

ENVS S422: Earth's Climate System

Modeling Exercise 1: Introduction to systems modeling

This modeling exercise is designed to give you familiarity with systems modeling in STELLA. The handout consists of (i) background on modeling, which you should use as a reference for future modeling exercises, and (ii) a series of simple model experiments that you will build and explore to gain an understanding of some important systems concepts. For each model experiment, you should submit a graph (with the exception of Section 2.2) and a brief 1-paragraph response to the questions that are being explored in the exercise.

Due date: 30 January 2019

1 BACKGROUND

In order to illustrate some fundamental aspects of modeling and the behavior of dynamic systems, we'll start with the simplest system imaginable — a tub of water with a faucet and drain.

1.1 First steps

The first step in modeling is to define and consider the system as it actually exists in the real world. This involves identifying the components of the system, what material or entity is moving through the system, what processes are involved in moving this material or entity, and what kinds of things these processes are likely to depend on. This step is often facilitated by drawing a cartoon of the system, as shown in the schematic of the water tub (Figure 1). The purpose of a schematic is to

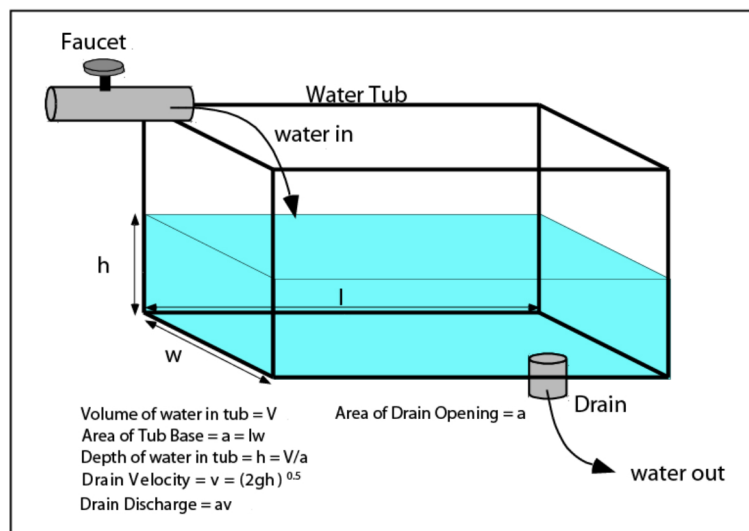


Figure 1: A simple sketch of the water tub system, consisting of a faucet, a drain, and tub that contains water. The faucet flow rate is independently controlled, but the rate of flow through the drain is a function of the water depth and the area of the drain opening.

clarify what we are modeling, the components of the system, and the relationships between these

In the model that we will be exploring, the faucet can be adjusted to supply water at whatever flow rate we choose. In contrast, the rate of flow through the drain is going to be a function of the size of the drain opening and how much water is in the tub because the weight of the water overlying the drain determines the amount of pressure that is forcing the water through the drain. So, the outflow is dependent on the amount of water in the tub. A more precise description of this dependence is provided by Torricelli's Law, which states that the velocity of water flowing out of a drain is equal to the square root of two times the gravitational acceleration times the depth of the water above the drain. The velocity is multiplied by the area of the drain opening to give an outflow in volume of water per unit time.

The diagram illustrates a bathtub model, a common representation in system dynamics. It shows the flow of water into and out of a bathtub (the stock).

- Source (infinite):** Represented by a cloud icon, it is the origin of the water flow.
- Flow:** Indicated by a wavy arrow, it shows the rate of water entering the bathtub.
- Faucet:** A circle with a tap symbol, representing the control element that regulates the inflow.
- water tub:** A square box representing the bathtub, which is the stock of water.
- Flow:** Indicated by a wavy arrow, it shows the rate of water leaving the bathtub.
- Sink (infinite):** Represented by a cloud icon, it is the destination of the water flow.
- drain:** A circle with a drain symbol, representing the control element that regulates the outflow.
- CONNECTORS:** Arrows that link the variables in the model.
- area of tub base:** A circle representing the area of the bathtub's base.
- depth of water:** A circle representing the current level of water in the tub.
- CONVERTERS:** Circles with a gear symbol that perform calculations on the inputs.
- gravity:** A circle representing the constant acceleration due to gravity.
- drain velocity:** A circle representing the speed at which water is leaving the tub.
- drain area:** A circle representing the area of the drain.

Numerical Values & Expressions:

- INIT water tub = 1 (m³)
- faucet = 1 (m³/sec)
- drain = drain velocity * drain area (m³/sec)
- drain area = .05 (m²)
- area of tub base = .5 (m²)
- depth of water = water tub / area of tub base (m)
- gravity = 9.8 (m/sec²)
- drain velocity = SQRT(2*gravity*depth of water) (m/sec)

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1.2 A word on units

A very important step in setting up a model is to make sure that your units agree. Otherwise you won't really know what is being calculated when the model is being run. For this simple model of a water tub, we'll use units of meters and seconds. When you specify the model parameters in STELLA, you can also select the units for each reservoir, flow, and converter.

1.3 Mathematics and numerical algorithms

In mathematical terms, the flows in the STELLA models essentially represent rates of change. Thus, the models that you will be using throughout the semester consist of coupled differential equations. For some simple systems you may be able to find an *analytical solution* (i.e., one that you derive by hand). As systems become more complex, analytical solutions become increasingly difficult/impossible to find. Computer methods for solving equations (and doing so efficiently) is an active area of research in scientific computing and numerical analysis. For our purposes, you should just know that STELLA uses a variety of finite difference schemes to integrate the model equations. In particular, you can choose between Euler's Method, Runge-Kutta 2, and Runge-Kutta 4. Due to the way that these algorithms work, you can get large numerical errors if you choose a time step that is "too large". One way to check whether your time step is too large is to run multiple simulations with different time steps and compare the results. A model that has time steps that are too large will likely produce erratic, nonphysical results. With the Runge-Kutta methods, computations take longer but you can get away with larger time steps.

