The Cost of Capital and Misallocation in the United States

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Research question and basic idea

Research question: How does dispersion in the cost of capital affect misallocation?

Traditional approach:

- Strong assumptions about production functions (homogeneous Cobb-Douglas)
- $\implies$ Measure heterogeneity in marginal products from cross-sectional data
- $\implies$ Measure misallocation

Our approach:

- Main idea: $r + \delta = \text{MPK}$
- Uses credit registry data + model to carefully measure cost of capital
- Use heterogeneity in cost of capital to infer the cost of misallocation coming from imperfect credit markets

Contribution and findings

Methodological contribution:

- Adapt a standard dynamic corporate finance model to enable measurement using micro data
- Derive a sufficient statistic for misallocation using credit registry data

Empirical Results (US):

- Average cost of capital tracks treasury rates, with a spread
- Measures of cost of capital correlate with traditional measures of $ARPK_i$
- Credit markets seem quite efficient in normal times (losses $\approx 1\%$ of GDP)
- Losses from misallocation increased to 1.6% of GDP in 2020-2021

Related literature

- **Measuring misallocation:**
  - Seminal work: Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Challenge: Standard methods rely on strong assumptions (Haltiwanger et al., 2018).
  - Recent advances: Experimental/quasi-experimental methods to recover marginal products directly (Carrillo et al., 2023; Hughes and Majerovitz, 2025).
  - Contribution: use heterogeneity in funding costs to measure dispersion in MRPK
- **Heterogeneity in the cost of capital:**
  - Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
  - US: Gilchrist, Sim, and Zakrajsek (2013), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
  - Contribution:
    - Estimate firm cost of capital using credit registry data, correcting for loan characteristics, etc.
    - Derive and estimate sufficient statistic for misallocation

Model

Model Summary

- Time discrete and infinite
- Continuum of firms, each matched with a lender
- No aggregate risk

Borrowers

- Produce output $f(k_i, z_i)$
- Invest in capital k_i
- Long-term debt b_i
- Limited liability

Lenders

- Discount rate ρ_i
- Recover $\phi_i k_i$ in default
- Break-Even Pricing (NPV $= 0$)

Key question: how do heterogeneity in ρ_i and financial frictions distort the allocation of capital?

Model Equations

Firm Value Function:

$$V_i\left(k_i, b_i, z_i\right) = \max_{k_i', b_i'} \pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) + \beta \mathbb{E} \overbrace{\left[\max\left\{V_i\left(k_i', b_i', z_i'\right), 0\right\} \middle| z_i\right]}^{\text{Limited liability}}$$

Firm profits:

$$\pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) = f\left(k_i, z_i\right) + (1-\delta) k_i - k_i' - \theta b_i + Q_i\left(k_i', b_i', z_i\right)\left[b_i' - (1-\theta_i) b_i\right]$$

Price of debt:

$$Q_i\left(k_i', b_i', z_i\right) = \frac{\mathbb{E}\left\{\mathcal{P}_i\left(k_i', b_i', z_i'\right)\left[\theta_i + (1-\theta_i) Q_i\left(k_i'', b_i'', z_i'\right)\right] + \left(1 - \mathcal{P}_i\left(k_i', b_i', z_i'\right)\right) \overbrace{\frac{\phi_i k_i'}{b_i'}}^{\text{recovery}} \middle| z_i\right\}}{\underbrace{1 + \rho_i}_{\text{lender discount rate}}}$$

Firm's cost of capital

Firm has limited liability; does not care about states of the world where it defaults

Define the implicit interest rate paid by the firm as

$$1 + r_i^{firm} = \frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}{Q_i}$$

Lemma 1 (Firm's cost of capital)

The firm's cost of capital is:

$$1 + r_i^{firm} = \frac{1+\rho_i}{1+\Lambda_i} \qquad \Lambda_i := \frac{\mathbb{E}\left[(1-\mathcal{P}_i')\,\phi_i k_i'/b_i' \middle| k_i', b_i', z_i\right]}{\mathbb{E}\left[\mathcal{P}_i'\,(\theta + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}$$

