Linear Algebra HW 1

Vector spaces

1. To be a Vector Space, all operations on its vectors need to be *closed*, meaning that the result is also a member of the vector space.
   a. If the numbers used for scalars and elements of tuples in the vector space (we speak of the *field over which* the vector space is defined) are restricted to the integers, is the vector space closed?
   b. If we define a vector space over the non-negative rational numbers is it closed?
2. For the vector v in a vector space, suppose both the vectors $u \text{ and } u\prime$ are additive inverses to $v, \text{ e.g.} v + u = \mathbf{0}$. Prove that the additive inverse to v is unique.
   a. To prove uniqueness you want to show that $u = u\prime$. Use axioms from the definition of a vector space to show this algebraically.

Linear combinations

1. For some given values of the b_is, and the vectors $A_1 = (1,0,0), A_2 = (1,1,0), A_3 = (1,1,1)$ is there a linear combination (e.g. a tuple of the *xs),* such that $x_1A_1 + x_2A_2 + x_3A_3 = (b_1, b_2, b_3)$? What happens if all b_is, are zero?
2. Is there a linear combination of these vectors **with non-zero coefficients** that equals the zero vector? What if we add a fourth vector $A_4 = (0,0,1)$ to the linear combination?
3. For the 2-tuples $B_1 = (a,7), B_2 = (7,b)$, such that $x_1B_1 + x_2B_2 = \mathbf{0}$, for what values of a and b do $x_1 \text{ and } x_2$ have a non-zero solution?
4. What do the 3 problems above look like in matrix notation?