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Design Considerations for Pixelated Antennas Regarding Pixel Size and Symmetry

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Abstract— **Pixelated structures offer a unique possibility to design and optimize antennas tailor-made for very specific and challenging application scenarios, where conventional antenna design may lead to unsatisfactory results. Several different optimization techniques can be applied to these structures, depending on the desired goals and applications, making pixelated antennas very versatile fundamental structures. In this study, we investigate the optimization of pixelated antennas with a focus on the effects of pixel size and asymmetrical design approaches on gain. By simulation techniques and optimization algorithms, including genetic algorithms and multi-objective frameworks, we analyze how these design parameters influence the antenna's performance in terms of gain. Our findings reveal that smaller pixel sizes enhance the gain of large antennas, while asymmetrical designs offer advantages over symmetrical configurations.**

Keywords— **Antennas, Genetic algorithm, Particle swarm optimization, Evolutionary computation, Fragmented**

I. Introduction

The evolution of antenna technology has witnessed significant advancements in the past decades, to satisfy the demands of modern radio frequency (RF) applications. A particularly interesting subset of antennas are pixelated antennas, which are characterized by their modular structure composed of discrete elements (pixels). They offer an innovative approach to antenna design, fabrication, and performance optimization. This concept utilizes the power of various optimization algorithms and advanced manufacturing techniques, such as additive manufacturing, to create antennas with highly customized radiation patterns, improved efficiency, and enhanced operational bandwidths.

One of the most compelling aspects of pixelated antennas is their adaptability. Through the manipulation of individual pixels or voxel elements, designers can tailor the antenna's properties to meet specific application requirements, from telecommunications to sensor networks and beyond. This level of control enables the fine-tuning of performance characteristics, including impedance matching, gain, and directivity, which are parameters for the efficiency and reliability of wireless communication systems.

Furthermore, the integration of optimization algorithms like genetic algorithms and deep neural networks with pixelated antenna design signifies a paradigm shift in how antennas are conceived and realized. By leveraging genetic algorithms and other optimization strategies, the design process transcends traditional limitations, exploring a vast solution space to discover antenna configurations that offer optimal performance. This automated, intelligent exploration of design possibilities not only accelerates the development cycle but also unveils innovative antenna structures that might not be intuitive through conventional design methodologies.

The manufacturing of pixelated antennas, particularly through laser powder bed fusion (LPBF) and other additive manufacturing techniques, underscores another dimension of their potential. Additive manufacturing enables the production of complex, three-dimensional structures with precision and flexibility, allowing for the realization of pixelated antenna designs that would be challenging, if not impossible, to fabricate using traditional methods. This capability opens up new avenues for integrating antennas into a variety of substrates and devices, including those with intricate shapes or limited space, further broadening the application spectrum of pixelated antennas [1].

In the past different algorithms in the realm of artificial intelligence were used to optimize pixelated or voxelated structures. Zechmeister et. al. focused on a novel algorithm like Binary Ink Stamp Optimization [2]. The ability of the Binary Ink Stamp Optimization was demonstrated over fifteen benchmark functions and compared with the results of six state-of-the-art optimization algorithms like genetic algorithms (GA), Binary Particle Swarm Optimization (BPSO), Binary Covariance Matrix Adaptation Evolution Strategy (BCMAES), Binary Bat Algorithm (BBA), Binary Dragonfly Algorithm (BDA) and Multi-Verse Optmization (MVO). However, they did not optimize the parameters of the other algorithms (like mutation and recombination rate of the GA) and the algorithm was only tested for the very specific case of a pixelated horn antenna. Mair et. al. used different kinds of genetic algorithms to optimize antennas for structural health monitoring inside concrete and to optimize three-dimensional volxelated antennas [3], [1]. Gjokaj et. al. optimized a pixelated patch with the help of genetic algorithms [4], Ranjan et. al. optimized a pixelated metamaterial absorber with the help of binary wind driven optimization algorithm [5], Li et. al. used multi objective BPSO to optimize pixelated patches [6], Ghadimi et. al. used BPSO to optimize pixelated structures for reducing mutual coupling between two patch antennas [7], Jacobs et. al. used deep neural network regression models to predict the resonance frequency of pixelated patches [8], Lee et. al. used genetic learning particle swarm optimization

