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$G(s) \rightarrow G(z)$

(e.g.) $G(s) = \dfrac{0.1}{s(s+0.1)}$ $\quad T = 1$

$$G(z) = (1 - z^{-1})\, \mathcal{Z}\left[\frac{G(s)}{s}\right] \qquad (\text{ZOH})$$

$$= (1 - z^{-1})\, \mathcal{Z}\left[\frac{0.1}{s^2(s+0.1)}\right]$$

$$\frac{0.1}{s^2(s+0.1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+0.1} \qquad (\text{partial fractions})$$

$$C = \frac{0.1}{(-0.1)^2}$$

$$C = 10$$

Let $s = 1$

$$\frac{0.1}{1.1} = A + B + \frac{10}{1.1}$$

$$A + B = -\frac{9.9}{1.1}$$

$$A + B = -9$$

$$A = -9 - B$$

Let $s = 2$

$$\frac{0.1}{4(2.1)} = \frac{A}{2} + \frac{B}{4} + \frac{10}{2.1}$$

$$\frac{A}{2} + \frac{B}{4} = -4.75$$

$$2A + B = -19$$

$$2(-9 - B) + B = -19$$

$$-18 - 2B + B = -19$$

$$-B = -1$$

$$B = 1$$

$$A + B = -9$$

$$A = -10$$

(cont)

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(cont)

$$G(z) = (1-z^{-1})\, \mathcal{Z}\left[-\frac{10}{s} + \frac{1}{s^2} + \frac{10}{s+0.1} \right]$$

$$= (1-z^{-1}) \left[\frac{-10}{1-z^{-1}} + \frac{T \cdot z^{-1}}{(1-z^{-1})^2} + \frac{10}{1-e^{-0.1T} z^{-1}} \right]$$

$$= -10 + \frac{T \cdot z^{-1}}{(1-z^{-1})} + \frac{10(1-z^{-1})}{1-e^{-0.1T} z^{-1}}$$

Let $T = 1$

$$= -10 + \frac{z^{-1}}{1-z^{-1}} + \frac{10(1-z^{-1})}{1-0.905 \cdot z^{-1}}$$

$$-10(1-z^{-1})(1-0.905 z^{-1}) + z^{-1}(1-0.905 z^{-1}) + 10(1-z^{-1})(1-z^{-1})$$

$$(-10+10z^{-1})(1-0.905 z^{-1}) + z^{-1} - 0.905 z^{-2} + (10-10z^{-1})(1-z^{-1})$$

$$-10 + 19.05 z^{-1} - 9.05 z^{-2} + z^{-1} - 0.905 z^{-2} + 10 - 20 z^{-1} + 10 z^{-2}$$

$$= \frac{0.05 z^{-1} + 0.045 z^{-2}}{(1-z^{-1})(1-0.905 z^{-1})}$$

$$\boxed{G(z) = \frac{0.05 z^{-1} + 0.045 z^{-2}}{1 - 1.905 z^{-1} + 0.905 z^{-2}}} \quad \checkmark$$

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System From Specs

$T_r = 0.5$

$\%OS = 25$

$$T_r = \frac{2.22}{\omega_n} = 0.5$$

$$\omega_n = 4.44$$

$$\%OS = \left(1 - \frac{\zeta}{0.6}\right) \cdot 100 = 25$$

$$-\frac{\zeta}{0.6} = \frac{25}{100} - 1$$

$$\zeta = -0.6\left[\frac{25}{100} - 1\right]$$

$$\zeta = 0.450$$

$$H(s) = \frac{\omega_n^2}{s^2 + 2 \cdot \zeta \cdot \omega_n + \omega_n^2}$$

$$H(s) = \frac{19.714}{s^2 + 3.996 \cdot s + 19.714}$$

pole placement

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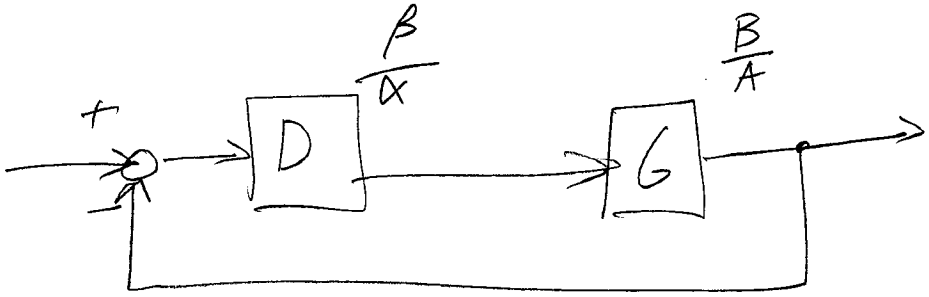

