PHYS S211: General Physics I
Lab 5: Torque
10/8/24 (due 10/15/24)

Name(s):

Topics:

1. Relationship between torque, moment of inertia, and rotational motion
2. Combination motion (rotational motion plus translational motion)

Introduction:
In this lab you will explore the concepts of rotational motion, rolling motion, and moment of inertia. In the first part of the lab, you will use a rotational motion apparatus to measure the moment of inertia for three objects. In the second part, you will compare the speed at which objects of different shapes, and therefore different moments of inertia, roll down an incline.

Newton's second law, written for linear motion, has the familiar form $\sum \vec{F} = m\vec{a}$, and states that the linear acceleration that an object receives depends directly on the force acting upon it and inversely upon its mass. For the case of angular motion, we have seen that Newton's second law takes the form $\sum \tau = I\alpha$. Here the angular acceleration depends directly on the torque acting on a body and inversely on its moment of inertia. Moment of inertia is the property of an object that measures its ease or resistance to change in angular velocity, just as mass is a measure of an object's ease or resistance to change in linear velocity. The moment of inertia depends on the mass of a body and how that mass is distributed — its shape, size, and location of the rotational axis.

What you should turn in:
You may submit a group report. Submit the report as a single document separate from this hand-out, and clearly indicate if you are submitting it as a group report.

1. Part 1: A free-body diagram [2 pts], the force balance equations and derivation you used to arrive at an equation for the moment of inertia [4 pts], and your calculated and estimated moments of inertia [6 pts]. How much error do you introduce when treating the cylinders as point masses? [2 pts]
2. Part 2: A brief description of the experiment and qualitative observations [2 pts], a derivation of the time it takes the object to roll down the ramp [4 pts], a table of the expected and observed travel times [2 pts], and a discussion of how "rolling friction" influences the motion of an object (i.e., discuss what sort of object's are most influenced by rolling friction) [2 pts].

Equipment

- Rotating apparatus and disks, rings, and "point" masses
- String and hanging masses
- Scale
- Rolling objects (disk and ring)
- 2-m stick

Additional background
The moment of inertia can be calculated analytically for objects that have a lot of symmetry. In the table below, m indicates the mass of the object, r is the distance of the object from the axis of rotation, and R is the radius of the object.

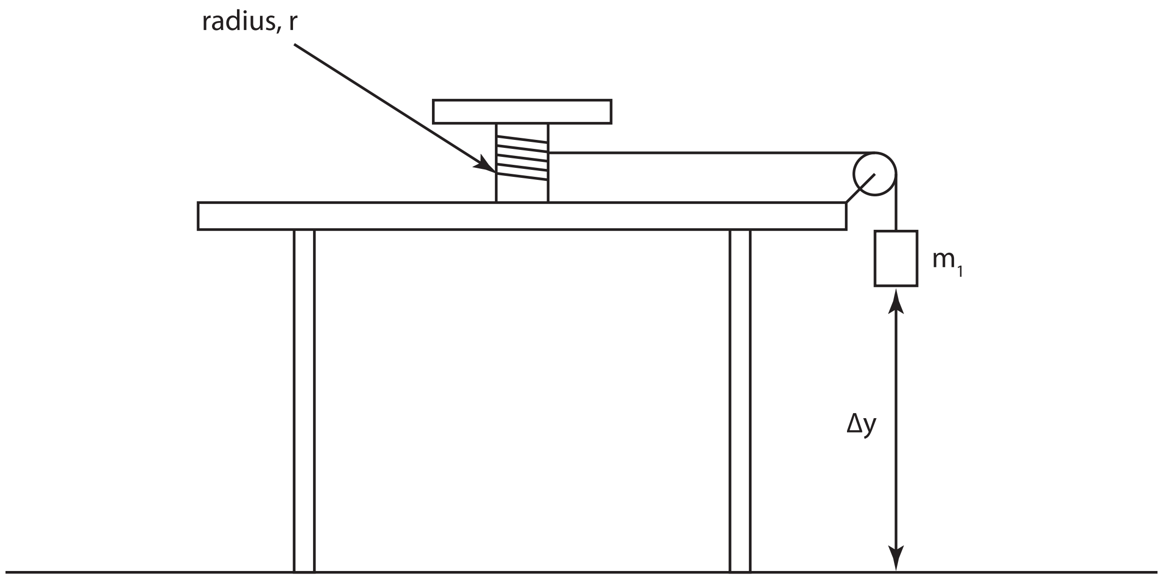
Object	Moment of inertia
Point mass	mr^2
Cylinder	$\frac{1}{2}mR^2$
Thick-walled cylinder	$\frac{1}{2}m\left(R_{\text{inner}}^2 + R_{\text{outer}}^2\right)$
Solid sphere	$\frac{2}{5}mR^2$
Spherical shell	$\frac{2}{3}mR^2$

