

Name(s):

**Topics:**

1. Relationship between torque, moment of inertia, and rotational motion
2. Combination motion (rotational motion plus translational motion)

**Introduction:**

In this lab you will explore the concepts of rotational motion, rolling motion, and moment of inertia. In the first part of the lab, you will use a rotational motion apparatus to measure the moment of inertia for two objects. In the second part, you will compare the speed at which objects of different shapes, and therefore different moments of inertia, roll down an incline.

Newton's second law, written for linear motion, has the familiar form  $\sum \vec{F} = m\vec{a}$ , and states that the linear acceleration that an object receives depends directly on the force acting on it and inversely on its mass. For the case of angular motion, we have seen that Newton's second law takes the form  $\sum \tau = I\alpha$ . Here the angular acceleration depends directly on the torque acting on a body and inversely on its moment of inertia. Moment of inertia is the property of an object that describes its resistance to changes in angular velocity, just as mass is a measure of an object's resistance to changes in linear velocity. The moment of inertia depends on the mass of a body and how that mass is distributed — its shape, size, and location of the rotational axis.

**What you should turn in:**

You may submit a group report.

1. Part 1: A free-body diagram [2 pts], the force balance equations and derivation you used to arrive at an equation for the moment of inertia [4 pts], and your measured and theoretical moments of inertia [4 pts].
2. Part 2: A derivation of the time it takes the object to roll down the ramp [4 pts], a table of the expected and observed travel times [2 pts], and a discussion of how "rolling friction" influences the motion of an object (i.e., discuss what sort of object's are most influenced by rolling friction) [2 pts].

**Equipment**

- Rotating apparatus and disks and rings
- String and hanging masses
- Scale
- Rolling objects (disk and ring)
- 2-m stick

### Additional background

The moment of inertia can be calculated analytically for objects that have a lot of symmetry. In the table below,  $m$  indicates the mass of the object,  $d$  is the distance of the object from the axis of rotation,  $R$  is the radius of the object, and  $L$  is the length of a rod.

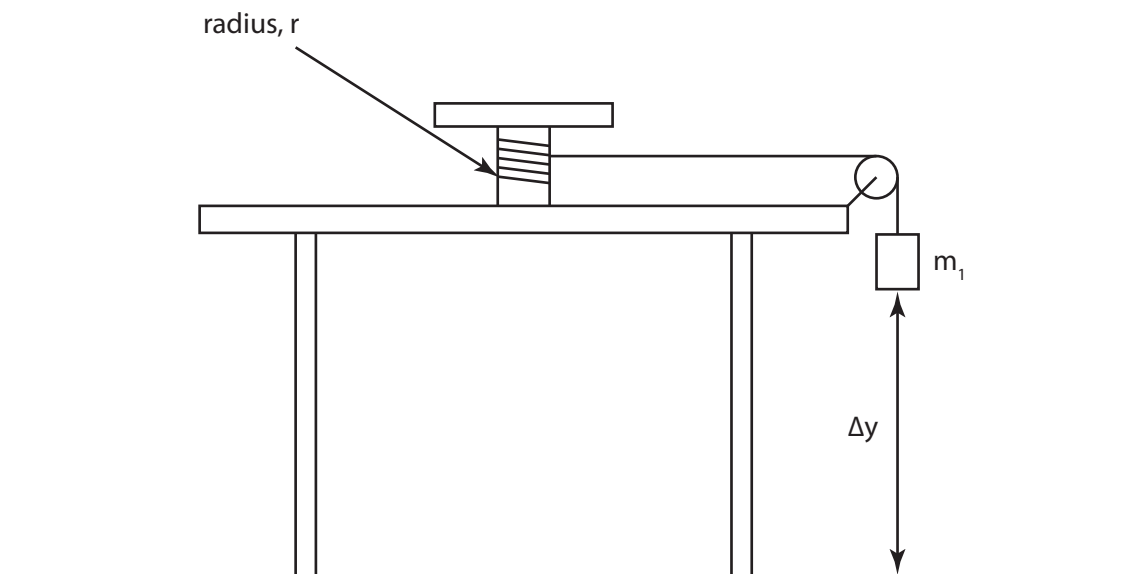
Object	Moment of inertia
Point mass	$md^2$
Cylinder	$\frac{1}{2}mR^2$
Thick-walled cylinder	$\frac{1}{2}m(R_{\text{inner}}^2 + R_{\text{outer}}^2)$
Solid sphere	$\frac{2}{5}mR^2$
Spherical shell	$\frac{2}{3}mR^2$
Rod (around the center)	$\frac{1}{12}mL^2$

