Linear Algebra HW 2

Assignment

Read Strang, Ch 1.4 on matrix multiplication.

Read Evans, 1.2.1, on Venn Diagrams, and 1.3 on event algebra and probability axioms.

Red Boyd, Ch 6 on Matrices

1. How does the term "dimension" apply to (i.e. what is the dimension of) the vector space of a singular matrix? To a non-singular matrix?
2. What is $\det|-\mathbf{A}|$? If one negates all the elements in a matrix how does that change the value of the determinant?
3. For the types of matrices - diagonal, triangular, symmetric, permutation - is multiplication closed? Is it commutative?
   Note, a permutation matrix consists of just 1s and 0s, with no more than a single 1 in any row or column. The identity matrix -- 1s along the diagonal and 0s elsewhere is the identity for permutations. Pre-multiplying by a permutation matrix swaps rows, and post-multiplying swaps columns. There are just two 2X2 permutation matrices:

   Question: In general, for the class of n X n matrices how many different permutation matrices are there?

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

4. For these three vectors, $(3,5,0),(-2,-2,0),(0,0,1)$of length 3, not scalar multiples of each other:
   a. Show they span $\mathbb{R}^3$, e.g. conventional 3-space.
   b. Show they are linearly independent
   c. Show that any non-zero linear combination of them, can replace one of the two vectors and preserve the "span" and "linearly independent" properties.
5. Show that the common formula for a 2 X 2 determinant $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$ is consistent with the 8 properties of determinants in the class notes.
6. Prove from the 8 properties that $0 = \det|\mathbf{M}|$
   a. For a matrix ***M*** with a column of zeros.
   b. For a matrix ***M*** with two columns that are equal.

Restate these results as properties of a set of vectors in a vector space.

8. Why does a permutation matrix have a non-zero determinant? How does one determine it’s determinant?