1a) $\overset{1\times 2}{(x_1, x_2)} \overset{2\times 2}{\begin{pmatrix} a_{00} & a_{01} \\ a_{01} & a_{11} \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 25$

$= (a_{00}x_1 + a_{01}x_2 ; a_{01}x_1 + a_{11}x_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$= a_{00}x_1^2 + a_{01}x_2x_1 + a_{01}x_1x_2 + a_{11}x_2^2$

$= a_{00}x_1^2 + 2a_{01}x_1x_2 + a_{11}x_2^2 = 25$

$a_{00} = 14 \quad a_{01} = -2 \quad a_{11} = 11$

$A = \begin{pmatrix} 14 & -2 \\ -2 & 11 \end{pmatrix}$

b) $(14-\lambda)(11-\lambda) - 4 = 0$

$\lambda^2 - 25\lambda + 150 = 0$

$(\lambda - 15)(\lambda - 10) = 0$

$\boxed{\lambda = 15 \quad \lambda = 10}$

$\lambda = 15$

nullspace of $A - \lambda I$

$$\left(\begin{array}{cc|c} 14-15 & -2 & 0 \\ -2 & 11-15 & 0 \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{cc|c} -1 & -2 & 0 \\ -2 & -4 & 0 \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{cc|c} -1 & -2 & 0 \\ 1 & 2 & 0 \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$x_1 = -2x_2 \qquad x_2 = x_2$

$$\vec{x} = x_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\boxed{\frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \end{pmatrix}}$$

$\lambda = 10$

$$\left(\begin{array}{cc|c} 4 & -2 & 0 \\ -2 & 1 & 0 \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ -2 & 1 & 0 \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$2x_1 = x_2$

$x_2 = x_2$

$x_1 = \frac{1}{2} x_2$

$$\vec{x} = x_2 \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$= x_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\boxed{\frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

d)

$$\frac{1}{\sqrt{5}}\begin{pmatrix}-2 & 1\\ 1 & 2\end{pmatrix}\begin{pmatrix}14 & -2\\ -2 & 11\end{pmatrix}\frac{1}{\sqrt{5}}\begin{pmatrix}-2 & 1\\ 1 & 2\end{pmatrix}$$

$$=\frac{1}{5}\begin{pmatrix}-30 & 15\\ 10 & 20\end{pmatrix}\begin{pmatrix}-2 & 1\\ 1 & 2\end{pmatrix}=\frac{1}{5}\begin{pmatrix}75 & 0\\ 0 & 50\end{pmatrix}$$

$$=\begin{pmatrix}15 & 0\\ 0 & 10\end{pmatrix}$$ It's the eigenvalues as expected.

$$\frac{1}{5}\begin{pmatrix}-2 & 1\\ 1 & 2\end{pmatrix}\begin{pmatrix}-2 & 1\\ 1 & 2\end{pmatrix}=\frac{1}{5}\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Identity as expected for an orthonormal eigenbasis.

$$B = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} = = \begin{pmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\left(90 - \cos\left(\frac{-2}{\sqrt{5}}\right) \cdot \frac{180}{\pi}\right) = 26.56 \text{ deg}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = 26.56 \text{ deg} \checkmark$$

g) Ran out of time/confused

a) $\kappa = \dfrac{\max(|\lambda(A)|)}{\min(|\lambda(A)|)}$ $\max = 1.75\ldots$

$\min = 1.09 \cdot 10^{-13}$

κ is $= 16025571824157.9$

$\approx 10^{13}$

$n \times n$ $m \times n$

$n \times n$

b) Max error $= 9.155 \cdot 10^{-5}$

using 1st method

c) Max error using 2nd method

14745100337917

- Picture of code for part 2 included.

Note: Its quite messy as it was just a calculator

Homework3 > HW3_calculator.py > ...

```python
import numpy as np
import scipy.linalg as lag
hil10 = lag.hilbert(10)
hil10eig, eigenvectors = lag.eigh(hil10)
print("The array is", hil10eig)
print("K is", np.max(hil10eig)/np.min(hil10eig))
print("Max eigenvalue:", np.max(hil10eig))
print("Min eigenvalue:", np.min(hil10eig))
hil10inv = lag.inv(hil10)
product = hil10inv @ hil10
#print("Inv*hil", product)
error = np.absolute(product - np.identity(10))
print("Max error 1st method is:", np.max(error))
hil10inv2 = eigenvectors @ np.reciprocal(hil10eig) @ np.transpose(eigenvectors)
product2 = hil10inv2 @ hil10
error2 = np.absolute(product2 - np.identity(10))
print("Max error 2nd method is:", np.max(error2))

```

PROBLEMS 2 OUTPUT DEBUG CONSOLE TERMINAL 2: Pyth

```
The array is [1.09320259e-13 2.26674561e-11 2.14743883e-09 1.22896774e-07
 4.72968929e-06 1.28749614e-04 2.53089077e-03 3.57418163e-02
 3.42929548e-01 1.75191967e+00]
K is 16025571824158.016
Max eigenvalue: 1.7519196702651791
Min eigenvalue: 1.0932025948828975e-13
Max error 1st method is: 9.1552734375e-05
Max error 2nd method is: 1474510037917.5474

C:\Users\Jeff\Coding\Python\P427>
```