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Linear Algebra unit 5a

Date: @today

Lane: *Linear Algebra & Statistics*

Topic: Gram-Schmidt normalization, polynomial regression

What it covers:

- Creating orthonormal basis from any basis vectors
- Applying this to non-linear regression

Requires:

Definition of inner product and Vector spaces unit 1, linear independence, matrix operations.

Required by, used by:

The statistics of Linear regression

Review - why do we want orthonormal vectors for regression?

The Gram-Schmidt algorithm

Gram-Schmidt is an iterative process for generating an orthonormal basis set from any basis. It reduces a basis to an orthonormal basis. The idea is to subtract off the “non-orthogonal” components of vectors, *v*, leaving vectors *u*, that are orthogonal.

Algorithm:

1. Start with any vector in the basis. Normalize it, $u_1 \leftarrow v_1/\|v_1\|$.
2. Pick a second vector, and subtract off the projected component, leaving the perpendicular component:

$$u_2 \leftarrow v_2 - < v_2, u_1 > u_1$$
$$u_2 \leftarrow u_2/\|u_2\|$$

3. Continue, subtracting the projected components from the prevous set of vectors v:

$$u_3 \leftarrow v_3 - < v_3, u_2 > u_2 - < v_3, u_1 > u_1$$
$$u_3 \leftarrow u_3/\|u_3\|$$

4. Finish when all n basis vectors have been transformed. If any vector in the set was <u>not</u> linearly independent (e.g. not part of the basis), then there will be no orthogonal part left and the resulting vector will be zero.

Orthogonal polynomials

One can start with the data, then orthogonalize it, or first derive a set of orthogonal transformations, then apply it to the data.

Legendre polynomials

Regression with orthogonal “features”

Project assignment

You will simulate a dataset of a continuous non-linear function, then create an orthogonal set of feature vectors to recover the function.

Make these assumptions:

1. Come up with a 3rd degree polynomial - be sure to include a constant term.
2. Choose an domain interval for the function on the real line
3. Choose an arbitrary CDF for the data error - a function that you can invert.

Steps:

1. Simulate the polynomial for 30, 300, 3000 samples within the domain
2. Build the 4 column regression (”design”) matrix for each of the sample sets.
3. Apply Gram-Schmidt to orthogonalize the columns.   (Does one need to solve for the implicit polynomial for this orthogonalization?)
4. Run the regression - the projection onto the orthogonal columns
5. How well does the regression recover the original function? Can you recover the error CDF? Can you visualize it?
6. Build another regression matrix using the first 4 Legendre polynomials and compare.

References

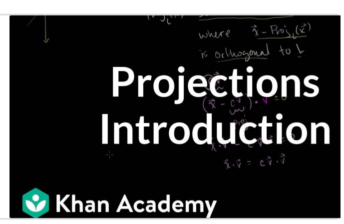
Read Brunton, Ch 1.

Here are some videos:

Khan Academy - Linear Algebra projection operators.

Introduction to projections | Matrix transformations | Linear Algebra | Khan Academy
Determining the projection of a vector on s line

Watch the next lesson: https://www.khanacademy.org/math/linear-
https://www.youtube.com/watch?v=27vT-NWuw0M



Here’s G. Stang’s lecture on projections:

15. Projections onto Subspaces
MIT 18.06 Linear Algebra, Spring 2005
Instructor: Gilbert Strang
View the complete course: http://ocw.mit.edu/18-06S05
https://www.youtube.com/watch?v=Y_Ac6KiQ1t0