▷ *Proof*

In general, $r_i^{firm} < \rho_i$, since bank recovers something in default, but firm puts a zero on default

Marginal revenue product of capital (MRPK)

The firm's cost of capital pins down its MRPK

$$\underbrace{(1+r_i^{firm})\mathcal{M}_i}_{\text{cost of capital}} = \underbrace{\mathbb{E}[\mathcal{P}_i'(f_k(k_i', z_i') + 1 - \delta)|\, k_i', b_i', z_i]}_{\text{expected marginal revenue product of capital}}$$

where $\mathcal{M}_i$ captures the *price impact* of the firm's actions

$$\mathcal{M}_i := \frac{1 - \gamma_i \times \frac{Q_i \cdot b_i'}{k_i'} \times \frac{\partial \log Q_i}{\partial \log k_i'}}{1 + \gamma_i \times \frac{\partial \log Q_i}{\partial \log b_i'}}, \qquad \gamma_i := \frac{b_i' - (1-\theta_i)b_i}{b_i'}$$

- With low default, $\mathcal{M}_i$ will be very close to 1
- We will set $\mathcal{M}_i = 1$, and do robustness where we estimate heterogeneous $\mathcal{M}_i$

Welfare and the Social Return on Capital

Which is the "right" cost of capital, r^{firm} (firm) or ρ (lender)?

Planner cares about total welfare: adds up payoffs to lender and firm

Social MPK is expected MPK plus derivative of expected recoveries

Define the social marginal product of capital at firm i:

$$1 + r^{social}_{i,t}(k_{i,t+1}) := \mathbb{E}\left[\underbrace{\mathcal{P}_{i,t+1}\left(f_k\left(k_{i,t+1}, z_{i,t+1}\right) + 1 - \delta\right)}_{r^{firm}_i} + \left(1 - \mathcal{P}_{i,t+1}\right)\phi_i\right]$$

Planner Optimality: **Equalize** $r^{social}_{i,t}(k^*_{i,t+1})$ across firms

Inefficient Equilibrium: Dispersion in $r^{social}_{i,t} \rightarrow$ misallocation

Misallocation

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r_i^{social}\right]$ *and* $Var\left(r_i^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{Var\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

▷ *Proof*

- Extends Hughes and Majerovitz (2025) to a dynamic economy with default
- Measures intensive-margin misallocation only; lower bound on full misallocation (see paper)
- Set $\mathcal{E}=\frac{1}{2}$ (elasticity of output w.r.t. $r^{social}+\delta$) and $\delta = 0.06$ ▷ Calibration
- **Next:** show how to measure r_i^{social} using credit registry data

Measurement with credit registry data

Data: FR Y-14Q (Schedule H.1)

▷ cleaning details

- Quarterly loan-level panel on universe of loan facilities $> \$1$M
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- 91% of C&I undertaken by top 25 banks/ 55% of C&I undertaken by all commercial banks
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans issued to non-government, non-financial US companies

Summary Statistics

▷ time series

	Mean	St. Dev.	p10	p50	p90
Interest rate	4.17	1.69	2.21	3.93	6.59
Maturity (yrs)	6.85	4.64	3.00	5.00	10.25
Real interest rate	2.38	1.24	0.88	2.33	3.99
Prob. Default (%)	1.42	2.37	0.19	0.82	2.85
LGD (%)	34.50	13.20	16.00	36.00	50.00
Loan amount (M)	10.77	68.82	1.11	2.55	22.64
Sales (M)	1,254.75	5,923.57	2.17	58.79	1,556.69
Assets (M)	1,770.85	8,956.85	1.06	35.51	1,792.00
Leverage (%)	72.03	24.57	42.57	71.17	100.00
Return on assets (%)	27.19	55.25	4.58	15.78	47.58
N Loans	62,686				
N Firms	38,586				
N Fixed Rate	31,540				
N Variable Rate	31,146				

