$s \Rightarrow z$ mapping $(z = e^{sT})$

10/23/12

$$D(s) = \frac{10(s+0.1)}{s+1} \qquad T = 1$$

$$D(s) = \frac{10s+1}{s+1}$$

zeros: $-\frac{1}{10}$

poles: -1

$$z_z = e^{(-\frac{1}{10})T}$$

$$z_p = e^{-T}$$

$$D(z) = K \cdot \frac{z - e^{(-\frac{1}{10})T}}{z - e^{-T}}$$

$$= K \cdot \frac{z - 0.904}{z - 0.368}$$

(to find K, use limits)

$$\lim_{s \to 0} D(s) = \lim_{s \to 0} \frac{10 \cdot s + 1}{s+1} = 1$$

$$\lim_{z \to 1} D(z) = K \cdot \frac{z - 0.904}{z - 0.368} = 1$$

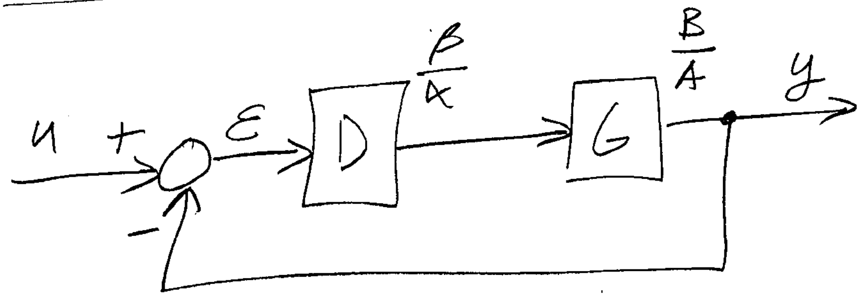
$$K = \frac{1 - 0.368}{1 - 0.904} = 6.58$$

$$\boxed{D(z) = (6.58)\frac{z - 0.904}{z - 0.368}}$$

Pole Placement

9/17/12

$$G(s) = \frac{1}{s(s+1)} \qquad H(s) = \frac{20}{s^2+4s+20} \qquad \omega_n = 4.44$$



$$\frac{y}{u} = H = \frac{DG}{1+DG}$$

$$= \frac{\beta B}{\alpha A + \beta B}$$

place the poles

$$\alpha A + \beta B = s^2 + 4s + 20$$

$$(\alpha_1 s + \alpha_0) \cdot s(s+1) + (\beta_1 s + \beta_0) = s^2 + 4s + 20$$

3rd order

2nd order

Add root $(\omega_n \cdot 10)$ to correct order

$$(\alpha_1 s + \alpha_0) s(s+1) + \beta_1 s + \beta_0 = (s^2+4s+20)(s+44.4)$$

$$\alpha_1 s^3 + (\alpha_0 + \alpha_1) s^2 + (\alpha_0 + \beta_1) s + \beta_0$$

$$= s^3 + 48s^2 + 196s + 880$$

(cont)

(cont)

$$\alpha_1 s^3 + (\alpha_0 + \alpha_1)s^2 + (\alpha_0 + \beta_1)s + \beta_0 = s^3 + 48s^2 + 196s + 880$$

(match polynomials)

$\alpha_1 = 1$

$\alpha_0 + \alpha_1 = 48$

$\alpha_0 = 47$

$\beta_0 = 880$

$\alpha_0 + \beta_1 = 196$

$\beta_1 = 196 - 47$

$\beta_1 = 149$

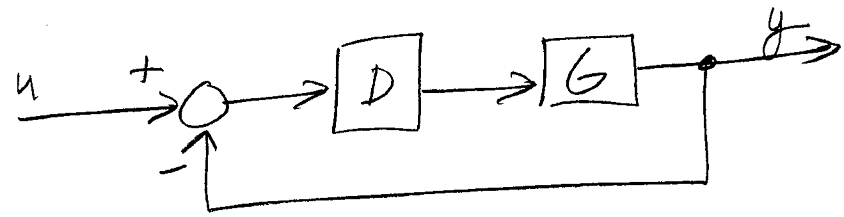
$$D(s) = \frac{\beta}{\alpha} = \frac{\beta_1 s + \beta_0}{\alpha_1 s + \alpha_0}$$

$$\boxed{D(s) = \frac{149s + 880}{s + 47}}$$

Algebraic

11/22/13

$$G(s) = \frac{1}{s(s+1)} \qquad H(s) = \frac{20}{s^2+4s+20}$$



$$\frac{y}{u} = H(s) = \frac{DG}{1+DG}$$

(solve for D)

$$D = \frac{H}{G - HG}$$

$$= \frac{H}{G} \cdot \frac{1}{1-H}$$

$$= \frac{20 \cdot s(s+1)}{(s^2+4s+20)} \cdot \frac{1}{\left(1 - \frac{20}{s^2+4s+20}\right)}$$

$$= \frac{(20s^2+20s)\cancel{(s^2+4s+20)}}{\cancel{(s^2+4s+20)}(s^2+4s+\cancel{20}-\cancel{20})}$$

$$\boxed{D(s) = \frac{20s^2+20s}{s^2+4s}}$$