Probability HW 3

Expectation

0. Compute the expected cost of serving a person using the “vegan / non-vegan” tree from HW one. Assume $v = P(\text{vegan}), c = P(\text{cheesecake} \mid \text{non-vegan}), \text{cheesecake} = \$10, \text{sherbert} = \$1, \text{vegan} = \$12, \text{non-vegan} = \$20$ (See the class notes)

1. For the sum of two dice, *X,* compute these expectations: (you may want to do this in a notebook)
   The last term
   σ_x^2 is called the *variance of X.*

$$E[X], E[X-7], E[X^2], (E[X])^2, E[X^2] - (E[X])^2$$

- Do the same for the conditional expectations, $E[X \mid X \leq 7]$
  - For the average of *n* dice (e.g. the sum divided by *n*) compute the mean $\overline{x}$ and variance σ_x^2 for n = 1,10,100 for both conditional and unconditioned expectations.

2. Consider a simplified recurrent game tree for “monopoly”, with 3 squares, “go”, “rent”, & “jail”. In any of the squares the corresponding probabilities of transitioning to each of the squares is m, m, and 1 - 2m. Landing on “go” earns you $5, on rent -$2. If you land in jail your turn ends.

- Draw a tree for one move of the game. It will recur, since landing on “go” or “rent” brings you back to the state you started in .
- Compute the expected number of moves before landing in jail as a function of m.
- Compute the expected value in dollars of moves before landing in jail as a function of m.

Optionally, you may solve any of these problems by writing a python notebook to simulate the solution.