HW9

1. $h(t) = \sin(20\pi t) - 3\sin(140\pi t) + \cos(160\pi t)$

$a\sin(b(x-\phi))+k$ ↑ cosine is just sine with a phase

$p = \frac{2\pi}{b}$ $\quad \nu = \frac{1}{p} = \frac{b}{2\pi}$

a) $\frac{20\pi}{2\pi} = 10\,Hz$ $\quad \frac{140\pi}{2\pi} = 70\,Hz$ $\quad \frac{160\pi}{2\pi} = 80$

b) It has a Nyquist Frequency of 80Hz so it must be sampled at at least $\frac{160\,\text{samples}}{1\,\text{sec}}$

So a time step of, $\frac{1000\,ms}{160\,\text{sample}} = \frac{6.25\,ms}{1\,\text{sample}}$ is the max.

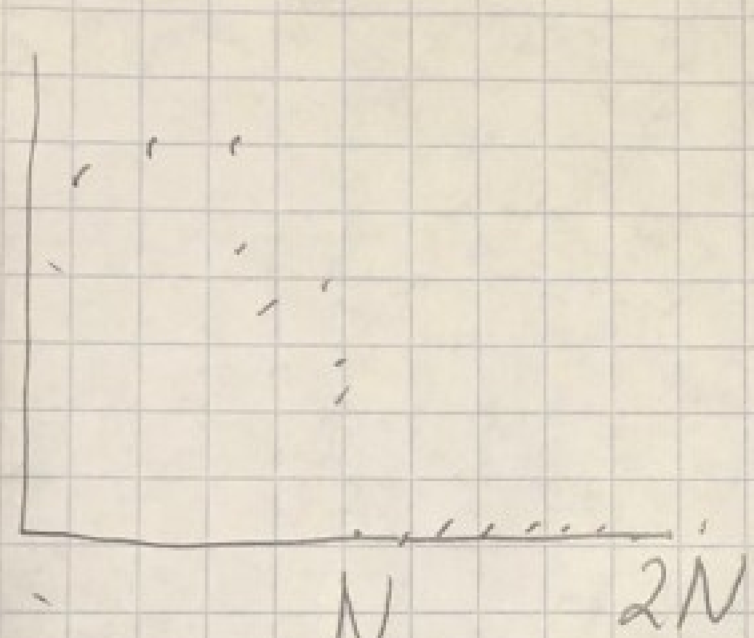
c) The largest period beat is 10Hz so $\frac{1}{10\,Hz} = 0.1\,s$ is the periodicity of the full signal.

2) $h(t_n)$ at N values

$H(\nu_m)$

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn}$$

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{2\pi i kn}{N}}$$



Fourier transform for 0 to 2N

$$g(t_n) = \frac{1}{2N}\left(\sum_{k=0}^{N-1} G(\nu_k) e^{\frac{2\pi i kn}{2N}} + \underbrace{\sum_{k=N}^{2N-1} G(\nu_k) e^{\frac{2\pi i kn}{2N}}}_{0}\right)$$

$G(\nu_N)$ through $G(\nu_{2N-1})$ is zero

$$g(t_n) = \frac{1}{2N}\sum_{k=0}^{N-1} G(\nu_k) e^{\frac{2\pi i kn}{2N}}$$

Sub $n \to 2n$, also means the width coefficient is $\frac{1}{2}$ so $\frac{1}{2N} \to \frac{1}{N}$ in addition to other changes.

$$g(t_{2n}) = \frac{1}{N}\sum_{k=0}^{N-1} G(\nu_k) e^{\frac{3\pi i kn}{N}}$$

$$h(t_n) = \frac{1}{N}\sum_{k=0}^{N-1} H(\nu_k) e^{\frac{2\pi i kn}{N}}$$

In this range $G(\nu) = H(\nu)$

So these are equal