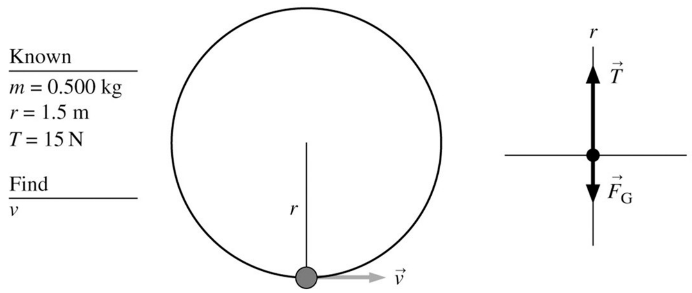
8.25. **MODEL:** Model the ball as a particle which is in a vertical circular motion.

VISUALIZE:

Pictorial representation



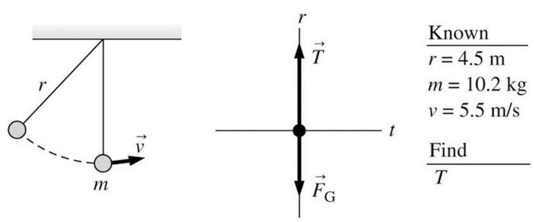
SOLVE: At the bottom of the circle,

$$\Sigma F_r = T - F_G = \frac{mv^2}{r} \Rightarrow (15\text{ N}) - (0.500\text{ kg})(9.8\text{ m/s}^2) = \frac{(0.500\text{ kg})v^2}{(1.5\text{ m})} \Rightarrow v = 5.5\text{ m/s}$$

8.26. **MODEL:** The ball is a particle on a massless rope in circular motion about the point where the rope is attached to the ceiling.

VISUALIZE:

Pictorial representation



Bottom of swing

SOLVE: Newton's second law in the radial direction is

$$(\Sigma F_r) = T - F_G = T - mg = \frac{mv^2}{r}$$

Solving for the tension in the rope and evaluating,

$$T = m\left(g + \frac{v^2}{r}\right) = (10.2\text{ kg})\left(9.8\text{ m/s}^2 + \frac{(5.5\text{ m/s})^2}{4.5\text{ m}}\right) = 168\text{ N}$$

REVIEW: The tension in the rope is greater than the gravitational force on the ball in order to keep the ball moving in a circle.

8.49. MODEL: Assume uniform circular motion.

VISUALIZE: We expect the centripetal acceleration to be very large because ω is large. This will produce a significant force even though the mass difference of 10 mg is so small.

A preliminary calculation will convert the mass difference to kg: $10\text{ mg} = 1.0 \times 10^{-5}\text{ kg}$. If the two samples are equally balanced then the shaft doesn't feel a net force in the horizontal plane. However, the mass difference of 10 mg is what causes the force.

We'll do another preliminary calculation to convert $\omega = 70{,}000\text{ rpm}$ into rad/s.

$$\omega = 70{,}000\text{ rpm} = 70{,}000\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 7330\text{ rad/s}$$

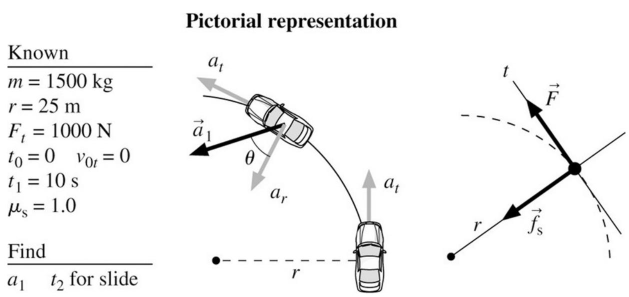
SOLVE: The centripetal acceleration is given by Equation 6.9 and the net force by Newton's second law.

$$F_{\text{net}} = (\Delta m)(a) = (\Delta m)(\omega^2 r) = (1.0 \times 10^{-5}\text{ kg})(7330\text{ rad/s})^2(0.12\text{ m}) = 64\text{ N}$$

REVIEW: As we expected, the centripetal acceleration is large. The force is not huge (because of the small mass difference) but still enough to worry about. The net force scales with this mass difference, so if the mistake were bigger it could be enough to shear off the shaft.