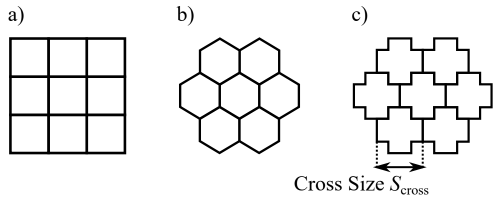

Fig. 1. Examples of different pixel shapes with a) rectangular, b) hexagonal, and c) cross-shaped pixels.

to generate a dielectric resonator antenna consisting out of individual bars [9], Karahan et. al. used deep learning based modeling for an inverse design of a pixelated patch antenna [10], Sun et. al. published the optimization of pixelated metamaterial antennas with elite-preserving genetic algorithms (EGAs) [11], Qiao et. al. used perturbation sensitivity analysis for the design of a reconfigurable pixelated patch antenna [12]. Although pixelated antennas were used in the past extensively and different kinds of algorithms have been employed, they rarely compared their algorithm to others and if comparisons were made the other algorithms parameters were not optimized. Also, the performance of pixelated antennas in general is rarely covered in literature [13], [14]. Therefore, this paper will highlight the dependency of gain - for pixelated antennas employing shifted cross-shaped elements - on different antenna and cross sizes as well as the influence of symmetry restrictions. These antennas are optimized with genetic algorithms. An introduction to genetic algorithms and pixelated antennas is given and new results - that show the dependency of gain and cross size can be eliminated by employing multi-objective optimization techniques - are presented.

II. Optimization Procedure

A. Pixelation

At the heart of pixelated antennas is the pixelation or - in the case of three-dimensional antennas - voxelation process, i.e. subdividing a pre-defined area into individual segments, i.e. pixels or voxels. This can be achieved - in principle - with any geometrical shape, depending on the employed simulation tool and specifically, imposed restrictions of the mesh. Three types of pixels commonly used in studies are shown in figure 1.

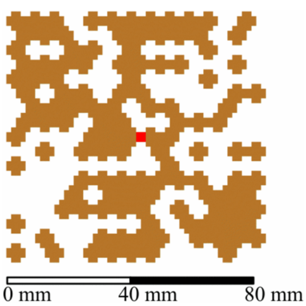

Fig. 2. IoT Antenna optimized for the 5G N20 band [15].

The user has to make sure that no singularities arise between individual pixels as they would occur with rectangular pixels in Figure 1 a). Pixels positioned diagonally adjacent to each other are limited to point contact, which compromises the accuracy of both simulations and fabrication outcomes. This configuration can introduce specific anomalies, resulting in connections that are poorly defined, exhibiting issues related to either excessive or insufficient conduction. Therefore, hexagons (Figure 1 b)), cross shapes (Figure 1 c)), or overlaps of the rectangular pixels are commonly used. Additionally, in the case of three-dimensional structures (or planar structures without a substrate), no floating elements can occur. In that case, region-growing algorithms have to be employed to guarantee manufacturability [1]. By tiling a predefined area with these pixels and switching individual elements "on" and "off", i.e. assigning them a Boolean operator "1" and "0", a (interconnected) antenna structure can be created as depicted in figure 2. In the case of planar structures a conductive element, or "1", can be analogous to a $35\,\mu\text{m}$ thick copper layer on top of the substrate. Therefore, an antenna structure can be encoded into a single bitstring which presents its phenotype. By changing the bitstring one can therefore change the antenna's structure and simulate it. Consequently by changing the bitstring and observing its properties, one can apply binary optimization techniques as presented in the introduction. The authors of this paper focus on their contribution to pixelated antennas regarding binary genetic algorithms which is why they are described in more detail in Section II-B.

