Jeff Anters P427 Exam 2

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1. $$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d}{d\xi}\theta(\xi)\right)+\theta(\xi)^n=0$$

when $n=0$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\xi^2\frac{d}{d\xi}\theta(\xi)+1=0$$

In more common variables:

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2y'\right)+1=0$$

$$\frac{1}{x^2}\left(x^2y''+2xy'\right)+1=0$$

$$\cancel{xy''+2y'=-x}$$

$$y''+\frac{2y'}{x}+1=0$$

$$xy''+2y'=-x$$

$$y=y_g+y_s$$

$$xy''+2y'=0 \qquad xy''+2y'=-x$$

$$y_g=c_1+\frac{c_2}{x} \qquad y=\frac{-x^2}{6}$$

$$\boxed{y = c_1 + \frac{c_2}{x} - \frac{x^2}{6} \quad \text{General solution}}$$

· Found this by transforming into standard form and solving for ~~the~~ homogenous and specific solutions.

$y \Rightarrow \theta(\xi) \quad x \Rightarrow \xi$

$$\boxed{\theta(\xi) = C_1 + \frac{C_2}{\xi} - \frac{\xi^2}{6}}$$

$$\theta(0) = 1 = C_1 + \frac{C_2}{0} - \frac{0^2}{6}$$

$$\Rightarrow C_1 + \frac{C_2}{0}$$ C_2 must be 0

$$= C_1 = 1$$

$\frac{d\theta}{d\xi} = 0$ when $\xi = 0$

$$\frac{d\theta}{d\xi} = \frac{-C_2}{\xi^2} - \frac{\xi}{3}$$

$\xi = 0$ $$\left.\frac{d\theta}{d\xi}\right|_{\xi=0} = \frac{-C_2}{0} - 0$$

$\cancel{\left.\theta(\xi)\right|_{\xi=0}}$

$$\boxed{\theta(\xi) = 1 + 0 - \frac{\xi^2}{6}}$$

For the given conditions.

C_2 must be 0 again

1b. $\varepsilon\theta'' + 2\theta' = -\varepsilon$

$\cancel{x_1 = \theta(t)}$

$\cancel{x_2 = \theta'(t)}$

$\cancel{x_3 = \theta''(t)}$

It's easier to think about in x and y

$xy'' + 2y' = -x$

$y'' + \frac{2y'}{x} = -1$

$y'' = \frac{-2y'}{x} - 1$

$x_1 = y$

$x_2 = y'$ $\quad x_1' = y' = x_2$

$\cancel{x_3 = y''}$ $\quad x_2' = y'' = \frac{-2y'}{x} - 1$

$= \frac{-2x_2}{x} - 1$

$x_1' = x_2$

$x_2' = \frac{-2x_2}{x} - 1$

$$\boxed{\begin{aligned} x_1' &= x_2 \\ x_2' &= \frac{-2x_2}{\varepsilon} - 1 \end{aligned}}$$

2 a.

Numerically, It can depend on the form of the equations.

An integration method might be more stable for parabolic vs. hyperbolic vs. elliptic and vice versa.

When numerically solving, time step and spatial step can affect this also.

In addition, foward, backward, and centered differencing can matter.

ie when we used asymmetric differencing we started seeing asymmetric astability.

A condition for stability can be if the ~~differnce~~ difference between two solutions, $y=c_1 e^{-x}$ and $y=c_2 e^{-x}$ for example, approaches 0 as c_1 approaches c_2.

2b.

Basically sample function at many different interval sizes.

By using 2 or more step sizes and combining them, you can cancel out the error.

For ~~ex~~ example, use a timestep Δx then use $\frac{\Delta x}{2}$ and compare the difference ~~and also~~ to get an idea of the error. Then, repeat.

c. Elliptic: No time dependence
ie Static systems
$\nabla^2 u = 0$ Laplace equation $\Rightarrow$ E&M

Parabolic: Single time deriv.
· Diffusion and advective behavior

$$\frac{\partial u}{\partial t} = \Delta u$$ Heat equation

Hyperbolic: 2 time derivatives
· Wave behavior

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$ wave equation

~~A system that has diffusive waves is not readily classifyable~~

~~ie.~~ $\cancel{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^3 u}{\partial x^2}}$

Higher order systems like $\frac{\partial^3 u}{\partial t} = \Delta u$
or combinations of order aren't immedietely identifyable.