Pricing term loans

The break-even condition for a lender with discount rate ρ_i is

$$1 = \sum_{t=1}^{T_i} \left\{ \frac{P_i^t \mathbb{E}_0 \left[r_{i,t} \right] + P_i^{t-1} (1 - P_i) \left(1 - LGD_i \right)}{\left(1 + \rho_i \right)^t \cdot \left(1 + \bar{\pi}_t \right)} \right\} + \frac{P_i^{T_i}}{\left(1 + \rho_i \right)^{T_i} \cdot \left(1 + \bar{\pi}_t \right)}$$

- T_i: maturity
- P_i: repayment probability (constant over time)
- $\mathbb{E}_0[r_{i,t}]$: fixed rate or spread over benchmark rate (Gürkaynak et al., 2007) ▷ forward rates
- LGD_i: loss given default (constant over time)
- $\bar{\pi}_t$: expected inflation, $1 + \bar{\pi}_t = \mathbb{E}_0 \left[\prod_{j=0}^{t} (1 + \pi_j) \right]$ (Cleveland Fed)
- $\Rightarrow$ Solve for lender's discount rate: ρ_i

Measuring Firm and Social Cost of Capital

Lemma 2 (Firm cost of capital)

We can write the firm cost of capital as

$$1 + r_i^{firm} = (1 + \rho_i) - (1 - P_i)(1 - LGD_i)$$

Lemma 3 (Social cost of capital)

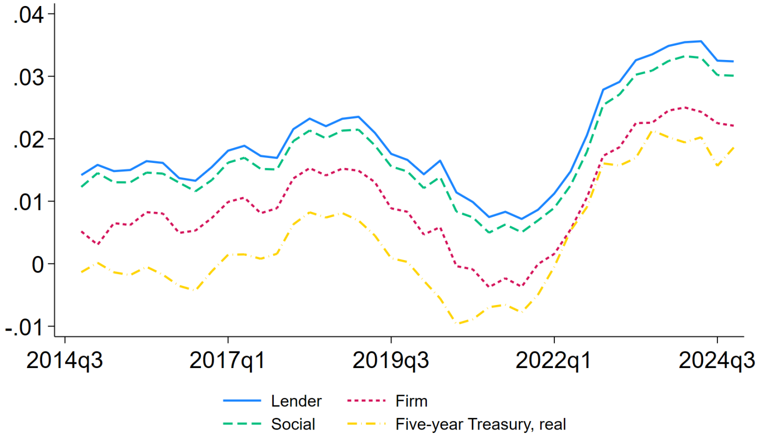
The social cost of capital can be written as:

$$\begin{aligned} 1 + r_i^{social} &= (1 + r_i^{firm})\mathcal{M}_i + (1 - P_i)(1 - LGD_i)lev_i \\ &= \underbrace{(1 + \rho_i)\,\mathcal{M}_i}_{\text{lender discount rate}} + \underbrace{(lev_i - \mathcal{M}_i) \cdot (1 - P_i) \cdot (1 - LGD_i)}_{\text{wedge due to financial frictions}} \end{aligned}$$

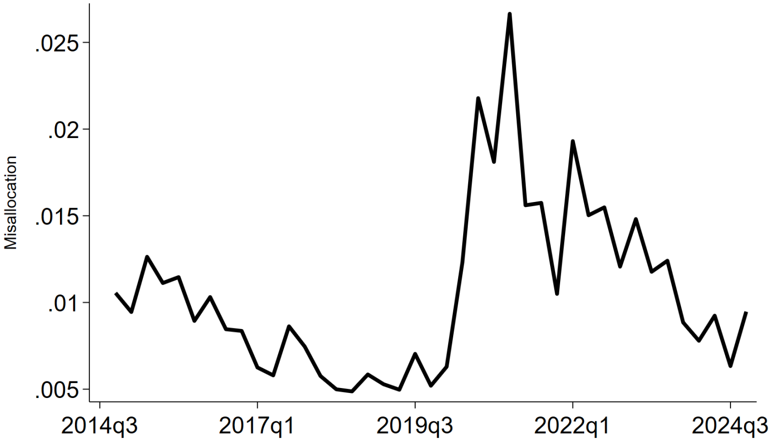
Note: In general, $r^{firm} \leq r^{social} \leq \rho$

