HW #7: Work and energy

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Due: 11:59pm on Wednesday, October 30, 2024

You will receive no credit for items you complete after the assignment is due. Grading Policy

Problem 9.16

Description: A 45 g bug is hovering in the air. A gust of wind exerts a force F_vec = (4.0 imath_unit-6.0 jmath_unit) * 10^-2 N on the bug. (a) How much work is done by the wind as the bug undergoes displacement Delta r_vec =(i imath_unit - i jmath_unit) m...

A 45 g bug is hovering in the air. A gust of wind exerts a force $\vec{F} = (4.0\hat{\imath} - 6.0\hat{\jmath}) \times 10^{-2}\ \mathrm{N}$ on the bug.

Part A

How much work is done by the wind as the bug undergoes displacement $\Delta\vec{r} = (5.0\hat{\imath} - 5.0\hat{\jmath})\ \mathrm{m}$?

Express your answer with the appropriate units.

ANSWER:

$W = \frac{i}{10} = 0.50\mathrm{J}$

Also accepted: $\frac{i}{10} = 0.500\mathrm{J}$, $\frac{i}{10} = 0.50\mathrm{J}$

Part B

What is the bug's speed at the end of this displacement? Assume that the speed is due entirely to the wind.

Express your answer with the appropriate units.

ANSWER:

$v = \sqrt{\frac{2\frac{i}{10}}{0.045}} = 4.7\frac{\mathrm{m}}{\mathrm{s}}$

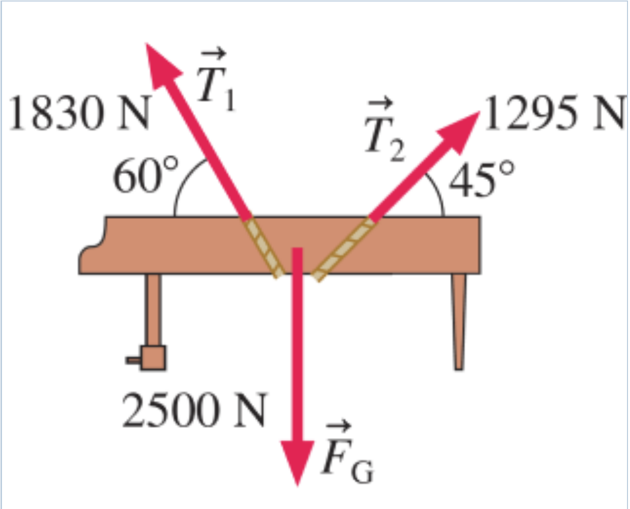
Also accepted: $\sqrt{\frac{2\frac{i}{10}}{0.045}} = 4.71\frac{\mathrm{m}}{\mathrm{s}}$, $\sqrt{\frac{2\frac{i}{10}}{0.045}} = 4.7\frac{\mathrm{m}}{\mathrm{s}}$

Problem 9.19 - Enhanced - with Expanded Hints

Description: [[Enhanced item has hints walking students through the problem-solving steps. All parts

and hints contain wrong answer feedback, with a worked solution available upon completing the part.]]
The two ropes seen in are used to lower a 255 kg piano...

The two ropes seen in are used to lower a 255 kg piano 5.80 m from a second-story window to the ground.



Part A

How much work is done by each of the three forces?

Express your answers in joules and separated by commas

Hint 1. How to approach the problem

Recall the expression for the work performed by a constant force for any specific coordinate system.

You may need to review The Vector Dot Product.

Hint 2. Simplify: Work done by force

What expression should we use to find the work of the force $\vec{F}$, by moving the particle distance $\Delta\vec{r}$, with the angle between the vector $\vec{F}$ and $\Delta\vec{r}$ equal to θ?