B. Genetic Algorithm

In the past, the authors used single and multi-objective genetic algorithms for optimization. These are heuristic optimization techniques that fall under the category of evolutionary algorithms, leveraging evolutionary mechanisms such as selection, recombination, and mutation to achieve one or multiple objectives. The core principle is "survival of the fittest," yet there exist elite genetic algorithms (GAs) that consistently prefer individuals with superior fitness values (e.g. the `@ga` function in Matlab), as well as controlled elite GAs (e.g. the matlab `@gamultiobj` function) that also consider individuals with lower fitness if they contribute to increasing the diversity of the population.

At the onset of the algorithm, objectives (such as minimum scattering parameters, maximum efficiency, maximum antenna gain) and constraints (like antenna size, pixel size, substrate used, casing) are specified. Subsequently, a generation of P randomly generated antennas (with P being the population size) is produced, which are then simulated using programs like Ansys HFSS, CST Microwave Studio, or Sonnet Software. The simulation results enable the calculation of an objective function. For simple elitist methods, this objective function might correspond to a single value. When optimizing for a desired bandwidth, an objective function to be minimized

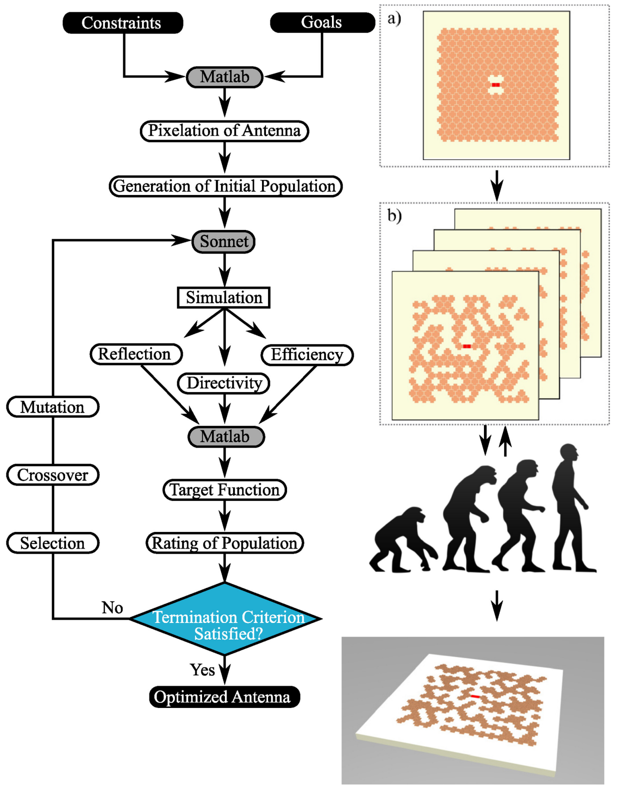

Fig. 3. Optimization flowchart. After the a) pixelation process the b) generation of initial antennas takes place followed by the genetic optimization until a termination criterion is satisfied [16].

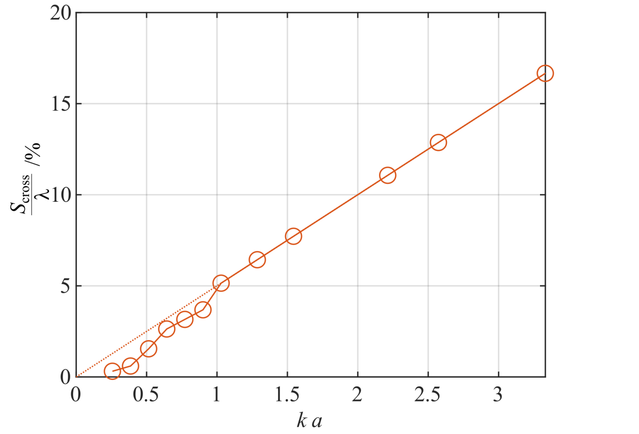

Fig. 4. Maximum possible cross size normalised to wavelength $S_{cross}/_{\lambda}$ for a successful optimisation in the course of five optimisations within 250 generations with respect to antenna size [13].