Empirical results

Time series for average discount rate, firm and social cost of capital



Misallocation in the US, 2014-2024



- About 0.8% before 2020
- ↑ to 1.6% in 2020-2021
- ↓ to 1.2% in 2022-2024

The 2020–2021 increase in misallocation

1. Predominantly explained by changes in dispersion in ρ_i, rather than financial frictions ▷ details

2. Sharp rise in the coefficient of variation of ρ_i ▷ details

3. ρ_i dispersion $\uparrow$ due to increased dispersion of expected losses ▷ details

Relation to measures of ARPK

	(1) $\log(ARPK)$, Sales	(2) $\log(ARPK)$, EBITDA	(3) $\log(ARPK)$, Sales	(4) $\log(ARPK)$, EBITDA	(5) $\log(ARPK)$, VA
$\log(r^{social} + \delta)$	0.17^{***}	0.26^{***}	0.17^{**}	0.15^{*}	0.37^{***}
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	56,908	55,029	4,041	3,933	3,315
Adj. R2	0.28	0.22	0.68	0.52	0.60
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

ARPK-based misallocation

▷ other measures

Focus on Compustat firms to make measures comparable

	$r^{social} + \delta$	$\frac{\text{Sales}}{\text{Capital}}$	$\frac{\text{EBITDA}}{\text{Capital}}$	$\frac{\text{Value Added}}{\text{Capital}}$
$Var(\log)$	0.01	0.18	0.24	0.20
Misallocation (%)	0.37	4.65	6.15	5.23

- Pros: does not require detailed data on firm financials (i.e., value added); applicable to most existing credit registries
- Cons: we measure the gain of reallocating capital only, holding fixed other inputs

Cross-country comparison

▷ details

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
$\mathbb{E}[r_i]$, %	66.8	8.00	83.0	12.4	1.1
$\mathrm{SD}(r_i)$, %	38.1	2.9	93.3	5.2	1.5
$\mathbb{E}[1 - P_i]$, %	2.7	16.9	4.0	8.9	1.4
$\mathbb{E}[1 - LGD_i]$, % (World Bank)	42.8	42.8	18.2	63.9	81.0
Implied misallocation, %	6.5	13.5	21.5	2.8	1.2

- Developing countries: higher mean and standard deviation of real interest rates
- U.S.: lower mean and standard deviation of interest rates, higher recovery
- Misallocation exercise: Assumes all firms have same P and LGD, uses ρ instead of r^{social}
- Brazil: most extreme misallocation: 21.5%.

Conclusion

- Develop a framework to measure misallocation using credit registry data
  1. Standard macrofinance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Relies on standard credit registry variables as inputs (r, P, LGD, T, etc.)
- Application to U.S. credit registry data (FR Y-14Q)
  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 1% in normal times
  3. Rise in 2020-21, driven by increase in variance of expected losses
- **Work in progress:** Adding in aggregate risk

Appendices

Proof: firm cost of capital

▷ back

$$\mathbb{E}_t \left[\frac{\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right)}{Q_t} \right] = (1+\rho) \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right) \right]}{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right) \right] + \mathbb{E}_t \left[\left(1 - \mathcal{P}_{t+1} \right) \phi k' / b' \right]}$$

$$= (1+\rho) \left(1 + \frac{\mathbb{E}_t \left[\left(1 - \mathcal{P}_{t+1} \right) \phi k' / b' \right]}{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right) \right]} \right)^{-1}$$

$$= (1+\rho) \left(1 + \Lambda \right)^{-1}$$

where

$$\Lambda \equiv \frac{\mathbb{E}_t \left[\left(1 - \mathcal{P}_{t+1} \right) \phi k' / b' \right]}{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right) \right]}$$

Full planner problem

▷ back

$$U^* = \max_{\left\{\{k_{i,t}(S^{t-1}),\omega_{i,t}(S^t)\}_i\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(Y_t - I_t\right)$$

$$\text{s.t.} \qquad \omega_{i,t}\left(S^t\right) \in \{0,1\}\ \forall i$$

$$\omega_{i,t+1}\left(S^{t+1}\right) \leq \omega_{i,t}\left(S^t\right)\ \forall S^t \subset S^{t+1}, \forall i$$

