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Ice-rafted debris associated with binge/purge oscillations of the Laurentide Ice Sheet

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Abstract. The North Atlantic sediment record suggests quasi-periodic (7000- to 12,000-year period) ice-rafted debris (IRD) depositions during at least the last glacial period. The cause of these Heinrich events, as they are commonly known, is not fully understood; however, they may point to surges of the ice stream that drained the Hudson Bay/Hudson Strait region of the Laurentide Ice Sheet. We investigate a simple conceptual model of ice stream instability (the binge/purge model) to suggest ways in which the ice stream could have entrained sufficient debris to account for the estimated mass of IRD associated with a typical Heinrich IRD layer in the North Atlantic ($1.0 \pm 0.3 \times 10^{15}$ kg). We find that freezing of debris-laden ice at the bed of the ice stream during the brief ($\approx$ 750 years) surge phase of the ice stream's hypothesized binge/purge cycle can incorporate up to 5.1×10^{15} kg. This amount is sufficient to meet the constraints of the North Atlantic sediment record but by no means verifies the binge/purge model as the cause of Heinrich events.

Introduction

Heinrich events of the glacial North Atlantic are episodes of rapid ice-rafted debris (IRD) deposition [*Heinrich*, 1988; *Bond et al.*, 1992; *Broecker et al.*, 1992] which are associated with possibly widespread patterns of climate change [*Bond et al.*, 1993; *Clark*, 1994; *Maslin et al.*, 1993; *Broecker*, 1993]. *Bond et al.* [1993], for example, suggest that Heinrich events are coordinated with long-term ($\approx$ 7000 years) cooling trends that modulate high-frequency ($\approx$ 1000 years) swings in the $\delta^{18}O$ record of the Greenland Ice-Core Project (GRIP) ice core [*Johnsen et al.*, 1992].

To determine the cause of Heinrich events and understand their possible climatic significance, it is necessary to address the sedimentological questions concerning how Heinrich event IRD was originally entrained into ice sheets and how it managed to survive transport aboard drifting icebergs [*Andrews et al.*, 1993]. The lithology, isotopic signature, and geologic age of much of the Heinrich event IRD have affinities to sediments released at the eastern margin of the Laurentide Ice Sheet (LIS). In particular, carbonate detritus similar to sediments found in Hudson Strait and central Hudson Bay appear to be dispersed widely across the North Atlantic basin [*Andrews and Tedesco*, 1992; *Bond et al.*, 1992]. These carbonate sediments may have been entrained in basal ice of an ice stream flowing through Hudson Strait. Periodic surges of this ice stream caused by an internal glaciological process or by external climate forcing may thus constitute a principal origin of debris-laden icebergs in the North Atlantic.

In this paper, we assess the sedimentological implications of a plausible, but yet unproven, model of Heinrich events, namely, the binge/purge model of the LIS [*MacAyeal*, 1993a, b]. According to this model, the volume of the LIS suffers cyclic perturbations. Long periods of slow growth (binge) alternate with short periods of rapid discharge (purge). During the approximately 7700-year binge phase of the cycle, ice builds up gradually over Hudson Bay while geothermal heat slowly warms the glacial bed. Once thawed, the glacial bed becomes lubricated by soft, water-charged subglacial till derived from underlying sedimentary rock. This lubrication facilitates the rapid flow of an ice stream in Hudson Strait, which drains the central part of the LIS. During the brief, 750-year purge of the ice stream, iceberg discharge into the Labrador Sea would account for IRD with affinities to bedrock conditions in Hudson Bay and Hudson Strait. The purge ends and a new binge begins when ice sheet thinning promotes the widespread refreezing of the glacial bed.

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An important characteristic of the binge/purge model which sets it apart from other, perhaps equally plausible, models of Heinrich events (e.g., G. Bond and R. Lotti, unpublished abstract, 1994) is its lack of external forcing. Our assessment of the sedimentological implications of the binge/purge model is thus motivated by a need to develop testable predictions.

Our immediate goal is to predict an IRD flux associated with a binge/purge cycle to compare with the North Atlantic sediment record [e.g., *Andrews and Tedesco,* 1992; *Bond et al.,* 1992; *Grousset et al.,* 1993]. To make this prediction, we propose several physical processes by which subglacial debris is entrained into an ice stream flowing through Hudson Strait. We consider three processes in particular: simple freeze-on of water-soaked basal muds, entrainment of basal debris by regelation (pressure-induced freezing and melting associated with obstacles at the glacial bed), and mechanically induced debris diffusion. Our focus is thus limited to the question of how to deliver icebergs containing IRD to the ocean, not how IRD remains aboard icebergs long enough to cross the North Atlantic [e.g., *Andrews et al.,* 1993]. Our analysis is further limited to the binge/purge model of Heinrich events. The plausibility of the binge/purge model in relation to other models of Heinrich events will not be addressed here.