In part 1 of the lab, you will estimate and measure the moment of inertia of a disk, a ring, and a system of two point masses. The masses of the disks and rings are shown in the following table.

Object	Mass [kg]
Gray ring	4.297
Gray disk	4.825
Blue ring	4.359
Blue disk	4.975
Brown disk	4.869
“Point” masses	2.703
Rod	0.832

## PART 1: MEASURING AN OBJECT'S MOMENT OF INERTIA

For this part of the lab, a body is set in rotation about a vertical axis. The applied torque is from a constant force produced by a falling mass. Using force and torque balances, you will derive an expression in which the moment of inertia of the rotating body is related to measurable quantities. Additionally, since the moments of inertia of certain bodies may be calculated from their masses and physical dimensions (using the equations in the textbook, which are also reproduced in Part 2), the theoretical moments of inertia may be computed to check the measured values.



This system is illustrated schematically in the figure above. The body on the table is free to rotate about a vertical axis. This rotation occurs as the mass  $m_1$  falls through a distance  $\Delta y$ . The cord that constrains the motion of the mass  $m_1$  is wound around the drum of the rotating body. The radius of the drum is  $r$  and the rotating body has a moment of inertia  $I_{\text{total}}$ . The rotating body will consist of the apparatus plus a disk or ring that you place on top. Assume that the pulley is massless and frictionless.

Before performing any experiments, **draw a free body diagram** and use Newton's Second Law to **derive an expression for the moment of inertia of the rotating object** that depends on  $m_1$ ,  $r$ ,  $\Delta y$ , and  $\Delta t$ . Be sure that your derivation is correct before proceeding.

Now perform experiments in which you measure the variables needed to calculate the moment of inertia.

### Procedures:

1. Measure the diameter of the drum around which the cord is wound.
2. Use a cord long enough so that the hanging weight can fall at least 1 meter. Wind the cord around the drum.
3. Make a practice run to make sure the apparatus is properly set up.

The rotating apparatus has a moment of inertia even without the ring or disk. You will need to take this into account when calculating the moment of inertia of the ring/disk. Because the apparatus and the ring/disk have the same axis of rotation, we can write

$$I_{\text{total}} = I_{\text{ring}} + I_{\text{apparatus}}$$

When you perform the experiments you will be calculating  $I_{\text{total}}$ . To calculate  $I_{\text{ring}}$ , you will therefore need to also know  $I_{\text{apparatus}}$ , which you can determine by running the experiment without the ring/disk. You will perform this experiment for two objects — a disk and a ring. Measure the physical mass and dimensions of the objects.

**Free body diagram and force balance derivation**

Sketch a free body diagram and use Newton's Laws to derive an expression for the moment of inertia of rotating apparatus.

### Measured and theoretical moments of inertia

Use the experimental values and the equation that you derived to calculate the moment of inertia of the ring and the disk.