Can separate into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t,\{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

Rewrite inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\{k_{i,t}\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left[\omega_{it} \cdot f\left(k_{it}; z_{it}\right) - \left(1-\omega_{it}\right) \cdot \left(\left(1-\delta\right) k_{it} - \phi_i k_{it}\right)\right] di$$

$$\text{s.t.} \qquad K_t = \int_0^1 k_{it} di$$

Proof of Misallocation

▷ back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i)\mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $(r^{social} + \delta)$

Calibration of $\mathcal{E}$

▷ back

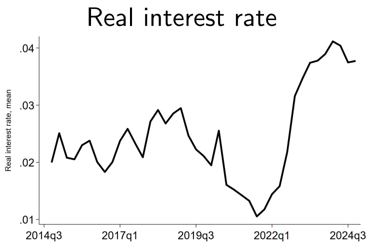
- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital
- Assume that $f(k, z) = z \cdot k^{\alpha}$ and there is no default, then

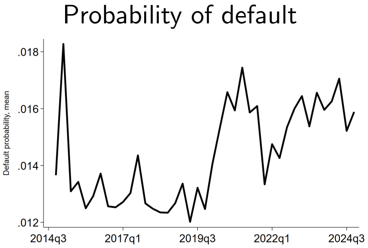
$$\mathcal{E} = \frac{\alpha}{1-\alpha}$$

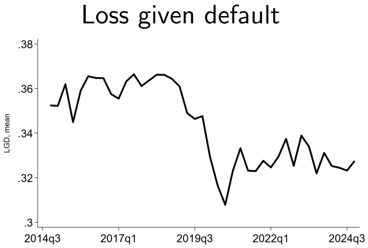
- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

Time series for averages: real interest rate, PD, LGD

▷ back







Data Cleaning and Sample Construction

▷ back

Sample period: We use FR Y-14Q Schedule H.1 data from 2014Q4 onward **Borrower Filters:**

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data Cleaning and Sample Construction

▷ back

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type $= 7$)
- Drop loans with:
  - Mixed-rate structures
  - Maturity outside 110 years
  - Implausible interest rates or spreads (outside 1st99th percentile, or $> 50\%$)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD $= 1$ (flagged as in default)

Forward interest rate expectations

▷ back

To estimate ρ_i for floating rate loans, need estimates of $\mathbb{E}_0[r_t] + s_i$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Average spread between SOFR and Treasury rates 2018-2025 $\simeq$ 2 basis points
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0[r_t] + s_i$ for each loan, using treasury forward rate plus loan's spread

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\,Q_{t+1}\right) + \left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho}$$

Note that

$$\begin{aligned}
Q_t &= Q_t^P + Q_t^D \\
Q_t^P &= \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\,Q_{t+1}\right)\right]}{1+\rho} \\
Q_t^D &= \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho}
\end{aligned}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\,Q_{t+1}\right)\right]} = \frac{Q_t^D}{Q_t^P}$$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}$$

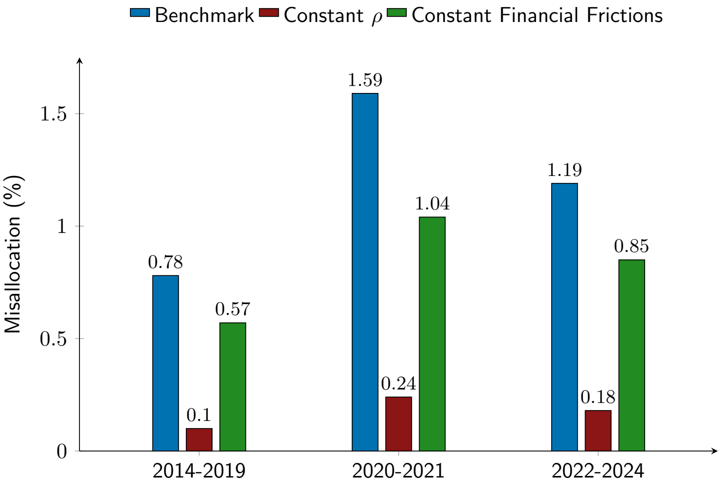
$$1 = \frac{(1-P)(1-LGD)}{1+\rho} + P\frac{\mathbb{E}_t\left[r_{t+1}\right]}{1+\rho} + \left(\sum_{s=2}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}\right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

