Eight-Stub Filter				
$10 \log_{10} K_8$	k_1	k_2	k_3	k_4
37.11	0.1	0.480	1.050	1.455
53.35	0.2	0.860	1.735	2.305
73.35	0.4	1.543	2.881	3.691
98.06	0.8	2.802	4.885	6.080
107.14	1.0	3.409	5.829	7.199
128.19	1.6	5.189	8.562	10.433
139.08	2.0	6.359	10.340	12.535

Nine-Stub Filter					
$10 \log K_9$	k_1	k_2	k_3	k_4	k_5
50.68	0.1	0.4974	1.138	1.689	1.898
69.02	0.2	0.8838	1.852	2.613	2.890
91.56	0.4	1.577	3.043	4.112	4.491
119.39	0.8	2.851	5.121	6.688	7.237
129.6	1.0	3.456	6.098	7.894	8.251
153.3	1.6	5.266	8.928	11.377	12.231
176.0	2.4	7.624	12.592	15.879	17.022

Ten-Stub Filter					
$10 \log K_{10}$	k_1	k_2	k_3	k_4	k_5
65.24	0.1	0.510	1.203	1.866	2.245
85.66	0.2	0.901	1.938	2.842	3.340
110.753	0.4	1.601	3.161	4.423	5.102
141.69	0.8	2.887	5.292	7.138	8.117
153.07	1.0	3.506	6.294	8.408	9.257
179.39	1.6	5.322	9.195	12.076	13.596
204.651	2.4	7.699	12.949	16.815	18.851

W. W. MUMFORD
Bell Telephone Labs., Inc.
Whippany, N. J.

A Simple Design Procedure for Small Percentage Bandwidth Round-Rod Interdigital Filters

The continued correspondence concerning interdigital filter [1] prompted me to submit this correspondence. The concepts and design procedures to be described have been found to be practically useful for determining the geometry of the coupled-rod structure, and because these procedures are apparently much simpler than those presently available, publication of the subject matter may be helpful to others. It will be apparent that some of this material is tutorial in nature—it is included to clarify the other information being presented.

I. CONCERNING THE OVERALL DEVICE

However else the device may be described, when used to produce small percentage, i.e., less than 10 per cent, pass bands, the first-order phenomenon involved in a so-called interdigital filter is identical to that occurring in direct coupled-resonator filters with groundings arranged so that the magnetic field coupling is in phase with the electric field coupling; i.e., the so-called "capacity aiding" connection in the $n=2$ IF interstages of old. Thus, the concepts and quantitative procedures described in [2] are directly applicable to the design, adjustment, and alignment of these filters. Specifically the device is adequately designed if, with all other resonances short-circuited, one correctly adjusts the coefficients-of-couplings between adjacent resonances; the singly loaded Q of the input and output resonances; and the resonant frequency of each resonance.

If the foregoing is true, the following helpful questions arise:

1) Why should one bother using different diameter rods (or different width rectangular bars) in the filter?

2) Why should one waste the space to put in rods #0 and #$(n+1)$ the only purpose of which is to properly couple the resistive generator and resistive load to the input and output resonances, respectively?

The answer to the first question is apparently that different diameter rods and different width bars are used because the presently available designs call for them. Actually there are an infinite number of correct diameter and spacing combinations, and there is no need to use different diameter rods (or different width bars) if the filter is correctly designed and adjusted for uniform diameter rods (or uniform width bars). Section III gives the simple design procedure involved.

The answer to the second question also is that apparently rods #0 and #$(n+1)$ are used because the presently available designs call for them. Actually in the bandwidth regions considered in this correspondence these rods are not required. In the less than one percent bandwidth region, probe or loop coupling of the generator to resonator #1, and the load to resonator #n is practical; and in the greater than one percent bandwidth region, tapping of the generator onto rod #1, and tapping of the load onto rod #n, is a practical way of producing the required singly loaded Q for the input and output resonators. Section IV gives the simple design procedure involved when tapping is used.

II. THE COEFFICIENT-OF-COUPLING BETWEEN ADJACENT UNIFORM DIAMETER RODS

The definition of coefficient-of-coupling (K) applicable to all small percentage bandwidth coupled-resonator filters is given in Section IV of [2]; also given there is a straightforward experimental procedure for accurately measuring and/or adjusting K. This procedure applied to an actual filter for which $d/h=0.5$ resulted in the three cross-marked points on the graph of Fig. 1. The K measured was that between two adjacent rods which also had equally spaced rods on their other sides; thus, equal far-side coupling was involved.