ANSWER:

$W =$

- ○ $F\Delta r\cos(180^\circ - \theta)$
- ○ $F\Delta r\sin(\theta)$
- ◉ $F\Delta r\cos(\theta)$
- ○ $F\Delta r$

ANSWER:

W_G, W_1, W_2 = $255 \cdot 9.8d, \ -1830d\cos\left(\frac{30\pi}{180}\right), \ -1295d\cos\left(\frac{45\pi}{180}\right)$ = 1.4×10^4, -9200, -5300 J

Also accepted: $255 \cdot 9.8d, \ -1830d\cos\left(\frac{30\pi}{180}\right), \ -1295d\cos\left(\frac{45\pi}{180}\right)$ = 1.45×10^4, -9190, -5310, $255 \cdot 9.8d, \ -1830d\cos\left(\frac{30\pi}{180}\right), \ -1295d\cos\left(\frac{45\pi}{180}\right)$ = 1.4×10^4, -9200, -5300

SOLVE: Model the piano as a particle and use $W = \vec{F} \cdot \Delta\vec{r}$, where W is the work done by the force $\vec{F}$ through the displacement $\Delta\vec{r}$.

For the force $\vec{F}_{\mathrm{G}}$:

$$W = \vec{F} \cdot \Delta\vec{r} = \vec{F}_{\mathrm{G}} \cdot \Delta\vec{r} = (F_{\mathrm{G}})(\Delta r)\cos(0^\circ) = (255\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(5.80\ \mathrm{m})(1.00) = 1.4 \times 10^4\ \mathrm{J}$$

For the tension $\vec{T}_1$:

$$W = \vec{F} \cdot \Delta\vec{r} = \vec{T}_1 \cdot \Delta\vec{r} = (T_1)(\Delta r)\cos(150^\circ) = (1830\ \mathrm{N})(5.80\ \mathrm{m})(-0.8660) = -9200\ \mathrm{J}$$

For the tension $\vec{T}_2$:

$$W = \vec{F} \cdot \Delta\vec{r} = \vec{T}_2 \cdot \Delta\vec{r} = (T_2)(\Delta r)\cos(135^\circ) = (1295\ \mathrm{N})(5.80\ \mathrm{m})(-0.7071) = -5300\ \mathrm{J}$$

Problem 9.30

Description: A horizontal spring with spring constant 85 N/m extends outward from a wall just above floor level. A m box sliding across a frictionless floor hits the end of the spring and compresses it x before the spring expands and shoots the b...

A horizontal spring with spring constant 85 $\mathrm{N/m}$ extends outward from a wall just above floor level. A 8.5 kg box sliding across a frictionless floor hits the end of the spring and compresses it 6.5 cm before the spring expands and shoots the box back out. How fast was the box going when it hit the spring?

Part A

How fast was the box going when it hit the spring?

Express your answer with the appropriate units.

ANSWER:

v = $x\sqrt{\frac{85}{m}}$ = 0.21 $\frac{\mathrm{m}}{\mathrm{s}}$

Also accepted: $x\sqrt{\frac{85}{m}}$ = 0.206 $\frac{\mathrm{m}}{\mathrm{s}}$, $x\sqrt{\frac{85}{m}}$ = 0.21 $\frac{\mathrm{m}}{\mathrm{s}}$

Problem 9.35 - Enhanced - with Expanded Hints

Description: [[Enhanced item has hints walking students through the problem-solving steps. All parts

and hints contain wrong answer feedback, with a worked solution available upon completing the part.]]
An ## kg crate is pulled ## m up a 30 degree(s) incline by a...

An 8.5 kg crate is pulled 5.1 m up a 30° incline by a rope angled 17° above the incline. The tension in the rope is 140 N and the crate's coefficient of kinetic friction on the incline is 0.27.

For help with math skills, you may want to review: The Vector Dot Product

Part A

How much work is done by tension, by gravity, and by the normal force?

Express your answers in joules to two significant figures. Enter your answers numerically separated by commas.

Hint 1. How to approach the problem

Draw a picture showing the block on the incline with an appropriate coordinate system having the directions of the positive axes clearly indicated. Then, draw all the forces acting on the block, and use the definition of the work done by a force to separately calculate the work done by each of these three forces while paying careful attention to the angle between each force and the direction of displacement.

