Risky Business and the Process of Development

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Introduction

Financial markets much less developed in poor countries

- How much does this matter for development?

Quantitative models study equilbrium effects of financial frictions, in dynamic environment

- Buera, Kaboski, and Shin (2011), Midrigan and Xu (2014), Moll (2014)

These models feature strong self-financing channel (Banerjee and Moll 2010)

- Even in financial autarky, entrepreneurs rapidly save until $MPK \to \rho + \delta$
- Models with concave production functions deliver small misallocation

Rapid self-financing happens because entrepreneurs fully lever up

- Previous models are missing investment risk!

This paper: Introduce investment risk by making investment partially irreversible

Summary

- Irreversibility creates investment risk
  - Investment may not pay off if productivity falls, but costly to undo investment
  - Risk is idiosyncratic, so planner would provide insurance
- Slow investment dynamics and misallocation across firms
  - Poor entrepreneurs demand high risk premium
  - Wealthy entrepreneurs can self-insure
  - Productive firms grow slowly as owner builds wealth
  - Wealth inequality $\implies$ Heterogeneous MRPK $\implies$ Misallocation
- Investment Risk has complicated effects on savings and investment
  - Entrepreneurs demand risk premium above risk-free rate
  - Precautionary savings pushes risk-free rate down
  - Share of assets in fixed supply (e.g. land) mediates risk premium vs. precautionary savings
- Beneficial Role for Defaultable Debt?
  - State-contingent repayment insures against investment risk

Self-Financing and Irreversibility

Self-Financing in the Standard Model

Standard model of financial frictions:

$$\max_{c_t, i_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t) dt$$

$$da = \big(\pi(k_t, z_t) + w + r a_t - c_t - i_t\big) dt, \quad dk = (i_t - \delta k_t) dt$$

$$\pi(k_t, z_t) = \max_l z_t k_t^\alpha l_t^\beta - wl$$

$$a_t \geq -\lambda k_t, \quad k_t \geq 0$$

Result: If $MPK > r + \delta$, then $k = -\lambda a$

Maximum leverage $\implies$ rapid self-financing $\implies$ MPK close to $r + \delta$

Modest output losses, especially if production function is concave

Why do we get this extreme result? How can we break it?

Irreversibility Makes Investment Risky

In standard model, portfolio choice problem (k vs. a) is static

- Productivity, z_t, is known when investment is made
- No dynamic implications: If z changes, can costlessly liquidate
- Portfolio choice is also riskless, for similar reasons

In reality, firms do not recover full value of capital after liquidation

- Capital is specialized to specific firm
- Machines damaged/break if moved to a new firm
- Lemons market in used capital goods
- Kermani and Ma (2023) find liquidation value is 35% of book value

This makes investment risky

- If productivity falls, must accept lower return or costly liquidation
- No insurance $\implies$ low return is also when marginal utility is high!
- Entrepreneurs will demand a risk premium

Note: We treat irreversibility as technological, rather than a "friction"

Model Setup

Entrepreneur's Problem with Partial Irreversibility

Entrepreneur gets utility from consumption, c, and (rented) housing, h

$$\max_{c_t, h_t, i_t, i_{n,t}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t, h_t) dt$$

Continuous-time Markov process for productivity $z_t \in \{z_1, z_2, z_3, z_4, z_5\}$

Three assets: bank deposits/debt, b, capital, k, and land, n

Collateral-based credit constraint:

- Capital backs up to λ units of debt, land backs one-to-one
- $\implies$ constraint on net liquid assets, $a_t := b_t + pn_t$

$$\underbrace{b_t + pn_t}_{a_t} \geq -\lambda k_t, \quad k_t \geq 0, \quad n_t \geq 0$$

