Physical bounds and automatic design of antennas above ground planes

Casimir Ehrenborg*, and Mats Gustafsson*

*Electrical and Information Technology, Lund University, Sweden
e-mail: casimir.ehrenborg@eit.lth.se

***Abstract*—Physical bounds for antennas above ground planes are calculated by optimizing the antenna current. The bounds are compared with antennas modeled as fragmented patches and optimized using genetic algorithms. Monopole structures over ground planes are modeled with image theory and optimized. The monopole over ground plane structure simplifies experimental verification of the bounds.**

I. Introduction

Physical bounds for antennas establish theoretical performance standards in terms bandwidth and gain for restricted design volumes [1]. The bounds have been verified numerically for spherical and planar structures in [2]–[7], [18]. Experimental verification is more challenging [2], [3] and is the goal of this contribution. The physical bounds are defined for strict volumes, thus it has hitherto been problematic to establish satisfactory verification beyond simulation results.

In [9] physical bounds for antennas over ground planes are analyzed. The ground plane solves many measurement related issues by shielding the antenna from cables and other components, letting the results portray only the volume of interest. Thus this contribution investigates optimizing monopole structures over ground planes.

To reach the physical bounds, the presented antennas are designed and optimized with genetic algorithms [10], [11]. The antennas are planar and built in a 'pixelated' structure, *i.e.*, constructed from small metal squares. Corner to corner connections, which can be an issue between Method of Moments (MoM) and other simulation techniques, are resolved by chamfering connections according to [12].

II. Stored Energy

Stored electromagnetic energy is the energy of the fields which are generated by the antenna structure but never escape its vicinity. Calculating this energy is notoriously difficult due to the ambiguity in the definition of the radiated energy in the near field of the antenna. However, the stored energy is an essential component in small antenna optimization through its connection to the Q-factor, and thus bandwidth [13], [14],

$$Q = \frac{2\omega\max\{W_\mathrm{e}, W_\mathrm{m}\}}{P_\mathrm{d}},$$

where W_e is the stored electric energy, W_m is the stored magnetic energy, and P_d is the dissipated power. The stored energy is calculated from the impedance matrix of the antenna, derived through the MoM formulation of the electrical field integral equation (EFIE) [15]. The impedance matrix, $\mathbf{Z} = \mathbf{R} + \mathrm{j}\mathbf{X}$, can be divided as,

$$\mathbf{Z} = s\mu\mathbf{L} + \frac{1}{s\epsilon}\mathbf{C}_\mathrm{i} \quad \text{and} \quad \mathbf{Z}\mathbf{I} = (s\mu\mathbf{L} + \frac{1}{s\epsilon}\mathbf{C}_\mathrm{i})\mathbf{I} = \mathbf{B}V_\mathrm{in},$$

where the matrices $\mathbf{L}$, $\mathbf{C}_\mathrm{i}$ depend on the frequency $s = \mathrm{j}\omega$. A voltage state $\mathbf{U} = \frac{1}{s\epsilon}\mathbf{C}_\mathrm{i}\mathbf{I}$ is introduced to write the state space formulation,

$$s\begin{pmatrix}\mu\mathbf{L} & \mathbf{0}\\ \mathbf{0} & \epsilon\mathbf{C}\end{pmatrix}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\end{pmatrix} = \begin{pmatrix}\mathbf{0} & -\mathbf{1}\\ \mathbf{1} & \mathbf{0}\end{pmatrix}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\end{pmatrix} + \begin{pmatrix}\mathbf{B}\\ \mathbf{0}\end{pmatrix}V_\mathrm{in}.$$

The stored energy of the state space representation is calculated by the quadratic form of the first matrix,

$$\begin{aligned} W &= \frac{\mathrm{Re}}{4}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\end{pmatrix}^\mathrm{H}\begin{pmatrix}\mu_0(\mathbf{L}+\mathrm{j}\omega\mathbf{L}') & \mathbf{0}\\ \mathbf{0} & \epsilon_0(\mathbf{C}+\mathrm{j}\omega\mathbf{C}')\end{pmatrix}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\end{pmatrix}\\ &= \frac{\mathrm{Re}}{4}\big(\mu_0\mathbf{I}^\mathrm{H}(\mathbf{L}+\mathrm{j}\omega\mathbf{L}')\mathbf{I} + \epsilon_0\mathbf{U}^\mathrm{H}(\mathbf{C}+\mathrm{j}\omega\mathbf{C}')\mathbf{U}\big)\\ &= \frac{\mathrm{Re}}{4}\mathbf{I}^\mathrm{H}\big(\mu_0(\mathbf{L}+\mathrm{j}\omega\mathbf{L}') + \frac{1}{\omega^2\epsilon_0}(\mathbf{C}_\mathrm{i}-\mathrm{j}\omega\mathbf{C}_\mathrm{i}')\big)\mathbf{I} = \frac{1}{4}\mathbf{I}^\mathrm{H}\frac{\partial\mathbf{X}}{\partial\omega}\mathbf{I}\end{aligned}$$

where the matrices are differentiated with regards to frequency. Thus giving the classical estimation based on differentiation of the reactance matrix as the stored energy [15]–[17].

