Basic definitions for linear motion

- displacement: $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$
- average velocity: $\vec{v} = \dfrac{\Delta\vec{x}}{\Delta t}$
- average acceleration: $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$
- if $a \cdot v > 0$, object is speeding up

Kinematic equations, $\vec{v} = \text{constant}$ (i.e., $\vec{a} = 0$)

- $\vec{v} = \dfrac{\Delta\vec{x}}{\Delta t}$
- $\vec{x}_f = \vec{x}_i + \vec{v}\Delta t$

Kinematic equations, $\vec{a} \neq 0$ and $\vec{a} = \text{constant}$

- $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$
- $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$
- $\Delta\vec{x} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$
- ${v_f}^2 - {v_i}^2 = 2a\Delta x$
- common example of $a = \text{constant}$ is $g = 9.81\ \text{m/s}^2$

Motion of object A relative to object C

- $\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$

Basic definitions for circular and rotational motion

- angle: $\theta(\text{radians}) = \dfrac{s}{r}$; $s =$arclength, $r =$radius
- angular displacement: $\Delta\theta = \theta_f - \theta_i$
- angular velocity: $\omega = \dfrac{\Delta\theta}{\Delta t} = 2\pi f = 2\pi\dfrac{1}{T}$
- angular acceleration: $\alpha = \dfrac{\Delta\omega}{\Delta t}$
- if $\alpha \cdot \omega > 0$, object is speeding up

Kinematic equations

- $\Delta\theta = \omega_i\Delta t + \dfrac{1}{2}\alpha\Delta t^2$; if $\alpha = 0$, $\omega_f = \omega_i = \text{constant}$.
- $\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

Speed, acceleration, and forces

- speed: $v = \omega r$
- centripetal acceleration: $a_c = \dfrac{v^2}{r} = \omega^2 r$

- centripetal force: $F_c = ma_c = m\frac{v^2}{r} = m\omega^2 r$; points toward center of circle
- tangential acceleration: $a_t = \alpha r$

Newton's Laws

1. if $\vec{F}_{net} = 0 \Rightarrow \vec{a} = 0$
2. $\sum \vec{F} = m\vec{a}$
3. $\vec{F}_{1\,on\,2} = -\vec{F}_{2\,on\,1}$

Types of forces

- gravitational force (aka, weight) at the Earth's surface, $\vec{F}_g = m\vec{g}$
- Newton's Law of Gravity (gravitational force between two objects): $F_{1\,on\,2} = F_{2\,on\,1} = \frac{Gm_1m_2}{r^2}$
- Gravitational constant: $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
- normal force, $\vec{F}_n$, is perpendicular to surface and prevents objects from penetrating the surface
- frictional force, $\vec{F}_f$, depends on $\vec{F}_n$ and $\vec{v}$
  - if $\vec{v} = 0$, $\vec{F}_f$ balances other forces as long as $\left|\vec{F}_f\right| \leq \mu_s \left|\vec{F}_n\right|$
  - if $\vec{v} \neq 0$, $\left|\vec{F}_f\right| = \mu_k \left|\vec{F}_n\right|$
- tensional force, $\vec{F}_t$, is transmitted through a rope and around pulleys
- spring force / Hooke's 'Law': $F = -k\Delta x$; k is empirically determined spring constant

Torque and moment of inertia

- Torque: $\tau = rF_\perp$; $F_\perp$ is force perpendicular to radial axis
- Newton's Second Law for rotation: $\sum \tau = I\alpha$
- Moment of inertia: I, indicates how difficult it is to rotate an object
- Rolling constraint: $v = \omega R$; rolling object, with perfect friction, moves forward with this velocity

Static equilibrium and elasticity

- $\sum F_x = 0$; $\sum F_y = 0$; $\sum \tau = 0$
- Choose convenient pivot point
- Before coming to equilibrium, objects change shape to accommodate the forces being exerted on them.
- The relationship between force and deformation for elastic materials is given by $\frac{F}{A} = Y\left(\frac{\Delta L}{L}\right)$, where Y is Young's modulus (a material property). Sometimes this is written as stress $= Y \times$ strain

Impulse and momentum

- impulse, $\vec{J} = \vec{F}_{avg}\Delta t$
- momentum, $\vec{p} = m\vec{v}$

- Impulse-Momentum Theorem, $\vec{J} = \Delta\vec{p}$
- total momentum, $\vec{P} = \vec{p}_1 + \vec{p}_2 + ...$
- conservation of momentum, $\Delta P = 0$ if $\vec{F}_{net} = 0$ or if $\vec{F}_{net}$ is small compared to other forces for short Δt

