyfitz's entropy $\mathcal{H}$. Life expectancy is made up of a combination of age-specific mortality rates so its change might be driven by an offsetting effect of improvements in old mortality and a reversal in middle-age mortality. In such way that the relationship between life expectancy and dispersion at death is not clear. I found that the negative relationship between life expectancy and lifespan variation holds even in atypical periods of stagnation. Furthermore, the relationship between both measures seems to be consistent over time simultaneously. Although Eastern European countries have experienced improvements in life expectancy during the last decade, high levels of life disparity remain. Pointing to a high prevalence of premature deaths, potentially avoidable.

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Introduction

The 20th century was marked by sizable improvements in mortality and health in most countries in the world (World Health Organization 2000). However, these improvements were shattered in the second half of the past century, as Eastern European countries experienced an unprecedented period of stagnation and, in some countries, decreases in life expectancy at birth after 1979 (Chenet et al. 1996). Although improvements in infant mortality were documented during this period, an offsetting effect driven by increases in young and middle-aged mortality (from circulatory diseases, lung cancer, cirrhosis and accidents) led to a substantial deterioration in the health status of the populations in Czechoslovakia, Hungary and Poland (Chenet et al. 1996). Three of the wealthiest former socialist countries. Similarly, Russia experienced a brief rise in life expectancy in the 1980s, followed by a pronounced decline in males' life expectancy in the 1990s (over 5 years) (Cockerham 1997) mainly caused by premature adult mortality associated with alcohol consumption (Rehm et al. 2007).

Importantly, this period of increase in Russian longevity coincided with the implementation of Gorbachev's anti-alcohol campaign in the mid-1980s that lasted until 1987. Empirical evidence suggests that the rise in Russian life expectancy has been attributed to the success of this campaign (Bobadilla et al. 1997). Nevertheless, after the break down of the former Soviet Union, Russian's health status continued worsening and by 1994, life expectancy was even lower than their Soviet counterparts in 1960, for both men and women (Cockerham 1997). Although women also experienced deterioration in life expectancy in these nations, men were specially susceptible to dying prematurely, leading to large sex-differences in Poland, Hungary, Russia, and the European post-Soviet Republics (McKee and Shkolnikov 2001).

National trends in life expectancy are important and informative. Nonetheless, they conceal heterogeneity and variation at age at death. Several studies have found a negative correlation between life expectancy and the variation in the ages at death across time and countries in the context of improvements in averting premature deaths (e.g. Vaupel et al. (2011)) and the rectangularization of the survivorship (Wilmoth and Horiuchi 1999). In spite of this fact, some countries, like the U.S., showed an unanticipated high variation in age at death due to mortality at very young and older ages compared with the average lifespan (Shkolnikov et al. 2003).

Variation at age at death depends on the interaction of saving lives at younger ages and at old ages simultaneously (Vaupel 1986). The balance between both drives the compression (or expansion) of ages at death. Given the atypical patterns in life expectancy and the burden of premature mortality observed in Eastern European countries, understanding trajectories of the lifespan variation and its relationship with life expectancy is an important step toward extend prior work regarding the *longevus* framework. Does the correlation between life expectancy and variation at age at death holds for populations in periods of stagnation?, in periods when life expectancy declines?. If so, is this correlation stronger or weaker in these particular periods?. Is variation in lifespan driven by a change in life expectancy, or the other way around?, is one lagging behind the other? This essay addresses such questions immersed in the *longevus* perspective.

Data & Methods

I used life tables from the Human Mortality Database (2015) for 12 countries from 1947 to the most recent year available. The countries included in the study are Belarus, Bulgaria, Czech Republic, Hungary, Poland, Russia, Slovakia, Ukraine, Slovenia, Estonia, Latvia and Lithuania. These data contain information on life table's measures (e.g. d_x, l_x, e_x,q_x) by single age, sex and country.

Lifespan dispersion measures

Life disparity ($e^\dagger$) and Keyfitz's entropy ($\mathcal{H}$) are used to measure the dispersion in age at death. Life disparity, $e^\dagger$, is defined as the average remaining life expectancy when death occurs; or life years lost due to death (Vaupel 1986). This measure was further developed by Vaupel and Canudas-Romo (2003) and Zhang and Vaupel (2009) recently. It can be expressed as

$$e_x^\dagger = \frac{1}{l_x} \int_x^\infty l(a)\mu(a)e(a)da \tag{1}$$

where $l(a)$, $\mu(a)$ and $e(a)$ are the survival function, the force of mortality, and life expectancy, respectively.

Its application is justified because of its easy interpretation relative to others life dispersion measures. When death is very variable, some people will die before their expected age at death, contributing many lost years to life disparity. When people survive to older ages , the difference between the age at death and the expected remaining years decreases, and life disparity gets smaller.

In order to compare consistency in the results obtained with life disparity Keyfitz's life table entropy $\mathcal{H}$ is also used, which is a measure also related to lifespan variation (Vaupel 1986, Vaupel and Canudas-Romo 2003). However, the interpretation is not as easy as life disparity. It is defined as:

$$\mathcal{H} = \frac{\int_0^\infty l(a)\mu(a)e(a)ds}{e^o(0)} \tag{2}$$

Note that for $l_0 = 1$, $e^\dagger = \mathcal{H} * e_0^o$ (Vaupel and Canudas-Romo 2003).