Hint 2. Simplify: work done by tension

Using T for the tension in the rope and Δx for the displacement of the crate, what is the work done by tension?

ANSWER:

- ○ $W_T = T\Delta x$
- ○ $W_T = T\Delta x\cos(30^\circ)$
- ◉ $W_T = T\Delta x\cos(17^\circ)$
- ○ $W_T = T\Delta x\cos(47^\circ)$

Hint 3. Simplify: work done by gravity

Using m and Δx for the mass and displacement of the crate, respectively, what is the work done by gravity?

ANSWER:

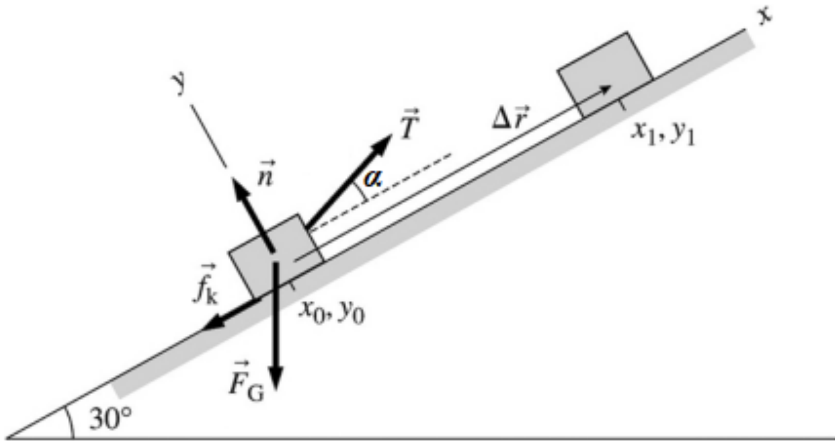
- ○ $W_\mathrm{G} = mg\Delta x\cos(90^\circ)$
- ◉ $W_\mathrm{G} = mg\Delta x\cos(120^\circ)$
- ○ $W_\mathrm{G} = mg\Delta x\cos(60^\circ)$
- ○ $W_\mathrm{G} = mg\Delta x\cos(30^\circ)$

ANSWER:

W_T, W_G, W_n = $T l \cos(\alpha)$, $m \cdot 9.8 l \cos(90+30)$, 0 = 680, -210, 0 J

MODEL: Use the particle model, the definition of work $W = \vec{F} \cdot \Delta\vec{r}$, and the model of kinetic friction.

VISUALIZE: We place the coordinate frame on the incline so that its x-axis is along the incline.



Known
m = 8.5 kg
T = 140 N
μ_k = 0.27
α = 17°
x_0 = 0 m
x_1 = 5.1 m

SOLVE:

$$W_T = \vec{T} \cdot \Delta\vec{r} = T\Delta x\cos(17^\circ) = (140\ \text{N})(5.1\ \text{m})\cos(17^\circ) = 680\ \text{J}$$

$$W_G = \vec{F}_G \cdot \Delta\vec{r} = mg\Delta x\cos(120^\circ) = (8.5\ \text{kg})(9.8\ \text{m/s}^2)(5.1\ \text{m})\cos(120^\circ) = -210\ \text{J}$$

$$W_n = \vec{n} \cdot \Delta\vec{r} = n\Delta x\cos(90^\circ) = 0\ \text{J}$$

Part B

What is the increase in thermal energy of the crate and incline?

Express your answer in joules to two significant figures.

Hint 1. How to approach the problem

Draw a picture showing the block on the incline with an appropriate coordinate system having the directions of the positive axes clearly indicated, along with all the forces acting on the block. Since the change in thermal energy depends directly on the force of kinetic friction, you need to know how large this force is. However, the force of kinetic friction in turn depends on the normal force, so apply Newton's second law in the direction perpendicular to the incline to solve for the normal force, noting that there is motion in this direction.

Hint 2. Simplify: thermal energy

The increase in thermal energy is due to the dissipative force of friction. Choose the correct expression for the change in thermal energy in terms of the displacement of the crate, Δx, normal force acting on it, n, and the coefficient of friction, μ_k.