8.61. MODEL: Model the car as a particle on a circular track.

VISUALIZE:



SOLVE: (a) Newton's second law along the t-axis is

$$\Sigma F_t = F_t = ma_t \Rightarrow 1000\text{ N} = (1500\text{ kg})a_t \Rightarrow a_t = 2/3\text{ m/s}^2$$

With this tangential acceleration, the car's tangential velocity after 10 s will be

$$v_{1t} = v_{0t} + a_t(t_1 - t_0) = 0\text{ m/s} + (2/3\text{ m/s}^2)(10\text{ s} - 0\text{ s}) = 20/3\text{ m/s}$$

The radial acceleration at this instant is

$$a_r = \frac{v_{1t}^2}{r} = \frac{(20/3\text{ m/s})^2}{25\text{ m}} = \frac{16}{9}\text{ m/s}^2$$

The car's acceleration at 10 s has magnitude

$$a_1 = \sqrt{a_t^2 + a_r^2} = \sqrt{(2/3\text{ m/s}^2)^2 + (16/9\text{ m/s}^2)^2} = 1.90\text{ m/s}^2 \qquad \theta = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left(\frac{2/3}{16/9}\right) = 21^\circ$$

where the angle is measured from the r-axis.

12.20. MODEL: The pulley acts as if all of its mass were concentrated at its center of mass.

VISUALIZE: There are three torques: two clockwise and one counterclockwise. All of the forces that produce torques act straight down.

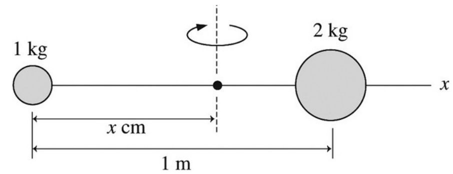
SOLVE:

$$\Sigma\tau = r_1F_1 + r_2F_2 + r_3F_3 = (0.075\text{ m})(15\text{ kg})(9.8\text{ m/s}^2) - (0.075\text{ m})(5.0\text{ kg})(9.8\text{ m/s}^2) - (0.225\text{ m})(10\text{ kg})(9.8\text{ m/s}^2)$$
$$= -14.7\text{ N m}$$

REVIEW: The answer is negative because the clockwise torques are greater than the counterclockwise torques.

12.26. **MODEL:** Two balls connected by a rigid, massless rod are a rigid body rotating about an axis through the center of mass. Assume that the size of the balls is small compared to 1 m.

VISUALIZE:



We placed the origin of the coordinate system on the 1.0 kg ball.

SOLVE: The center of mass and the moment of inertia are

$$x_{cm} = \frac{(1.0\text{ kg})(0\text{ m}) + (2.0\text{ kg})(1.0\text{ m})}{(1.0\text{ kg} + 2.0\text{ kg})} = 0.667\text{ m} \quad \text{and} \quad y_{cm} = 0\text{ m}$$

$$I_{\text{about cm}} = \Sigma m_i r_i^2 = (1.0\text{ kg})(0.667\text{ m})^2 + (2.0\text{ kg})(0.333\text{ m})^2 = 0.667\text{ kg m}^2$$

We have $\omega_f = 0$ rad/s, $t_f - t_i = 5.0$ s, and $\omega_i = -20\text{ rpm} = -20(2\pi\text{ rad/60 s}) = -\frac{2}{3}\pi\text{rad/s}$, so

$\omega_f = \omega_i + \alpha(t_f - t_i)$ becomes

$$0\text{ rad/s} = \left(-\frac{2\pi}{3}\text{rad/s}\right) + \alpha(5.0\text{ s}) \Rightarrow \alpha = \frac{2\pi}{15}\text{rad/s}^2$$

Having found I and α, we can now find the torque τ that will bring the balls to a halt in 5.0 s:

$$\tau = I_{\text{about cm}}\alpha = \left(\frac{2}{3}\text{kg m}^2\right)\left(\frac{2\pi}{15}\text{ rad/s}^2\right) = \frac{4\pi}{45}\text{ N m} = 0.28\text{ N m}$$

The magnitude of the torque is 0.28 N m, applied in the counterclockwise direction.

12.29. **MODEL:** The object is in equilibrium, so $\Sigma F = 0$, and $\Sigma\tau = 0$.

VISUALIZE: There are three torques:

SOLVE:

$$\Sigma F_y = 40\text{ N} + F_2 - F_1 = 0 \Rightarrow F_1 = 40\text{ N} + F_2$$

Take the torques around the left end.

$$\Sigma\tau = (3.0\text{ m})F_2 - (2.0\text{ m})F_1 = 0$$
$$(3.0\text{ m})F_2 - (2.0\text{ m})(40\text{ N} + F_2) = 0$$
$$(3.0\text{ m} - 2.0\text{ m})F_2 = 80\text{ N m} \Rightarrow F_2 = 80\text{ N}$$

Plug back into the force equation to get $F_1 = 120$ N

REVIEW: This can be checked by taking torques around another convenient axis.