Overall strategy

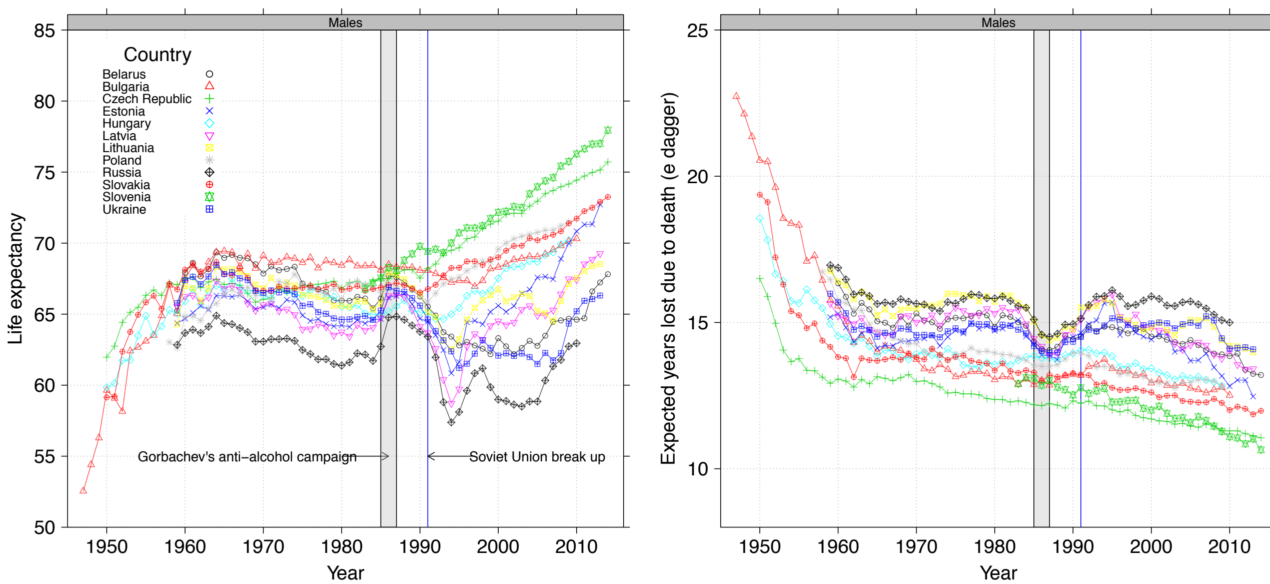
The aim is to analyze trends in lifespan dispersion and life expectancy and their relationship. First, $e^\dagger$ and $\mathcal{H}$ are calculated for all Eastern Europe countries and their available data. Second, patterns in life expectancy and lifespan dispersion are simultaneously explored. Third, the relationship between the observed results is described. Finally, I performed microsimulation processes to examine patterns and their relationship in differences among different periods to include the time variable in the study. Everything was carried out using R (Ripley 2001).

Results

Figure 1 shows male's life expectancy at birth e_0 (panel a) and lifespan disparity $e^\dagger$ panel (b) for Eastern European countries from 1947 to 2014. All countries experienced marked increases in life expectancy before 1960. In contrast, from 1960 to 1984 life expectancy stagnated for most of the countries, some of them even experienced decreases (e.g. Russia, Latvia, Estonia, Ukraine). This period was followed by a notable increase in life expectancy in the mid-1980s. However, in 1987 life expectancy among these countries started to diverge. Slovenia and the Czech Republic exhibited a continuous increase from that point up to now. Hungary, Poland, Bulgaria stagnated for a short period and then continued an upward trend until 2014. The rest of the countries (Russia, Latvia, Estonia, Ukraine, Belarus and Lithuania) experienced a marked decrease in life expectancy from 1988 to 1993. From that point on, all of them have experienced improvements life expectancy. The trends for both male and females are similar (figure with females' results are shown in the Appendix). Yet, the magnitude of the changes is shorter for women and the level of life expectancy is significantly higher than men.

Figure 1: Trends in males' e_0 and $e^\dagger$ for 12 Eastern European countries, 1960-2014

(a) e_0 (b) $e^\dagger$



Source: own calculations based on Human Mortality Database (2015) data.