$$H(s) = \frac{DG}{1+DG} \qquad G(s) = \frac{B}{A} = \frac{1}{s(s+0.4)}$$

$$= \frac{\frac{\beta}{\alpha}\cdot\frac{B}{A}}{1+\frac{\beta}{\alpha}\frac{B}{A}} \qquad D(s) = \frac{\beta}{\alpha} = \frac{\beta_1 s+\beta_0}{\alpha_1 s+\alpha_0}$$

$$= \frac{\beta\cdot B}{\alpha A+\beta B} = \frac{19.714}{s^2+3.996\cdot s+19.714}$$

$$\alpha A+\beta B = s^2+3.996\cdot s+19.714 \qquad \omega_n\cdot 10$$

$$(\alpha_1 s+\alpha_0)\cdot s(s+0.4)+(\beta_1 s+\beta_0) = (s^2+3.996\cdot s+19.714)(s+44.4)$$

$$(\alpha_1 s+\alpha_0)(s^2+0.4\cdot s)+\beta_1 s+\beta_0 = s^3+48.396\,s^2+197.14s+875$$

$$\alpha_1 s^3+(\alpha_0+0.4\alpha_1)s^2+(0.4\alpha_0)s+\beta_1 s+\beta_0 =$$

$$\alpha_1 s^3+(\alpha_0+0.4\alpha_1)s^2+(0.4\alpha_0+\beta_1)s+\beta_0 = s^3+48.396s^2+197.14s+875$$

$\beta_0 = 875$

$\alpha_1 = 1$

$\alpha_0+0.4 = 48.396$

$\alpha_0 = 47.996$

$0.4(47.996)+\beta_1 = 197.14$

$\beta_1 = 177.94$

$$D(s) = \frac{177.94\cdot s+875}{s+47.996}$$ ✓

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PID

$$G(s) = \frac{B}{A} = \frac{b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0} = \frac{1}{s^2 + s(0.4)}$$

(roots of $H(s)$)

$$(s^2 + 3.996 \cdot s + 19.714)(s + 44.4) = d_3 s^3 + d_2 s^2 + d_1 s + d_0$$

$$s^3 + 48.396 s^2 + 197.14 s + 875 = d_3 s^3 + d_2 s^2 + d_1 s + d_0$$

(special PID case, $b_1 = 0$)

$$k_i = \frac{d_0}{b_0} = \frac{875}{1}$$

$$k_i = 875$$

$$k_p = \frac{d_1 - a_0}{b_0} = \frac{197.14 - 0}{1}$$

$$k_p = 197.14$$

$$k_d = \frac{d_2 - a_1}{b_0} = \frac{48.396 - 0.4}{1}$$

$$k_d = 47.996$$

$$D(s) = \frac{k_d s^2 + k_p \cdot s + k_i}{s} \quad \checkmark$$

$$D(s) = \frac{47.996 s^2 + 197.14 \cdot s + 875}{s}$$

$G(s) \Rightarrow G(z)$ discrete

$$G(s) = \frac{1}{s(s+0.4)}$$

$$G(z) = (1 - z^{-1})\, Z\left[\frac{G(s)}{s}\right] \qquad \text{(ZOH)}$$

$$= (1 - z^{-1})\, Z\left[\frac{1}{s^2(s+0.4)}\right]$$

$$\frac{1}{s^2(s+0.4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+0.4} \qquad \text{(partial fractions)}$$

$$\vdots$$

$$= \frac{-6.25}{s} + \frac{2.5}{s^2} + \frac{6.25}{s+0.4}$$

$$= \frac{-6.25}{1 - z^{-1}} + \frac{2.5 \cdot T z^{-1}}{(1 - z^{-1})^2} + \frac{6.25}{1 - e^{-0.4 \cdot T} \cdot z^{-1}}$$

$$G(z) = (1 - z^{-1}) \left[\frac{-6.25}{1 - z^{-1}} + \frac{2.5\, T \cdot z^{-1}}{(1 - z^{-1})^2} + \frac{6.25}{1 - e^{-0.4 \cdot T} z^{-1}}\right]$$

$$= -6.25 + \frac{2.5 \cdot T \cdot z^{-1}}{(1 - z^{-1})} + \frac{6.25(1 - z^{-1})}{1 - e^{-0.4 \cdot T} \cdot z^{-1}}$$