1. The 2020-21 rise in misallocation was driven by $\{\rho_i\}$

▷ details



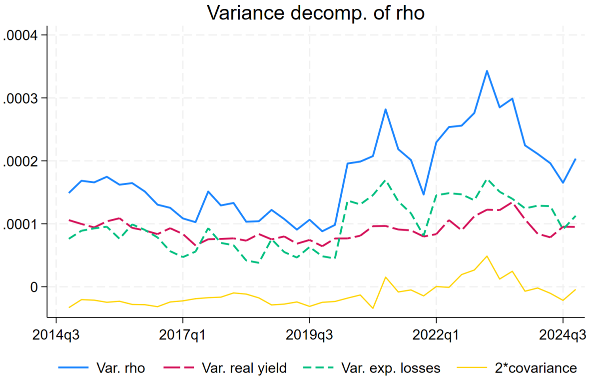
2. The CV of ρ_i increased during 2020-21

- As policy rates decreased in 2020-21, so did mean ρ_i
- Standard deviation of ρ_i increased during this period

3. Variance of ρ related to variance of expected losses

▷ details

$$\rho_i = \underbrace{\rho_i(P_i = 1)}_{\text{real yield}} + \underbrace{[\rho_i - \rho_i(P_i = 1)]}_{\text{exp. losses}}$$



- $\sigma(\rho) \uparrow$ due to $\sigma(\text{exp. losses}) \uparrow$
- $\sigma(\text{exp. losses}) \uparrow$ without $\sigma(r) \uparrow$
- Possibly tied to underpricing of risky loans, implicit guarantees, etc.

Decomposing misallocation

▷ back

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r_{social}^{cf,I} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to financial frictions

Counterfactual II: what if we equalize financial frictions?

$$1 + r_{social}^{cf,II} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to heterogeneous cost of capital

Details: ρ decomposition

▷ back

- The "real yield" is the implied ρ_i^* when $P_i = 1$

$$1 = \sum_{t=1}^{T_i} \left\{ \frac{\mathbb{E}_0\left[r_{i,t}\right]}{\left(1+\rho_i^*\right)^{t} \cdot \mathbb{E}_0\left[\prod_{j=0}^{t}(1+\pi_j)\right]} \right\} + \frac{1}{\left(1+\rho_i^*\right)^{T_i} \cdot \mathbb{E}_0\left[\prod_{j=0}^{T_i}(1+\pi_j)\right]}$$

- Real yield independent of P_i, LGD_i

- Only affected by losses through the contractual rate r

Variance decomposition

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities

	Time	Bank	Firm	Loan
Contractual rate	71.88	1.63	13.45	13.04
Lender discount rate, ρ	61.94	3.08	14.02	20.96
Firm cost of capital, r^{firm}	33.23	4.25	20.12	42.4
Social cost of capital, r^{social}	53.84	3.87	16.21	26.08
N Firms	1681			
N Loans	14738			

Table: Variance decomposition of interest rates and cost of capital (ρ, r^{firm}, and r^{social})

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function Q, γ, and firm leverage Qb'/k' we can compute $\mathcal{M}$

1. Loans are modeled as perpetuities that decay at a geometric rate θ, we can write Q as the present value of all future payments, discounted at the real interest rate r:

   $$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$

2. Guess a functional approximation $Q(z, k, b, \rho)$

3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives

4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

- We approximate (the log of) Q as a polynomial of firm capital, borrowing, productivity and ρ
- Capital: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets
- Approximation:

$$\begin{aligned}\log Q_i = {} & \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ & + \beta_{k,k} (\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ & + \beta_{b,b} (\log b_i)^2 \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ & + \beta_{z,z} (\log z_i)^2 \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho} (\rho_i)^2 \epsilon_i\end{aligned}$$

- Compute the partial derivatives of $\log Q$ with respect to investment and borrowing.