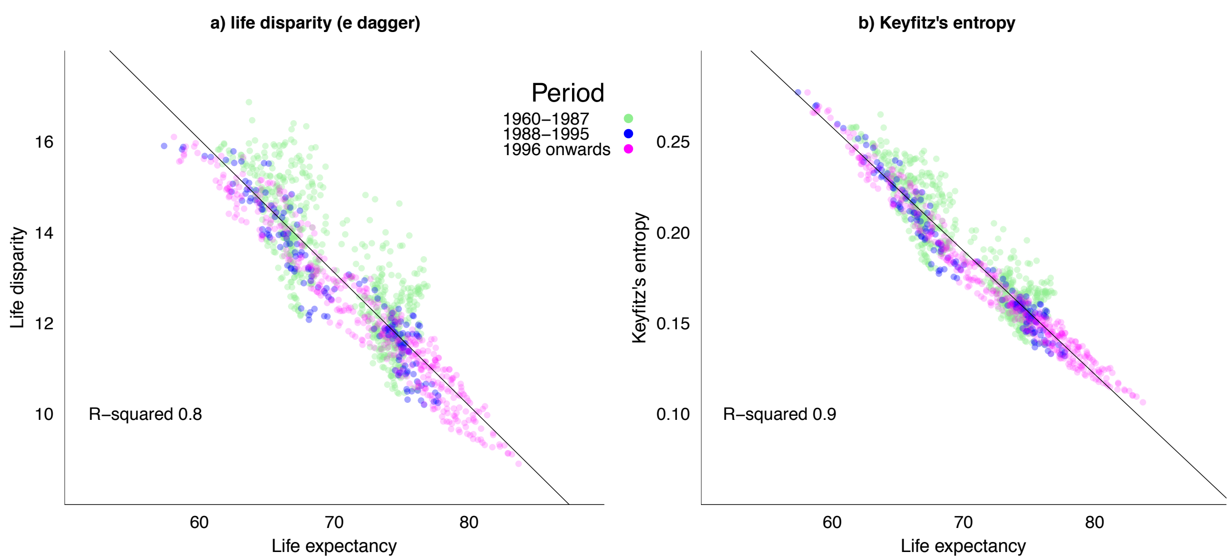
Opposing the trends in life expectancy, life disparity, $e^\dagger$ (panel b), exhibited similar tendencies for all countries but in an inverted manner. Bulgaria, Slovakia, Hungary and the Czech Republic experienced a remarkable decrease in life disparity before 1960, from 22 to 13 years. Followed by a period of stagnation until the mid 1980's. From that point on, changes are similar to the ones observed in life expectancy with a negative scale. Russia, Lithuania, and Latvia presented the highest levels of lifespan disparity during the whole period. In contrast, the Czech Republic and Slovenia showed the lowest levels since 1950. It is worth noting that although Slovenia is the record holder in the lowest life disparity throughout the entire period, it was just the life expectancy record holder in the 1950's.

Figure 2 shows the relationship between life disparity $e^\dagger$ and Keyfitz's entropy $\mathcal{H}$ with life expectancy at birth for Eastern European countries. The scatter diagrams include data from 1960 to 2014 for both females and males together. Green dots relate to data between 1960 and 1987, blue dots to 1988-1995, and magenta dots are associated with data from 1996 to 2014. Both measures, $e^\dagger$ and $\mathcal{H}$ exhibit a similar pattern. However, $\mathcal{H}$'s dots are more concentrated in a linear trend. The R^2 coefficient between life expectancy and $e^\dagger$ is 0.81 from a linear model, while for $\mathcal{H}$ is 0.92.

Data related to years between 1960 and 1987 exhibit higher variation compared with the most recent data (panel a). Nevertheless, they account for the largest changes in both measures, bigger improvements in life expectancy correspond to bigger reductions in life disparity. The dots that correspond to the decreasing period (1988-1995) in life expectancy are spread over the black line. Finally, more recent data show higher levels of life expectancy, which correspond to the lowest levels of life disparity.

Similar results are observed in Keyfitz's entropy (panel b). However, dots that correspond to this measure are more assembled through the linear line. Likewise to $e^\dagger$, the largest variation concern data in the period 1960-1987 (green dots). This was expected from the equations described in the methods' section ((1),(2)), as each measure can be expressed in terms of the other (Vaupel and Canudas-Romo 2003).

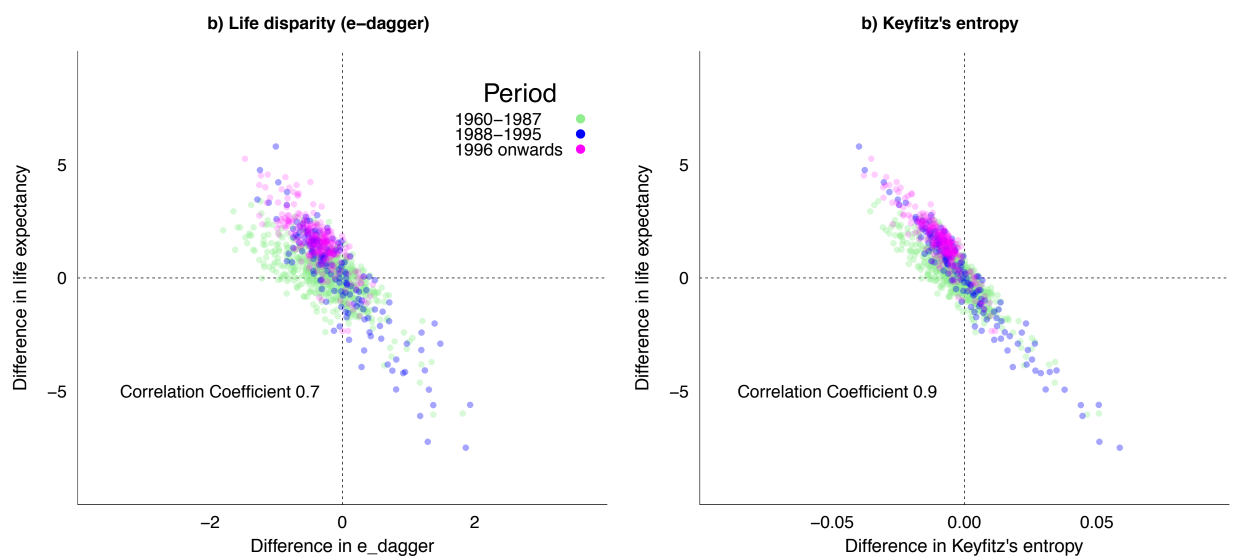
Figure 2: Relationship between $e^\dagger$ and $\mathcal{H}$ with e_0 for Eastern European countries with a lag of 6 years, 1960-2014



Source: own calculations based on Human Mortality Database (2015) data.

