Problem Tutorial: "Sappers"

Let's divide our table in half at first. First $m/2$ columns belong to the 1_{st} sapper and last $m/2$ columns belong to the 2_{nd} sapper. For the sake of convenience, let the $n = 3$ and $m = 4$, and the number of mines in the first or second half is cnt_1 or cnt_2, and the overall number of mines is k. Now we'll prove that it's always possible to achieve the best answer, 0 or 1 when k is even or odd, unless $n = 1$ (corner case). Let's analyze what happens if we add cells for each component one by one:

1122	1112	1111	1111	1111	2111	2211
1122	1122	1122	1212	2211	2211	2211
1122	1222	2222	2222	2222	2221	2211

At first, the numbers of mines in each part are cnt_1 and cnt_2 respectively. But at the end, the numbers became cnt_2 and cnt_1 respectively. Because each time we either increasing or decreasing the number of mines in each component at most by 1, there will be a moment when the difference is minimum (when difference is at most 1 depending on the parity of k).

Note that, when we'll be adding the last row to the 1_{st} component, we'll start adding it starting from the end. Otherwise the component might become not connected. And don't forget the case when n is 1. For example:

1 6

* * *...

In this case, the best achievable answer would be 3.