ANSWER:

- ○ $\Delta E_{\text{th}} = \frac{n\Delta x}{\mu_k}$
- ◉ $\Delta E_{\text{th}} = \mu_k n\Delta x$
- ○ $\Delta E_{\text{th}} = \frac{n}{\mu_k \Delta x}$
- ○ $\Delta E_{\text{th}} = \frac{\mu_k n}{\Delta x}$

Hint 3. Simplify: normal force

Apply Newton's second law and determine the correct equation for the normal force acting on the crate.

ANSWER:

- ◉ $n = mg\cos(30^\circ) - T\sin(17^\circ)$
- ○ $n = mg - T\sin(47^\circ)$
- ○ $n = mg\sin(30^\circ) - T\cos(17^\circ)$
- ○ $n = mg - T\cos(47^\circ)$

ANSWER:

ΔE_{th} = $\mu l\,(m\cdot 9.8\cos(30) - T\sin(\alpha))$ = 43 J

The amount of energy transformed into thermal energy is $\Delta E_{\text{th}} = f_{\text{k}}\Delta x = \mu_{\text{k}} n\Delta x$.

To find n, we write Newton's second law as follows:

$\sum F_y = n - F_{\text{G}}\cos(30^\circ) + T\sin(17^\circ) = 0\text{ N} \Rightarrow n = F_{\text{G}}\cos(30^\circ) - T\sin(17^\circ)$

$n = mg\cos(30^\circ) - T\sin(17^\circ) = (8.5\text{ kg})(9.8\text{ m/s}^2)\cos(30^\circ) - (140\text{ N})\sin(17^\circ) = 31.2\text{ N}$

Thus, $\Delta E_{\text{th}} = (0.27)(31.2\text{ N})(5.1\text{ m}) = 43\text{ J}$.

REVIEW: Any force that acts perpendicular to the displacement does no work.

Drag on a Skydiver

Description: Use conservation of energy to determine the work done by air drag on a skydiver. Then find the power delivered by the drag force after the skydiver reaches terminal velocity.

A skydiver of mass m jumps from a hot air balloon and falls a distance d before reaching a terminal velocity of magnitude v. Assume that the magnitude of the acceleration due to gravity is g.

Part A

What is the work W_d done on the skydiver, over the distance d, by the drag force of the air?

Express the work in terms of d, v, m, and the magnitude of the acceleration due to gravity g.

Hint 1. How to approach the problem

If no nonconservative forces were acting, then the total mechanical energy (kinetic plus potential) of the skydiver upon leaving the plane would be equal to the total mechanical energy of the skydiver after falling a distance d.

Now consider the drag force, which is nonconservative. The drag force opposes the motion of the skydiver, which means that it does negative work on the skydiver. Thus, the final mechanical energy of the skydiver will be smaller than the initial mechanical energy by an amount equal to the work done by the drag force.

Hint 2. Find the change in potential energy

Find the change in the skydiver's gravitational potential energy, after falling a distance d.

Express your answer in terms of given quantities.

ANSWER:

ΔU_g = $-mgd$

Hint 3. Find the change in kinetic energy

Find the change in the skydiver's kinetic energy, after falling a distance d.

Express your answer in terms of given quantities.

ANSWER:

ΔK = $\frac{1}{2}mv^2$

ANSWER:

W_d = $\frac{1}{2}mv^2 - mgd$

Part B

Find the power P_d supplied by the drag force after the skydiver has reached terminal velocity v.

Express the power in terms of quantities given in the problem introduction.

Hint 1. How to approach the problem

One way to approach this problem would be to apply the definition of power as the time derivative of the work done. A simpler method that works for this problem is to use the formula for the power delivered by a force $\vec{F}$ acting on an object moving with velocity $\vec{v}$:

$$P = \vec{F} \cdot \vec{v}.$$

Hint 2. Magnitude of the drag force

Find the magnitude $F_{\rm d}$ of the drag force after the skydiver has reached terminal velocity.

