

# Probability HW 2

## Probability algebra


Problems about events.

1. Show that if  $E_2 \subset E_1$  then  $P(E_1 \setminus E_2) = P(E_1) - P(E_2)$ . (The backslash is the set difference operator, not the probability conditioning symbol.)
2. The law of *inclusion - exclusion* for 2 events is

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

- The expression for the union of 3 events has seven terms. Derive it from the basic laws of probability.
  - Do you see the pattern? Explain how one derives the number of terms in the case of the union of 10 events.
3. The probability of an earthquake in the Bay Area in the next year assuming we don't have one this year is (say, for sake of argument) 1%. The probability of an earthquake in the year following a year with an earthquake is lower, say, 0.5%. What is the probability of no earthquake in 2 years?  
For a detailed look at actual earthquake probabilities see

Calculating the Earthquake Odds

 <https://pubs.usgs.gov/fs/old.1999/fs152-99/calcodds.html>

4. Bayes' Rule is sometimes expressed as a ratio between two events (i.e. two hypotheses). For two hypotheses,  $H_1, H_2$  whose likelihoods, in theory, generate the same observable data,  $D$ , write the expression for the ratio of the posteriors of the hypotheses given the data. Why might this be more convenient to apply than the conventional form of Bayes Rule? What other probabilities need to be assumed?

## Optional open-ended programming assignment

### Fair random assignments

Derive a fair algorithm for choosing students “at random” to present in class so that no student is called on twice before all other students have been called on once. Write a procedure to test its fairness.