

$$1a) \quad f(x+2\Delta x) \approx f(x) + f'(x)2\Delta x + \frac{f''(x)(4\Delta x^2)}{2} + \frac{f'''(x)(8\Delta x^3)}{6} + O(\Delta x^4)$$

$$f(x+\Delta x) \approx f(x) + f'(x)\Delta x + \frac{f''(x)\Delta x^2}{2} + \frac{f'''(x)\Delta x^3}{6} + O(\Delta x^4)$$

$$f(x-\Delta x) \approx f(x) - f'(x)\Delta x + \frac{f''(x)\Delta x^2}{2} - \frac{f'''(x)\Delta x^3}{6} + O(\Delta x^4)$$

$$f(x-2\Delta x) \approx f(x) - f'(x)2\Delta x + \frac{f''(x)4\Delta x^2}{2} - \frac{f'''(x)8\Delta x^3}{6} + O(\Delta x^4)$$

$$8f(x+\Delta x) - 8f(x-\Delta x) =$$

$$8(f(x+\Delta x) - f(x-\Delta x)) = 8(2f'(x)\Delta x + 2f'''(x)\Delta x^3)$$

$$= 16f'(x)\Delta x + \frac{8}{3}f'''(x)\Delta x^3$$

$$f(x-2\Delta x) - f(x+2\Delta x) = -4f'(x)\Delta x - \frac{8}{3}f'''(x)\Delta x^3$$

addition $= 12f'(x)\Delta x$

$$\frac{12f'(x)\Delta x}{12\Delta x} = f'(x)$$

$$f(x+\Delta x) + f(x-\Delta x) = 2f(x) + f''(x)\Delta x^2 + O(\Delta x^4)$$

$$\frac{2f(x) + f''(x)\Delta x^2 - 2f(x)}{\Delta x^2} = f''(x)$$

2b) Maybe because negative
 c would mean a negative
 wave group velocity? Or the linked
 range.