Laws of motion for assets:

$$db/dt = \underbrace{\pi(k_t, z_t) + w + rb_t + r_n n_t}_{\text{Profits+Wages+Interest}} - \underbrace{c_t - r_n h_t - i_t - p \cdot i_{n,t}}_{\text{Consumption+Rent+Investment}} - \underbrace{\boldsymbol{\Phi(i_t, k_t)}}_{\text{Adjustment Cost}}$$

$$dk = (i_t - \delta k_t)dt, \quad dn = i_n dt$$

Adjustment Costs and Partial Irreversibility

When the firm invests, needs to pay adjustment cost, $\Phi(i, k)$, in addition to cost of investment

When investment is negative (selling off capital), only gets back $\phi \leq 1$ dollars for each dollar of capital sold

Also pays small quadratic adjustment cost: this is just to make the problem smooth

$$\Phi(i, k) = \begin{cases} \frac{\kappa}{2}\left[\frac{i}{k+\bar{k}}\right]^2 (k+\bar{k}) & i \geq 0 \\ -(1-\phi)i + \frac{\kappa}{2}\left[\frac{i}{k+\bar{k}}\right]^2 (k+\bar{k}) & i < 0 \end{cases}$$

Closing the Model: Housing, Banking, and Labor

Household consumes a Cobb-Douglas composite of consumption and housing, with CRRA preferences over the composite

$$u(c,h) = \frac{x^{1-\sigma} - 1}{1-\sigma}, \quad x = c^{1-\omega_h} h^{\omega_h}$$

Housing sector converts fixed supply of land one-to-one into housing

$$\int h dG(b,n,k,z) = \int n dG(b,n,k,z) = 1$$

Households supply one unit of labor inelastically

$$\int l dG(b,n,k,z) = 1$$

Banks take deposits and make loans: zero net supply

$$\int b dG(b,n,k,z) = 0$$

All markets competitive $\implies r = \frac{r_n}{p}$, and $w = MPL$ for all firms

Planner’s Problem

Planner's Problem

We compare to a utilitarian planner's problem:

- maximize unweighted average utility $\int_{(a,k,z)} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u(c,h) dt\right] dG(b,h,k,z)$
- Subject to technological and informational constraints
  - Technological: Planner still needs to pay to liquidate capital
  - Informational: Planner does not know future productivity

Market failures/incompleteness: Credit constraint and lack of insurance

Key idea of the paper: Interaction of irreversibility (technology) with lack of insurance (incomplete markets)

Characterizing the Planner's Solution

What happens under planner's solution?

How can we solve the planner's problem?

Can implement the planner's solution (theoretically and on the computer) as a competitive equilibrium (second welfare theorem)

Allows us to separate production and consumption sides of problem

Consumption: Full insurance and all households have same consumption

- Housing allocation is simple: $h^* = 1$

Production: Entrepreneur $\rightarrow$ risk-neutral firm, maximizes NPV of profits

Planner's Solution (Static)

Let $\Omega(k, z)$ denote the distribution of (k, z) across firms

Given Ω, labor allocation is static problem with analytic solution

$$\max_{l} \int zk^{\alpha} l^{\beta} d\Omega \quad s.t \quad \int l d\Omega = 1$$

$$l(k, z) = \frac{\left(zk^{\alpha}\right)^{\frac{1}{1-\beta}}}{\int \left(zk^{\alpha}\right)^{\frac{1}{1-\beta}} d\Omega}, \quad y(k_t, z_t) = \frac{z_t^{\frac{1}{1-\beta}} k_t^{\frac{\alpha}{1-\beta}}}{\left[\int \left(zk^{\alpha}\right)^{\frac{1}{1-\beta}} d\Omega\right]^{\beta}}$$

$$w^* = \left(\int \left(\beta zk^{\alpha}\right)^{\frac{1}{1-\beta}} d\Omega\right)^{1-\beta}$$

However, planner must solve dynamic investment/liquidation problem...

