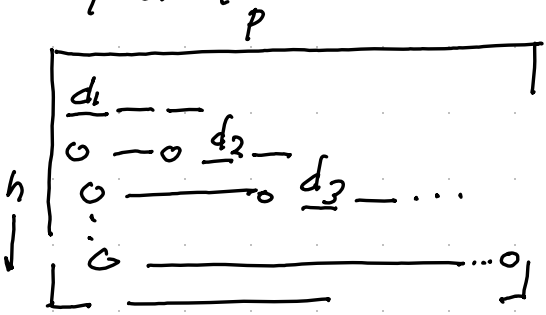
Four fundamental subspaces

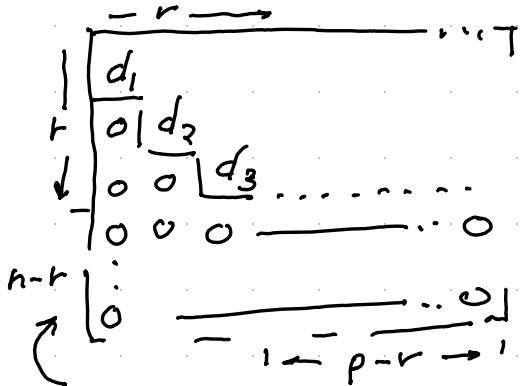
Consider the linear equation $Ax = 0$ where A has n rows and p columns, $p > n$.

To find the subspaces, reduce A to U with pivots d_i



with zeros in columns below the pivots & any numbers above.

Reorder the columns to put all pivots along the diagonal:



subspaces of $\mathbb{R}^p$

row space
$r = \dim R(A^T)$
$\oplus$
null space
$p - r = \dim N(A)$

$n - r = \dim(N(A^T))$ left nullspace
$\oplus$
$r = \dim(R(A))$ column space

subspaces of $\mathbb{R}^n$