III. Ground Plane

When measuring small antenna performance experimentally it is challenging to distinguish between different sources of the field. However, physical bounds for antennas are defined for strict volumes, thus measuring contributions from cables and other sources would invalidate the measurement. In this contribution the antennas are situated above a ground plane to screen the measurement from unwanted radiation. The ground plane is modeled with image theory, simulating a mirrored dipole to reflect the monopole structure over a ground plane. The equivalent dipole structure retains the same Q-factor of the monopole over a ground plane [4].

IV. Physical Bounds

Antenna performance can be bounded in regards to size of the design volume. This represents the theoretically optimal current constellation for the given design volume [18]. To calculate the bound a convex optimization problem is solved,

$$\begin{aligned} &\text{minimize} \quad \max\{\mathbf{I}^\mathrm{H}\mathbf{X}_\mathrm{e}\mathbf{I}, \mathbf{I}^\mathrm{H}\mathbf{X}_\mathrm{m}\mathbf{I}\}\\ &\text{subject to} \quad \mathbf{F}\mathbf{I} = 1,\end{aligned}$$

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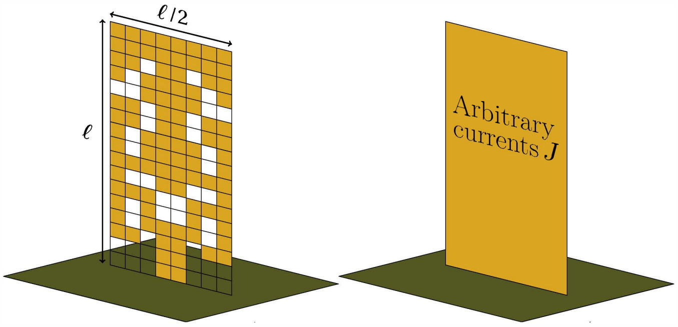

Fig. 1. To the left, a conceptual example of an antenna designed using the pixelized methodology. To the right, the equivalent problem solved to find the performance bounds.

where $\mathbf{X}_\mathrm{e}$ is electric reactance matrix, $\mathbf{X}_\mathrm{m}$ is magnetic reactance matrix and $\mathbf{F}$ is the farfield [17], [19]. By minimizing the stored energy expressions subject to a fixed radiation intensity the G/Q is maximized. The bound can also be calculated for a monopole using sum rules, see [4].

V. Genetic Algorithm

The genetic algorithm [10], [11] implemented is an adaptation of Holter's code in [20], see also [5], [6]. The code utilizes a performance based tournament selection, where 40% of the population compete to be one of two reproducing parents. For every coupling two children are produced and returned to the population pool, to keep a constant population rate the two least fit individuals are removed from the pool. Crossover at pairing happens in 10% of bits with a probability of 0.8 and mutation rate is 0.4. Individuals are constructed as matrices of ones and zeros where each one represents a square of metal in the planar structure, creating a pixel antenna structure see Fig 1. The structures are evaluated with a MoM code.

A. Example

By optimizing the antenna for bandwidth the resulting structures reach the performance bounds over a wide frequency band, see Fig 2. Changing the evaluation metric of the genetic algorithm, other quantities can be optimized for as well.

In MoM corner to corner metal structures are treated as unconnected. In other solvers, such as FDTD, and in reality a small electric connection exists. To manufacture an antenna in accordance with the MoM solutions these connections have been severed. By chamfering the corners the capacitive loading is maintained but no current flows across [12].

VI. Conclusions

GA optimized antennas have been used in the past to numerically verify physical bounds [4], [5]. Investigating similar structures, but shielded by a ground plane, this contribution enables further verification of this well established concept.

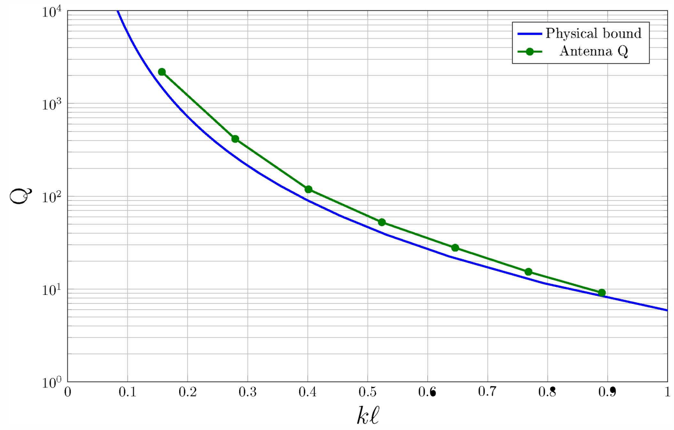

Fig. 2. Q values of optimized structures in relation to the Q-bound for planar structures.