2 SYSTEMS CONCEPTS

A number of important concepts connected to dynamic systems can easily be illustrated with a simple model of a water tub. Begin by building the model presented in Figure 2.

2.1 Steady state

If you run the model for 100 s, you should observe the system evolving to a *steady-state*, which means that the amount of water in the tub remains constant. Generate and submit a single plot that shows the temporal evolution of the inflow from the faucet, the outflow from the drain, and the volume of water in the tub. Include with the plot a brief description of the relationship between these model outputs.

2.2 Residence time

Once the system is at (or at least very near to) steady state, we can calculate the *residence time*. The residence time is the average length of time that an entity, in this case a water molecule, remains in a reservoir. By definition, the residence time is the amount of material in the reservoir, divided by either the inflow or the outflow (they are equal when the reservoir is in steady-state). If there are multiple inflows or outflows, then we use the sum of the inflows or outflows to determine the residence time.

What is the residence time for the model run that you did in Section 2.1? Does the residence time for this simple system depend on the inflow? (To answer this, double the inflow and the model duration and run the model a second time.)

Residence time is an important concept in problems of pollutants in ground water or surface water reservoirs, and also in understanding the long-term effects of greenhouse gases added to the atmosphere.

2.3 Response time

A concept that is closely related to the residence time is the *response time* of a system, which measures how quickly a system recovers and returns to its steady state after some perturbation. This concept can be illustrated by running several simulations in which you vary the initial state of the reservoir and observing how long it takes the system to reach a steady state.

Set the model inflow back to $1 \text{ m}^3/\text{s}$ and the model duration to 100 s. Run five simulations in which you vary the initial volume of the tub from 0–20 m^3 in increments of 5 m^3 .

Submit a graph showing how the volume of water in the tub varies with time for each of the five simulations. To include all of the model results in one plot, you will need to set-up the graph before running any of the model simulations. Change the graph properties by setting the range of the graph from 0–20 m^3 and clicking the box labelled “Comparative”; the latter holds the results from previous simulations. How does a perturbation to a volume of water in the tub affect the system’s response time?

The concept of response time is important in many processes in Earth science, such as the evolution of the global carbon cycle. If we halt the anthropogenic emissions of CO_2 , the response time of the system tells us how long it would take for the carbon cycle to return to a more natural state.

In this example, you should have always observed the system returning to the same steady state, which indicates that the steady-state of the system is primarily determined by the nature of the inflows and outflows. Later in the semester, we will explore systems that have multiple steady states.

2.4 Feedback mechanisms

This particular system returns to a steady state because it contains a *negative feedback mechanism* in the connection between the drain flow rate and the amount of water in the tub. A negative feedback mechanism is a controlling mechanism that tends to counteract some kind of initial imbalance or perturbation. Note that the word negative, as used here, does not mean that it is bad feedback; it just means that the feedback mechanism acts to reverse the change that set the feedback mechanism into operation. So if our tub is in its steady state, knocking the system out of its steady state by suddenly dumping in more water will cause a response — the drain will increase its flow rate, thus decreasing the volume of water in the tub and bringing it back towards the steady state value. If we instead decrease the amount in the tub, the negative feedback associated with the drain forces the amount of water in the tub to increase until the steady state is returned. The important thing to remember is that negative feedback mechanisms tend to have stabilizing effects on systems.

In contrast, a *positive feedback mechanism* is one that exacerbates some initial change from the steady state, leading to a runaway condition — it acts to promote an enhancement of the initial change. A simple way to modify the tub system in order to create a positive feedback system is to alter the inflow and outflow. This time, set the outflow to $1 \text{ m}^3/\text{s}$ and let the inflow equal 0.2

times the volume of water in the tub. (To do this, you will need to delete most of the converters and connectors in the model.) For this experiment, run the simulation for 50 s.

Does this model have a steady-state? If so, what is the volume of water in the tub when it is in steady state? What happens if you increase or decrease the initial volume of water? Submit a comparative plot that illustrates how the water volume evolves for different initial volumes.

Positive feedback mechanisms, like negative feedback mechanisms, are not necessarily good or bad. Epidemics and infections have positive feedback mechanisms associated with them, but so does the growth of money in a bank account with compounded interest. The Earth contains a wide variety of both positive feedbacks and negative feedbacks. Depending on the conditions, either kind may dominate. The fact that we exist, and that our planet has water and an atmosphere, is compelling evidence to suggest that the Earth system is dominated by negative feedback mechanisms over long time scales. However, over time periods that matter to humans, positive feedback mechanisms may be very important and have the potential to produce dramatic changes.

2.5 Lag time

You will now build a slightly more complex system to investigate the concept of *lag time*. In this model, two tubs are connected such that the water from the first tub drains into the second tub, which then drains out of the system. Set the initial volume of the tubs to 10 m^3 and the outflow from both tubs to 0.1 times the water volume. The inflow will vary with time, which you can describe in STELLA using a “graphical function”. Set the inflow equal to “TIME”, and then click the graph button below the equation definition. Modify the graph so that the inflow starts with a value of $1 \text{ m}^3/\text{s}$, then rapidly increases to a peak of $4 \text{ m}^3/\text{s}$ and returns back to $1 \text{ m}^3/\text{s}$. Generate and submit a plot illustrating the variation in inflow and the water volume of both tubs. Describe the behavior that you observe and compare it to what you might expect for a flood propagating down a river.

The concept of a lag time is also relevant to systems such as the global carbon cycle. If we halt CO_2 emissions today, the climate will continue to warm.