11/25/13

$G(s) \to G(z)$ ZOH

General T

$$G(s) = \frac{1}{s(s+1)}$$

$$G(z) = (1 - z^{-1})\, Z\left[\frac{G(s)}{s}\right]$$

$$= (1 - z^{-1})\, Z\left[\frac{1}{s^2(s+1)}\right] \qquad \text{(ZOH)}$$

$$\frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

Let $s = -1$ Let $s = 0$

$C = 1$ $B = 1$

Let $s = 1$

$$\frac{1}{2} = A + B + \frac{C}{2}$$

$$0 = A + 1$$

$$A = -1$$

$$G(z) = (1 - z^{-1})\, Z\left[\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}\right]$$

(cont)

(cont)

$$G(z) = (1-z^{-1})\left[\frac{-1}{1-z^{-1}} + \frac{Tz^{-1}}{(1-z^{-1})^2} + \frac{1}{1-e^{-T}z^{-1}}\right]$$

$$= -1 + \frac{Tz^{-1}}{(1-z^{-1})} + \frac{1-z^{-1}}{1-e^{-T}z^{-1}}$$

Let

$$\gamma = 1-z^{-1}$$
$$\tau = 1-e^{-T}z^{-1}$$

$$= \frac{-\gamma\cdot\tau + T\cdot z^{-1}\tau + \gamma^2}{\gamma\tau}$$

(numerator)

z^{-2}: $1 - T\cdot e^{-T} - e^{-T}$

$1-e^{-T}(T+1)$

z^{-1}: $-2 + T + 1 + e^{-T}$

$-1 + T + e^{-T}$

z^{0}: $1 - 1 = 0$

$$\gamma^2 = (1-z^{-1})(1-z^{-1})$$
$$= 1 - 2z^{-1} + z^{-2}$$
$$T\cdot z^{-1}\cdot\tau = T(z^{-1} - e^{-T}z^{-2})$$
$$= T\cdot z^{-1} - T\cdot e^{-T}z^{-2}$$
$$-\gamma\cdot\tau = (z^{-1}-1)(1-e^{-T}z^{-1})$$
$$= z^{-1} - 1 - e^{-T}z^{-2} + e^{-T}z^{-1}$$
$$= -1 + z^{-1}(1+e^{-T}) - e^{-T}z^{-2}$$

$$\boxed{\begin{aligned} G(z) &= \frac{(T+e^{-T}-1)z^{-1} + [1-e^{-T}(T+1)]z^{-2}}{(1-z^{-1})(1-e^{-T}z^{-1})} \\ &= \frac{(T+e^{-T}-1)z^{-1} + [1-e^{-T}(T+1)]z^{-2}}{1-(1+e^{-T})z^{-1}+e^{-T}z^{-2}} \end{aligned}}$$

substitute for T to find a specific version

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Direct Design

$$G(s) = \frac{1}{s(s+1)} \qquad T = 0.2$$

roots: $z = 0.6 \pm 0.4i$

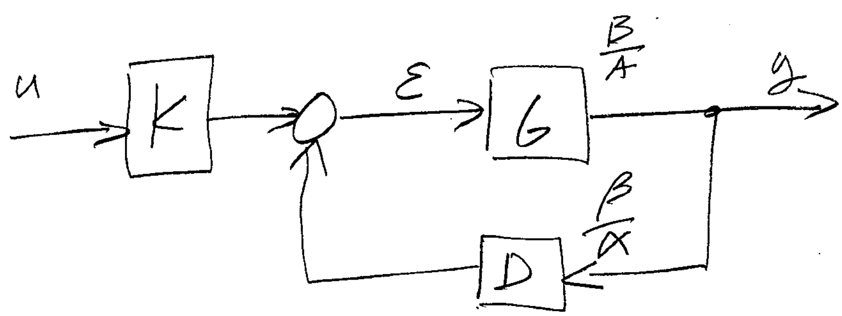
$$G(z) = (1 - z^{-1})\, \mathcal{Z}\left[\frac{G(s)}{s}\right]$$

$$= (1 - z^{-1})\, \mathcal{Z}\left[\frac{1}{s^2(s+1)}\right]$$

$$= \frac{(T + e^{-T} - 1) z^{-1} + [1 - e^{-T}(T+1)] z^{-2}}{1 - (1 + e^{-T}) z^{-1} + e^{-T} z^{-2}}$$