could be represented as used in [1]:

$$O(S) = \frac{1}{N}\sum_{i=1}^{N} \max\left(1, \left|\frac{S_{11,\text{Max,dB}}}{S_{11,i,\text{dB}}}\right|\right) \quad (1)$$

This objective function calculates the average over N frequencies within the desired frequency band of the inverted reflection coefficient and scales it with $S_{11,\text{Max}}$. The scaling ensures that optimization halts at a value of $O = 1$. Should the reflection coefficient at any frequency surpass the target value $S_{11,\text{Max}}$, the value in the objective function calculation is kept at 1 by using $\max$, aiming for a uniform optimization across the entire frequency range.

The computed values are then scaled in MATLAB using rank-based fitness scaling (via the `@fitscalingrank` function), which assigns ranks to antennas based on their objective function values. Lower ranks (better objective function values) receive higher fitness scores, and higher ranks (worse objective function values) receive lower fitness scores. Applying this rank-based fitness scaling fine-tunes the adjustment of objective function values for parent selection in a genetic algorithm. By doing so, the selection pressure on the most efficient individuals is increased, making them more likely to be chosen as parents. This can accelerate the convergence speed of the genetic algorithm and enhance the efficiency of the search in the solution space.

Following the assignment of fitness values to each antenna, selection occurs. In our case, stochastic universal sampling was employed to choose antennas that move to the next generation for recombination or mutation. Each antenna is allocated a segment on a line proportional to its fitness within the total fitness F, starting from a random point $r \in [0, F/K]$ and selecting K antennas at equidistant steps of F/K.

Determining the number of antennas K to select involves considering the size of the new generation, which should also comprise P antennas. The number of elite individuals E dictates how many antennas carry over to the new generation unchanged. Thus, $P - E$ antennas must be generated for the new generation. The crossover rate C determines the percentage of the new generation to consist of offspring, thus $2 \times (P - E) \times C$ calculates the number of antennas selected for crossover. The remaining $(P - E) \times (1 - C)$ antennas are mutated, making the total $K = (P - E) \times (C + 1)$ individuals to be selected. The selected individuals are then randomly permuted and queued for recombination. During crossover, two antennas are combined into a new antenna for the next generation, with entries from the bitstrings of either parent randomly chosen. After forming $(P - E) \times C$ antennas, the remaining $(P - E) \times (1 - C)$ in the list are mutated. The mutation rate determines the likelihood of inverting a bit. The parameters used in our study are summarized in the following table.

Table 1. Parameters used for the genetic algorithm function within Matlab [17].

Property	Value
Population size P	50
Crossover Function	@crossoverscattered
Crossover Rate	0.8
Mutation Function	@mutationadaptfeasible
Mutation Rate	0.01
Elite Count	2
MaxGenerations	250

Note, that these values are not optimal however they were chosen as they were found to be sufficient. Further studies on the optimum of these values are necessary. However, the population size should not get too large. Goldberg [18] highlights the importance of carefully selecting the population size in genetic algorithms to avoid inefficiencies in the optimization process, akin to the 'gambler's problem,' where overly large populations can lead to diminished returns in search efficiency.

After the new generation is built, the antennas are again simulated and selection, crossover, and mutation repeat iteratively until the target - a reflection coefficient of $S_{11,\mathrm{Max}}$ within the full specified band - is reached.

As it will be discussed in Section IV, it is beneficial to use multi-objective optimization for a large gain and a low reflection or bandwidth. In our case for this purpose, we used the `@gamultiobj` function within Matlab.

It utilizes a controlled, elitist genetic algorithm, specifically a variant of NSGA-II, to optimize multiple objectives simultaneously, aiming to find a set of points on the Pareto front. The algorithm begins by generating an initial population like the `@ga` function does [19].

The core of the algorithm unfolds through iterative steps where parents are selected from the current population using a binary tournament selection process. This process emphasizes diversity and fitness by considering both rank and crowding distance among individuals.