Observed Ice-Rafted Debris Distribution

To evaluate possible sedimentological processes associated with Heinrich events, it is necessary to estimate the net mass m of IRD deposited in the North Atlantic during each event. This estimate is difficult to make, because most of what is known about Heinrich event IRD deposits comes from magnetic susceptibility flux measurements and lithic grain counts [e.g., *Grousset et al.* 1993; *Bond et al.,* 1992] which do not directly indicate IRD mass. We estimate the net (area integrated) IRD mass associated with a typical Heinrich event using a magnetic susceptibility flux time series, $\mu(t)$, where t is time, derived from a typical sediment core (SU-9008 of *Grousset et al.,* [1993]) and the total IRD mass, M, deposited in the North Atlantic over the period between 125,000 and 13,000 years ago (isotopic stages 2-5) estimated by *Ruddiman* [1977].

We assume that M accounts for all Heinrich event and non-Heinrich event IRD deposition and that $\mu(t)$ is a proxy indicator of the schedule with which this IRD mass M is laid down. Under these assumptions, the fraction of M attributed to Heinrich events is the ratio of two graphical areas shown in Figure 1. The first is the area under the observed $\mu(t)$ curve within the time period 125,000 to 13,000 years ago. The second is the area under a "background" magnetic susceptibility flux curve (denoted by $\tilde{\mu}(t)$) over the same time period. As shown in Figure 1, we synthesize $\tilde{\mu}(t)$ from $\mu(t)$ by removing the magnetic susceptibility spikes associated with the four most prominent Heinrich layers

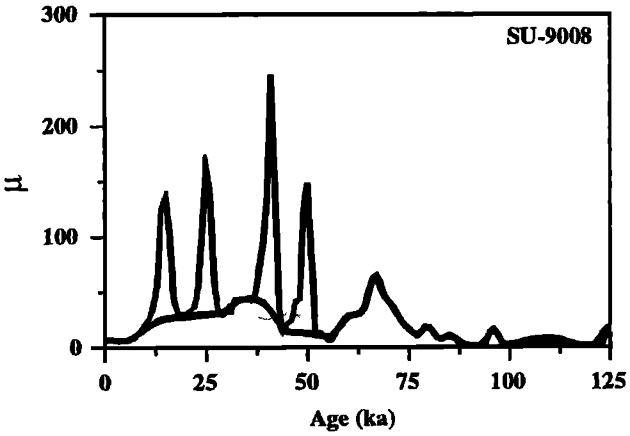

Figure 1. Magnetic susceptibility flux $\mu(t)$ (thin line) of sediment core SU-9008 (10^{-6} electromagnetic units per centimeter squared per thousand years). The $\mu(t)$ is a spline interpolation of data points digitized from Figure 3 of *Grousset et al.* [1993]. The heavy line represents the estimated background magnetic susceptibility flux $\tilde{\mu}(t)$ used in the estimation of ice-rafted debris (IRD) mass associated with each Heinrich event. We assume $\mu(t)$ to be a proxy schedule for the delivery of IRD to the North Atlantic ocean. The ratio of the areas under the curves of $\tilde{\mu}(t)$ and $\mu(t)$ (in the time period 13,000 to 125,000 years ago) is assumed to give the fraction of total IRD delivered to the North Atlantic that is not associated with Heinrich events.

(H1, H2, H4, and H5). In computing the areas under the two curves, we integrate over the time period between 125,000 and 13,000 years ago.

The final step in our calculation of m is to divide the fraction of M attributed to Heinrich events by N, the number of Heinrich events seen in $\mu(t)$. In other words, we assume

$$m = \left(1 - \frac{\int \tilde{\mu}(t)dt}{\int \mu(t)dt}\right)\frac{M}{N} \tag{1}$$

where the limits of integration are between 125,000 and 13,000 years ago (isotopic stages 2-5).

There are two advantages to estimating m according to the above procedure. First, it allows an associated estimate of uncertainty. We can compare the magnetic susceptibility flux measurements between cores, and between Heinrich events in individual cores, to estimate what we assume to be a random variance of the mixing ratio of IRD with oceanically derived sediment. Our casual (qualitative) inspection of the published magnetic susceptibility flux measurements [e.g., *Grousset et al.* 1993, Figure 2] suggests that this random variance translates to an uncertainty in m of approximately (1/3) m. The uncertainty of M is also important but is not easy to quantify (see *Ruddiman's* [1977] discussion of contouring principles). We discuss the factors which contribute to uncertainty in M below.

The second advantage to estimating m according to the above procedure is that it facilitates a retracing of steps to identify incorrect assumptions should our estimate prove, after further study, to be grossly inadequate. Other procedures (e.g., performing a census of IRD content using data from a marine sediment core repository) would undoubtedly yield greater accuracy in the estimate of m. Our simple method provides sufficient accuracy, however, to meet the goals of our study.

The most serious disadvantage to our method for estimating m concerns three assumptions that are difficult to justify. The first assumption is that μ is linearly proportional to the mixing ratio between IRD and other sediment. This assumption introduces inaccuracy, because both biogenic sediment and sediment porosity can appreciably affect magnetic susceptibility of sediment cores. The second assumption is that the background magnetic susceptibility flux, $\tilde{\mu}(t)$, is that portion of the observed $\mu(t)$ that lies below the four largest sharp peaks (as shown in Figure 1). Peaks which might correspond to other Heinrich events have been disregarded. The third assumption is that *Ruddiman*'s [1977] estimate of M accounts for all IRD deposited during isotopic stages 2-5.