The relationship between changes in life disparity (panel a) and Keyfitz's entropy (panel b) with changes in life expectancy are shown in figure 3. I performed micro-simulation processes to study the relationship between the differences of both dispersion measures and the changes in life expectancy with a maximum lag of 15 years between them. This process allowed me to identify the higher correlation between them out of 225 possible combinations. The results shown in figure 3 correspond to the lag that maximizes the R^2 of a linear model without the intercept. The value for both, life disparity and Keyfitz's entropy, is 6 years. The interesting fact of these results is that out of all the possible combinations the lags coincide with changes in life expectancy and lifespan variation. Suggesting that both measures evolve simultaneously.[1] The consistency in life disparity and in Keyfitz's entropy reinforce the result. However, these results should not be interpreted as if life expectancy changes are followed by changes in life dispersion by lag of $n = 6$ years. The importance of the length n is diminished and it is not meaningful since it could be explained by random noise in the time series.

Figure 3: Relationship between differences in $e^\dagger$ and $\mathcal{H}$ with e_0 for Eastern European countries, 1960-2014



Source: own calculations based on Human Mortality Database (2015) data.

The idea behind the above exhibit is that if life disparity and life expectancy are negatively related, then a positive change in life expectancy (y-axis) should correspond to a negative change in life disparity ($e^\dagger$ and $\mathcal{H}$, x-axis). Therefore, the differences should be concentrated in the upper-left and lower-right axes.

Green dots, which correspond to the period of stagnation between 1960 and 1987 are concentrated around zero, while blue dots (1988-1995) are spread out from around zero to the lower-right quadrant. Finally, magenta dots, that account for the most present period, are located just up to the left of the zero, suggesting improvements in life expectancy with a simultaneous decrease in life disparity. The results are similar with Keyfitz's entropy $\mathcal{H}$ (panel b).

Importantly, results in figure 3 show a high correlation between changes in life dispersion measures ($e^\dagger$ and $\mathcal{H}$) and variation in life expectancy (e_0) since the majority of the dots are gather in the upper-left and

[1] I performed exactly the same procedure incorporating the data before 1960 as a sensitivity analysis. The lag that maximizes the correlation of both measures also coincide.

lower-right quadrant of the graph. The Pearson's correlation coefficient are 0.81 for $e^{\dagger} \mid e_0$ and 0.92 for $\mathcal{H} \mid e_0$.

Discussion

Life expectancy and disparity trends

Life expectancy at birth experienced an atypical pattern in Eastern Europe since 1950 relative to the trend observed in other European regions and in the record life expectancy (Oeppen and Vaupel 2002). During which, these countries experienced considerable improvements on the average age at death before 1960, followed by a fairly large period of stagnation (1960-1990) with a mean life expectancy around 66 years. After Gorbachev's anti-alcohol campaign was implemented and the Soviet Union broke up, some of these countries exhibited and unprecedented decrease in life expectancy (figure 1). Russia and Latvia's male life expectancy declined from 64 in 1991 to 57 and 58 respectively. To put this in perspective, Russia and Latvia were having the same level of life expectancy as Slovakia used to have in 1959 and contradicting the best practice upward tendency of 2.5 years every decade (Oeppen and Vaupel 2002). This reversal in life expectancy was mainly driven by mortality at younger ages (15-75) caused by hazardous alcohol consumption (Shkolnikov et al. 2001, Leon 2011). Nevertheless, in the last decade life expectancy has showed significantly improvements in all the Eastern European countries, yet high levels of inequality between and inside the countries remain (Leon 2011).

In parallel, results show an astounding decline in life disparity before 1960, from 23 to less than 15 years, almost a 10-year decrease in a 13 year period. This improvements were reversed as Eastern European countries experienced a slowdown in life disparity reduction until the mid-1980's. Shkolnikov et al. (2003) attribute the decrease in lifespan variation in Russia after 1987 (figure 1, panel b) to the success of Gorbachev's anti-alcohol campaign. This could possibly be the reason to explain reductions in life disparity in all Easter European countries in the same period. After 1991, life disparity increased and then started to decrease in the early 2000s. Although, with a lower rate, not comparable with the period prior to 1960. Nutritional patterns are likely to have contributed to the changes in life disparity after 1991. The standard diet in the region is characterized by a high consumption of cholesterol-rich foods, sugar, salt and bread. Which, may account for the high mortality patterns caused by circulatory diseases and cancer in premature ages (McKee and Shkolnikov 2001). These trends in life disparity suggest a relationship with those observed in life expectancy since they coincide over the period in an inverse way.

