Dynamics of life expectancy and lifespan equality

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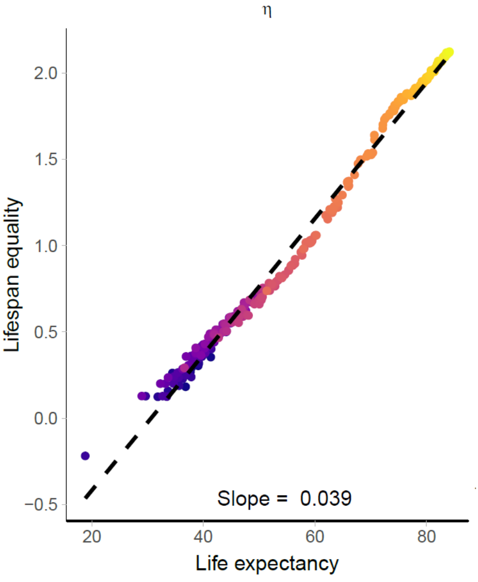
October 2018

Life expectancy vs lifespan equality



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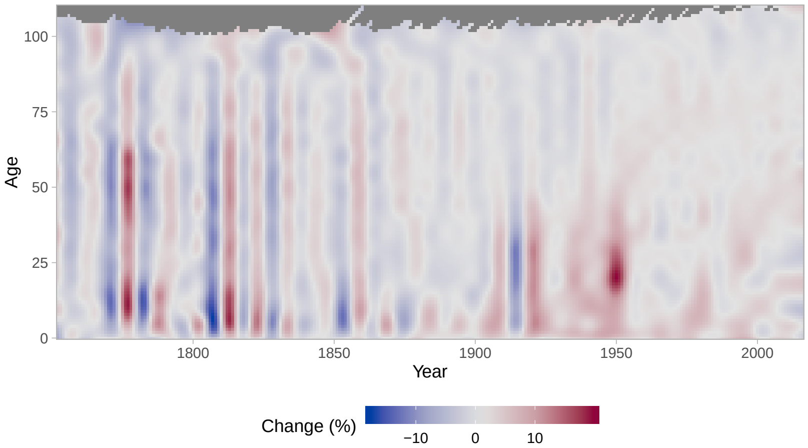
where

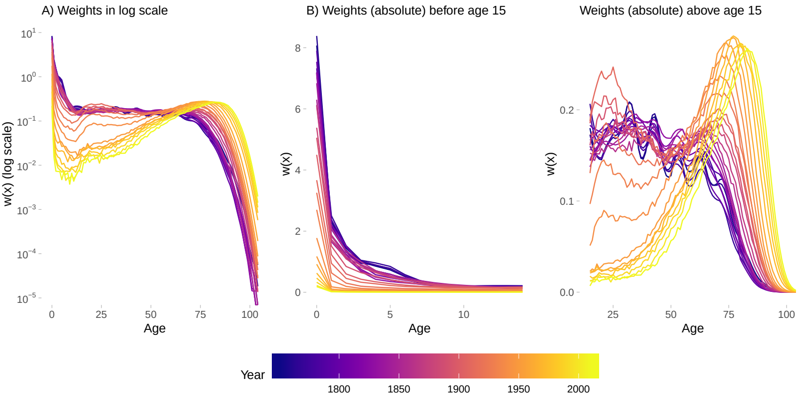
$$\rho(x) = -\frac{\partial \mu(x)/dt}{\mu(x)}$$

$\dot{e_o} =$ **is a weighted total of rates of mortality improvement** ρ (Vaupel & Canudas Romo, 2003)

Mortality improvements, Swedish females, 1751–2016



Life expectancy weights, Swedish females, 1751–2016



Three lifespan equality indicators

- Logarithm of the inverse of **Keyfitz' entropy**

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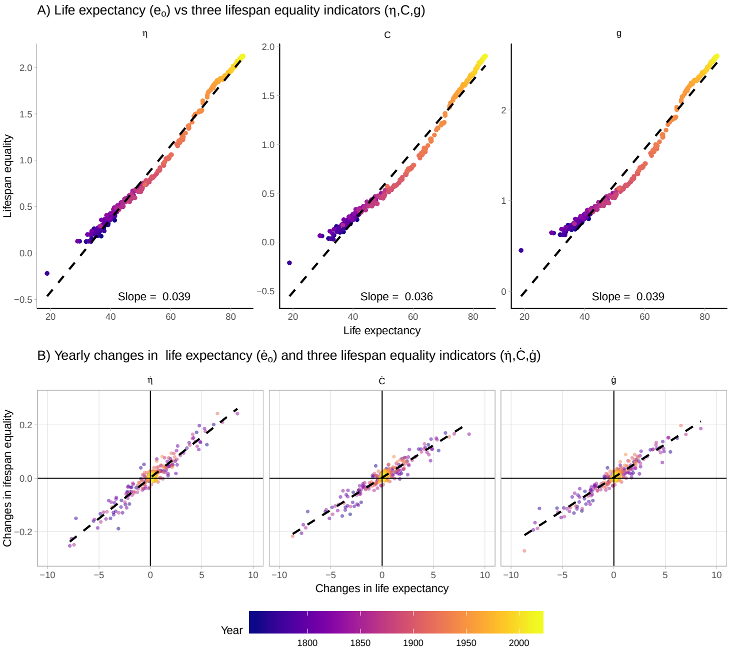
- Variant of the **coefficient of variation**

