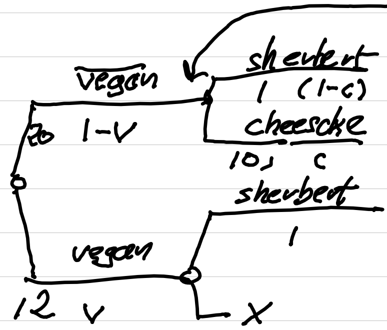
ISE 201 15 Feb '24

class 3 HW solutions

Expectation

Compute the expected cost of serving a person using the “vegan / non- vegan” tree from HW one. Assume $v = P(\text{vegan})$, $c = P(\text{cheesecake} \mid \text{non-vegan})$, cheesecake = \$10, sherbert = \$1, vegan = \$12, non-vegan = \$20



Note: This node $= (1-c) + 30(1-v)c$

$21 \cdot (1-v)(1-c)$

$30 \cdot (1-v)c$

$(12+1)v$

$= 1 - c + 30(c - cv)$

$= 1 + 29c - 30cv$

$$E(\text{serving}) = 21(1 - v - c + vc) + 30(c - vc) + 13 \cdot v$$

$$= 21 + c(30-21) + v(13-21) + vc(21-30)$$

$$= 21 + 9c + (-8)v + (-9)vc$$

For the sum of two dice, *X*, compute these expectations: (you may want to do this in a notebook)

The last termσ_x^2 is called the *variance of X*.

E[*X*], *E*[*X* − 7], *E*[*X*2], (*E*[*X*])2, *E*[*X*2] − (*E*[*X*])2

We can make some simplifications
Since X is a symmetric distribution, it's mean = mode = 7. This is also the 'center of mass' if we think in physical terms.

Further, since $E(x+y) = E(x) + E(y)$ and for a constant c $E[c] = c$

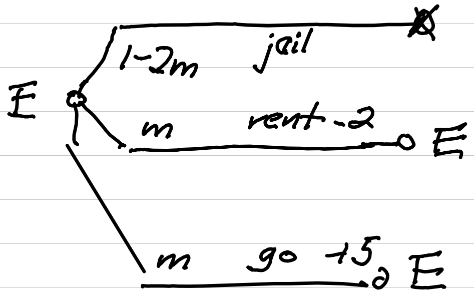
$$E[X-7] = E[X] - 7 = 0$$

For the remaining calculations - see the notebook!

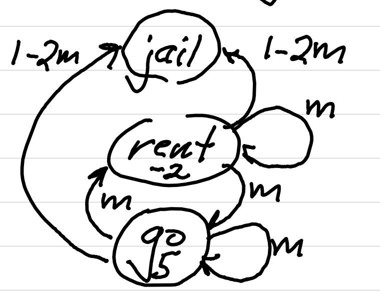
Probability 2:

- Consider a simplified recurrent game tree for "monopoly", with 3 squares, "go", "rent", & "jail". In any of the squares the corresponding probabilities of transitioning to each of the squares is m, m, and 1 - 2m. Landing on "go" earns you \$5, on rent -\$2. If you land in jail your turn ends.
- Draw a tree for one move of the game. It will recur, since landing on "go" or "rent" brings you back to the state you started in .
- Compute the expected number of moves before landing in jail as a function of m.
- Compute the expected value in dollars of moves before landing in jail as a function of m.

The tree:



As a state diagram



Probability recursion:

$$E = 1\cdot(1-2m) + (1+E)m + (1+E)m$$

$$E = 1-2m + 2m(1+E) = 1 + 2m(1+E-1)$$

$$E = 1 + 2mE$$

$$E - 2mE = (1-2m)E = 1, \quad E = \frac{1}{1-2m}$$

Expection recursion

$$E = 0\cdot(1-2m) + (E+5)m + (E-2)m$$

$$E = Em + 5m + Em - 2m = 2Em - 3m$$

$$E - 2Em = -3m$$

$$E(2m-1) = 3m, \quad E = 3m/(2m-1)$$

Linear Algebra HW3.

- For the "barnyard problem - see the pre-course assignment:
  - Form the A matrix for the problem and compute its LU decomposition
  - Solve the linear equations using the LU decomposition
  - What is the column space, and null space of the A matrix?
  - Compute the inverse of the A matrix. What is the determinant of A? The determinant of it's inverse?

Barnyard Problem

c - chickens
r - rhinos
g - goats

$$\begin{array}{cccc} c & r & g & \\ 1 & 1 & 1 & 12 \\ 2 & 4 & 4 & 38 \\ 0 & 1 & 2 & 10 \end{array}$$

$$\begin{array}{ccccl} c & r & g & & \\ 1 & 1 & 1 & 12 & \\ 0 & 1 & 1 & 7 & (-2R_1+R_2)/2 \\ 0 & 1 & 2 & 10 & \end{array}$$

$$\begin{array}{ccccl} c & r & g & & \\ 1 & 1 & 1 & 12 & \\ 0 & 1 & 1 & 7 & (-2R_1+R_2)/2 = R_2' \\ 0 & 0 & 1 & 3 & R^3 - R_2' \end{array}$$

$$\begin{array}{cccc|l} c & r & g & & \\ 1 & 1 & 1 & 12 & 5 = 12-4-3 \\ 0 & 1 & 1 & 7 & 4 = 7-3 \\ 0 & 0 & 1 & 3 & 3 \end{array}$$

The matrix is full rank, so
dim(column space) = dim(row space) = 3
dim(null space) = 0

I think trace U = I, but det(A) = 2
and det(A^{-1}) = 1/2

$$A = \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{vmatrix}$$

$$\begin{array}{cc} & L \\ & \begin{vmatrix} 1 & & \\ 0 & 1 & \\ -1 & 0 & 1 \end{vmatrix} \end{array} \quad \begin{array}{c} U \\ \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \end{array}$$

$R_3 - R_1$

(b - no linear equation)

c - column space A, A

- null space - 2 cols w/out pivots ⇒ dim = 2

Solve
$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

.. set $c = 1$ & $d = 0$ & solve

$$\begin{aligned} a + 2b \quad &= 0 \\ b + c &= 0 \end{aligned} \Rightarrow a + b - c = 0$$

set $c = 0$ & $d = 1$

$$\begin{aligned} a + 2b + d &= 0 \\ b &= 0 \end{aligned} \Rightarrow a + d = 0$$

d - no inverse since its not full rank