Life expectancy and its relationship with life disparity

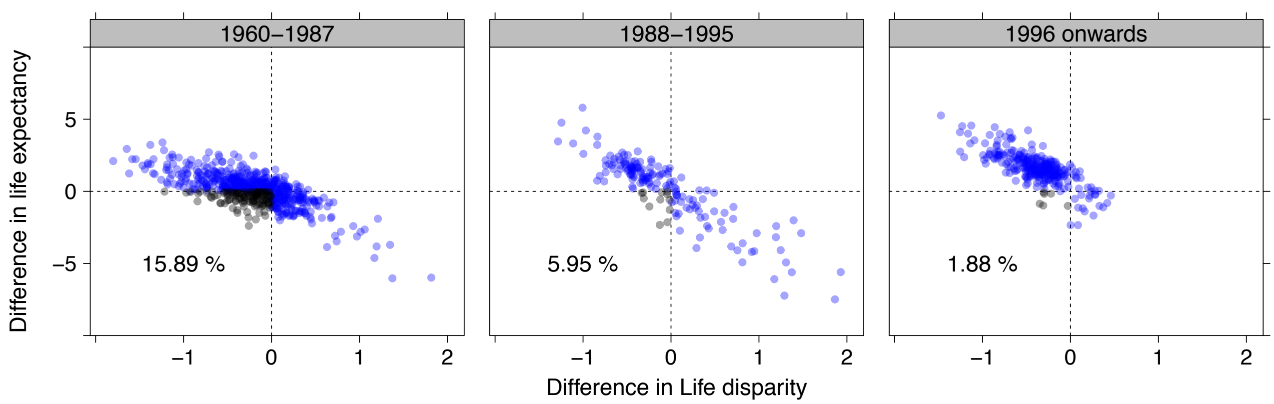
This relationship has been previously studied in diverse contexts and a bouquet of measures have been proposed (Shkolnikov et al. 2003, Van Raalte and Caswell 2013). For instance, Edwards and Tuljapurkar (2005) found that achieving the best practice means reducing inequalities as opposed to pushing the aging boundary among industrialized countries. Similarly, Vaupel et al. (2011) performed a big scale study to determine the contribution of progress in avoiding premature deaths to the improvements in life expectancy and life disparity. They found that the countries that have successfully averted premature deaths have the higher life expectancy and the lower life disparity levels. Recently, other authors have studied the variability withing age groups and by socieconomic status (Engelman et al. 2010, van Raalte et al. 2014). However, no study addresses the relationship between life disparity and life expectancy in the context of life expectancy stagnation/decrease focusing in the Eastern European case. This research has the potential to shed some light on this regard by showing that the life expectancy-life disparity relationship ($e_0 \mid e^\dagger$) holds for these group of countries.

Life expectancy and life disparity are highly correlated and the relationship can be explained with a straight line ($R^2 = 0.81$ for $e_0 \mid e^\dagger$ and $R^2 = 0.92$ for $e_0 \mid \mathcal{H}$), as shown in figure 2. This result is consistent with previous research and was expected (Vaupel et al. 2011). Albeit the strong relationship, results surprisingly show heterogeneity when looking by period. This can be explained by the pattern observed in life expectancy relative to life disparity. The variation among the age at death and the mortality distribution (not shown here) between Eastern European countries can account for this inconsistency in the results compared with previous research (Vaupel et al. 2011). The high proportion of deaths at younger ages and the rickety pattern of mortality over the time can justify the variation by period observed in this figure. In addition to alcohol-related mortality and circulatory diseases, this might be due to the lack of determination of policymakers to avoid deaths due to injuries and violence in the central and Eastern European region (McKee and Shkolnikov 2001). Although this findings strengthen the relationship between life disparity and life expectancy, the fact that the record holder in low life disparity is not always the record holder in life expectancy over the period suggests an underlying process that drives some part of both measures independently. Therefore, looking into the changes over time in both measures and their relationship is an step forward to understand clearer life expectancy and disparity.

Changes in life expectancy and its relationship with changes in life disparity

There are three possible scenarios. First, if life expectancy and disparity are truly related then an increase in life expectancy should be followed by a decrease in life disparity. Second, it is also possible the inverse order, meaning that changes in life expectancy are driven by life disparity variation. Finally, the changes might respond simultaneously. If one change is followed by the other, is there a lag between this changes? a change in life disparity is driven by a change in life expectancy that occurred one, two, n years before? Fluctuations and the unsteady pattern of life expectancy and disparity could be caused by data quality and conceal real processes underlying both measures. Therefore linear models were performed to 225 possible combinations of differences with a maximum lag of 15 years between each measure. This means, that at the maximum, a change in life expectancy could be a response to a life disparity change 15 years before, or the other way around. Two important results came out of the analysis. First, changes in life expectancy and disparity are related (Pearson correlation coefficient of 0.75 for $e^{\dagger}$ and 0.90 for $\mathcal{H}$), reinforcing previous research adding the changes through time as part of the analysis (figure 3). Second, out of every combination with all the data available for the 12 countries (females and males), the maximum for life disparity coincides with the maximum lag for life expectancy (also when I performed the analysis with Keyfitz's entropy). This evidence suggests that the changes in both measures are driven by some underlying process at the same time. Results also show that bigger changes in life expectancy correspond to bigger changes in life disparity. Nevertheless, periods of stagnation and decrease in life expectancy are more spread out in the graph, suggesting that the reverse pattern in life expectancy (e.g. decreasing) might not result always in an increase in life disparity. For instance, between 1960 and 1987, 15.9% of the declines observed in life expectancy correspond to decreases in life disparity (Figure 4). While for 1988-1995 the corresponding value is 5.9 percent. Opposing this, since 1996 results show that the negative relationship between the changes in both measures holds for almost every data dot.