We do not know whether *Ruddiman*'s [1977] sediment cores were sampled with sufficient resolution to account for all the IRD deposited during Heinrich events. As is evident from the magnetic susceptibility flux measurements of typical cores [e.g., *Grousset et al.* 1993, Figure 2] a systematic failure to sample a particular Heinrich layer in *Ruddiman*'s [1977] census could lead to a 10-20 % shortfall in M. While we do not have the expertise to comment further on Ruddiman's census, we consider such a systematic sampling error to be unlikely.

The accuracy of *Ruddiman*'s [1977] estimate of M also depends on the geographic coverage of his sediment cores and the type of IRD (sand) he quantified in his cores. The IRD census area includes some regions of the North Atlantic where Heinrich layers are absent [*Bond et al.* 1992], but excludes the region near the mouth of Hudson Strait where Heinrich layers are especially thick [*Andrews et al.* 1994]. The effects of geographic coverage on the estimate of M is difficult to quantify. We believe, however, that non-Heinrich layer IRD derived from areas included in the census largely offsets Heinrich layer IRD missed near Hudson Strait. The focus of *Ruddiman*'s [1977] census was on ice-rafted, noncarbonate sand with a correction added to account for finer fractions, thus excluding all gravel and all carbonate IRD, both of which occur in Heinrich layers. The M we use is what *Ruddiman* [1977] suggests is applicable to all types of IRD, including types not explicitly analyzed by his census. We accept this figure without modification. We suspect, however, that what has been learned about nonsand forms of IRD in the North Atlantic since *Ruddiman*'s [1977] census implies that the value of M we use slightly underestimates the influx associated with Heinrich events.

The estimation procedure outlined above gives $m = 1.0 \pm 0.3 \times 10^{15}$ kg. Assuming that IRD has a density of 2700 kg m^{-3}, the m we estimate corresponds to an IRD volume of $3.7 \pm 1.2 \times 10^{11}$ m^3. For this estimate, we used $M = 9.8 \times 10^{15}$ kg, *Ruddiman*'s [1977] figure for "total" IRD (i.e., sand and nonsand components of IRD), and we took $\mu(t)$ to be the value reported by *Grousset et al.* [1993, Figure 3] for core SU-9008 (located at 43°30′N, 30°24′W, 3.1 km depth). We took $N = 4$ from the fact that $\mu(t)$ for core SU-9008 displays four major Heinrich event peaks (associated with H1, H2, H4, and H5).

Grousset et al. [1993, Figure 1] (see also *Bond et al.* [1992]) show that IRD associated with the major Heinrich events (e.g., H1, H2, H4, and H5) is distributed over an area of roughly 5×10^{12} m^2. This region extends east/west across the North Atlantic and corresponds roughly to the band of high IRD deposition reported by *Ruddiman* [1977]. The thickness of IRD-rich layers ranges from almost 1 m near the mouth of Hudson Strait [*Andrews et al.*, 1994] to as little as 1 cm off Ireland [*Grousset et al.*, 1993]. An average thickness of 10 cm over the affected region appears to be of the correct magnitude, depending somewhat on the method used to define the IRD layer and the IRD event considered [e.g., *Bond et al.*, 1992; *Grousset et al.*, 1993]. Our estimate of m yields an average thickness of 7.4 ± 2.5 cm of rock equivalent. Allowing for porosity, this average thickness translates to approximately 10 cm of wet sediment.

Simple Freeze-on Model of Debris Entrainment

The quantitative model of the binge/purge oscillator [*MacAyeal*, 1993a, b] may be modified to predict IRD flux by assuming that basal till simply freezes to the base of the ice stream as a result of upward heat conduction. These modifications are described in the Appendix. A qualitative description of these modifications is presented as follows: During the initial portion of the purge, frictional heating at the ice stream bed due to rapid ice flow far exceeds the upward conduction of heat through the ice. This excess heat is used to melt (possibly debris laden) basal ice. We assume that meltwater thus produced is stored as pore fluid in a subglacial till. (Subglacial tills below West Antarctic ice streams can contain approximately 40% water by volume [*Blankenship et al.*, 1987; *Engelhardt et al.*, 1990].) As the purge progresses, upward heat conduction through the ice gradually increases, while frictional heating at the bed gradually reduces as the ice stream thins. Eventually, upward conduction exceeds heating, and water-soaked basal sediments begin to freeze to the base of the ice stream. (Ice sediment segregation analogous to that observed in permafrost might occur early in the freeze-on [e.g., *Smith*, 1985], but in the absence of a large water source from bedrock below the till, segrega-