Within their applicable ranges, either Honey's closed-form equations for Z_{oe} and Z_{oo} [3], or Cristal's graphs for C_m/ϵ and C/ϵ [4] can be used to calculate the coefficient-of-coupling resulting from a given geometry d/h and c/h. It should be noted that the small percentage coupling case herein considered is very forgiving of a multitude of approximation sins, and perhaps a proof of this is the fact that for the less than 10 percent coupling case considered herein, and for normalized rod diameters less than $d/h=0.5$, both procedures give "exactly," i.e., within about 2 per cent, the same numerical answers. If Honey's approximation is used, (1) gives the K between a pair of rods

$$K = \frac{4}{\pi}\left\{\frac{\ln \coth\left(\dfrac{\pi}{2}\dfrac{c}{h}\right)}{\ln \coth\left(\dfrac{\pi}{4}\dfrac{d}{h}\right)}\right\}. \qquad (1)$$

If Cristal's graphs are used, (2) is used to obtain the K between a pair of rods

$$K = \frac{4}{\pi}\left\{\frac{1}{\dfrac{C_g/\epsilon}{C_m/\epsilon}+2}\right\}. \qquad (2)$$

The graph of Fig. 1, giving the coefficient-of-coupling between two adjacent rods as a function of normalized center to center rod spacing c/h, and normalized rod diameter d/h, results from an application of either (1) or (2). The very excellent agreement on Fig. 1 between the three experimentally determined cross-marked points, and the line for $d/h=0.5$, is gratifying.

As Fig. 1 indicates, in this region of interest, the log of the coefficient-of-coupling is almost a linear function of both the normalized center to center spacing (c/h) and the normalized rod diameter (d/h). A linear equation approximating the straight, almost equally-spaced lines of Fig. 1 is given in (3).

$$\log K \doteq \left\{-1.37\left(\frac{c}{h}\right) + 0.91\left(\frac{d}{h}\right) - 0.048\right\}. \qquad (3)$$

As presented, the equations and graph of Fig. 1 apply exactly only to the case of equal (C_m/C) on each side of every equal diameter rod. In a "modern network-theory filter" this is, of course, not the case; however, for small percentage couplings the foregoing simple equations and graph can still be used to design such an unequal C_m filter, using equal diameter rods, because of the following fundamental fact: Fringing phenomenon is such that, when the spacing between rods is changed, the Y_0 of any one line with all other lines short-circuited, e.g. $(Y_{0_{23}}+Y_{e_3}+Y_{0_{34}})$, changes negligibly; e.g., even with spacing changes such that the resultant coefficient of coupling changes from zero up to 10 percent, there is less than 3 percent change in this Y_0, even for d/h as large as 0.5. The value of this essentially constant Y_0 is very closely that for a single rod between parallel ground planes given by (4) (page 592 of [5]).

$$Y_0 \doteq 1/138 \log\left(\frac{4}{\pi}\frac{h}{d}\right) \qquad (4)$$

Thus, up to 10 percent couplings, the factor $(C_g/\epsilon\ 2C_m/\epsilon)$ in the denominator of (2) is essentially independent of changes in rod spacing; and to a first-order the numerical value for the corresponding numerator of (2), i.e. (C_m/ϵ), is still obtainable from Cristal's Fig. 2 [4], even for unequally spaced rods on each side of a given pair.

It should be noted that a graph similar to Fig. 1 but differing by the factor $4/\pi$, has

Manuscript received May 24, 1965; revised June 2, 1965.