Figure 4: Proportion of observations corresponding to decreases in life expectancy and disparity for Eastern European countries, 1960-2014



Source: own calculations based on Human Mortality Database (2015) data.

Both life expectancy and life disparity are made up of a combination of age-specific mortality rates. Therefore, the atypical pattern observed in Figure 4 (black dots) could be explain by an offsetting effect of improvements in infant and old-age mortality that made life disparity decline, and an increase in mortality rates in middle-ages that brought life expectancy down. To get a deeper understanding of what is driving this process I analyze the following relationships.

Concluding remarks regarding the Eastern European case

The analysis proposed in the present study can be used in different levels to help policymakers strengthen their strategies to improve health in Eastern European countries. Premature deaths can be avoided if public health interventions are successful and well performed. Although the most recent data confirms an upward trend in life expectancy, there still exist life disparity at age at death. Therefore, efforts to reduce inequalities at age at death is a step towards achieving the best practice life expectancy. The public health framework should focus its endeavors in minimizing injuries and violence in premature deaths to lessen mortality at young ages. Regulations on building, equipment and health care access should be improved as they still remain in a poor quality level (McKee and Shkolnikov 2001). Eastern European countries have a characteristic lifestyle that contributes to the risk of premature mortality. Thus, particular policies should be implemented to reduce this risk. Since lifestyle is influenced by social circumstances, the lifespan disparity reduction and the increase in life expectancy should be a goal approached from different perspectives. The field of biodemography, which gathers efforts from demographers, epidemiologists and other biomedical researchers, should be a great tool to help policymakers plan the best strategies to help their populations to achieve longer and healthier lives (Vaupel 2010, Baudisch 2011).

Formal relationships

A proportional increase in μ at all ages and its relationship with changes in life expectancy

I first analyze the effect of a change in the force of mortality in life expectancy as Keyfitz and Caswell (2005, p. 78) proposed and then on life disparity following a similar procedure. Let $\mu(x)$ be the force of mortality at age x. The probability of surviving from birth to age x is, with a radix of $l(0) = 1$, is

$$l(x) = e^{-\int_0^x \mu(a)da}$$

so that life expectancy at age x is given by

$$e^o(x) = \int_x^\infty e^{-\int_0^s \mu(a)da} ds \tag{3}$$

I am interested in life expectancy at birth, which means that x is fixed at zero and in making $\mu(a)$ vary in the age-dimension. Therefore, life expectancy at birth can be re-expressed as

$$e_0^o(\mu(a)) = \int_0^\infty e^{-\int_0^s \mu(a)da} ds. \tag{4}$$

Consider a constant increase in μ of $\delta > 0$. Then, the new mortality function is $(1 + \delta)\mu(a)$ and the new probability of surviving from birth to age x is

$$l_x((1+\delta)\mu(a)) = e^{-\int_0^x (1+\delta)\mu(a)da} = \left[e^{-\int_0^x \mu(a)da}\right]^{1+\delta} = [l_x(\mu(a))]^{1+\delta}.$$

Then, the new life expectancy at birth is

$$e_0^{o*}((1+\delta)\mu(a)) = \int_0^\infty l(s)^{1+\delta} ds. \tag{5}$$

A relative increase in mortality should cause a relative decrease in life expectancy and a relative increase in life disparity. The derivative of (5) with respect to δ will show the effect of a small change δ on life expectancy. That is

$$\frac{\partial e_0^{o*}}{\partial \delta} = \int_0^\infty \frac{\partial}{\partial \delta} e^{\ln(l(s)^{1+\delta})} ds = \int_0^\infty \frac{\partial}{\partial \delta} e^{(a+\delta)\ln(l(s))} ds = \int_0^\infty [\ln l(s)] l(s)^{1+\delta} ds \tag{6}$$

If δ is small, Keyfitz and Caswell (2005) approximated a relative change in life expectancy at birth as

$$\frac{\Delta e_0^o}{e_0^o} = \left[\frac{\int_0^\infty l(s)\ln(l(s))ds}{\int_0^\infty l(s)ds}\right]\delta = \left[\frac{-\int_0^\infty l(s)\mu(s)e(s)ds}{\int_0^\infty l(s)ds}\right]\delta = -\mathcal{H}\delta = \left[\frac{-e^\dagger}{e_0^o}\right]\delta \tag{7}$$

Since $0 < l(x) < 1$, the ratio in (7) is negative, causing a decrease in life expectancy. The negative of this ratio is the equivalent of Keyfitz's entropy $\mathcal{H}$ in (2). This means that a proportional increase in mortality rates at all ages result in a proportional decrease in life expectancy of δ times $\mathcal{H}$.