Estimating $\mathcal{M}$: results

▷ back

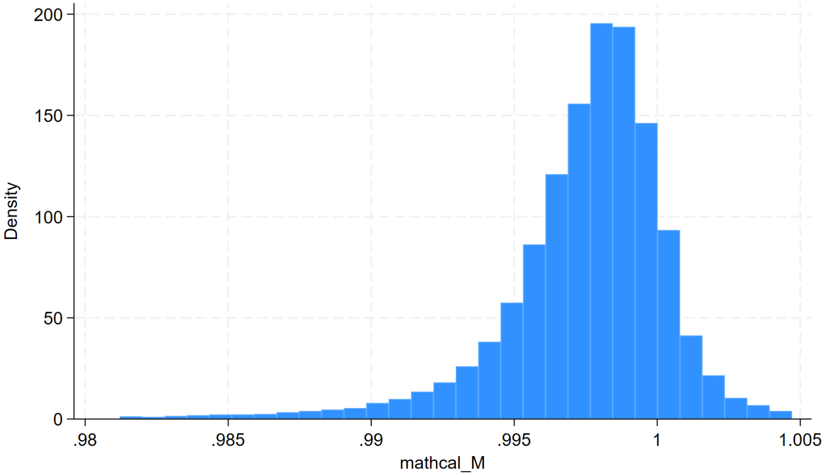


Figure: Histogram for estimated $\mathcal{M}_i$

Risk Premia & Aggregate Shocks

▷ back

- Alternative hypothesis: Rise in ρ reflects higher **risk premia** as lenders demand extra compensation amid extreme uncertainty (e.g. COVID-19).
- Firms differ in exposure to aggregate shocks $\Rightarrow$ heterogeneous risk premia need not imply misallocation (David et al., 2022).
- Our framework is steady-state $\Rightarrow$ cannot model time-varying aggregate shocks or risk-premium spikes.
- Data contradict the risk-premia story:
  - Average ρ **falls** from 3.6% (2014-19) to 2.7% (2020-21).
  - Skewness becomes **more negative**: $-2.6 \rightarrow -3.5$ (left tail thickens).
- Interpretation: Risk premia likely **declined**, perhaps owing to explicit/implicit policy guarantees.

ARPK-based misallocation measures

▷ back

	(1) $\log(ARPK)$, sales	(2) $\log(ARPK)$, EBITDA	(3) $\log(ARPK)$, sales	(4) $\log(ARPK)$, EBITDA
$\log(r^{social} + \delta)$	0.19^{***}	0.26^{***}	0.20^{***}	0.17^{**}
	(0.03)	(0.04)	(0.08)	(0.09)
Observations	56,912	55,033	4,064	3,963
Adj. R2	0.25	0.18	0.62	0.46
NAICS3, Quarter FE	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat
$Var[\log(ARPK)]$	2.17	1.72	0.31	0.37
Misalloc., $ARPK$, %	72.21	53.77	8.03	9.73
$Var[\log(r^{social} + \delta)]$	0.04	0.04	0.02	0.02
Misalloc., r^{social}, %	1.09	1.09	0.41	0.41

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Details on cross-country comparison

▷ back

- Recovery rates and inflation rates from the World Bank
- For a fixed real interest rate, ρ has a closed-form:

$$1 + \rho_i = P_i \left(1 + r_i\right) + \left(1 - P_i\right)\left(1 - LGD_i\right)$$

- Assume all loans have the same maturity:
  1. Obtain mean real rate by subtracting average realized inflation from mean nominal rate
  2. Inflation should not affect standard deviation of nominal rates (or spreads)
- Assume all loans have the same P_i, LGD_i, equal to the average
- Approximate $r_i^{social} \simeq \rho_i$ and compute misallocation using our formula:

$$\log(Y^*/Y^{DE}) = \frac{1}{2}\mathcal{E}\log\left(1 + \frac{Var(\rho_i)}{(\mathbb{E}[\rho_i] + \delta)^2}\right)$$

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