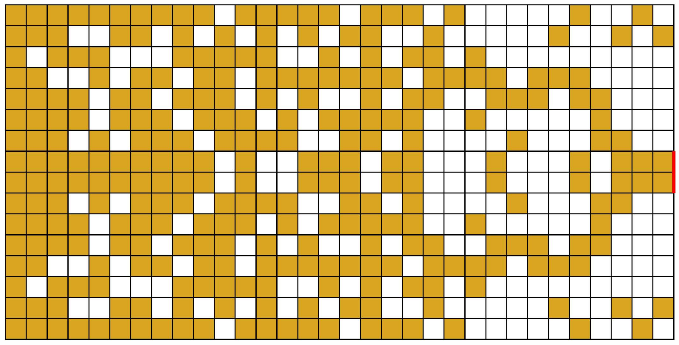

Fig. 3. Example of a monopole antenna generated by the genetic algorithm to reach the Q-bound in Fig 2. The red line illustrates the feeding edge.

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References

[1] M. Gustafsson, D. Tayli, and M. Cismasu, *Physical bounds of antennas. In: Handbook of Antenna Technologies.* Springer-Verlag, 2016, pp. 1–32.

[2] S. R. Best, "The radiation properties of electrically small folded spherical helix antennas," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 4, pp. 953–960, 2004.

[3] ——, "Electrically small resonant planar antennas: Optimizing the quality factor and bandwidth." *Antennas and Propagation Magazine, IEEE*, vol. 57, no. 3, pp. 38–47, 2015.

[4] M. Gustafsson, C. Sohl, and G. Kristensson, "Illustrations of new physical bounds on linearly polarized antennas," *IEEE Trans. Antennas Propagat.*, vol. 57, no. 5, pp. 1319–1327, May 2009.

[5] M. Cismasu and M. Gustafsson, "Antenna bandwidth optimization with single frequency simulation," *IEEE Trans. Antennas Propagat.*, vol. 62, no. 3, pp. 1304–1311, 2014.

[6] ——, "Multiband antenna Q optimization using stored energy expressions," *IEEE Antennas and Wireless Propagation Letters*, vol. 13, no. 2014, pp. 646–649, 2014.

[7] M. Shahpari, D. V. Thiel, and A. Lewis, "An investigation into the gustafsson limit for small planar antennas using optimization," *Antennas and Propagation, IEEE Transactions on*, vol. 62, no. 2, pp. 950–955, 2014.

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[8] M. Capek and L. Jelinek, “Optimal composition of modal currents for minimal quality factor q,” *ArXiv*, 2016.

[9] D. Tayli and M. Gustafsson, “Physical bounds for antennas above a ground plane,” *IEEE Antennas and Wireless Propagation Letters*, 2016.

[10] Y. Rahmat-Samii and E. Michielssen, *Electromagnetic Optimization by Genetic Algorithms*, ser. Wiley Series in Microwave and Optical Engineering. John Wiley & Sons, 1999.

[11] J. M. Johnson and Y. Rahmat-Samii, “Genetic algorithms and method of moments GA/MOM for the design of integrated antennas,” *IEEE Trans. Antennas Propagat.*, vol. 47, no. 10, pp. 1606–1614, oct 1999.

[12] D. V. Thiel, M. Shahpari, J. Hettenhausen, and A. Lewis, “Point contacts in modeling conducting 2d planar structures,” *IEEE Antennas and Propagation Letters*, 2015.

[13] A. D. Yaghjian and S. R. Best, “Impedance, bandwidth, and Q of antennas,” *IEEE Trans. Antennas Propagat.*, vol. 53, no. 4, pp. 1298–1324, 2005.

[14] J. Volakis, C. C. Chen, and K. Fujimoto, *Small Antennas: Miniaturization Techniques & Applications*. New York: McGraw-Hill, 2010.

[15] G. A. E. Vandenbosch, “Reactive energies, impedance, and Q factor of radiating structures,” *IEEE Trans. Antennas Propagat.*, vol. 58, no. 4, pp. 1112–1127, 2010.

[16] R. F. Harrington and J. R. Mautz, “Control of radar scattering by reactive loading,” *Antennas and Propagation, IEEE Transactions on*, vol. 20, no. 4, pp. 446–45, 1972.

[17] M. Gustafsson, D. Tayli, C. Ehrenborg, M. Cismasu, and S. Nordebo, “Tutorial on antenna current optimization using MATLAB and CVX,” Lund University, Tech. Rep., 2015. [Online]. Available: http://lup.lub.lu.se/record/8412159/file/8412166.pdf

[18] L. Jelinek and M. Capek, “Optimal currents on arbitrarily shaped surfaces,” *arXiv*, 2016.

[19] M. Gustafsson and S. Nordebo, “Optimal antenna currents for Q, superdirectivity, and radiation patterns using convex optimization,” *IEEE Trans. Antennas Propagat.*, vol. 61, no. 3, pp. 1109–1118, 2013.

[20] B. Thors, H. Steyskal, and H. Holter, “Broad-band fragmented aperture phased array element design using genetic algorithms,” *IEEE Trans. Antennas Propagat.*, vol. 53, no. 10, pp. 3280 – 3287, oct. 2005.

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