$$C = -\ln \frac{\sigma}{e_o}.$$


A) Life expectancy (e$_o$) vs three lifespan equality indicators (η,C,g)
η
C
g
Lifespan equality
Slope = 0.039
Slope = 0.036
Slope = 0.039
Life expectancy
B) Yearly changes in life expectancy (ė$_o$) and three lifespan equality indicators (η̇,Ċ,ġ)
η̇
Ċ
ġ
Changes in lifespan equality
Changes in life expectancy
Year
1800
1850
1900
1950
2000

The change over time in lifespan equality $\eta = -\ln\left(\frac{e^\dagger}{e_o}\right)$ is given by

$$\frac{\partial \eta}{\partial t} = \dot{\eta} = \frac{\dot{e}_o}{e_o} - \frac{\dot{e}^\dagger}{e^\dagger} \tag{2}$$

$$\dot{e}^{\dagger} = \int_0^{\infty} \rho(x) w(x) \Big(H(x) + \bar{H}^{+}(x) - 1 \Big) dx$$

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$\dot{e}^{\dagger}$ = **weighted total of rates of mortality improvement** ρ

$$\dot{\eta} = \int_0^\infty \rho(x) w(x) W(x) dx$$

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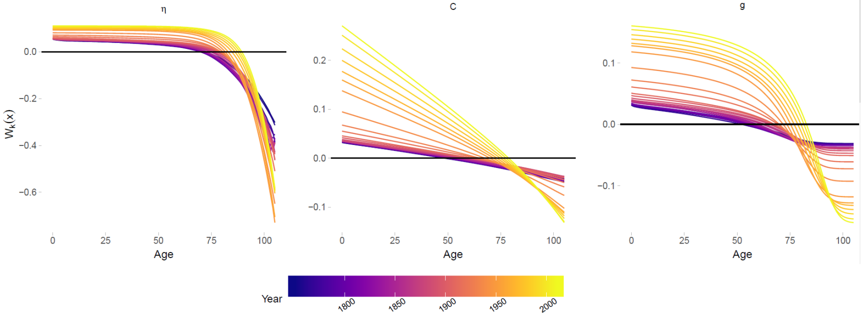
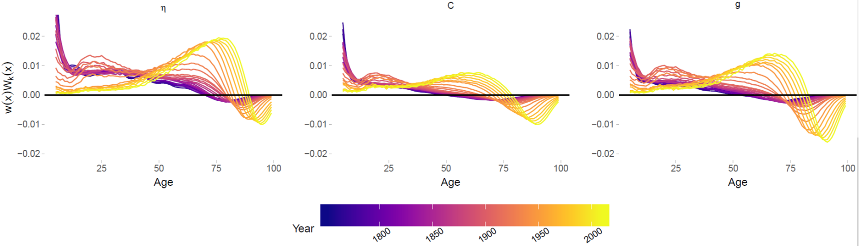
Key point: change in η over time is a weighted total of ρ

A) Weights for three lifespan equality indicators



C) Weights for three lifespan equality indicators from age 20



An application

Let $\tilde{\rho}$ denote the average value of the age-specific rate of mortality improvements $\rho(x)$, weighted by $w(x)$ and defined as

$$\tilde{\rho} = \frac{\int_0^\infty \rho(x)\, w(x)\, dx}{\int_0^\infty w(x)\, dx} \, .$$

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Note that $\tilde{\rho}$ can be interpreted as the loss-weighted average pace of mortality improvement.