Planner's Solution (Dynamic)

Solving for capital requires solving the firm's problem

In steady-state, $r = \rho$, so firm's problem is:

$$\max_{i_t} \; \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(y(k_t, z_t) - i_t - \Phi(i_t, k_t) - w^* \cdot l(k_t, z_t) \right) dt$$

$$dk = (i_t - \delta k_t) dt$$

Note that none of these prices depend on the wealth distribution

$\implies$ Complete-markets competitive equilibrium coincides with planner's solution on the production side of the economy

Sufficient to solve firm's problem + distribute consumption equally

Results

Calibration

Description	Parameter	Value	Target
Discount Rate	ρ	0.09	—
Risk Aversion	σ	1	Standard
Depreciation Rate	δ	0.06	Standard
Production Function	α, β	0.3,0.49	Buera, Kaboski, Shin (2011)
Liquidation Value	ϕ	0.35	Kermani and Ma (2023)
Adjustment Cost	$\kappa, \bar{k}$	0.1,0.1	Negligible (Avoid Jumps)
LTV Constraint	λ	0.75	—
Housing Weight	ω_h	0.25	Housing Expenditure Share

High discount rate, ρ, speeds up numerical convergence

LTV constraint, λ, does not seem to matter much

Calibration is still a work-in-progress... think of this as a numerical example with not-too-crazy parameters!

Calibration: Productivity Process

Productivity, z, can take one of five values; $z = z_1 = 0$ corresponds to firm exit

Continuous-time Markov process, with transition intensities below

Either exits or otherwise gets reshuffled (equal probability on z_2, z_3, z_4, z_5)

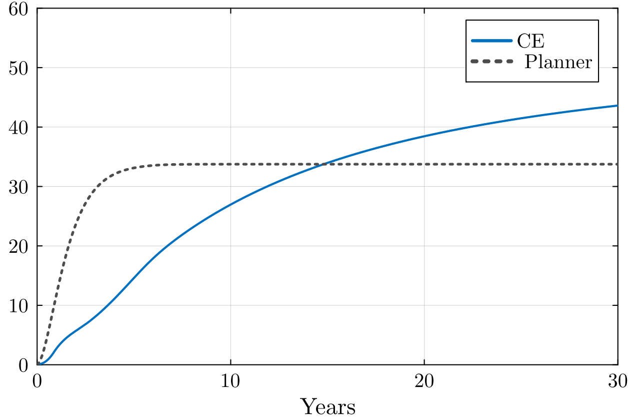
Transition Rate (Row→Column)	z_1	z_2	z_3	z_4	z_5
$z_1 = 0$	—	0.00375	0.00375	0.00375	0.00375
$z_2 = \frac{1}{4}$	0.07	—	0.02	0.02	0.02
$z_3 = \frac{1}{2}$	0.07	0.02	—	0.02	0.02
$z_4 = 1$	0.07	0.02	0.02	—	0.02
$z_5 = 2$	0.07	0.02	0.02	0.02	—
Stationary Mass	82%	4.4%	4.4%	4.4%	4.%4

Annually: Exit rate = 7%, SD($\log z$)=0.78, and autocorrelation = 0.85

These targets are within range of estimates (Peters and Zilibotti, 2021; David et al., 2020)

Investment Risk Slows Down Self-Financing

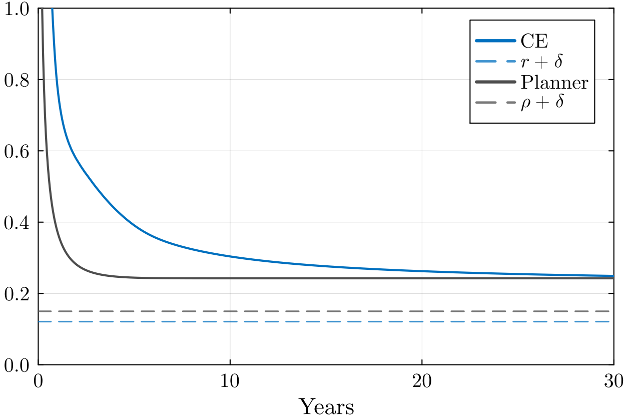
Figure 1: $k(t)$ for $z = z_5$, starting at $a = 0$, $k = 0$