Express your answer in terms of the skydiver's mass m and other given quantities.

Hint 1. Definition of terminal velocity

Once terminal velocity is reached, the skydiver's acceleration goes to zero. (The drag force exactly balances the downward acceleration due to gravity.)

ANSWER:

$F_{\rm d}$ = mg

Hint 3. Relative direction of the drag force and velocity

When you find $\overrightarrow{F_d} \cdot \vec{v}$, remember that the direction of the drag force is opposite to the direction of the skydiver's velocity.

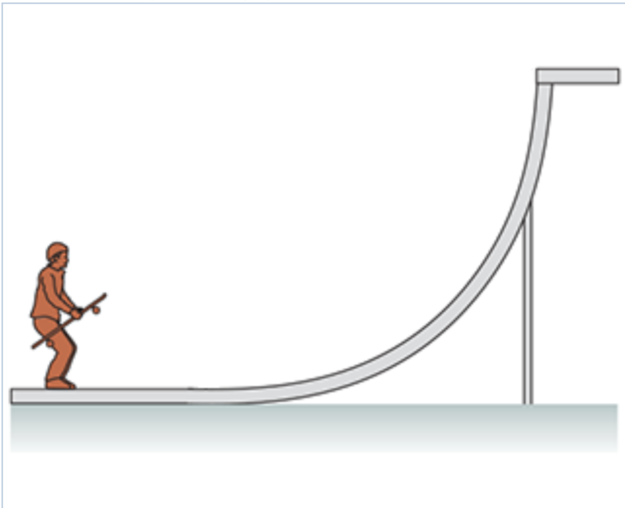
ANSWER:

$P_{\rm d}$ = $-mgv$

Problem 10.6

Description: A ## kg skateboarder wants to just make it to the upper edge of a "quarter pipe," a track that is one-quarter of a circle with a radius of ## m. (a) What speed does he need at the bottom?

A 52.0 kg skateboarder wants to just make it to the upper edge of a "quarter pipe," a track that is one-quarter of a circle with a radius of 3.20 m.



Part A

What speed does he need at the bottom?

Express your answer with the appropriate units.

ANSWER:

$v_0 = \sqrt{(2 \cdot 9.8r)}$ = 7.92 $\frac{\mathrm{m}}{\mathrm{s}}$

Problem 10.16 - Enhanced - with Expanded Hints

Description: [[Enhanced item has hints walking students through the problem-solving steps. All parts and hints contain wrong answer feedback, with a worked solution available upon completing the part.]] A student places her ## g physics book on a...

A student places her 300 g physics book on a frictionless table. She pushes the book against a spring, compressing the spring by x = 5.90 cm, then releases the book.

Part A

What is the book's speed as it slides away? The spring constant is 1250 $\mathrm{N/m}$.

Express your answer with the appropriate units.

Hint 1. How to approach the problem

Treat the book as a particle. Choose a coordinate system. Then determine the direction of velocity in your chosen coordinate system. Then use the law of conservation of energy to find the speed. Take into account the change in the potential and kinetic energies when the book moves.

Hint 2. Simplify: the law of conservation of energy

Choose the correct expression for speed from the law of conservation of energy.

