Pre-course assignment

Four math problems to as a “warm-up” for the class. This is to assess your problem-solving memory. This should take you about an hour. If you don’t recognize how to do the problem, just say so. Answers should be written showing how you solved it. If you resort to help from a large language model (e.g. you look up the answer in “chat-GPT”), please also provide the answer it gave you.)

1. The “barnyard problem” (simultaneous equations)

In the barnyard there are c chickens, r unicorns, and g goats. You need to solve for the number of each, with these 3 clues:

- Among all the animals, there are 10 horns total. (Note goats have two horns and unicorns one.)
- Counting all the feet all the animals have totals 38.
- There are 12 animals total.

2. 20-sided (icosahedron) dice. (counting)

We assume when a die is thrown that each face is equally likely to appear.

- With two typical 6-sided dice, what is the number of times out of all possible outcomes that the least likely combination appears? The most likely?
- What about for two 20-sided dice?

3. Matrix operations (linear algebra)

Compute the matrix:

$$(M^T M)^{-1}$$

for the matrix:

$$M = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$$

4. Extreme points of a non-linear function (calculus)

This function is defined on the real line:

$$f(x) = x^3 e^{-x}$$

- Solve for the function’s extreme and inflection points
- How would you solve this if the function values sampled at regular intervals had random noise added to the values? (See the notebook file *extremes.ipynb* if you want to experiment with this.