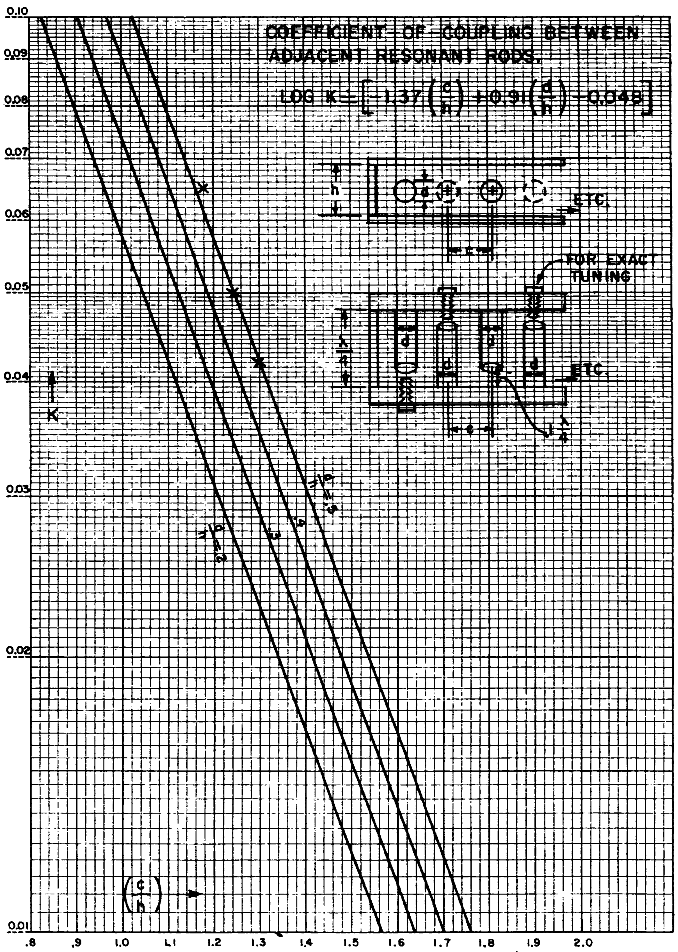

Fig. 1. Coefficient-of-coupling between adjacent resonant rods.

already been presented in [3], but its possible application to an insertion loss type of filter design was not presented.

III. Effect of End Wall Spacing e/h

Because of the method of generator and load coupling to be described in a later section, the end walls can be placed immediately adjacent to the first and last resonators.

With closely spaced end walls the z_0 of the first and last rods with all other rods grounded is given closely by later presented (9).

To obtain the required c_{12}/h, substitution in the basic coefficient of coupling definition gives (5)

$$K_{m_{12}} = K_{12}\sqrt{\frac{Y_{01}}{Y_0}} \tag{5}$$

where $K_{m_{12}}$ is the K of Fig. 1, and K_{12} is the value called for by the synthesis procedure, and Y_0 and Y_{01} are given by (4) and (9). It is worth noting that so long as e/h is less than 0.7, the effect of the square root factor is well less than 2 percent for all values of d/h up to 0.5.

IV. Application of Fig. 1 to a Design Example

All design procedures require that a previously accomplished synthesis give the required normalized coefficients-of-coupling, or their equivalents. In Chapter 7 of [5], and in [6], tables of these required, normalized-K's (k) are given for Butterworth, Chebyshev, maximally linear-phase, and Gaussian-amplitude responses; and in Dishal [7] closed-form design equations are given for the normalized-K's required for any n to produce the Butterworth and Chebyshev responses with "lossless" resonators. It is important to realize that in the above three references, the required normalized-K's (k) are normalized to the 3-dB down fractional bandwidth, and to obtain the required actual K from them they are multiplied by $(BW_{3\,\mathrm{dB}}/f_0)$, i.e.,

$$K = k\left(\frac{BW_{3\,\mathrm{dB}}}{f_0}\right). \tag{6}$$

Once one knows the various K's required for a filter, the graph of Fig. 1 [or (1) or, less exactly, (3)] is all that is needed to design the center to center rod spacings in most interdigital filters.

It is apparently standard operating procedure to present a design example for Matthaei's 6 resonator, 10 percent bandwidth, 0.1-dB Chebyshev response shape [8], [4], [1], and we will follow this procedure.

First, let us realize that the 10 percent bandwidth referred to in [8], [4], and [1], is not the 3-dB down-bandwidth, but is the so-called "ripple-bandwidth," which, as Curve 5 on Fig. 9 in Chapter 7 of [5] shows, is 0.914 times the 3-dB down-bandwidth.[1] Thus, the filter involved is to have a 3-dB down percentage bandwidth of 10.95 percent. Next, Fig. 29 in Chapter 7 of [5] shows that for the 0.1-dB ripple response shape, the required 3-dB down normalized-K's are $k_{12}=0.716$, $k_{23}=0.539$, and $k_{34}=0.518$.

The part of the design which is the subject of this section, is then accomplished by the three steps given in the following Table I.

If, the exact K values produced by this design are calculated, we find that the foregoing extremely simple procedure has resulted in errors in K_{23} and K_{34} of less than one percent, errors which are completely tolerable in practice.

Because in Cristal's example only rods 2 and 3 end up with very closely the same diameter, the normalized center-to-center rod spacings (c_{23}/h) and (c_{34}/h) are the ones which should be compared—his values are 1.114 and 1.125, respectively.