- Figure shows "take-off" path of capital for an entrepreneur that starts with nothing and suddenly draws maximum productivity
- Planner rockets to efficient level quickly; entrepreneur ascends much more slowly
- Why? Household self-insures as it gets wealthier (standard result for CRRA)
- Entrepreneur eventually overtakes planner, because $r < \rho$ in competitive equilibrium

Path of MPK

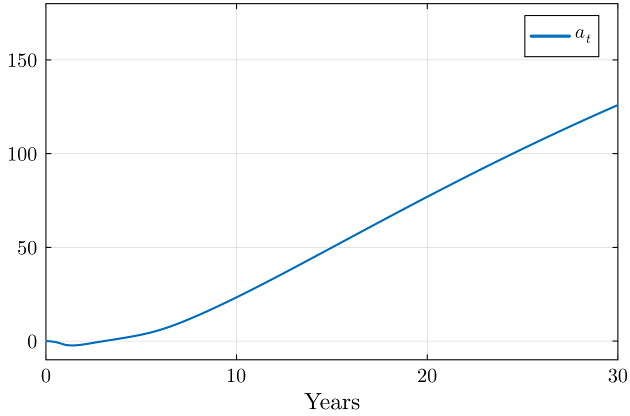
Figure 2: $MPK(t)$ for $z = z_5$, starting at $a = 0$, $k = 0$



- Figure shows path of $MPK(t)$
- Again, planner is quick but entrepreneur is slow
- Wages are lower in competitive equilibrium, so MPK higher even when CE overtakes planner
- Entrepreneur approaches risk neutrality as wealth grows
- MPK asymptotes to level above $r + \delta$, because of liquidation risk

Irreversibility Attenuates Leverage

Figure 3: $a(t)$ for $z = z_5$, starting at $a = 0$, $k = 0$



- Figure shows path of liquid assets, $a_t := b_t + pn_t$
- In standard model, entrepreneur chooses maximum leverage until firm is fully funded
- In our model, liquid assets are almost always positive, even though MPK is high
- Breaks the maximum leverage result from the standard model
- Entrepreneur prefers to accumulate buffer of safe assets

Self-Financing is Rapid with Full Reversibility

Figure 4: Path of $k(t)$ for $z = z_5$; $\phi = 1$

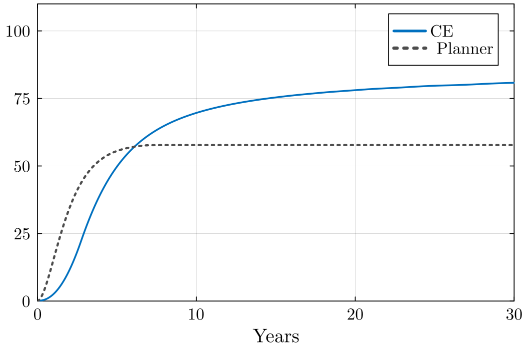
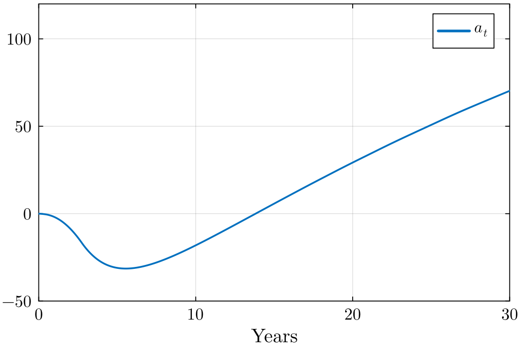