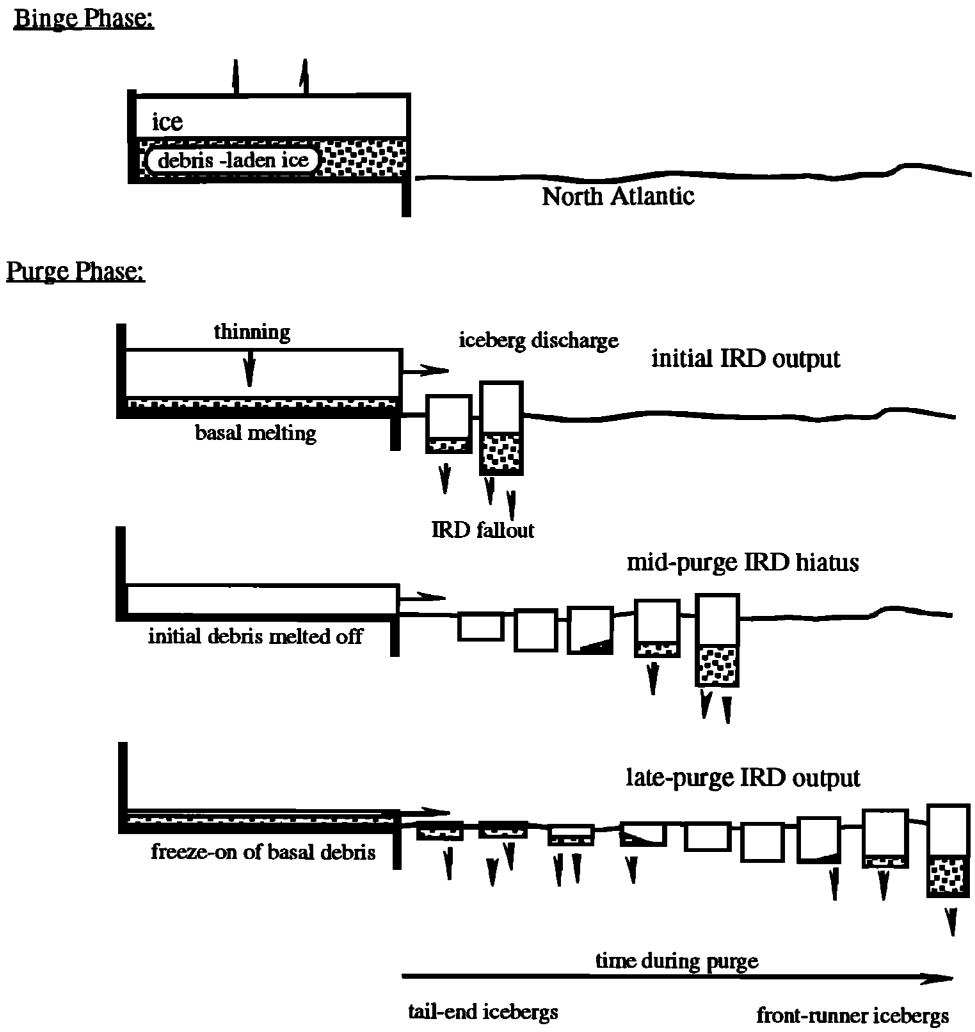

Figure 2. A schematic diagram of a Heinrich event displaying the IRD flux associated with a binge/purge cycle of the Laurentide Ice Sheet when debris is entrained into the ice stream by a simple freeze-on mechanism. Other possible debris entrainment mechanisms, such as might occur when sticky spots exist at the glacial bed, may reduce or eliminate the midpurge IRD flux hiatus.

tion will be limited and till freeze-on will occur.) When all available water-soaked basal sediment has frozen to the bed (at least over a majority of the bed), latent heat is no longer available to balance upward heat conduction, the bed freezes, and the purge ends.

In the absence of complications discussed in the next sections, this simple conceptual model suggests that IRD flux to the North Atlantic is divided between two distinct pulses associated with each Heinrich event. As shown in Figure 2, debris-laden basal ice left over from the freeze-on portion of the previous purge is transported across the grounding line to provide an initial pulse of IRD to the North Atlantic. When debris-laden basal ice is completely melted off the ice stream sometime during the early part of the purge, the IRD flux goes to zero. This becomes the midpurge IRD flux hiatus (Figure 2). After freeze-on commences, but before the purge ends, IRD flux resumes and continues to the end of the purge. To the best of our knowledge, the marine sediment record has not revealed such "twinning" within individual Heinrich layers. If observations fail to confirm this prediction of the freeze-on mechanism, we can still appeal to more complex models of debris entrainment, such as the mixed bed mechanism described in the next section, before having to abandon the binge/purge model altogether.

To illustrate the above reasoning, we incorporated a basal till layer and a debris-laden ice layer into the low-order model described by *MacAyeal* [1993a]. As described in the Appendix, the condition in the model which determines the transition from purge to binge was modified to account for latent heat stored in the bed as pore water in subglacial till. This modification lengthens the period of the binge/purge oscillation to 7730 years from its previous (non-IRD producing) periodicity of 7260 years. The amplitude of the ice thickness cycle is also increased (here, the net change in ice thickness of the idealized model ice sheet is 1460 m; in *MacAyeal's* [1993a] study, it is 1228 m).