The derivative of life expectancy over time can be decomposed as

$$\dot{e}_o = \int_0^\infty \rho(x)\, w(x)\, dx = \frac{\int_0^\infty \rho(x)\, w(x)\, dx}{\int_0^\infty w(x)\, dx} \int_0^\infty w(x)\, dx$$
$$= \tilde{\rho}\, e^\dagger$$

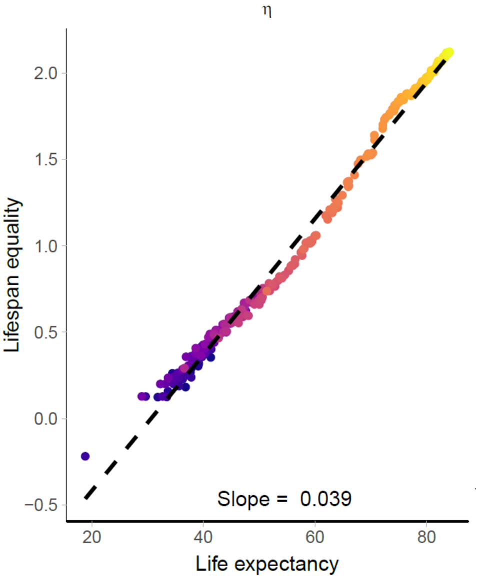
For each of the three measures of lifespan equality η, g and C define the average of the age-specific rate of mortality improvement $\rho(x)$, weighted by the corresponding weights,

$$\bar{\rho}_k = \frac{\int_0^\infty \rho(x)\, w(x)\, W_k(x)\, dx}{\int_0^\infty w(x)\, W_k(x)\, dx} \qquad \text{for } k \in \{\eta, C, g\} \; .$$

For instance, if $k = \eta$ the change over time in η can be decomposed as

$$\dot{\eta} = \bar{\rho}_\eta \int_0^\infty w(x)\, W_\eta(x)\, dx \; ,$$

Life expectancy vs lifespan equality



Slope is given by

$$\frac{\dot{\eta}}{\dot{e}_o} = \frac{\bar{\rho}_\eta}{\tilde{\rho}}\,\overline{W}_\eta = \frac{\bar{\rho}_\eta}{\tilde{\rho}}\,\frac{\int_0^\infty w(x)\,W_\eta(x)\,dx}{\int_0^\infty w(x)\,dx}\,. \tag{3}$$

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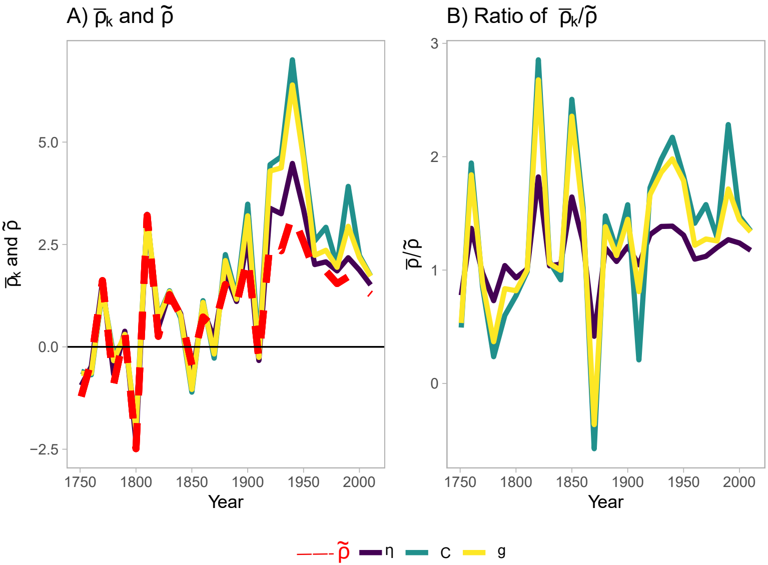
$$\frac{\dot{\eta}}{\dot{e}_o} = \frac{\bar{\rho}_\eta}{\tilde{\rho}} \overline{W}_\eta = \frac{\bar{\rho}_\eta}{\tilde{\rho}} \frac{\int_0^\infty w(x) \, W_\eta(x) \, dx}{\int_0^\infty w(x) \, dx} \ . \tag{3}$$

Key point: slope of the relationship between η and e_o is given by the quotient of the average mortality improvements and the lifespan equality weights $\overline{W}_\eta$.

Slope analysis of mortality improvements, Swedish females, 1751–2016



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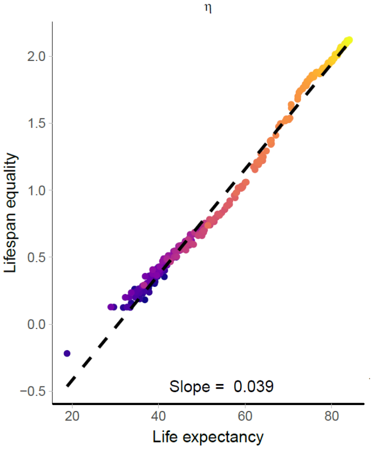

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