Basic definitions of kinematics

- position: $\vec{x} = \langle x, y \rangle$
- displacement: $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$ for finite displacement
- instantaneous velocity: $\vec{v} = \dfrac{d\vec{x}}{dt}$
- instantaneous acceleration: $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{x}}{dt^2}$

Kinematic equations, $\vec{v} =$ constant (i.e., $\vec{a} = 0$)

- $\vec{v} = \dfrac{\Delta\vec{x}}{\Delta t}$
- $\vec{x}_f = \vec{x}_i + \vec{v}\Delta t$

Kinematic equations, $\vec{a} \neq 0$ and $\vec{a} =$ constant

- $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$
- $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$
- $\Delta\vec{x} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$
- $(v_{x,f})^2 - (v_{x,i})^2 = 2a_x\Delta x$ and $(v_{y,f})^2 - (v_{y,i})^2 = 2a_y\Delta y$
- common example of $a =$ constant is $g = 9.81$ m/s^2

Motion of object A relative to object C

- $\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$

Basic definitions for circular and rotational motion

- angular position: $\theta(\text{radians}) = \dfrac{s}{r}$; $s =$arclength, $r =$radius
- angular displacement: $\Delta\theta = \theta_f - \theta_i$
- angular velocity: $\omega = \dfrac{d\theta}{dt} = 2\pi f = \dfrac{2\pi}{T}$; $f =$frequency, $T =$period
- angular acceleration: $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Kinematic equations for constant angular acceleration

- $\omega_f = \omega_i + \alpha\Delta t$
- $\Delta\theta = \omega_i\Delta t + \dfrac{1}{2}\alpha\Delta t^2$; if $\alpha = 0$, $\omega_f = \omega_i =$ constant.
- $\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

Speed, acceleration, and forces

- speed: $v = \omega r$
- centripetal acceleration: $a_c = \dfrac{v^2}{r} = \omega^2 r$
- centripetal force: $F_c = ma_c = m\dfrac{v^2}{r} = m\omega^2 r$; points toward center of circle
- tangential acceleration: $a_t = \alpha r$

Newton's Laws

1. if $\vec{F}_{net} = 0 \Rightarrow \vec{a} = 0$
2. $\sum \vec{F} = m\vec{a}$
3. $\vec{F}_{12} = -\vec{F}_{21}$

Types of forces

- Newton's Law of Gravity: $F_{12} = F_{21} = \dfrac{Gm_1 m_2}{r^2}$; points from one object to another; this can also be expressed as $\vec{F}_{12} = -\dfrac{Gm_1 m_2}{r^2}\hat{r}_{12}$ where $\hat{r}_{12}$ is the unit vector that points from object 1 to object 2.
- Gravitational constant: $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
- For objects near the Earth's surface, $F_g = mg$
- normal force, $\vec{F}_n$, is perpendicular to surface and prevents objects from penetrating the surface
- frictional force depends on $\vec{F}_n$ and $\vec{v}$
  - if $\vec{v} = 0$ use static friction; $\vec{F}_s$ balances other forces as long as $\left|\vec{F}_s\right| \leq \mu_s \left|\vec{F}_n\right|$
  - if $\vec{v} \neq 0$ use kinetic friction; $\left|\vec{F}_k\right| = \mu_k \left|\vec{F}_n\right|$
- tensional force, $\vec{F}_t$, is transmitted through a rope and around pulleys

Impulse and momentum

- momentum, $\vec{p} = m\vec{v}$
- Newton's second law, $\sum \vec{F} = \dfrac{d\vec{p}}{dt}$
- Impulse-Momentum Theorem, $\vec{J} = \Delta\vec{p} = \int_{t_0}^{t_1} \vec{F}_{net}\, dt = \vec{F}_{avg}\Delta t$
- total momentum, $\vec{P} = \vec{p}_1 + \vec{p}_2 + ...$
- conservation of momentum, $\Delta\vec{P} = 0$ if $\vec{F}_{net} = 0$ or if $\vec{F}_{net}$ is small compared to other forces for short Δt

Orbits and gravity

- orbital velocity: $v = \sqrt{\dfrac{GM}{r}}$ at a distance of r from the center of a planet with mass M

Torque and moment of inertia

- Torque: $\tau = rF_\perp$; $F_\perp$ is force perpendicular to radial axis; depending on what angles are given, you may find that $F_\perp = F \sin\phi$
- Newton's Second Law for rotation: $\sum \tau = I\alpha = \dfrac{dL}{dt}$; $L = I\omega =$ angular momentum.
- Moment of inertia: I, indicates how difficult it is to rotate an object
- Conservation of angular momentum: if $\tau_{ext} = 0$ then $\Delta L = 0$.
- Rolling constraint: $v = \omega R$ and $a = \alpha R$; rolling object, with perfect friction, moves forward with this velocity

Static equilibrium

- $\sum F_x = 0$; $\sum F_y = 0$; $\sum \tau = 0$
- Choose convenient pivot point

Stability and balance

- Does torque restore object to its original position?
- Critical angle: $\theta_c = \tan^{-1}\left(\dfrac{t}{2h}\right)$; $t =$width of object's base, h is the height to the center of mass

Springs and elasticity

- Hooke's 'Law': $F = -k\Delta x$; k is empirically determined spring constant

Energy and work

- Total energy: $E = K + U_g + U_s + E_{th} + ...$
- Conservation of energy: for an isolated system, $F_{ext} = 0$ and therefore and therefore $W = \Delta E = 0$.
- Work: $W = F_\| \cdot d$; force $\times$ displacement
- Translational kinetic energy: $K_{trans} = \dfrac{1}{2}mv^2$; scalar quantity (note that v is speed, not velocity)
- Rotational kinetic energy: $K_{rot} = \dfrac{1}{2}I\omega^2$
- Total kinetic energy: $K = K_{trans} + K_{rot}$
- Gravitational potential energy: $\Delta U_g = mg\Delta y$; choose convenient reference height

- Elastic potential energy: $U_s = \dfrac{1}{2}k(\Delta x)^2$; k is the spring constant
- Thermal energy (from friction): $\Delta E_{th} = F_f \Delta x$
- Thermal energy (from collision in which one object is initially stationary): $\Delta E_{th} = K_i \left(\frac{m_2}{m_1+m_2}\right)(1 - C_r^2)$, where C_r is the coefficient of restitution for the collision and is given by $C_r = (v_{2,f} - v_{1,f})/v_{1,i}$
- Conservation of mechanical energy: for isolated system with no friction, $\Delta K + \Delta U_g + \Delta U_s = 0$
- Power: $P = \dfrac{dE}{dt}$