A proportional increase in μ at all ages and its relationship with changes in life disparity

From (1) and (7), we have that

$$e_0^\dagger = e^\dagger = \int_0^\infty l(s)\mu(s)e(s)ds = -\int_0^\infty l(s)\ln(l(s))ds. \tag{8}$$

Evaluating (8) with the new force of mortality $(1+\delta)\mu(a)$ yields

$$e^{\dagger*} = -\int_0^\infty l_s((1+\delta)\mu(a))\ln(l_s((1+\delta)\mu(a)))ds. = -\int_0^\infty l(s)^{1+\delta}\ln[l(s)^{1+\delta}]ds \tag{9}$$

Similarly, as in (6), the derivative of $e^{\dagger*}$ with respect to δ will show the effect of a small change δ on life disparity.

$$
\begin{aligned}
\frac{\partial}{\partial \delta} e^{\dagger *} &= \frac{-\partial}{\partial \delta} \int_0^{\infty} l(s)^{1+\delta} \ln[l(s)^{1+\delta}] ds \\
&= \frac{-\partial}{\partial \delta} (1+\delta) \int_0^{\infty} l(s)^{1+\delta} \ln l(s) ds \\
&= \frac{-\partial}{\partial \delta} \left[(1+\delta) \frac{\partial e_0^{o*}}{\partial \delta} \right] \\
&= -\frac{\partial e_0^{o*}}{\partial \delta} - (1+\delta) \frac{\partial^2 e_0^{o*}}{\partial \delta^2} \qquad (10)
\end{aligned}
$$

Two results are important from (10). First, that an increase in the force of mortality results in an increase in life disparity. The first term in (10) cannot be negative since the first derivative of e_0^{o*} is always negative (Keyfitz and Caswell 2005). We know from (7) that $\frac{\partial e_0^{o*}}{\partial \delta}$ can be approximated (for small δ) as

$$
\frac{\partial e_0^{o*}}{\partial \delta} \approx \delta \int_0^{\infty} l(s) \ln(l(s)) ds.
$$

Thus, the second derivative in (10) can be approximated as

$$
\frac{\partial^2 e_0^{o*}}{\partial \delta^2} \approx \frac{\partial}{\partial \delta} \delta \int_0^{\infty} l(s) \ln(l(s)) ds \approx \int_0^{\infty} l(s) \ln(l(s)) ds, \qquad (11)
$$

which is always negative since $0 < l(x) < 1$. Therefore, the second term in (10) is always positive for small δ.

The second main result from equation (10) is that it shows that life expectancy and life disparity change in opposite directions when a variation in the survival function happens.

The relationship between relative changes in life expectancy and disparity over time

Consider the equality (Vaupel and Canudas-Romo 2003, p. 205, eq. 18):

$$
e^{\dagger}(t) = \mathcal{H}(t) e_0^o(t) \qquad (12)
$$

Since I am interested in relative changes over time I calculate the derivative (indicated by a dot over the functions) of (12) respect to t divided by $e^{\dagger}(t)$ applying Leibniz law, that is

$$
\begin{aligned}
\frac{\dot{e^{\dagger}(t)}}{e^{\dagger}(t)} &= \frac{\dot{[\mathcal{H}(t) e_0^o(t)]}}{\mathcal{H}(t) e_0^o(t)} \\
&= \frac{\dot{\mathcal{H}(t)} e_0^o(t) + \mathcal{H}(t) \dot{e_0^o(t)}}{\mathcal{H}(t) e_0^o(t)} \\
&= \frac{\dot{\mathcal{H}(t)}}{\mathcal{H}(t)} + \frac{\dot{e_0^o(t)}}{e_0^o(t)}. \qquad (13)
\end{aligned}
$$

Which implies that relative changes in life disparity are driven by relative changes in life expectancy and in the life table entropy.

Moreover, from equation (13) follows that

$$\dot{e^\dagger}(t) = e^\dagger(t)\left[\frac{\dot{\mathcal{H}(t)}}{\mathcal{H}(t)} + \frac{\dot{e_0^o(t)}}{e_0^o(t)}\right] \tag{14}$$

which implies that $e^\dagger$ and e_0^o do not have a linear link. For example, if these to measures would have a linear link, then they could be expressed as $e^\dagger = \alpha e_0^o + \beta$, and the changes over time then could be expressed as $\dot{e^\dagger} = \alpha \dot{e_0^o}$, like figures 2 and 3 suggested. However, although life expectancy and disparity react in opposite ways to changes, equation (14) states that variation on $e^\dagger$ depends on variation of both entropy and life expectancy.

Incorporating $a^\dagger$

Zhang and Vaupel (2009) stated that reducing deaths may increase or reduce $e^\dagger$. Whether the effect is positive or negative depends on the age at which the deaths are averted. They showed that depending on Keyfitz's entropy, there is an age before which if deaths are reduced also is life disparity, with the opposite after this age, called $a^\dagger$. I focus in this case where Keyfitz's entropy is less than one, since for the countries studied in this paper $\mathcal{H}$ does not exceed .35.