To compute F (cubic meters per second), the model IRD flux, we use the equation $F = WUh_d$, where $W = 10^5$ m is the width of Hudson Strait, U is the average ice stream velocity at the calving margin, and h_d is the thickness of pure debris distributed as an ice/debris mixture in the ice stream. We assumed a 60/40 volumetric ratio between detritus and ice in the debris-laden ice, as would be expected if a subglacial till of 40% porosity were to freeze en masse to the ice stream bottom. Subglacial till porosity of 40% is similar to that found below ice streams in Antarctica [*Blankenship et al.,* 1987; *Engelhardt et al.,* 1990]. For U, the horizontal ice velocity at the iceberg-calving margin, we used the expression $U = -LH_t/H$, where $L = 10^6$ m is the length of the Hudson Strait ice stream, $H_t = dH/dt$ is the ice thinning rate, H is the total thickness of the ice stream, and $-H_t/H = \tau_{is}^{-1}$ is the horizontal strain rate of the ice stream [*MacAyeal,* 1993a]. This expression for U stems from the incompressibility condition which relates vertical thinning rates with horizontal extension rates. The numerical value of τ_{is}, the ice stream time constant, is chosen to be 250 years based on the assumed e-folding timescale of ice stream thinning.

Figure 3 displays the time series of F generated using the same initial and climatic conditions described by *MacAyeal* [1993a]. As anticipated, the purge phase of each cycle contains two maxima of F. A close-up view of the double-peaked IRD flux for a single purge phase of the low-order model is displayed in Figure 4. The debris hiatus between the two maxima is approximately 250 years in duration. It may be possible to detect such a hiatus in the microstructure of marine sediment cores from locations where the background sediment flux is large.

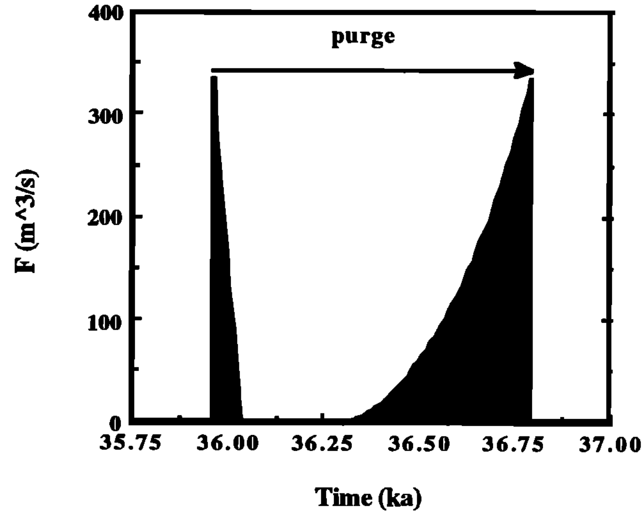

Figure 4. A close-up of modeled F during an individual Heinrich event (see Figure 3). Two peaks occur because debris-laden basal ice is completely melted from the base of the ice stream during the middle portion of the purge (as suggested schematically in Figure 2). The values of F at the beginning and end of the Heinrich event are equal and correspond to the maxima of the two peaks. This equivalence is due to the cyclic behavior of the model (the debris-laden ice load at the start of the purge is the same as that at the end of the previous purge) and the constant ice velocity during the purge phase of the cycle. The constant ice velocity is a simplification associated with the low-order description of ice stream dynamics [*MacAyeal,* 1993a]. With a more realistic glaciological model, the ice velocity would be expected to decrease during a purge. Greater ice velocity at the beginning of the purge would increase the height, but reduce the width, of the first peak.

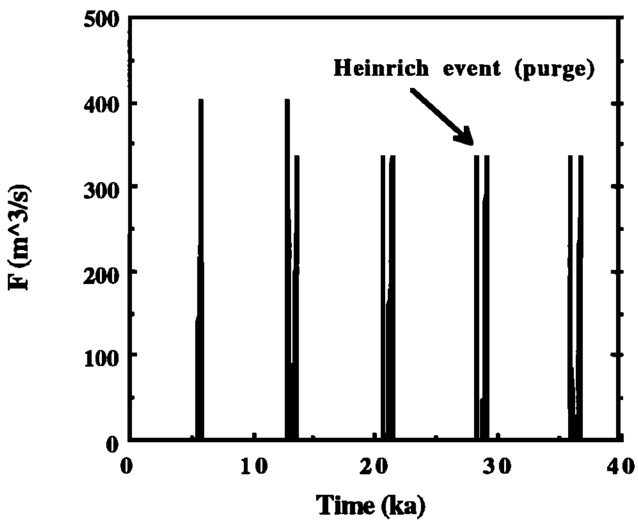

Figure 3. Modeled IRD flux F predicted by the low-order binge/purge model of *MacAyeal* [1993a], assuming basal till simply freezes on to the base of the ice. The starting point of the modeled Heinrich event chronology depicted here is arbitrary. The periodicity of F, however, illustrates the plausibility of the binge/purge cycle as a possible origin of Heinrich events.

The magnitudes of the F maxima exceed 300 $m^3\ s^{-1}$ (to convert to kilograms per second, multiply by an assumed density of 2700 kg m^{-3}). The integral of F over a single purge cycle is 1.9×10^{12} m^3, which corresponds to an IRD mass of 5.1×10^{15} kg per event. Assuming the density of IRD to be 2700 kg m^{-3}, the model gives an average Heinrich layer thickness of 38 cm of rock equivalent, assuming a 5×10^{12} m^2 area of IRD deposition in the North Atlantic. The model IRD mass discharge per event is thus approximately 5 times larger than m, the value suggested by the marine sediment record (estimated above).

