CS171 Problem Set 4

Justin Mac
861086907

Problem 2: If there are m items/features, prove there are $3^m - 2^{m+1} + 1$ different association rules possible.

I will use combinatorics to prove this. If there are m items, we choose k of them, $\binom{m}{k}$ ways for forming the left side. To choose the remaining, there are $\binom{m-k}{i}$ ways.

$$\text{Total} = \sum_{k=1}^{m} \binom{m}{k} \sum_{i=1}^{m-k} \binom{m-k}{i}$$

Since $\sum_{i=1}^{m-k} \binom{m-k}{i} = 2^{m-k} - 1 \Rightarrow \text{Total} = \sum_{k=1}^{m} \binom{m}{k}(2^{m-k} - 1)$

Distribute the sigma: $\text{Total} = \sum_{k=1}^{m} \binom{m}{k} 2^{m-k} - \underbrace{\sum_{k=1}^{m} \binom{m}{k}}_{2^m - 1}$

$$\text{Total} = \underbrace{\sum_{k=1}^{m} \binom{m}{k} 2^{m-k}}_{-2^m} - (2^m - 1) + 3^m$$

↑ from LHS

$$\text{Total} = 3^m - 2^m - 2^m + 1$$

There are a total of $3^m - 2^{m+1} + 1$ different association rules possible ∎