The change in life expectancy can be broken in two parts (Zhang and Vaupel 2009):

$$\dot{e_0^o}(t) = \dot{e}^o_{0,0<s<a^\dagger}(t) + \dot{e}^o_{0,a^\dagger<s<\infty}(t), \tag{15}$$

where $0 < s < a^\dagger$ expresses the changes in life expectancy that correspond to ages before $a^\dagger$ and $a^\dagger < s < \infty$ afterwards. Then, equation (13) can be re-written as:

$$\frac{\dot{e^\dagger(t)}}{e^\dagger(t)} = \frac{\dot{\mathcal{H}(t)}}{\mathcal{H}(t)} + \frac{\dot{e}^o_{0,0<s<a^\dagger}(t)}{e_0^o(t)} + \frac{\dot{e}^o_{0,a^\dagger<s<\infty}(t)}{e_0^o(t)}, \tag{16}$$

which implies that relative changes in life disparity are driven by relative changes in Keyfitz's entropy along with changes in early and late mortality.

In the particular case of stagnation in life expectancy, like the first panel in figure 4, changes in life expectancy are very little. Mathematically, this means that $\dot{e_0^o} \longrightarrow 0$ or that $\dot{e}_{0,0<s<a^\dagger} = -\dot{e}_{0,a^\dagger<s<\infty}$. Therefore equation (14) can be approximated as

$$\dot{e^\dagger}(t) \approx e^\dagger(t)\left[\frac{\dot{\mathcal{H}(t)}}{\mathcal{H}(t)} + \epsilon\frac{\dot{e_0^o(t)}}{e_0^o(t)}\right], \epsilon \to 0_+, \tag{17}$$

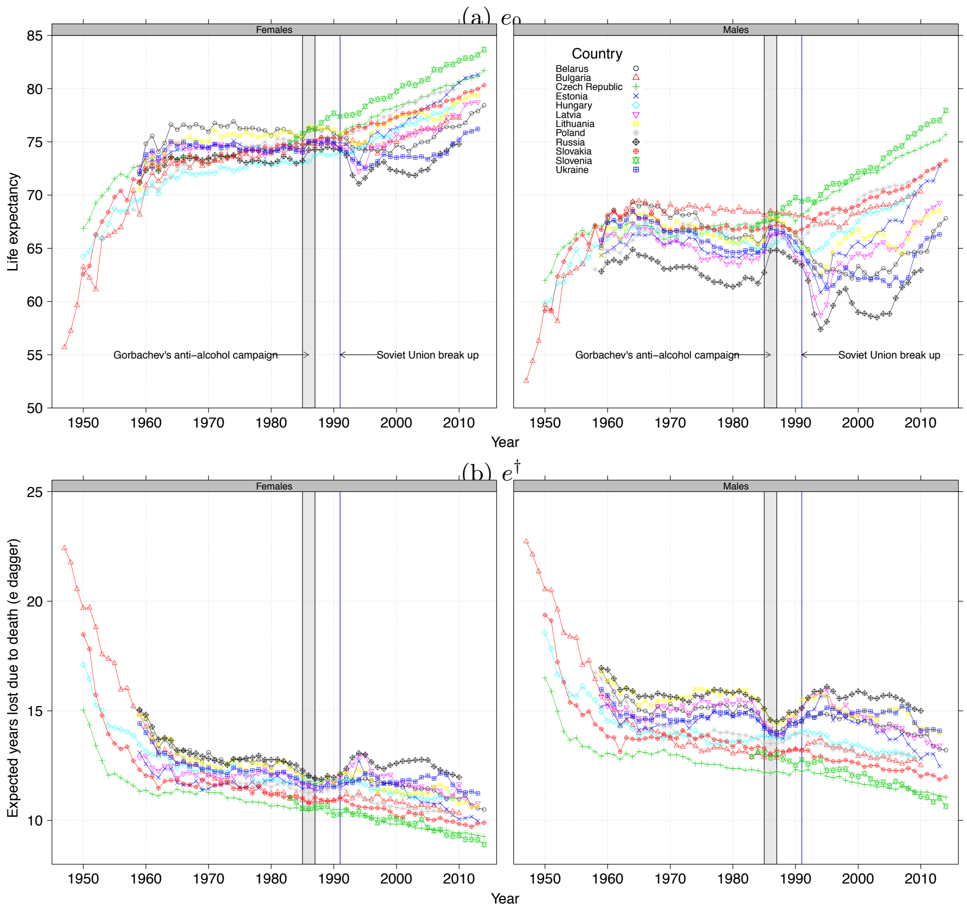
in the neighborhood of $\epsilon = 0$, the previous equation is

$$\dot{e^\dagger(t)} = e^\dagger(t)\left[\frac{\dot{\mathcal{H}(t)}}{\mathcal{H}(t)}\right], \tag{18}$$

which indicates that when life expectancy does not changes, variation in $e^\dagger$ is entirely driven by relative changes in the life table entropy weighted by the life disparity. This is how the black dots in figure 4 could be explained. By changes in the overall survival $\mathcal{H}$.

Appendix

Figure 5: Trends in e_0 and $e^\dagger$ for 12 Eastern European countries by sex, 1946-2014



Source: own calculations based on Human Mortality Database (2015) data.

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