Debris Entrainment From a Mixed Soft and Hard Glacial Bed

The analysis presented above assesses the thermal effects of an idealized "soft" bed: deformable till exists everywhere beneath the ice sheet, and production of rock material is easy and does not limit the till thickness. The binge/purge analyses described by *MacAyeal* [1993a, b], in contrast, did not include thermal effects of a soft bed and so estimated hard bed behavior only.

A growing body of evidence suggests the common occurrence of mixed glacier beds, in which well-lubricated regions of thick, soft till are interspersed with less well lubricated regions of thin or zero till [e.g., *Kamb,* 1991; *MacAyeal,* 1992; *Alley,* 1993]. This gives rise to "sticky spot" models of ice flow, with most of the driving force for ice motion supported on limited, sticky regions of the bed.

No single mixed bed or sticky spot model has yet emerged; however, our assessment of possible candidates indicates that the effects of a mixed bed model on the binge/purge oscillator can crudely be taken as a linear mixture of the hard bed and soft bed end members. The ice thickness amplitude and cycle length are similar for hard bed and soft bed models, so the effects of a mixed bed are only significant for prediction of the IRD flux.

Some possible effects of the mixed bed model on IRD flux are envisioned as follows: Suppose, for example, that till thickness is limited by bedrock erosion rather than by water availability. Some of the water produced during a purge then will flow directly to the ocean through an ice contact drainage system rather than by saturating newly eroded rock. The thickness of unconsolidated debris available to freeze to the ice sheet will be less than for the soft bed end-member, thus reducing the debris flux during purges, and the reduction in debris thickness will scale directly with the fraction of water diverted to the water drainage system. A purge will end sooner than for the soft bed end-member, because the freezing front will have less water-saturated sediment to pass through before freezing to bedrock. But the presence of some till will cause the purge to last longer than for the hard bed end-member, and the till that does freeze on will provide some debris flux during purges.

As a second example, suppose that till is readily available below most of the ice sheet, but that in local regions, sticky bedrock is in direct contact with the ice. The purge timescale should fall between the hard bed and soft bed end-members, because the purge should stop when the ice freezes to the bedrock high. In the absence of lateral stress transmission or lateral heat flow, this would occur at the same time as for the hard bed case. However, shear stress will be transmitted laterally from the lubricated regions to the sticky spots, causing strain heating on the sticky spots to be larger than if the entire bed were rock. Freeze-on to the sticky spot thus will take longer than for the hard bed model, but in no case can it exceed the time needed to freeze all of the basal till, which is the soft bed timescale. If the ice freezes to the sticky spots before much debris has frozen to the ice between the sticky spots, the second IRD peak in the soft bed model will be weakened or eliminated.

During the binge phase of this sticky spot ice sheet, heat flow into the basal ice will be larger over the till regions than over the bedrock because of latent heat of water in the till. This will warm the basal ice over till regions compared to rock regions, causing the till regions to switch from freezing on to melting off of till before the bedrock regions thaw and initiate a purge. As a result, less debris will remain in the ice when the purge starts than for the soft bed model. (If we ignore lateral heat and stress transmission, the timescales for binges and purges are identical to those of the hard bed model, and the thermal model over the till regions is the same as for the hard bed model except that a constant temperature rather than a constant gradient boundary condition applies at the bed between sticky spots. We estimate in this case that about 1/3 of the till frozen on during early stages of the binge will melt before purge initiation.)

Clearly, we could construct a wide range of mixed bed models, depending on factors such as till availability, lubricating ability of till, bedrock tenacity, and sticky spot distribution. Differences in what these models would predict, as stated above, are likely to fall between the hard bed and soft bed end-members; thus it is not productive at this stage to proceed further along these lines of inquiry. What is important, however, is the realization that the soft bed model (i.e., what we describe in the previous section) predicts an IRD flux that is a few times larger than that needed to explain a Heinrich event, whereas the hard bed model [*MacAyeal,* 1993a] predicts an IRD flux that is too small. Thus any mixed bed model with significant soft bed character can produce the IRD flux of a Heinrich event. However, in some mixed bed models, the second peak of the two-peaked IRD flux associated with the soft bed only model will be weakened or eliminated.

Pressure-Induced Basal Freezing

So far we have focused solely on vertical heat flux through the ice as a cause of basal freezing and debris entrainment. Basal freezing can also occur in response to pressure melting effects associated with ice flow (as around bumps in bedrock topography) or with the large-scale gradients in ice thickness. In this section, we consider the possibility of basal water migration from widespread regions of basal melting to a localized basal freezing site at the bedrock sill which constitutes the possible terminus of the Hudson Strait ice stream.