In part 1 of the lab, you will estimate and measure the moment of inertia of a disk, a ring, and a system of two point masses. The masses of the disks and rings are shown in the following table.

Object	Mass [kg]
Gray ring	4.297
Gray disk	4.825
Blue ring	4.359
Blue disk	4.975
Brown disk	4.869

PART 1: MEASURING AN OBJECT'S MOMENT OF INERTIA

For this part of the lab, a body is set in rotation about a vertical axis. The applied torque is from a constant force produced by a falling mass. Using force and torque balances, you will derive an expression in which the moment of inertia of the rotating body is related to measurable quantities. Additionally, since the moments of inertia of certain bodies may be calculated from their masses and physical dimensions (see Additional background), the theoretical moments of inertia may be computed to check the measured values.



This system is illustrated schematically in the figure above. The body on the table is free to rotate about a vertical axis. This rotation occurs as the mass m_1 falls through a a distance Δy. The cord that constrains the motion of the mass m_1 is wound around the drum of the rotating body. The radius of the drum is r and the rotating body has a moment of inertia I_{total}. The rotating body will consist of the apparatus plus a disk or ring that you place on top. Assume that the pulley is massless and frictionless.

Before performing any experiments, **draw a free body diagram** and use Newton's Second Law to **derive an expression for the moment of inertia of the rotating object**. Be sure that your derivation is correct before proceeding.

Now perform experiments in which you measure the variables needed to calculate the moment of inertia.

Procedures:

1. Measure the diameter of the drum around which the cord is wound.
2. Use a cord long enough so that the hanging weight can fall at least 0.5 m. Wind the cord around the drum.
3. Make a practice run to make sure the apparatus is properly set up.

The rotating apparatus has a moment of inertia even without anything being placed on top of it. You will need to take this into account when calculating the moment of inertia of the object. Because the apparatus and the object have the same axis of rotation, we can write

$$I_{\text{total}} = I_{\text{object}} + I_{\text{apparatus}}$$

When you perform the experiments you will be calculating I_{total}. To calculate I_{object}, you will therefore need to also know $I_{\text{apparatus}}$, which you can determine by running the experiment without the ring/disk. You will perform this experiment for three objects — a disk, a ring, and a system of two point masses. Be sure to measure the dimensions of the objects, which you will need for calculating the theoretical moments of inertia.

Free body diagram and force balance derivation
Sketch a free body diagram and use Newton's Laws to derive an expression for the moment of inertia of rotating apparatus.

Calculated and estimated moments of inertia

Use the experimental values and the equation that you derived to calculate the moment of inertia of the ring and the disk. Finally, measure the mass and diameter/radius of the objects. **Use these values to calculate the theoretical moments of inertia** using the equations given in Part 2.