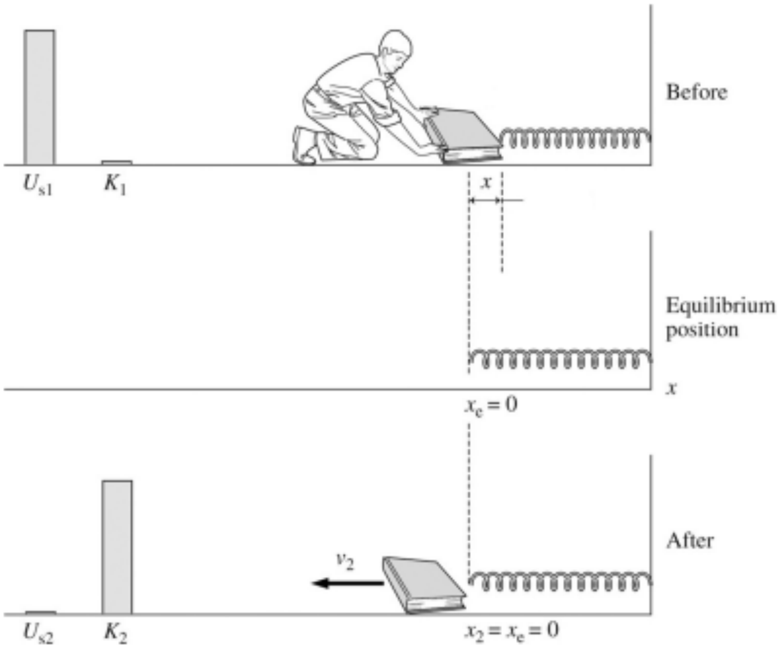
ANSWER:

- ○ $v = \sqrt{\frac{kx}{m}}$
- ○ $v = \sqrt{kxm}$
- ○ $v = \sqrt{kx^2m}$
- ◉ $v = \sqrt{\frac{kx^2}{m}}$

ANSWER:

$x\sqrt{\frac{k}{m}}$ = 3.81 $\frac{\mathrm{m}}{\mathrm{s}}$

Assume an ideal spring that obeys Hooke's law. There is no friction, so the mechanical energy $K + U_s$ is conserved. Also model the book as a particle.



The figure shows a before-and-after pictorial representation. The compressed spring will push on the book until the spring has returned to its equilibrium length. We put the origin of our coordinate system at the equilibrium position of the free end of the spring. The energy bar chart shows that the potential energy of the compressed spring is entirely transformed into the kinetic energy of the book.

The conservation of energy equation $K_2 + U_{s2} = K_1 = U_{s1}$ is

$$\frac{1}{2}m{v^2}_2 + \frac{1}{2}k(x_2 - x_{\mathrm{e}})^2 = \frac{1}{2}m{v^2}_1 + \frac{1}{2}k(x_1 - x_e)^2$$

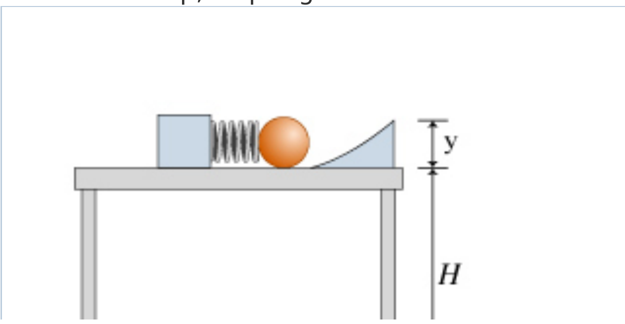
Using $x_2 = x_e = 0$ and $v_1 = 0\ \mathrm{m/s}$, this simplifies to

$$\frac{1}{2}m{v^2}_2 = \frac{1}{2}k(x_1 - 0\ \mathrm{m})^2 \rightarrow \mathrm{v}_2 = \sqrt{\frac{\mathrm{k{x^2}_1}}{\mathrm{m}}} = \sqrt{\frac{(1250\ \mathrm{N/m})(0.059\ \mathrm{m})^2}{0.300\ \mathrm{kg}}} = 3.81\ \mathrm{m/s}$$

Spring and Projectile

Description: A spring gun is used to launch a ball off of a table with a ramp at the edge. Problem contains a number of multiple choice questions about changes in kinetic and potential energy (including an energy diagram), then asks for analytic expressions for the speed of the ball when it leaves the ramp and when it hits the floor.

A child's toy consists of a block that attaches to a table with a suction cup, a spring connected to that block, a ball, and a launching ramp. The spring has a spring constant k, the ball has a mass m, and the ramp rises a height y above the table, the surface of which is a height H above the floor.



Initially, the spring rests at its equilibrium length. The spring then is compressed a distance s, where the ball is held at rest. The ball is then released, launching it up the ramp. When the ball leaves the launching ramp its velocity vector makes an angle θ with respect to the horizontal.

