Infinite Phased Array Analysis Using FDTD Periodic Boundary Conditions - Pulse Scanning in Oblique Directions

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Abstract

A novel technique to implement periodic boundary conditions for unit cell analysis of infinite phased arrays in the finite difference time domain (FDTD) is presented. It allows for pulse excitation and oblique scan directions in both the cardinal and inter cardinal planes. The ordinary Yee lattice is used which makes the algorithm easy to incorporate in an already existing FDTD code

The Problem of Unit Cell Analysis in FDTD

Unit cell analysis in the time domain is difficult for the following reason. Consider the unit cell in Fig. 1a, and assume that the array is scanned in the y=0 plane to an angle θ in the positive x-direction. The fields at boundary C and D are then identical at every point of time. Therefore, simple time independent boundary conditions can be applied. However, at boundaries A and B the tangential fields are related according to $f(x_B,y,z,t)=f(x_A,y,z,t-\tau_x)$ and $f(x_A,y,z,t)=f(x_B,y,z,t+\tau_x)$, where $\tau_x=D_x\sin\theta/c$ is the excitation time delay between adjacent unit cells and c is the speed of light. The first relation implies that tangential field values at boundary B are obtained from time delayed values at boundary A, which are simply saved in a buffer for later use. The second relation implies that time advanced field values from boundary B are used at boundary A. This poses a major problem, since the time advanced values at time $t+\tau_x$ are not known at time t.

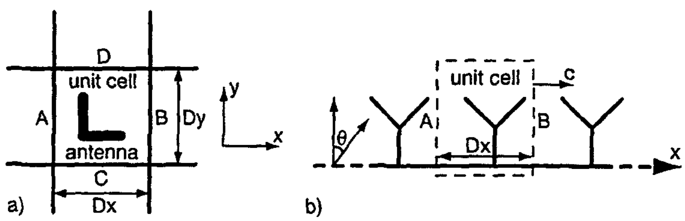

Figure 1. a) Top view of one element in a 2-D infinite periodic array. b) Side view of a 1-D infinite periodic array.

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Formulation of the Technique

In this section, the technique for oblique pulse scanning is formulated. The first subsection treats scanning in the cardinal planes [1]. In the second subsection, inter cardinal plane scanning is considered [2].

Cardinal Scanning

Consider the 1-D linear infinite array in Fig. 1b and assume that the array is scanned to an angle θ so that the pulse excitation appears to move with the phase velocity $c/\sin\theta$ along the array. The dashed rectangle represents a 3-D unit cell, modelled in FDTD. The idea is now to move this cell with the speed of light in the x-direction. This is accomplished by shifting the FDTD volume in the computer memory. The fields radiating from the infinitely many array elements to the left of boundary A will never reach the moving cell. Reflections from boundary A will never catch up with the moving unit cell, so boundary A needs not be considered. The effect is that we do not need time advanced field values on boundary A. At boundary B time delayed values obtained from an earlier time point in the FDTD computation are applied.

At the start of the FDTD simulation all fields are zero in the moving unit cell. The phase velocity $c/\sin\theta$ of the antenna excitation is always larger than the speed c when $\theta<90^0$ and as time goes on, the pulse excitation will pass the moving unit cell. Element currents and voltages are calculated for the element, which happens to be in the moving cell. Therefore, at the end of the FDTD simulation, a simple post processing of the stored element currents and voltages must be performed in order to obtain the currents and voltages for a particular element in the array.

Example: With reference to Fig. 1b, assume that D_x=0.5 m, FDTD-cell size $\Delta x=\Delta y=\Delta z=D_x/10=0.05$ m and time step $\Delta t=\Delta x/(2c)$. The time for a signal to travel the distance D_x at the speed of light is therefore $20\Delta t$. The distance D_x is divided into 10 FDTD cells. After 20 time steps, the moving unit cell should have moved the distance D_x. Therefore, moving the unit cell with the speed of light is accomplished by shifting the FDTD volume in the computer memory every second time step. Further, let N be the excitation time delay τ_x between adjacent unit cells divided by the time step Δt used in FDTD $N=\tau_x\Delta t=(D_x\sin\theta/c)/(/\Delta x/2c)=20\sin\theta=N_0$ $\sin\theta$. In order to reuse earlier computed values the excitation time delay τ_x must be an integer multiple of the time step Δt. Possible scan angles θ are given by the requirement of N being an integer greater than zero and less than N_0=20. N=0 corresponds to broadside scanning, which is not possible with the current technique. However, this is not a limitation, since at broadside, simple time independent boundary conditions may be used with a stationary unit cell. The post processing that is necessary to obtain currents and voltages for a single element in the array is performed by erasing N time steps of the current and voltage in every N_0 time step cycle, because they contain no new information.

Extension to 2-D arrays is accomplished by using time independent boundary conditions. Consider Fig. 1a, the tangential field at corresponding points on boundary C and D are then the same, and therefore a simple time independent boundary condition applies. The time independent boundary condition is implemented in FDTD by wrapping the mesh around itself.

Inter Cardinal Plane Scanning

In this section, inter cardinal plane scanning is treated. Although the cardinal scanning technique could be used, it would work only for certain lattice dimensions. Therefore, we describe a modified version of the cardinal scanning technique which applies for arbitrary rectangular lattices, and probably also applies to any type of periodic lattices.

