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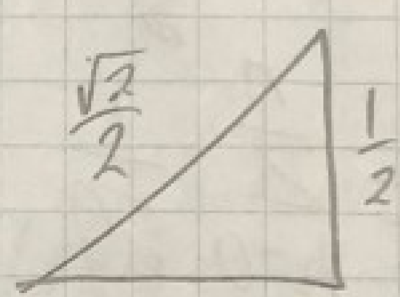
H1

· I did this whole assignment then accidentally threw it away so I'm doing this as quickly as possible/as simply as possible

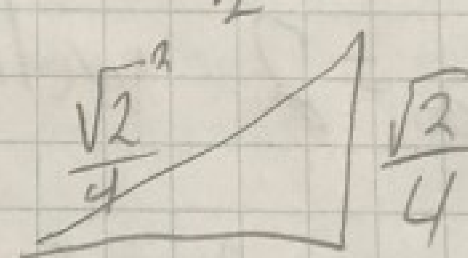
1a. individual perimeter

$$P_1 = 1 \cdot 4$$



$$P_2 = \frac{\sqrt{2}}{2} \cdot 4$$

$$P_3 = \left(\frac{\sqrt{2}}{2}\right)^2 \cdot 4$$



$$P_i = \left(\frac{\sqrt{2}}{2}\right)^{i-1} \cdot 4$$

Total perimeter

$$B_n = \sum_{i=1}^{n} \left(\frac{\sqrt{2}}{2}\right)^{i-1} \cdot 4 = \sum_{i=0}^{n} \left(\frac{\sqrt{2}}{2}\right)^{i} \cdot 4$$

agrees with manual addition

$4, 2\sqrt{2}, 2$

$$B_\infty = \text{geo. series} = \frac{a}{1-r} = \frac{4}{1-\frac{\sqrt{2}}{2}} \approx 13.66$$

b) $a_1 = 1^2 = 1$

$$a_2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$$

$$a_3 = \frac{1}{4}$$

$$a_4 = \frac{1}{8}$$

$$A_n = \sum_{n=0}^{n} \frac{1}{2^n}$$

$A_\infty = 2$ well known sum.

c) $|B_\infty - B_n| < \varepsilon$

$\downarrow$ Strictly increasing series

$B_\infty - B_n < \varepsilon$

$\downarrow$ First n to meet this

$B_\infty - B_n = \varepsilon$

$$B_\infty - \sum_{i=0}^{n} \left(\frac{\sqrt{2}}{2}\right)^i \cdot 4 = \varepsilon$$

$$S_n = \sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$$

$$B_\infty - 4 \cdot \frac{1-\frac{\sqrt{2}}{2}^{n+1}}{1-\frac{\sqrt{2}}{2}} = \varepsilon$$

$$\frac{B_\infty - \varepsilon}{4} = \frac{1-\left(\frac{\sqrt{2}}{2}\right)^{n+1}}{1-\frac{\sqrt{2}}{2}}$$

$$\left(1-\frac{\sqrt{2}}{2}\right)\cdot\frac{(B_\infty - \varepsilon)}{4} = 1-\left(\frac{\sqrt{2}}{2}\right)^{n+1}$$

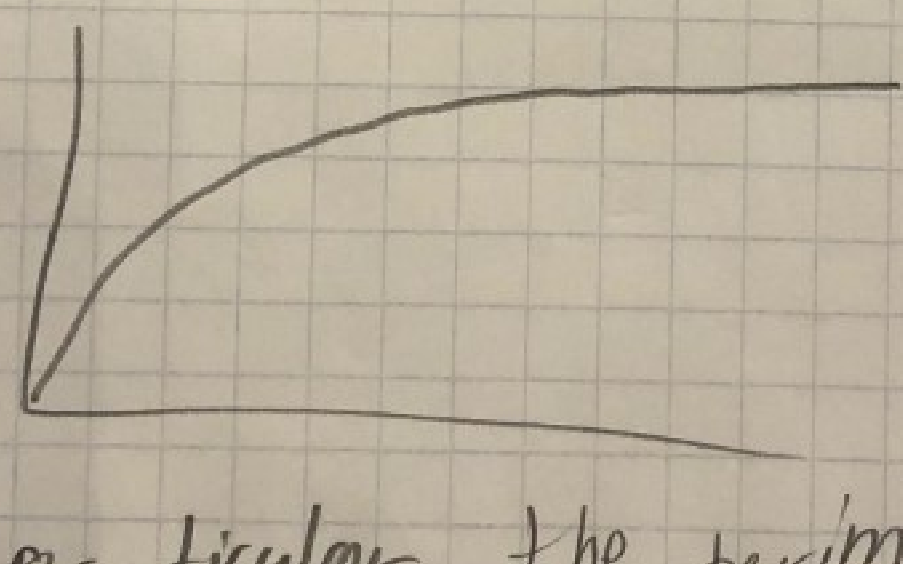
$$\ln\left(\left(1-\frac{\sqrt{2}}{2}\right)\frac{(B_\infty-\varepsilon)}{4} - 1\right) = \ln\left(\frac{\sqrt{2}}{2}\right)(n+1)$$

$$\frac{\ln\left(\left(1-\frac{\sqrt{2}}{2}\right)\frac{(B_\infty-\varepsilon)}{4} - 1\right)}{\ln\left(\frac{\sqrt{2}}{2}\right)} - 1 = n$$

Somethings off, plugged into calculator and this can't be right.

d) To get $\varepsilon = 10^{-7}$ you need around $n = 51$. I got this manually as my function is wrong.

$\varepsilon = 10^{-15}$ would take an extremely large n. This is because the sum approaches the final value asymptotically like so



In particular, the perimeter takes longer to converge than the area since $\frac{1}{2} < \frac{\sqrt{2}}{2}$. The higher order terms are larger in the perimeter series.