V. Correct Loading of the First and Last Resonators by "Tapping"

As indicated in [2] and [7], all coupled-resonator ladder filters of the type considered here require that the first resonator and the last resonator be resistively loaded so that their singly loaded-Q's are set at a value specified by the network synthesis part of the overall design procedure. Along with the required values for the normalized-K's, [2], [5]–[7] give the normalized required values for these singly loaded end Q's. The values given, (q) are normalized to the 3-dB down "fractional mid-frequency" ($f_0/BW_{3\,\mathrm{dB}}$), and to obtain the actual required Q (7) is used

$$Q = q\left(\frac{f_0}{BW_{3\,\mathrm{dB}}}\right). \tag{7}$$

For tap points which are located on the bottom 20 percent of a $\lambda/4$ transmission line, (8) gives the relationship between the tap point and the resulting singly loaded Q (page 576 of [5])

$$Q = \frac{\pi}{4}\left(\frac{R}{Z_0}\right)\left\{1/\sin^2\left(\frac{\pi}{2}\,\frac{l}{L}\right)\right\}. \tag{8}$$

For convenience this equation is graphed on Fig. 2.

It is thus necessary to know the Z_0 of resonator #1 and #n with all other resonators short-circuited. With small percentage couplings the effect of short-circuited rod #2, and short-circuited rod #$(n-1)$ on this Z_0 is, practically speaking, negligible. Thus, with the surface of the end walls a normalized distance e/h from the center of the end rods, (9) can be used to approximate these Z_0's (page 530 of [5]).

$$Z_{0_1} \doteq 138 \log\left\{\frac{4}{\pi}\left(\frac{h}{d}\right)\tanh\left(\frac{\pi e}{h}\right)\right\}. \tag{9}$$

[1] Because of the lack of divisions on these figures, three significant figures cannot be read—the exact value was obtained from (15b) of [9] which shows that the relationship between these two bandwidths is

$$\frac{BW_{3\,\mathrm{dB}}}{BW_\beta} = \cosh\left\{\frac{1}{n}\cosh^{-1} 1/[(V_p/V_v)^2 - 1]^{1/2}\right\}.$$

TABLE I

Required K's	Based upon max unloaded-Q, or availability, or etc., pick a normalized rod diameter (d/h): To compare with Cristal we will pick 0.350. Fick a normalized end wall spacing e/h: we will pick 0.6; therefore from (5) $(K_m/K)_{12} = 1.02$	From Fig. 1
$K_{12} = 0.716 \times 0.1095 = 0.0784 = K_{56}$		$\left(\frac{c}{h}\right)_{12} = 1.01 = \left(\frac{c}{h}\right)_{56}$
$K_{23} = 0.539 \times 0.1095 - 0.0589 = K_{45}$		$\left(\frac{c}{h}\right)_{23} = 1.11 = \left(\frac{c}{h}\right)_{45}$
$K_{34} = 0.518 \times 0.1095 = 0.0566$		$\left(\frac{c}{h}\right)_{34} = 1.12$

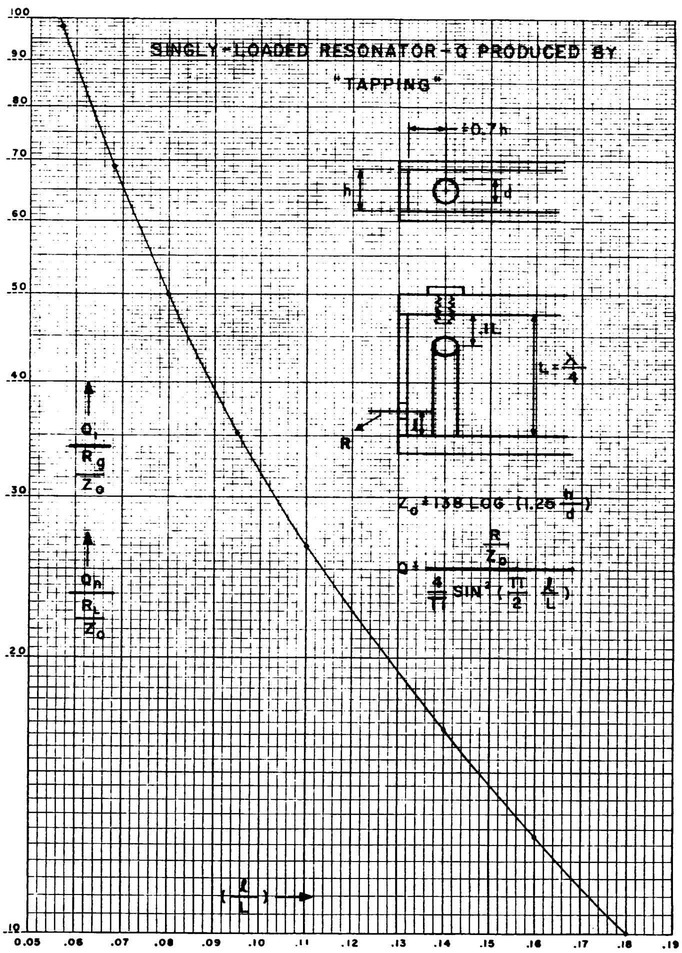

Fig. 2. Singly loaded resonator Q produced by "tapping."