Figure 5: Path of $a(t)$ for $z = z_5$; $\phi = 1$



Under full reversibility, accumulate capital rapidly, financed by debt

Asymptotes above planner because $r < \rho$

Note: Still have small quadratic adjustment costs

Investment Risk Creates Misallocation

Planner would separate efficiency and distribution: operate firms to produce as much as possible, regardless of owner

Inequality and Misallocation: Entrepreneurs with same (z, k) will invest different amounts, depending on their wealth

Slow dynamics imply that MRPK is elevated for a while after a shock

- We have dampened the rapid self-financing that kept MPK constant across firms

Slow Dynamics/Failure of Separation $\implies$ Excess dispersion of MRPK $\implies$ Misallocation

Stationary Equilibrium with and without Irreversibility

Table 1: Aggregates in Stationary Equilibrium

Variables	$\phi = 0.35$	$\phi = 1$
r	6.1%	6.9%
Y/Y^*	0.87	0.97
K/K^*	0.74	1.00
Z/Z^*	0.96	0.96
C/C^*	0.91	0.98
Welfare	0.52	0.46
$\mathbb{E}$[MPK]	0.23	0.16
SD(MPK)	0.21	0.13

- Welfare: Proportional increase in c that would make average utility same as in planner's solution
- Aggregate productivity: $Z := Y/(K^\alpha L^\beta)$
- Model with irreversibility has lower output, capital, and consumption than planner
- Partly due to misallocation, partly due to underinvestment
- Welfare losses are much larger; incorporate benefits of insurance and redistribution
- *Note: With other housing share, K/K^* in second column would not equal one.*

Risk Premium vs. Precautionary Savings

How does investment risk affect the level of capital?

Two competing forces:

- Entrepreneurs demand risk premium over risk-free rate
- But, risk increases precautionary savings, lowering the risk-free rate
- In principle, can get over or under accumulation!

Key role of land (asset in fixed supply):

- Elastic assets (capital) respond to demand through higher quantities
- Inelastic assets (land) respond by bidding up the price
- Land can soak up demand for assets, dampening precautionary savings effect
- Bank loans/deposits cannot play this role because they are in zero net supply

Related mechanism in other papers:

- Angeletos (2007): HHs can trade (fixed) present value of labor income ("Human Wealth")
- Di Tella (2020): Money instead of land
- Trouvain (2025): Land, but in an Aiyagari economy (labor income risk)

Fixed Asset (Land) Mediates Effect of Investment Risk

Table 2: Aggregates in Stationary Equilibrium: Different Housing Shares

Housing Share	0	0.01	0.10	**0.25**	0.375	0.5
r	-1.0%	1.1%	4.1%	6.1%	7%	7.7%
p	—	0.82	2.32	4.64	7.16	10.9
Y/Y^*	0.96	0.92	0.89	0.87	0.87	0.88
K/K^*	1.38	1.03	0.83	0.74	0.72	0.72
Z/Z^*	0.87	0.91	0.94	0.96	0.96	0.97
C/C^*	0.97	0.94	0.92	0.91	0.91	0.91
Welfare	0.32	0.34	0.42	0.52	0.62	0.72
$\mathbb{E}$[MPK]	0.16	0.19	0.22	0.23	0.23	0.23
SD(MPK)	0.17	0.19	0.20	0.21	0.20	0.20

As housing share increases, more of precautionary savings is "soaked up" by land

Apparent tradeoff between misallocation vs. underaccumulation

Discontinuity at zero land! Rent on housing must be positive, and $r = \frac{r_n}{p}$, so $r > 0$ Conclusion

Extension: Model with Default

Financial Markets Can Diversify Risk

So far, little demand for credit. How does credit enter the picture?

Credit interacts with risk if the entrepreneur can default.

The option to default creates a state-contingent contract:

- If things get very bad, entrepreneur can default and not pay off debts
- Makes investment less risky (can even cause overinvestment)

Punishment for default is loss of capital and temporary loss of access to credit.