As calculated above, a purge produces about $V_w = 10^{13}$ m^3 of water, or just over 12 m of water thickness per square meter of ice involved in the purge. The mouth of Hudson Strait is about 400 m shallower than the deepest part of the overdeepening behind it [*Laughton and Monahan,* 1978], so as an upper limit, we can assume that all of the water produced in a purge undergoes this maximum 400-m rise to clear the sill at the mouth of the Strait. The pressure drop of the water is 4 MPa. The pressure also drops because of changes in the ice surface elevation. We shall disregard this second cause of pressure drop, because its effects are comparable to, and perhaps balanced by, the heat generated by viscous dissipation in the basal water flow [*Rothlisberger and Lang,* 1987].

The pressure dependence of the freezing point of ice falls between the air-saturated value, $C_t = -9.8 \times 10^{-8}$ K Pa^{-1}, and the pure-water value, $C_t = -7.4 \times 10^{-8}$ K Pa^{-1}. Taking the air-saturated value as an upper limit, the water must warm about 0.4 K in rising over the sill. The specific heat, C_s, of water is about 4.2×10^6 J m^{-3} K^{-1}, and the latent heat of ice, C_f, is about 3.06×10^8 J m^{-3}. Thus about $C_s/C_f = 0.014$ m^3 of ice is expected to freeze for each degree of warming of a cubic meter of water. In other words, 0.005 m^3 of ice must freeze to warm 1 m^3 of water by 0.4 K. Using the value of V_w produced in a purge, about 5×10^{10} m^3 of ice is estimated as the upper limit for freeze-on from this mechanism. Assuming that the debris concentration of this frozen-on ice is 5% by volume, a typical value for regelation ice frozen onto glaciers as they slide over their beds [*Kamb and LaChapelle,* 1964; *Sugden and John,* 1976, p. 161], only about 2.5×10^9 m^3 of IRD per Heinrich event is delivered to the North Atlantic via this mechanism. This flux is more than 2 orders of magnitude too small to explain the observed amount of IRD in a typical Heinrich layer.

Ice Tectonics as a Debris Entrainment Mechanism

We know of few satisfactory models of the folding and cavitation process which might entrain debris as an ice stream flows across a rough, obstacle-strewn bed [*Clarke and Blake,* 1991]. We expect that whenever basal ice encounters an obstacle, there is some chance that the basal debris will move up from the bed into the ice. If a constant debris concentration is maintained at the bed, then this probabilistic debris entrainment might be modeled as an upward diffusion process with a constant-concentration boundary condition. In this section, we estimate a debris diffusivity, D_d, for basal ice flowing across a field of obstacles and use this diffusivity to estimate a debris load for icebergs calved into the North Atlantic.

We can make a first estimate of a debris diffusivity by realizing that significant basal debris penetrates only a few meters upward from the bed in most modern ice sheets. Furthermore, the few meters of debris is an upper limit for diffusion, because the debris may have been entrained by net freeze-on or during ice sheet formation, rather than by upward diffusion. We thus expect a debris eddy diffusivity $D_d < 10^{-9}$ m^2 s^{-1}, and probably $< 10^{-10}$ m^2 s^{-1}, although this might increase in regions of fast sliding where the basal ice encounters obstacles frequently. (In comparison, basal heat penetrates kilometers into the ice because of the thermal diffusivity of the order of 10^{-6} m^2 s^{-1}.)

Following the above reasoning, we estimate that about 12 m of ice melts off the base of the ice stream (or 30 m of till) during a purge, so the diffusion distance $(Dt)^{1/2}$ would need to be similar to, or larger than, 12 m over the time span $t \approx 750$ years of a purge to allow significant debris entrainment by this mechanism. An unreasonably large debris eddy diffusivity ($> 10^{-8}$ m^2 s^{-1}) is thus required to explain the Heinrich event IRD. In view of the D_d estimated above, we thus consider it unlikely that debris diffusion can be the significant source of debris entrainment needed to explain Heinrich events.

Conclusion

Our simple model suggests that an ice stream flowing through Hudson Strait is capable of delivering periodic bursts of IRD to the North Atlantic. The timing and magnitude of modeled IRD fluxes satisfy constraints imposed by the North Atlantic sediment record, and this lends support to the binge/purge model of Heinrich events. Other mechanims of debris entrainment that we have considered are unlikely to be sufficient to explain the observed IRD fluxes (Table 1). A test of the binge/purge model of Heinrich events is suggested by its prediction that IRD flux reaches a maximum twice during each Heinrich event. Discovery of a two-pulsed sedimentary signature in the North Atlantic Heinrich layers would provide strong support for the binge/purge model. Our study has addressed only the sedimentary processes which might accompany a binge/purge oscillation of the Laurentide Ice Sheet. We thus encourage the investigation of other plausible models capable of yielding quasi-periodic IRD deposition, such as might involve external climate forcing, as the origin of Heinrich events.