Offspring are then produced through mutation and crossover operations, inheriting characteristics from their parents. These children are evaluated to determine their objective function values and feasibility, blending them with the existing population to form an extended pool.

This extended population undergoes a trimming process based on individuals' rank, determined by their dominance in the objective space, and their crowding distance, which measures the density of solutions around a point. The algorithm prefers individuals with lower ranks (indicating they are less dominated) and higher crowding distances (indicating they are in less crowded areas of the solution space), fostering both the exploration of the objective space and the preservation of solution diversity. This trimmed population then serves as the basis for the next iteration, with the cycle repeating until a stopping condition—such as reaching a maximum number of generations or achieving a sufficient spread among the Pareto front solutions—is met.

III. Antenna Size

The antenna size, denoted as ka, is determined by the product of the wavenumber k and the radius a of the smallest sphere that fully encompasses the antenna. The wavenumber $k = \frac{2\pi}{\lambda}$, where λ is the wavelength of the electromagnetic wave, reflects the radians per unit distance along the wave's propagation direction. The radius a indicates the antenna's physical size in relation to the enclosing sphere. The ka product effectively quantifies the antenna's electrical size by comparing its physical dimensions to the operational wavelength.

This parameter is crucial as it provides a uniform metric to evaluate the characteristics of antennas of varying sizes and frequencies. An antenna with $ka \leq 1$ is typically deemed electrically small. This concept originates from Wheeler's foundational research [20], where the author proposes that the maximum dimension of a small antenna is a fraction of the radian wavelength, specifically $\frac{\lambda}{2\pi}$. This is derived from the near-far field approximation for a Hertzian dipole, indicating $\frac{\lambda}{2\pi}$ as the critical boundary for the transition from the near-field to the far-field. However, a detailed analysis of Wheeler work shows, that the actual threshold is $ka \leq 0.5$, marking a threshold beyond which the antenna's physical size substantially impacts its radiation efficiency and bandwidth. For electrically small antennas, especially when $ka \leq 0.5$, maintaining high efficiency and bandwidth becomes increasingly complex, necessitating creative design approaches to mitigate these fundamental challenges.

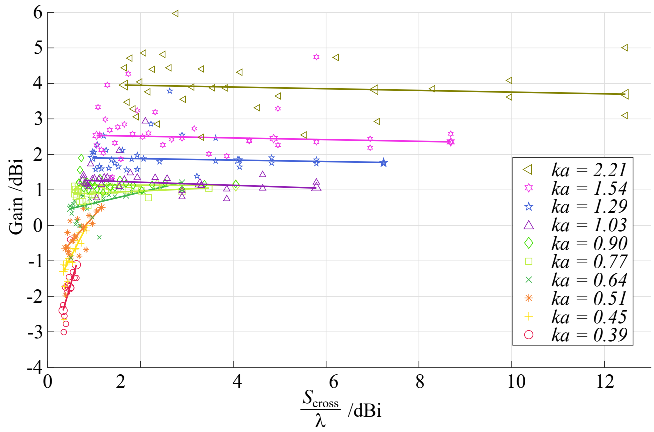

Fig. 5. Antenna gain as a function of the cross size normalised to wavelength S_{cross}/λ. Solid lines with markers are the respective linear fitted curves [13].

IV. Performance and Properties of Evolutionary Optimized Antennas

One of the fundamental questions regarding pixelated antennas is how various co-dependencies, such as antenna- and pixel size, influence the performance of such an antenna. This problem was tackled in a detailed study [13], were 400 antennas were optimized for an exemplary frequency of 868 MHz for a reflection coefficient of $-20\,\mathrm{dB}$. By scaling the cross size to the wavelength and by using the antenna's electrical size ka the obtained results are applicable for arbitrary resonance frequencies, as demonstrated in further studies [15].