(see ZOH general T)

$$G(z) = \frac{0.0187 z^{-1} + 0.0175 z^{-2}}{1 - 1.8187 z^{-1} + 0.8187 z^{-2}}$$



$$\frac{y}{u} = H = K \cdot \frac{G}{1 + DG}$$

$$= K \cdot \frac{B\alpha}{A\alpha + B\beta}$$

(cont)

(cont)

$$G(z) = \frac{B}{A} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

$$= \frac{0 + 0.0187 z^{-1} + 0.0175 z^{-2}}{1 - 1.8187 z^{-1} + 0.8187 z^{-2}}$$

$$E = \begin{bmatrix} a_2 & 0 & b_2 & 0 \\ a_1 & a_2 & b_1 & b_2 \\ a_0 & a_1 & b_0 & b_1 \\ 0 & a_0 & 0 & b_0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8187 & 0 & 0.0175 & 0 \\ -1.8187 & 0.8187 & 0.0187 & 0.0175 \\ 1 & -1.8187 & 0 & 0.0187 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = d_0 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}$$

$$= z^3 - 1.2 z^2 + 0.520 z + 0$$

$$D = \begin{bmatrix} d_3 \\ d_2 \\ d_1 \\ d_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.52 \\ -1.2 \\ 1 \end{bmatrix}$$

$$M = E^{-1} \cdot D \qquad M = \begin{bmatrix} \alpha_1 \\ \alpha_0 \\ \beta_1 \\ \beta_0 \end{bmatrix} = \begin{bmatrix} 0.242 \\ 1 \\ -11.30 \\ 20.1 \end{bmatrix}$$

(calc using Matlab)

(cont)

(cont)

Perform pole placement for the given roots,

$$z = 0.6 \pm 0.4i$$

$$(z - 0.6 + 0.4i)(z - 0.6 - 0.4i)$$

$$z^2 - 1.2z + 0.520 = D$$

$$H = K \cdot \frac{B\alpha}{A\alpha + B\beta}$$

$$\underset{(n-1)}{\alpha}\underset{(n)}{A} + \underset{(n-1)}{\beta}\underset{(n)}{B} = \underset{(2n-1)}{D} \qquad \text{(Diophantine)}$$

$$G(z) = \frac{B}{A} = \frac{0 + 0.0187z^{-1} + 0.0175z^{-2}}{1 - 1.8187z^{-1} + 0.8187z^{-2}} \qquad \begin{matrix} \text{order} \\ (2) \\ (2) \end{matrix}$$

For a second order $(n=2)$, D needs another root to satisfy the Diophantine equation. Another can be added at zero,

$$D = z^3 - 1.2z^2 + 0.520z \qquad (2n-1) \checkmark$$

(cont)

(cont)

$$D(z) = \frac{\beta}{\alpha} = \frac{\beta_0 z + \beta_1}{\alpha_0 z + \alpha_1}$$

$$\boxed{D(z) = \frac{20.1 z - 11.30}{z + 0.242}}$$

still need to find K

$$H = K \cdot \frac{B\alpha}{A\alpha + B\beta}$$

$$= K \cdot \frac{\overbrace{(0.0187z + 0.0175)}^{B}\overbrace{(z + 0.242)}^{\alpha}}{z^3 - 1.2z^2 + 0.52z}$$

desired poles w/root

Find K so that it goes to 1 for a step response.

$$\lim_{z \to 1} K \cdot \frac{(0.0187z + 0.0175)(z + 0.242)}{z^3 - 1.2z^2 + 0.52z} = 1$$

$$\boxed{K = 7.11}$$