No disk or ring:

Disk:

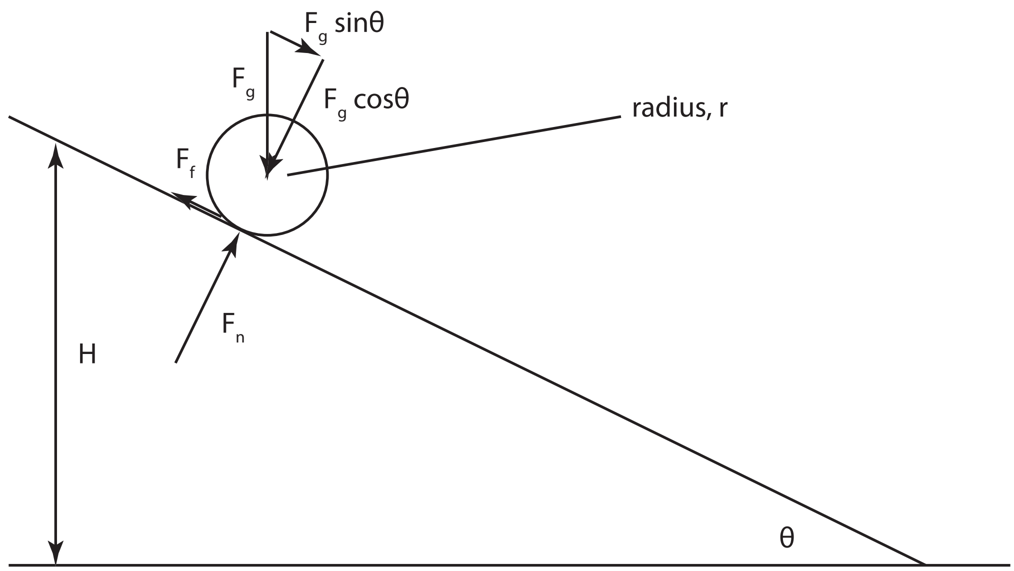
Ring:

System of two point masses:

When calculating the moment of inertia of the "point" masses, you assumed that the cylinders had all of their mass concentrated at a single point. How much error does this introduce? To answer this, calculate the moment of inertia of two cylinders rotating around shared axis. Note that the moment of inertia for the finite objects presented in the Additional background assumes that the objects rotate around their central axes. To shift to a different axis, you need to use the parallel-axis theorem (ie., add mr^2 to the moment of inertia of the object shown in the table). How much does your theoretical moment of inertia change when you do this?

PART 2: MOTION OF AN OBJECT ROLLING DOWN A SLOPE

For this part of the lab, you will compare the motion of two solid cylinders and two hollow cylinders as they roll down an inclined plane. For each object, measure the mass and radius so that you can compare the measured travel times with theoretically expected travel times.



First, derive an expression for the time that it takes the object to roll down the ramp. The result will depend on the object's moment of inertia, which can be computed analytically for some shapes.

Record the shape, mass, and radii of each of the four objects. For the hollow cylinders, you will need both the inner and outer radii.

	m [kg]	r_{outer} [m]	r_{inner} [m]	I [kg·m^2]	Expected Δt [s] (including friction)	Expected Δt [s] (ignoring friction)
Object 1						
Object 2						
Object 3						
Object 4						

Record the amount of time it actually takes the objects to roll down the ramp. Do three trials for each object and calculate the average Δt.

Compare these results to the expected values of Δt. For which objects did the rotational motion have the biggest effect on travel time? How much did including rolling friction change your results?

Note that you need to describe your observations both qualitatively and quantitatively. To help with interpreting the results (and as a check that your measurements and calculations are consistent),

	Δt_1 [s]	Δt_2 [s]	Δt_3 [s]	Δt_4 [s]
Trial 1				
Trial 2				
Trial 3				
AVERAGE:				

it may be helpful to do a series of head-to-head races with the different shapes. Which object accelerated most quickly? Least quickly?

In your report, you should be sure to include:

- a description of the experiment and qualitative observations,
- a derivation of the time that it takes the object to roll down the ramp, and
- a discussion of how the distribution of mass within an object influences how quickly it rolls.