Examination of this equation shows that even with d/h as large as 0.5, the end walls also have negligible practical effect, i.e., less than 2 percent, on Z_0, so long as the e/h used in Table I is greater than 0.75. Within this limit the Z_0 equation simplifies to that given on Fig. 2; if appreciably closer end-wall spacings are used, the full (9) is used. For most practical filter requirements, correction terms for (9) are not required.

Continuing with the $n=6$ design example: the synthesis information on Fig. 29 in Chapter 7 of [5] shows that for the 0.1-dB ripple shape, the 3-dB down singly loaded normalized end-Q's must be $q_1 = q_6 = 1.27$; therefore, $Q_1 = Q_6 = 1.27 \times 9.13 = 11.6$. Substitution of the normalized rod diameter used in our example, i.e., $d/h = 0.350$, in the Z_0 equation on Fig. 2, or in (9), gives $Z_0 = 76$ ohms. If the usual 50-ohm generator is to drive the filter then $Q_1/(R_1/Z_0) = 17.7$; and Fig. 2, or (8), shows that the tap point should be located at $0.136L$. Use of the known load resistance in the step just outlined will then give the required tap point on rod #6.

VI. Correct Tuning of Each Resonator

The final design parameter which must be correctly adjusted is the resonant frequency of each rod (with all other rods short circuited). For good performance each resonance should not differ from the correct frequency by more than 10 percent (approximately) of the 3-dB bandwidth. For small percentage bandwidth filters this is certainly the most important and most critical of the three parameters which must be adjusted. The tuning procedure given in [2] is, of course, directly applicable to interdigital filters. If it is undesirable to mount a loosely coupled alignment probe near the input resonator, it is worth pointing out the perhaps obvious fact that an exactly equivalent alignment procedure is as follows:

1) Feed the f_0 signal to the filter through a slotted line.
2) Short-circuit all rods by means of fine tuning screws.
3) Set the probe of the slotted line very accurately at the location of a zero on the slotted line, and lock the carriage in this position.
4) Tune the first rod for maximum output from the slotted line probe.
5) Tune the second rod for minimum output from the slotted line probe.
6) Third rod for maximum output. Fourth rod for minimum output, etc., etc.

If an interdigital filter having no fine tuning adjustments is desired, then in the engineering model the line lengths must be adjusted so that the preceding phenomenon occurs when the fine tuning screws are just flush with the inner surface of the wall on which they are mounted.

In conclusion I should add (with tongue in cheek?) that one can start, with no geometry calculations whatsoever, and successfully and exactly adjust each K and Q_1 and Q_n by the procedures described in Dishal [2]. In fact, if the pass band performance of the completed filter is not satisfactory, the experimental procedures of [2], or their equivalent, must be used to experimentally check each K and Q_1 and Q_n; this will at least guarantee that the physical filter is truly supplying the element values called for by the synthesis part of the overall design procedure.

M. Dishal
ITT Federal Labs.
Nutley, N. J.

References

[1] P. Vadopalas, with rebuttal by E. G. Cristal, "Coupled rods between ground planes," *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-13, pp. 254–255, March 1965.
[2] M. Dishal, "Alignment and adjustment of synchronously tuned multiple-resonant-circuit filters," *Proc. IRE*, vol. 39, pp. 1448–1455, November 1951. Also, *Elec. Commun.*, vol. 29, pp. 154–164, June 1952.
[3] J. T. Bolljahn, and G. L. Matthaei, "A study of the phase and filter properties of arrays of parallel conductors between ground planes," *Proc. IRE*, pp. 299–311, March 1962.
[4] E. G. Cristal, "Coupled circular cylindrical rods between parallel ground planes," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 428–439, July 1964.
[5] International Telephone and Telegraph Corp., *Reference Data for Radio Engineers*, 4th Ed. New York: American Book—Stratford Press, Inc.
[6] M. Dishal, "Gaussian-response filter design," *Elec. Commun.*, vol. 36, no. 1, pp. 3–26, 1959.
[7] ——, "Two new equations for the design of filters," *Elec. Commun.*, vol. 30, pp. 324–337, December 1952.
[8] G. L. Matthaei, "Interdigital band-pass filters," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 479–491, November 1962.
[9] M. Dishal, "Design of dissipative band-pass filters producing desired exact amplitude-frequency characteristics," *Proc. IRE*, vol. 37, pp. 1050–1069, September 1949. Also, *Elec. Commun.*, vol. 27, pp. 56–81, March 1950.