Throughout this problem, ignore friction and air resistance.

Part A

Relative to the initial configuration (with the spring relaxed), when the spring has been compressed, the ball-spring system has

ANSWER:

- ○ gained kinetic energy
- ◉ gained potential energy
- ○ lost kinetic energy
- ○ lost potential energy

Part B

As the spring expands (after the ball is released) the ball-spring system

ANSWER:

- ◉ gains kinetic energy and loses potential energy
- ○ gains kinetic energy and gains potential energy
- ○ loses kinetic energy and gains potential energy
- ○ loses kinetic energy and loses potential energy

Part C

As the ball goes up the ramp, it

ANSWER:

- ○ gains kinetic energy and loses potential energy
- ○ gains kinetic energy and gains potential energy
- ◉ loses kinetic energy and gains potential energy
- ○ loses kinetic energy and loses potential energy

Part D

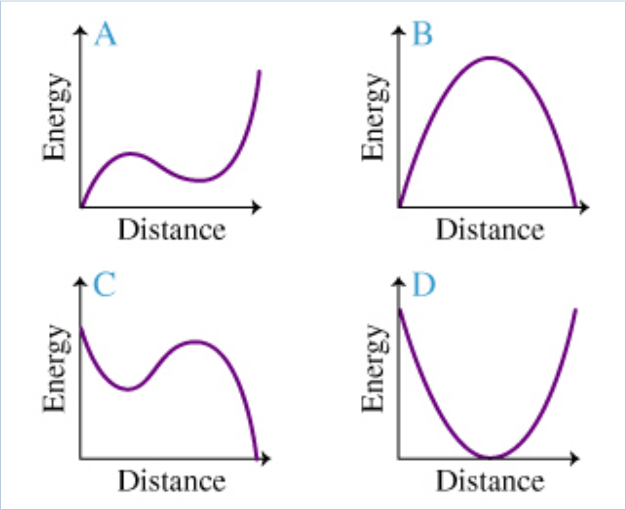
As the ball falls to the floor (after having reached its maximum height), it

ANSWER:

- (selected) gains kinetic energy and loses potential energy
- gains kinetic energy and gains potential energy
- loses kinetic energy and gains potential energy
- loses kinetic energy and loses potential energy

Part E

Which of the graphs shown best represents the potential energy of the ball-spring system as a function of the ball's horizontal displacement? Take the "zero" on the distance axis to represent the point at which the spring is fully compressed. Keep in mind that the ball is *not* attached to the spring, and neglect any recoil of the spring after the ball loses contact with it.



ANSWER:

- A
- B
- (selected) C
- D

Part F

Calculate v_{r}, the speed of the ball when it leaves the launching ramp.

Express the speed of the ball in terms of k, s, m, g, y, and/or H.

Hint 1. General approach

Find an expression for the mechanical energy (kinetic plus potential) of the spring and ball when the spring is compressed. Then find an expression for the mechanical energy of the ball when it leaves the launching ramp. (v_{r} will be an unknown in this expression.) Since energy is conserved, you can set these two expressions equal to each other, and solve for v_{r}.

Hint 2. Find the initial mechanical energy

Find the total mechanical energy of the ball-spring system when the spring is fully compressed. *Take the gravitational potential energy to be zero at the floor.*

Hint 1. What contributes to the mechanical energy?

The total initial mechanical energy is the sum of the potential energy of the spring, the gravitational potential energy, plus any initial kinetic energy of the ball.

ANSWER:

$E_{\mathrm{i}} = \frac{1}{2}ks^2 + mgH$

Hint 3. Find the mechanical energy at the end of the ramp

Find the total mechanical energy of the ball when it leaves the launching ramp. (At this point, assume that the spring is relaxed and has no stored potential energy.) Again, take the gravitational potential energy to be zero at the floor.

Express your answer in terms of v_{r} and other given quantities.

