1a) A computer has finite floating point precision. For example if h was arbitrarily small say $h = 10^{-100}$ then it wouldn't have an effect with normal floating point math

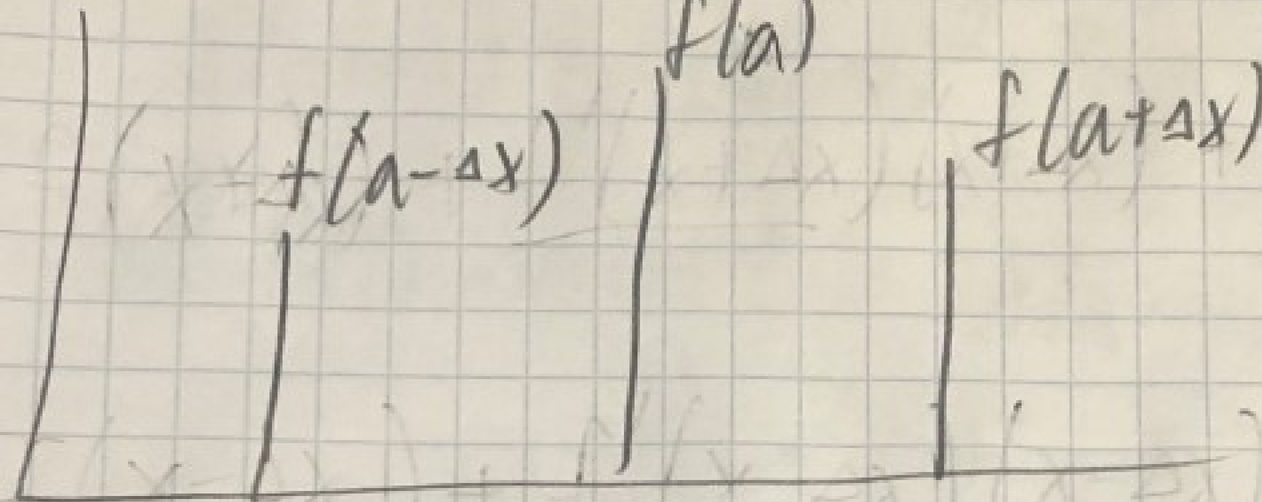
b) When using a closed region, you can use the intermediate value theorem. For example in the continuous region $[a,b]$ if a and b do not have the same sign then there must be a root.

Bracketing searches can also be faster/converge better

c) It can be useful when the 1st derivative is well defined and behaves nicely. Also if there are any non-root local extrema between the starting point and a root the method can diverge. Also if you have a good initial guess that can help the method quite abit.

2) $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

$= f(x) + \frac{f'(x)}{1}x + \frac{f''(x)}{2}x^2 \ldots$



relabelling to a because original label didn't make sense

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} \ldots$

$f(x+\Delta x) = f(a) + f'(a)(x+\Delta x-a) + \frac{f''(a)(x+\Delta x-a)^2}{2}$

$(x+\Delta x-a)^2 = (x+\Delta x-a)(x+\Delta x-a)$

$= x^2 + \Delta x x - ax + \Delta x x + \Delta x^2 - a\Delta x - ax - a\Delta x + a^2$

$= x^2 - 2ax - 2a\Delta x + 2\Delta x x + \Delta x^2 + a^2$

I don't see how you could remove $f''(a)$ considering the coefficient isn't odd or even so it won't be negative the same coefficient for $f(x-\Delta x)$

b)

$$f_{poly}(x) = Ax^2 + Bx + C$$

$$f(a) = Ax^2 + Bx + C = Aa^2 + Ba$$

$$f(a-\Delta x) = A(a-\Delta x)^2 + B(a-\Delta x)$$

don't need C

$$f(a+\Delta x) = A(a+\Delta x)^2 + B(a+\Delta x)$$

$$f(a) - f(a-\Delta x) = Aa^2 - A(a-\Delta x)^2 + Ba - Ba + \Delta x$$

$$= Aa^2 - A(a-\Delta x)^2 + \Delta x$$

$$f(a) - f(a-\Delta x) - \Delta x = Aa^2 - A(a^2 - 2a\Delta x + \Delta x^2)$$

$$= +2a\Delta x A - A\Delta x^2$$

$$= A(2a\Delta x - \Delta x^2)$$

$$A = \frac{f(a) - f(a-\Delta x) - \Delta x}{2a\Delta x - \Delta x^2}$$

$$f(a) = \left(\frac{f(a) - f(a-\Delta x) - \Delta x}{2a\Delta x - \Delta x^2}\right)a^2 + Ba$$

$$\frac{f(a)}{a} - \left(\frac{f(a) - f(a-\Delta x) - \Delta x}{2a\Delta x - \Delta x^2}\right)a = B$$

$$f'_{poly}(x) = 2Ax + B$$

$$f'_{poly}(a) = 2Aa + B$$

$$= 2a \cdot \left(\frac{f(a) - f(a - \Delta x) - \Delta x}{2a\Delta x - \Delta x^2} \right) + \frac{f(a)}{a}$$

$$- \left(\frac{f(a) - f(a - \Delta x) - \Delta x}{2a\Delta x - \Delta x^2} \right) a$$

c)

$$S = \sum_{i=1}^{3} 1 = 3$$

$$S_x = x + x^2 + x^3 \quad S_{xf} = x f(x) + x^2 f(x^2) + x^3 f(x^3)$$

$$S_f = f(x) + f(x^2) + f(x^3)$$

$$S_{xx} = x^2 + x^4 + x^6$$

$$b = \left(3x f(x) + x^2 f(x^2) + x^3 f(x^3) - (x + x^2 + x^3)(f(x) + f(x^2) + f(x^3))\right) \cdot \left(3(x^2 + x^4 + x^6) - (x + x^2 + x^3)^2\right)^{-1}$$