This study demonstrates that achieving a successful optimization or in other words, an antenna that meets the optimization criteria, requires a directly proportional relationship between cross-section and antenna size for electrically large antennas (where $ka \geq 1$) as seen in Figure 4. Thus, a minimum constant pixel number for any given antenna dimension for large antennas is needed. This result means, that electrically large antennas can be optimized using a small solution space, because the algorithms can optimize antennas using a minimum number of pixels of larger size. For electrically small antennas, starting from $ka < 1$ this linear dependency begins to deviate, with another change at $ka \approx 0.5$. This result can be explained by the fact, that small antennas need more pixels to lead to a satisfying result to still feature an input impedance of $50\,\Omega$. This is akin to conventional antenna structures, where decreasing sizes lead to more complicated design considerations to acquire e.g. matching. Furthermore, the influence of pixel size and antenna size on the maximum achievable antenna gain was observed as depicted in Figure 5. For large antennas, the pixelsize had a negligible impact on the maximum gain whereas for small antennas - especially with sizes $ka \leq 0.6$ it was found that smaller pixels lead to smaller gain. This result can be explained by the fact, that smaller pixels can lead to smaller connections which in turn lead to higher losses. In this study, the antennas were only optimized to feature a low reflection coefficient. Based on this, it was assumed, that optimization for antenna gain by multi-objective optimization could enhance the gain also for smaller pixels.

Indeed, we present new results regarding multi-objective optimization with the Matlab function `@gamultiobj` of pixelated antennas where antennas have been optimized for low reflection at 868 MHz and high gain. Antennas with the sizes of $ka = 1.03, 0.64$, and 0.39 were optimized for three different cross sizes. The results of this study are depicted in figure 6. As expected - due to the multi-objective

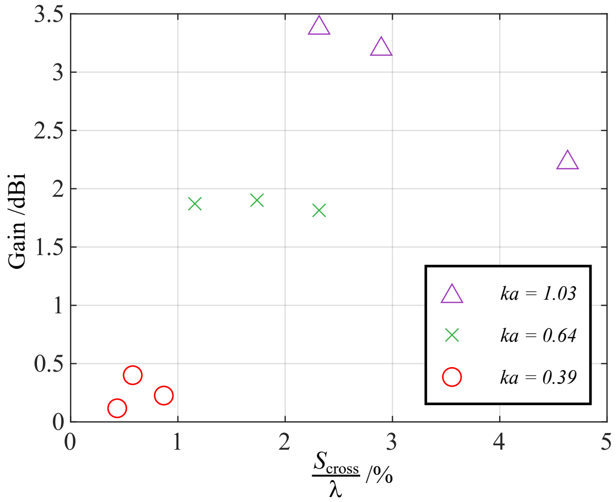

Fig. 6. Gain of pixelated Antennas with the sizes of $ka = 1.03$, $ka = 0.64$, and $ka = 0.39$.

optimization which also optimized for high gain - the gain of the individual antennas was higher as in the previous study and the dependency of gain concerning cross size vanished. This is due to the fact, that in principle infinitely small pixels can build the same structures as larger pixels do. Additionally, the solution space is larger for small pixel and the optimization can therefore optimize the antenna's main connections where losses are high to feature larger connections by accumulating pixels in these regions.

Furthermore, it can be seen in figure 6 for the antennas with a size of $ka = 1.03$ the gain decreases for larger crosses which means that it might be beneficial to use smaller crosses for large antennas. This makes sense as smaller crosses are needed for more complicated structures like antenna arrays.

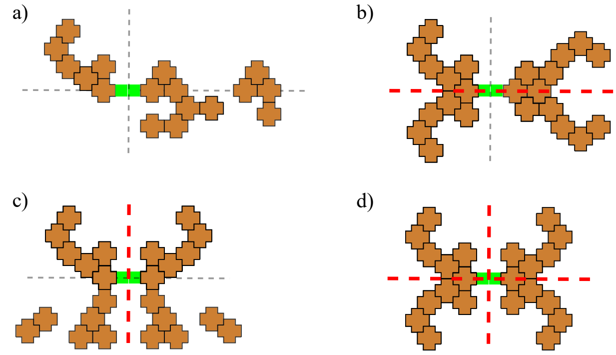

Fig. 7. Examples of the different antennas, where the dashed line indicates the symmetry axes and the antenna port in green: asymmetrical (a), symmetrical along the x-axis (b), symmetrical along the y-axis (c) and symmetrical along both x- and y-axes (d) [14].