ANSWER:

$E_{\mathrm{r}} = \frac{1}{2}m{v_{\mathrm{r}}}^2 + mg(H+y)$

Hint 4. Is energy conserved?

Because no nonconservative forces act on the system, energy is conserved: $E_{\mathrm{i}} = E_{\mathrm{r}}$.

ANSWER:

$v_{\mathrm{r}} = \sqrt{\left(\frac{ks^2}{m} - 2gy\right)}$

Part G

With what speed will the ball hit the floor?

Express the speed in terms of k, s, m, g, y, and/or H.

Hint 1. General approach

Find an expression for the initial mechanical energy (kinetic plus potential) of the spring and ball. Then find an expression for the mechanical energy of the ball when it hits the floor. (v_{f} will be an unknown in this expression.) Since energy is conserved, you can set these two expressions equal to each other, and solve for v_{f} .

Hint 2. Initial mechanical energy

For the initial mechanical energy, you can use either the expression you found for the mechanical energy of the ball at the top of the ramp or that for the total mechanical energy of the ball plus spring just before the ball was launched. These two expressions are equal.

Hint 3. Find the final mechanical energy

Find the total mechanincal energy E_{f} of the ball when it hits the floor.

Express your answer in terms of $v_{\rm f}$ and other given quantities.

ANSWER:

$E_{\rm f} = \frac{1}{2} m {v_f}^2$

Hint 4. Is energy conserved?

Only conservative forces (gravity, spring) are acting on the ball, so energy is conserved: $E_{\rm i} = E_{\rm r} = E_{\rm f}$.

ANSWER:

$v_{\rm f} = \sqrt{\left(\frac{ks^2}{m} + 2gH\right)}$

Problem 10.44

Description: A block of mass m slides down a frictionless track, then around the inside of a circular loop-the-loop of radius R. (a) From what minimum height h must the block start to make it around the loop without falling off? Give your answer as a multiple of R.

A block of mass m slides down a frictionless track, then around the inside of a circular loop-the-loop of radius R.

Part A

From what minimum height h must the block start to make it around the loop without falling off? Give your answer as a multiple of R.

ANSWER:

$h/R =$ 2.50

Problem 10.48

Description: A horizontal spring with spring constant ## N/m is compressed ## cm and used to launch a ## kg box across a frictionless, horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction of the...

A horizontal spring with spring constant 150 $\mathrm{N/m}$ is compressed 15 cm and used to launch a 2.1 kg box across a frictionless, horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction of the box on the surface is 0.15.

Part A

Use work and energy to find how far the box slides across the rough surface before stopping.

Express your answer with the appropriate units.

ANSWER:

$$l = \frac{\frac{\frac{kx^2}{2}}{0.15}}{\frac{m}{9.8}} \cdot 100 = 55 \, \mathrm{cm}$$

Problem 10.71 - Enhanced - with Hints and Feedback

Description: In a physics lab experiment, a compressed spring launches a ## g metal ball at a ## ! degree(s) angle. Compressing the spring ## cm causes the ball to hit the floor ## m below the point at which it leaves the spring after traveling ## m horizontally. ...

In a physics lab experiment, a compressed spring launches a 21 g metal ball at a 30° angle. Compressing the spring 17 cm causes the ball to hit the floor 1.8 m below the point at which it leaves the spring after traveling 5.1 m horizontally.

Part A

What is the spring constant?

Express your answer with the appropriate units.

Hint 1. How to approach the problem

Use kinematic equations of free-fall motion to find the launching speed of the ball. Then, apply the law of conservation of mechanical energy to find the spring constant.

ANSWER:

$$k = \frac{m \cdot 9.8 \left(\frac{0.5 l^2}{(l \tan(\alpha) + d)(\cos(\alpha))^2} + 2 \Delta x \sin(\alpha) \right)}{\Delta x^2} = 27 \, \frac{\mathrm{N}}{\mathrm{m}}$$

All Assignments

Physics for Scientists and Engineers with Modern Physics, 5e
Knight
amundson44156
Ends: 12/21/24