	$m_1$ [kg]	$r$ [m]	$\Delta y$ [m]	$\Delta t$ [s]	$I_{total}$ [kg·m <sup>2</sup> ]	$I_{object}$ [kg·m <sup>2</sup> ]
Platform						N/A
Disk						
Ring						
Rod						
2 point masses						

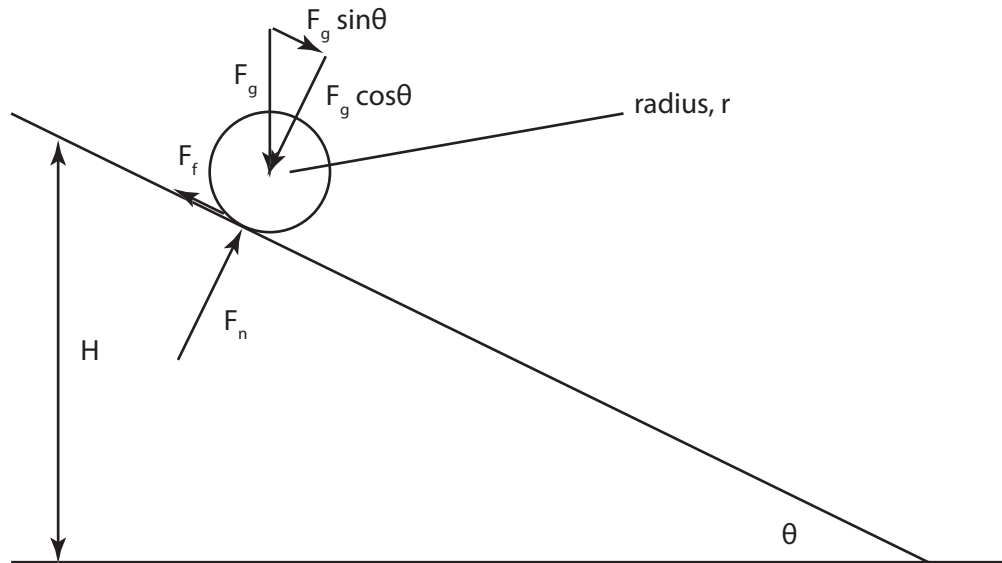
Next, calculate the theoretical moments of inertia using the formulas on page 2 of the handout.

	$I_{theoretical}$ [kg·m <sup>2</sup> ]
Disk	
Ring	
Rod	
2 point masses	

How do your measured and theoretical values compare?

## PART 2: MOTION OF AN OBJECT ROLLING DOWN A SLOPE

For this part of the lab, you will compare the motion of two solid cylinders and two hollow cylinders as they roll down an inclined plane. For each object, measure the mass and radius so that you can compare the measured travel times with theoretically expected travel times.



First, derive an expression for the time that it takes the object to roll down the ramp. Assume that the objects roll without slipping. The result will depend on the object's moment of inertia, which can be computed using the equations on page 2 of the handout.

**Derivation of the time that it takes the object to roll down the ramp**

Record the shape, mass, and radii of each of the four objects. For the hollow cylinders, you will need both the inner and outer radii.

	$m$ [kg]	$r_{\text{outer}}$ [m]	$r_{\text{inner}}$ [m]	$I$ [kg·m <sup>2</sup> ]	Expected $\Delta t$ [s] (including friction)	Expected $\Delta t$ [s] (ignoring friction)
Object 1						
Object 2						
Object 3						
Object 4						

Record the amount of time it actually takes the objects to roll down the ramp. Do three trials for each object and calculate the average  $\Delta t$ .

	$\Delta t_1$ [s]	$\Delta t_2$ [s]	$\Delta t_3$ [s]	$\Delta t_4$ [s]
Trial 1				
Trial 2				
Trial 3				
AVERAGE:				

**How does “rolling motion” influence the motion of an object? In other words, compare your results to what would happen with a frictionless surface, and discuss how the mass distribution affects your results.**

Compare these results to the expected values of  $\Delta t$ . For which objects did the rotational motion have the biggest effect on travel time? How much did including rolling friction change your results?

Note that you need to describe your observations both qualitatively and quantitatively. To help with interpreting the results (and as a check that your measurements and calculations are consistent), it may be helpful to do a series of head-to-head races with the different shapes. Which object accelerated most quickly? Least quickly?