- Leave autarky with probability χ_{dn}

Zero profits for banks $\implies$ default raises all borrowers' interest rate

Not today: Other financial products could also be very useful (e.g. issuing equity)

Bank's Problem with Default

Bank lends at r_b with loan-to-value constraint (requires $1/\lambda$ units of capital as collateral for each dollar of debt)

Same r_b for all borrowers: does not depend on (a, k, z). May result from information constraints and/or legal constraints.

If entrepreneur defaults, bank liquidates the firm and gets back $\phi_b \cdot k$. Bank has same liquidation technology as entrepreneur ($\phi_b = \phi$).

Bank borrows at rate r_s, perfectly elastic supply. Makes zero profit in equilibrium.

Calibration and Solution

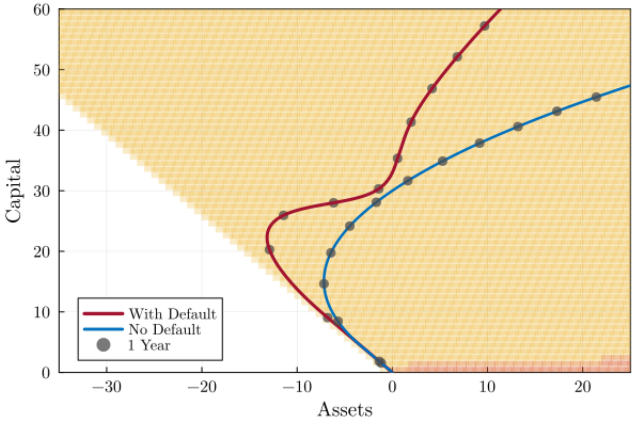
- Calibrate autarky exit rate $\chi_{dn} = 0.143$, for expected duration of seven years
- Takes seven years for a Chapter 13 bankruptcy flag to be removed (Dobbie et al. 2020)
- *Today: We don't have the model with housing and default solved yet, so will show illustrative results from older model (no housing, and $\sigma = 2$)*

Table 3: Aggregates with and without Default

Variables	No Default	Default
$r_b - r_s$	0	4.5%
Y/Y^*	1.02	1.05
K/K^*	2.52	2.40
Z/Z^*	0.78	0.80
Credit/Y	0.96	0.54
Welfare	0.23	0.24
$\mathbb{E}[MPK]$	0.13	0.15
SD(MPK)	0.15	0.15

Time Paths with and without Default

Figure 6: $a(t), k(t)$ for $z = z_5$, starting at $a = 0$, $k = 0$; Default vs. No Default



Time Paths with and without Default

Figure 7: Path of $k(t)$ for $z = z_5$

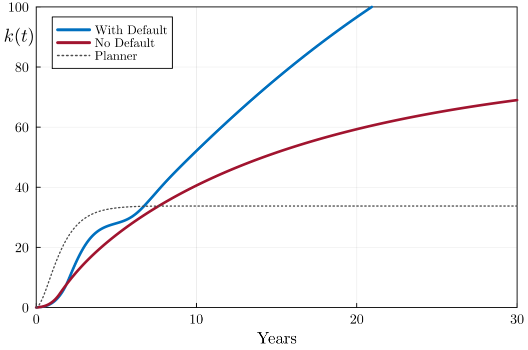
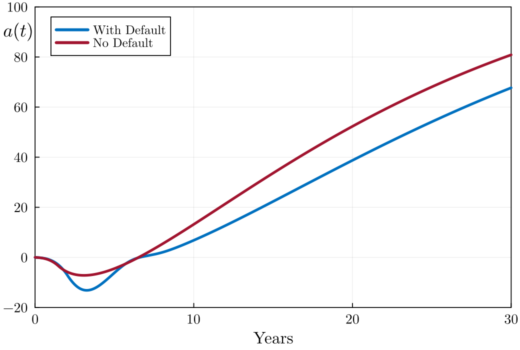
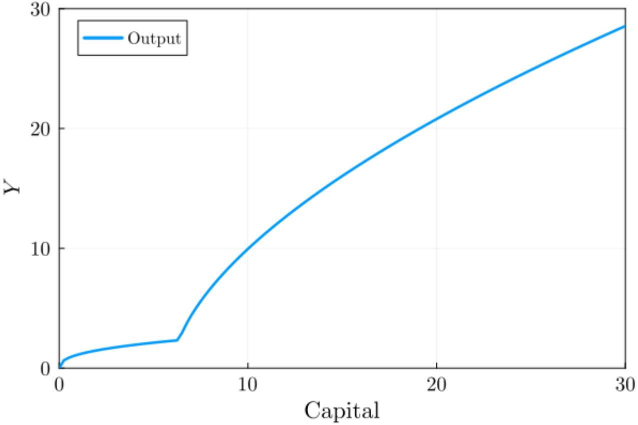