Following this analysis, symmetry considerations were investigated to gauge if restrictions in the geometrical space where the antenna is allowed to grow, influence the antenna's performance regarding radiated power[14]. For this reason, asymmetrical antennas (figure 7-a) were compared with horizontal symmetrical antennas (figure 7-b), vertical symmetrical antennas (figure 7-c) and horizontal and vertical symmetrical antennas (figure 7-d). 19 antennas were generated and all antennas featured an electrical size of approximately $ka = 1.9$. It was shown, that asymmetrical antennas were able to achieve better gain and matching than symmetrical antennas.

The results indicate that symmetry restrictions have no benefit for electrically large antennas. Figure 8 shows the gain and reflection coefficient for the simulated antennas, indicating that only asymmetrical antennas can outperform the Chu-Harrington Limit [21]. The nomenclature is *Date-Symmetry-Direction of Optimization*. If this is also the case for small antennas will be investigated in future studies. However, it is to be expected that in this case, multi-objective optimization will have to be employed.

V. Conclusion

Pixelated antennas are fascinating structures that enable tailor-made RF-interfaces, especially in highly demanding environments. Unlike earlier research that often focused on using a range of algorithms to tweak pixelated structures without a thorough comparison or comprehensive analysis on overall antenna performance, our study bridges this gap with a critical examination of the effects of pixel size and symmetry on antenna functionality, alongside the application of various optimization methods, including genetic algorithms and multi-objective optimization.

Our investigation underscores the remarkable adaptability of pixelated antennas, demonstrating their potential to significantly enhance antenna designs through the adjustment of individual pixel elements. This adaptability is harnessed through the application of optimization algorithms, allowing for the antenna's properties to be tailored to specific

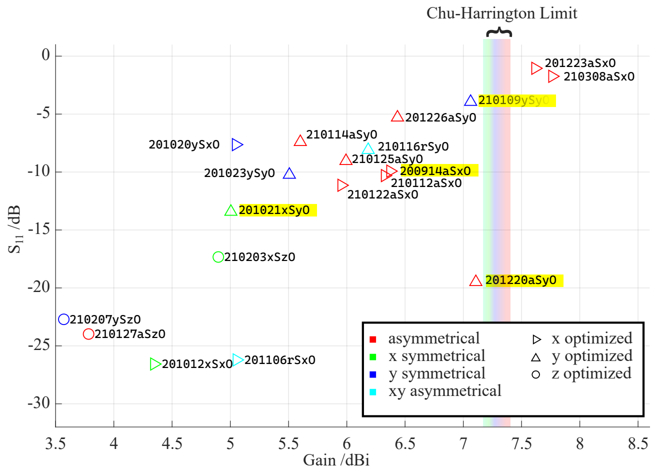

Fig. 8. Gain and S_{11} for all optimized antenna structures with their respective symmetry and direction of optimization. Highlighted antennas were measured to verify simulations [14].

requirements, thereby pushing the boundaries of traditional design methodologies.

A pivotal aspect of our contribution lies in the understanding of how pixel size impacts antenna performance. Our findings indicate that electrically larger antennas could see substantial benefits from employing smaller pixel sizes whereas smaller antennas seem to not benefit from smaller pixels. This revelation is significant, suggesting that reducing pixel size for larger antennas can lead to improved design flexibility and potentially higher performance in larger antennas.

Moreover, our exploration into symmetry restrictions sheds light on the complex dynamics between antenna design and performance, revealing that asymmetrical designs can potentially outperform their symmetrical counterparts.

In conclusion, our research contributes a perspective to the field of antenna design, particularly in the optimization and practical application of pixelated antennas.

However, the limited dataset of antennas optimized by multi-objective optimization must be extended to further examine the impact of reduced pixel size on large antennas.

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