(cont)

(cont)

$T = 0.1$

$$G(z) = -6.25 + \frac{0.25 \cdot z^{-1}}{1 - z^{-1}} + \frac{6.25\left(1 - z^{-1}\right)}{1 - 0.961 \cdot z^{-1}}$$

$$= \frac{-6.25(1-z^{-1})(1-0.961z^{-1}) + 0.25z^{-1}(1-0.961z^{-1}) + 6.25(1-z^{-1})^2}{(1-z^{-1})(1-0.961z^{-1})}$$

$$= \frac{-6.25\left[1 - 1.961z^{-1} + 0.961z^{-2}\right] + 0.25z^{-1} - 0.240z^{-2} + 6.25\left[1 - 2z^{-1} + z^{-2}\right]}{(1-z^{-1})(1-0.961z^{-1})}$$

$$G(z) = \frac{0.006\,z^{-1} + 0.004\,z^{-2}}{1 - 1.961\,z^{-1} + 0.961\,z^{-2}} \quad \checkmark$$

$s \rightarrow z$, backward

$T = 0,1$

$$D(s) = \frac{177,94 \cdot s + 875}{s + 47,996}$$

(from pole placement)

$$s = \frac{z-1}{z \cdot T}$$

backward

$$D(z) = \frac{177,94 \dfrac{z-1}{z \cdot T} + 875}{\dfrac{z-1}{zT} + 47,996}$$

$$= \frac{177,94(z-1) + 875 \cdot z \cdot T}{z-1 + 47,996 \cdot zT}$$

$$= \frac{(177,94 + 875 \cdot T)\, z - 177,94}{(1 + 47,996 \cdot T)\, z - 1}$$

$T = 0,1$

$$\boxed{D(z) = \frac{265,4 \cdot z - 177,94}{5,799 \cdot z - 1}}$$

$s \to z$, trapezoid

$T = 0.1$

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

trapezoid

$$D(s) = \frac{177.94 \cdot s + 875}{s + 47.996}$$

$$D(z) = \frac{177.94 \dfrac{2(z-1)}{T(z+1)} + 875}{\dfrac{2(z-1)}{T(z+1)} + 47.996}$$

$$= \frac{355.88(z-1) + 875 \cdot T \cdot (z+1)}{2(z-1) + 47.996 \cdot T(z+1)}$$

$$= \frac{(355.88 + 875 \cdot T)z + (875 \cdot T - 355.88)}{(2 + 47.996 \cdot T)z - 2 + 47.996 \cdot T}$$

$$\boxed{D(z) = \frac{443.3 \cdot z - 268.38}{6.799 \cdot z + 2.799}}$$

✓

$s \Rightarrow z$ mapping

$z = e^{sT}$

$T = 0.1$

$$D(s) = \frac{177.94 \cdot s + 875}{s + 47.996}$$

$$s_{z1} = \frac{-875}{177.94}$$

$$s_{z1} = -4.917$$

$$s_{p1} = -47.996$$

$$z_{z1} = e^{-4.917(0.1)}$$

$$z_{z1} = 0.6115$$

$$z_{p1} = e^{-47.996(0.1)}$$

$$z_{p1} = 0.00823$$

$$D(z) = k \cdot \frac{(z - 0.6115)}{(z - 0.00823)}$$

$$\lim_{s \to 0} D(s) = \lim_{s \to 0} \frac{177.94 \cdot s + 875}{s + 47.996} = 18.231$$

$$\lim_{z \to 1} D(z) = k \cdot \frac{(z - 0.6115)}{(z - 0.00823)} = 18.231$$

$$k = 46.54$$

$$D(z) = (46.54) \frac{(z - 0.6115)}{(z - 0.00823)}$$