Table 1. Ice-Rafted Debris Statistics

Model	Mass, 10^{15} kg	Volume, 10^{11} m^3
observed	1.0 ± 0.3	3.7 ± 1.2
freeze-on soft bed	5.1	19
freeze-on sticky spot	0 - 5	0 - 20
regelation	< 0.07	< 0.03
diffusion	< 0.03	< 0.1

Appendix: Low-Order Model With Basal Debris Entrainment

Here we quantify the simple freeze-on model of debris entrainment by considering the thermodynamics and kinematics of basal ice in the low-order model described by *MacAyeal* [1993a]. For simplicity, we treat the two-component mixture of water (or ice) and rock debris as if it were a single-component material consisting of water (or ice) only. This treatment allows us to develop a low-order view of the sedimentology of the Heinrich event oscillation without reference to the complex thermodynamic and rheological aspects of two-component mixtures. In effect, we assume that water melted from the base is stored in the bed (i.e., not drained away through subglacial conduits) as the pore fluid of an unconsolidated till. The erosion and grinding required to create the debris fraction of the unconsolidated till is not of interest here. In effect, we assume the ice to rest on an infinitely deep bed of unconsolidated sediment that is capable of storing water in its void space. When this water is frozen back on to the base of the ice sheet, it carries with it debris which, from our assumption, is treated as if it had zero heat capacity and an infinite conductivity (i.e., can be ignored when computing the temperature profile and melt rate of the dirty basal ice).

We make two modifications to the low-order model described by *MacAyeal* [1993a]. First, we assume that during the purge phase of the cycle, basal water is contained within the pore space of a subglacial till. The equivalent thickness of this basal water, if it were to be separated from the solid part of the subglacial till, is defined to be h. We assume that h is nonzero only during the purge phase of the cycle, for example, when the basal ice temperature is at the melting point, $\theta_b = 0$. (All notation not explicitly defined here is defined by *MacAyeal* [1993a].) Second, we assume that latent heat used to create the basal water layer is recoverable. Heat used to melt ice during the initial stage of the purge is returned to the ice in the final stages of the purge as the water layer refreezes to the base of the ice stream.

Basal water and till are considered immobile. Conservation of heat at the basal interface thus provides a means to determine $h(t)$:

$$h_t = \frac{G}{\rho\lambda} + \frac{k}{\rho\lambda}\frac{\partial\theta}{\partial z}\bigg|_{z=0} + \frac{gH^2}{\tau_{is}\lambda} \tag{A1}$$

where $\lambda = 3.3 \times 10^5$ J kg^{-1} is the latent heat of fusion for water, $G = 0.05$ W m^{-2} is the geothermal flux, $k = 2$ W C^{-1} m^{-1} is the thermal conductivity of ice, $\rho = 917$ kg m^{-2} is the density of ice, $g = 9.8$ m s^{-2} is the acceleration of gravity, H is the ice thickness, and $\tau_{is} =$ 250 years is the ice stream (fast) timescale suggested by *MacAyeal* [1993a]. The subscript t in equation (A1) and elsewhere denotes the time derivative.

Equation (A1) is interpreted as follows. The first term on the right-hand side represents the melting rate associated with the geothermal heat flux. The second term on the right-hand side represents the freezing rate (recall that $\partial\theta/\partial z|_{z=0} < 0$) due to the heat conducted up through the base of the ice stream. The third term on the right-hand side represents the melting rate driven by basal friction.

The equivalent thickness of basal water, $h(t)$, grows during the initial period of the purge phase of the cycle because basal friction dominates the vertical heat conduction. Near the end of the purge, vertical heat conduction dominates frictional heating, and the water layer vanishes by freezing back into the ice stream. Thus

$$\begin{aligned} h_t &> 0 \quad \text{when} \quad \frac{G}{\rho\lambda} + \frac{gH^2}{\tau_{is}\lambda} > -\frac{k}{\rho\lambda}\frac{\partial\theta}{\partial z}\bigg|_{z=0} \\ h_t &< 0 \quad \text{when} \quad \frac{G}{\rho\lambda} + \frac{gH^2}{\tau_{is}\lambda} < -\frac{k}{\rho\lambda}\frac{\partial\theta}{\partial z}\bigg|_{z=0} \end{aligned} \tag{A2}$$

At a point midway through the purge when $h_t = 0$, the basal water layer has reached its maximum thickness h_{max}. The cycle is assumed to be symmetric; thus by the end of the purge phase, an amount of water equal to h_{max} times the overall length of the ice sheet flow line (L) is frozen to the base of the ice sheet. This produces a basal ice layer containing till debris. The thickness of this debris-laden layer will depend on the ratio of debris volume to ice volume, which we take to be 60/40 from observations of subglacial till in West Antarctica [*Blankenship et al.*, 1987; *Engelhardt et al.*, 1990]. Defining h_d to be the thickness of pure debris in the debris-rich basal ice, h_d is governed by an equation similar to equation (1) of *MacAyeal* [1993a]:

$$\begin{aligned} (h_d)_t &= 0 && \text{when} \quad h_d = 0 \\ (h_d)_t &= \frac{-60}{40}h_t - \frac{h_d}{\tau_{is}} && \text{when} \quad h_d > 0 \end{aligned} \tag{A3}$$

The second term on the right-hand side of equation (A3) represents the effect of strain-thinning of the debris-laden ice layer. (Recall that during a purge, a layer of thickness d is assumed to thin exponentially with time, i.e., $d_t = -d/\tau_{is}$.) Equations (A1) and (A2) were incorporated into the low-order model decribed by *MacAyeal* [1993a] to generate the results we present in this study.

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