The scanning technique is based on the possibility to construct primitive cells, which are alternative unit cells in the periodic lattice. There are many ways to choose a primitive cell for a given lattice. Fig. 2 shows some primitive cells in a periodic rectangular lattice with dimension $D_x \times D_y$. The shape of the primitive cell determines our inter cardinal scan plane, which is perpendicular to the sides A and B. Thus φ denotes the angle between the x-axis and the scan plane. Instead of moving the unit cell with the speed of light c in the scan direction, the unit cell is now moved with a speed larger than c along the x-direction. The discrete set of angles φ is easily found to be $\varphi=\arcsin\sqrt{(1+(D_x/nD_y)^2)^{-1}}$, $n=1,2,3,\ldots$. Additional angles can be obtained by moving the unit cell along the y-axis.

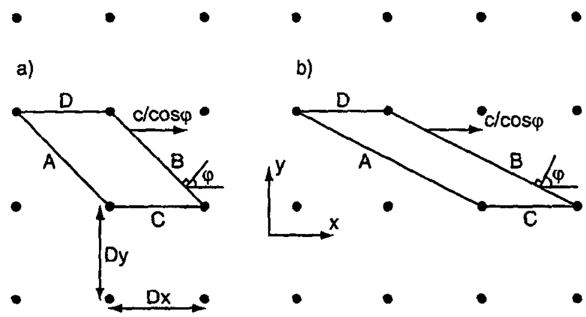

Figure 2. Example of unit cells in a periodic lattice with dimension $D_x \times D_y$. The array is scanned in the direction (θ, φ) by moving the unit cell with the speed $c/\cos\varphi$ along the positive x-direction.

Example: With reference to Fig. 2 and Fig. 1a, D_x=0.4 m, D_y=0.55 m. The FDTD cell-size Δx=0.04 m, Δy=0.055 m and Δz=0.05 m. The unit cell shown in Fig. 2a is used in the example and the analysis is for the scan direction (θ, φ). The excitation time delay between adjacent unit cells (in the x-direction) is $\tau_x=D_x\sin\theta\cos\varphi/c$. The FDTD timestep Δt should be chosen in a way which makes the moving of the unit cell in the computer memory as easy as possible, e.g. let the FDTD time step be $\Delta t=\Delta x\cos\varphi/2c$. This time step satisfies the CFL stability criterion. As before, define N as the excitation time delay divided by the time step $N=\tau_x/\Delta t=20\sin\theta=N_0\sin\theta$. Possible scan angles θ are again given by the requirement of N being an integer greater than zero and less than N_0=20.

The distance D_x comprises 10 FDTD cells and thus after N_0=20 time steps, the moving unit cell will have moved the distance D_x. Therefore the unit cell must be moved in the computer memory every second time step. The speed of the moving unit cell is independent of Δt and is given by $v=D_x/N_0\Delta t=c/\cos\varphi>=c$. This implies that the projection of v onto the φ plane is equal to the speed c. The case $v(\varphi=0)=c$ corresponds to cardinal scanning.

As before, moving of the unit cell is accomplished by shifting the FDTD volume in the computer memory and the boundary B is updated with saved field values. The tangential fields at corresponding points on boundary C and D are the same. Therefore, a simple time independent boundary condition is used at those boundaries. However, at first glance, one might suspect that the boundary A in Fig. 2a could be influenced by propagating fields coming from the negative y-direction because of the slope of the boundary A. However, this does not happen, since the projected unit cell speed v onto the φ-plane is equal to the speed of light.

Validation

Using the new technique we calculated the active impedances for an infinite periodic 2-D dipole array scanned in both cardinal and inter cardinal planes. Validation was performed via the 'element by element' approach, i.e. by a conventional FDTD simulation of a corresponding large, finite array. Non periodic boundaries were truncated by Berenger's perfectly matched layer [3]. The following data are common to all simulations: D_x=0.4 m, D_y=0.55 m. FDTD cell size Δx =0.04 m, Δy =0.055 m and Δz =0.05 m. The dipoles, pointing in the y-direction and fed at the midpoints, were modelled by setting nine collinear electric field components along subsequent FDTD cells equal to zero. The finite array contained 45×45=2025 dipoles. The active impedance is calculated for the center element of the array. A simple feed model, which incorporates a series resistance of 50 Ohm, was used for the dipoles [4]. Fig. 5 shows the active impedance for scan angles $(\theta, \varphi)=(64.2^0, 0^0)$ and $(\theta, \varphi)=(44.4^0, 71.0^0)$.

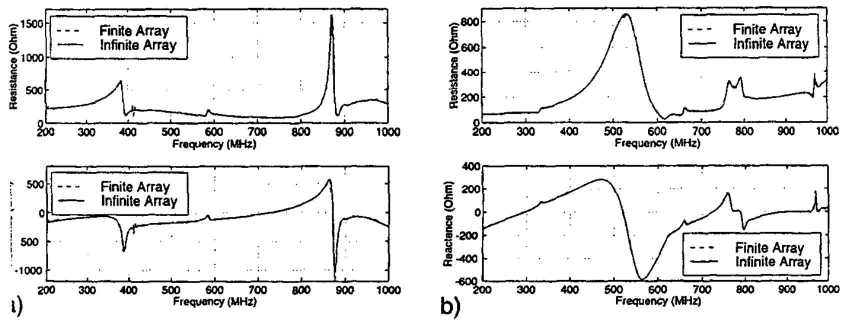

Figure 3. Active impedance for finite (45×45 antennas) and infinite array. a) Scan direction $\theta=64.2^0$, $\varphi=0^0$. b) Scan direction $\theta=44.4^0$, $\varphi=71.0^0$.

Summary

A new technique for FDTD analysis of obliquely scanning, pulsed array antennas has been presented. It reduces the computational volume to a single unit cell of the array, and therefore is computationally highly efficient. The technique has been successful applied to a 2-D array of dipoles scanned in both cardinal and inter cardinal planes.

References

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