Energy and work

- Total energy: $E = K + U_g + U_s + E_{ch} + E_{th} + ...$
- Work: $W = \Delta E$; mechanical transfer of energy into or out of a system
- Conservation of energy: for isolated system, $W = 0$ and therefore $\Delta E = 0$.
- Work: $W = F_{\|} \cdot d$; force $\times$ displacement
- Translational kinetic energy: $K_{trans} = \frac{1}{2}mv^2$; scalar quantity
- Rotational kinetic energy: $K_{rot} = \frac{1}{2}I\omega^2$
- Total kinetic energy: $K = K_{trans} + K_{rot}$
- Gravitational potential energy: $\Delta U_g = mg\Delta y$; choose convenient reference height
- Elastic potential energy: $U_s = \frac{1}{2}k(\Delta x)^2$; k is the spring constant
- Thermal energy (from friction): $\Delta E_{th} = F_f \Delta x$
- Conservation of mechanical energy: for isolated system with no friction, $\Delta K + \Delta U_g + \Delta U_s = 0$

Thermodynamics

- 1st Law: $Q + W = \Delta E_{th}$; Q is heat transferred into or out of the system
- 2nd Law: Entropy in a closed system (which describes disorder) can never decrease
- energy needed to change material's temperature: $Q = mc\Delta T$; c is the specific heat, a material property
- energy need to change a material's phase: $Q = mL_f$ to melt solid and $Q = mL_v$ to turn liquid into gas
- $L_v > L_f$ (latent heat of vaporization is greater than latent heat of fusion)
- conduction: $\frac{Q}{\Delta t} = \left(\frac{kA}{L}\right)\Delta T$; k is thermal conductivity
- advection/convection: need to solve fluid flow equations
- radiation: $\frac{Q}{\Delta t} = e\sigma AT^4$; e is emissivity and $\sigma = 5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2\text{K}^4}$ is the Stefan-Boltzmann constant
- for ideal gas in enclosed, insulated container, $\frac{PV}{T} = $ constant
- work done by expanding a gas at constant pressure: $W_{\text{gas}} = P\Delta V$
- note that for gases, specific heat at constant pressure c_p differs from specific heat at constant volume c_v. c_p is approximately 50% larger than c_v
- thermal energy: $\Delta E_{th} = \frac{3}{2}nR\Delta T$

Fluids

- density: $\rho = \frac{m}{V}$
- hydrostatic pressure: $P = \rho g h + P_0$; P_0 is pressure from above
- Archimede's principle (upward buoyant force): $F_b = \rho_f g V$; ρ_f is fluid density and V is volume displaced
- flux (continuity): $Q = vA$; Q is constant in a pipe for ideal fluid
- Bernoulli's equation: along a streamline, $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

Simple harmonic motion

- displacement: $x(t) = A\cos(2\pi f t)$
- velocity: $v(t) = -v_{\text{max}}\sin(2\pi f t)$
- acceleration: $a(t) = -a_{\text{max}}\cos(2\pi f t)$
- $v_{\text{max}} = 2\pi f A$ and $a_{\text{max}} = (2\pi f)^2 A$
- for a spring, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$; for a pendulum, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

Waves

- in general, $v = \lambda/T$, where λ is wavelength and T is period
- velocity of a wave on a string: $v = \sqrt{\frac{F_t}{\mu}}$, where μ is linear density
- velocity of sound waves in ideal gas: $v = \sqrt{\frac{\gamma R T}{M}}$, where γ is adiabatic index, $R = 8.314\frac{\text{J}}{\text{mol K}}$ is the gas constant, T is temperature in Kelvin, and M is the molar mass
- travelling wave displacement: $y(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$
- Doppler effect: $f = \frac{f_s}{1 \pm v_s/v}$; v_s is speed of source, v is wave speed. Use + if the source is moving away from the receiver, − if the source is moving toward the receiver.
- wave superposition: $y(x,t) = y_1 + y_2 + y_3$ where y_i are given by different amplitudes, wavelengths, and frequencies/periods.
- standing wave displacement: $y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right) + A\cos\left(2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right)$
- harmonics: $f_m = \frac{2L}{m}$ for a string that is pinned on both ends