We have $\omega_f = 0$ rad/s, $t_f - t_i = 5.0$ s, and $\omega_i = -20$ rpm $= -20(2\pi \text{ rad}/60 \text{ s}) = -\frac{2}{3}\pi \text{rad/s}$, so

$\omega_f = \omega_i + \alpha(t_f - t_i)$ becomes

$$0 \text{ rad/s} = \left(-\frac{2\pi}{3}\text{rad/s}\right) + \alpha(5.0 \text{ s}) \Rightarrow \alpha = \frac{2\pi}{15}\text{rad/s}^2$$

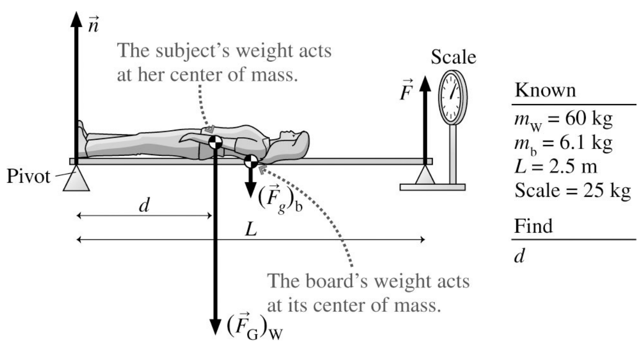
Having found I and α, we can now find the torque τ that will bring the balls to a halt in 5.0 s:

$$\tau = I_{\text{about cm}}\alpha = \left(\frac{2}{3}\text{kg m}^2\right)\left(\frac{2\pi}{15}\text{ rad/s}^2\right) = \frac{4\pi}{45}\text{ N m} = 0.28 \text{ N m}$$

The magnitude of the torque is 0.28 N m, applied in the counterclockwise direction.

12.58. MODEL: Assume the woman is in equilibrium, so $\Sigma F = 0$ and $\Sigma\tau = 0$.

VISUALIZE: Choose the axis to be the left end of the board.



SOLVE: Use $\Sigma_\tau = 0$.

$$\Sigma_\tau = L(\text{scalereading})g - d(m_w)g - \frac{L}{2}(m_b)g = 0 \text{ N}\cdot\text{m}$$

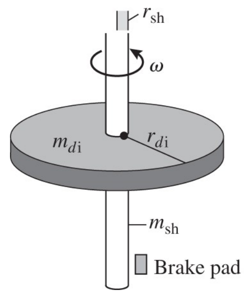
$$d = \frac{L(\text{scalereading})g - \frac{L}{2}(m_b)g}{m_w g} = \frac{L[(\text{scalereading}) - \frac{1}{2}m_b]}{m_w}$$

$$= \frac{(2.5 \text{ m})[(25 \text{ kg}) - \frac{1}{2}(6.1 \text{ kg})]}{60 \text{ kg}} = 0.91 \text{ m}$$

REVIEW: THIS IS A LITTLE MORE THAN HALFWAY UP THE BODY OF A WOMAN OF AVERAGE HEIGHT.

12.67. Model: Assume there are no other forces or torques other than the brake pad.

VISUALIZE: The friction force is tangent to the shaft. The initial angular momentum is $L_i = (I_{di} + I_{sh})\omega$.



Known
$r_{di} = 0.15$ m
$r_{sh} = 0.006$ m
$m_{di} = 1.2$ kg
$m_{sh} = 0.45$ kg
$\Delta t = 15$ s
$\omega = 33$ rpm $= 3.46$ rad/s

Find
f_k

SOLVE:

$$\frac{\Delta L}{\Delta t} = \vec{\tau} = \vec{r} \times \vec{F} = \vec{r}_{sh} \times \vec{f}_k \Rightarrow f_k = \left|\frac{1}{r_{sh}}\frac{\Delta L}{\Delta t}\right| = \frac{1}{r_{sh}}\frac{L_i}{\Delta t} = \frac{1}{r_{sh}}\frac{(I_{di} + I_{sh})\omega}{\Delta t} = \frac{1}{r_{sh}}\frac{(\frac{1}{2}m_{di}r_{di}^2 + \frac{1}{2}m_{sh}r_{sh}^2)\omega}{\Delta t}$$

$$= \frac{1}{0.006\text{ m}}\frac{\left(\frac{1}{2}(1.2\text{ kg})(0.15\text{ m})^2 + \frac{1}{2}(0.45\text{ kg})(0.006\text{ m})^2\right)(3.46\text{ rad/s})}{15\text{ s}} = 0.52\text{ N}$$

REVIEW: THIS IS NOT A LARGE FORCE, BUT IT TOOK 15 S TO SLOW DOWN THE DISK, SO IT IS REASONABLE.