

# Four fundamental subspaces

Consider the linear equation  $Ax = 0$  where  $A$  has  $n$  rows and  $p$  columns,  $p > n$ .

To find the subspaces, reduce  $A$  to  $U$  with pivots  $d_i$

$$\begin{array}{c} n \\ \downarrow \end{array} \left[ \begin{array}{cccc} \underline{d_1} & \_ & \_ & \_ \\ 0 & \_ & 0 & \underline{d_2} \_ \\ 0 & \_ & \_ & 0 \underline{d_3} \_ \dots \\ \vdots & & & \\ 0 & \_ & \_ & \dots 0 \end{array} \right]$$

with zeros in columns below the pivots & any numbers above.

Reorder the columns to put all pivots along the diagonal:

$$\begin{array}{c} \xrightarrow{r} \\ \downarrow r \\ n-r \end{array} \left[ \begin{array}{cccc} \underline{d_1} & \_ & \_ & \_ \\ 0 & \underline{d_2} & \_ & \_ \\ 0 & 0 & \underline{d_3} & \dots \\ 0 & 0 & 0 & \_ \dots 0 \\ \vdots & & & \\ 0 & \_ & \_ & \dots 0 \end{array} \right]$$

subspaces of  $\mathbb{R}^p$

row space

$r = \dim R(A^T)$

$\oplus$

null space

$p-r = \dim N(A)$

$n-r = \dim(N(A^T))$  left nullspace

$\oplus$   $r = \dim(R(A))$  column space

subspaces of  $\mathbb{R}^n$