Figure 8: Path of $a(t)$ for $z = z_5$

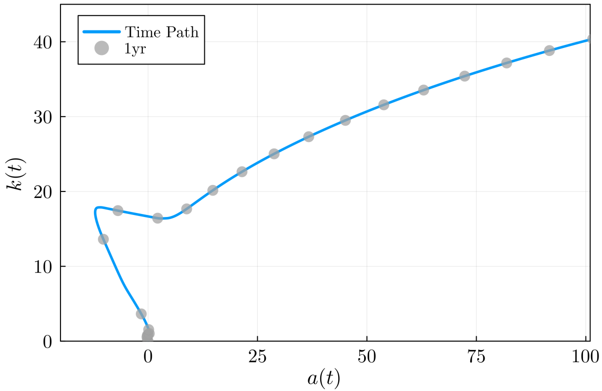


Role of Default Option: Skiba (1978) Technology



Skiba Technology: Time Paths

Figure 9: $z = z_5$ starting with $a = 0$, $k = 0$



Skiba Technology: Time Paths

Figure 10: Path of $k(t)$ for $z = z_5$; Skiba Technology

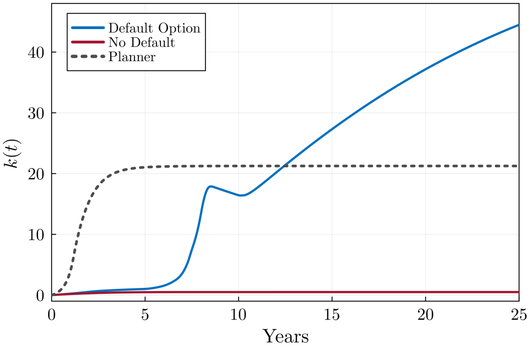
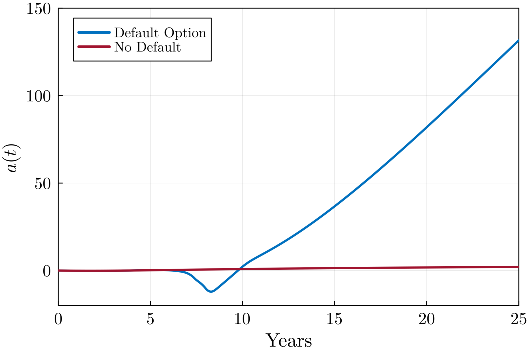


Figure 11: Path of $a(t)$ for $z = z_5$; Skiba Technology



Conclusion

Irreversibility makes investment risky

- $\implies$ Entrepreneurs do not fully lever up
- $\implies$ Slow self-financing, as entrepreneurs need to build wealth to self-insure
- $\implies$ Misallocation across firms: wealthier entrepreneurs invest more

Fixed assets mediate the effects on aggregate capital

- Precautionary savings vs. Risk Premium
- Fixed assets soak up demand for assets $\implies$ higher land share means risk premium dominates

Potential insurance role for defaultable debt

Thank you all for a great conference!