Basic definitions of kinematics

- position: $\vec{x} = \langle x, y \rangle$
- displacement: $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$ for finite displacement
- instantaneous velocity: $\vec{v} = \dfrac{d\vec{x}}{dt}$
- instantaneous acceleration: $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{x}}{dt^2}$

Kinematic equations, $\vec{v} =$ constant (i.e., $\vec{a} = 0$)

- $\vec{v} = \dfrac{\Delta\vec{x}}{\Delta t}$
- $\vec{x}_f = \vec{x}_i + \vec{v}\Delta t$

Kinematic equations, $\vec{a} \neq 0$ and $\vec{a} =$ constant

- $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$
- $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$
- $\Delta\vec{x} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$
- $(v_{x,f})^2 - (v_{x,i})^2 = 2a_x\Delta x$ and $(v_{y,f})^2 - (v_{y,i})^2 = 2a_y\Delta y$
- common example of $a =$ constant is $g = 9.81$ m/s^2

Motion of object A relative to object C

- $\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$

Basic definitions for circular and rotational motion

- angular position: $\theta(\text{radians}) = \dfrac{s}{r}$; $s =$arclength, $r =$radius
- angular displacement: $\Delta\theta = \theta_f - \theta_i$
- angular velocity: $\omega = \dfrac{d\theta}{dt} = 2\pi f = \dfrac{2\pi}{T}$; $f =$frequency, $T =$period
- angular acceleration: $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Kinematic equations for constant angular acceleration

- $\omega_f = \omega_i + \alpha\Delta t$
- $\Delta\theta = \omega_i\Delta t + \dfrac{1}{2}\alpha\Delta t^2$; if $\alpha = 0$, $\omega_f = \omega_i =$ constant.
- $\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

Speed, acceleration, and forces

- speed: $v = \omega r$
- centripetal acceleration: $a_c = \dfrac{v^2}{r} = \omega^2 r$
- centripetal force: $F_c = m a_c = m\dfrac{v^2}{r} = m\omega^2 r$; points toward center of circle
- tangential acceleration: $a_t = \alpha r$

Newton's Laws

1. if $\vec{F}_{net} = 0 \Rightarrow \vec{a} = 0$
2. $\sum \vec{F} = m\vec{a}$
3. $\vec{F}_{12} = -\vec{F}_{21}$

Types of forces

- Newton's Law of Gravity: $F_{12} = F_{21} = \dfrac{Gm_1m_2}{r^2}$; points from one object to another; this can also be expressed as $\vec{F}_{12} = -\dfrac{Gm_1m_2}{r^2}\hat{r}_{12}$ where $\hat{r}_{12}$ is the unit vector that points from object 1 to object 2.
- Gravitational constant: $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
- For objects near the Earth's surface, $F_g = mg$
- normal force, $\vec{F}_n$, is perpendicular to surface and prevents objects from penetrating the surface
- frictional force depends on $\vec{F}_n$ and $\vec{v}$
  - if $\vec{v} = 0$ use static friction; $\vec{F}_s$ balances other forces as long as $\left|\vec{F}_s\right| \leq \mu_s \left|\vec{F}_n\right|$
  - if $\vec{v} \neq 0$ use kinetic friction; $\left|\vec{F}_k\right| = \mu_k \left|\vec{F}_n\right|$
- tensional force, $\vec{F}_t$, is transmitted through a rope and around pulleys

Impulse and momentum

- momentum, $\vec{p} = m\vec{v}$
- Newton's second law, $\sum \vec{F} = \dfrac{d\vec{p}}{dt}$
- Impulse-Momentum Theorem, $\vec{J} = \Delta\vec{p} = \int_{t_0}^{t_1} \vec{F}_{net}\, dt = \vec{F}_{avg}\Delta t$
- total momentum, $\vec{P} = \vec{p}_1 + \vec{p}_2 + ...$
- conservation of momentum, $\Delta P = 0$ if $\vec{F}_{net} = 0$ or if $\vec{F}_{net}$ is small compared to other forces for short Δt

Orbits and gravity

- orbital velocity: $v = \sqrt{\frac{GM}{r}}$ at a distance of r from the center of a planet with mass M

Torque and moment of inertia

- Torque: $\tau = rF_{\perp}$; $F_{\perp}$ is force perpendicular to radial axis; depending on what angles are given, you may find that $F_{\perp} = F \sin\phi$
- Newton's Second Law for rotation: $\sum \tau = I\alpha = \frac{dL}{dt}$; L =angular momentum.
- Moment of inertia: I, indicates how difficult it is to rotate an object
- Conservation of angular momentum: if $\tau_{ext} = 0$ then $\Delta L = 0$.
- Rolling constraint: $v = \omega R$ and $a = \alpha R$; rolling object, with perfect friction, moves forward with this velocity

Static equilibrium

- $\sum F_x = 0$; $\sum F_y = 0$; $\sum \tau = 0$
- Choose convenient pivot point

Stability and balance

- Does torque restore object to its original position?
- Critical angle: $\theta_c = \tan^{-1}\left(\frac{t}{2h}\right)$; t =width of object's base, h is the height to the center of mass

Springs and elasticity

- Hooke's 'Law': $F = -k\Delta x$; k is empirically determined spring constant
- For elastic materials, $k = \frac{YA}{L}$; Y =Young's modulus
- Hooke's Law sometimes written $\left(\frac{F}{A}\right) = Y\left(\frac{\Delta L}{L}\right)$; stress = $Y \times$ strain

Energy and work

- Total energy: $E = K + U_g + U_s + E_{th} + ...$
- Work: $W = \int_{x_0}^{x_1} F_{ext}\, dx = \Delta E$; mechanical transfer of energy into or out of a system
- Conservation of energy: for an isolated system, $F_{ext} = 0$ and therefore and therefore $W = \Delta E = 0$.
- Work: $W = F_{\parallel} \cdot d$; force $\times$ displacement
- Translational kinetic energy: $K_{trans} = \frac{1}{2}mv^2$; scalar quantity (note that v is speed, not velocity)

- Rotational kinetic energy: $K_{rot} = \frac{1}{2}I\omega^2$
- Total kinetic energy: $K = K_{trans} + K_{rot}$
- Gravitational potential energy: $\Delta U_g = mg\Delta y$; choose convenient reference height
- Elastic potential energy: $U_s = \frac{1}{2}k(\Delta x)^2$; k is the spring constant
- Thermal energy (from friction): $\Delta E_{th} = F_f \Delta x$
- Conservation of mechanical energy: for isolated system with no friction, $\Delta K + \Delta U_g + \Delta U_s = 0$
- Power: $P = \frac{dE}{dt}$

Thermodynamics

- 1st Law: $Q + W = \Delta E$; Q is heat transferred into or out of the system
- 2nd Law: Entropy in a closed system (which describes disorder) can never decrease
- energy needed to change material's temperature: $Q = mc\Delta T$; c is the specific heat capacity, a material property
- energy need to change a material's phase: $Q = \pm mL_f$ to melt solid and $Q = \pm mL_v$ to turn liquid into gas; use negative sign if going from gas to liquid or liquid to solid
- $L_v > L_f$ (specific latent heat of vaporization is greater than specific latent heat of fusion)
- conduction: $\frac{Q}{\Delta t} = \left(\frac{kA}{L}\right)\Delta T$; k is thermal conductivity
- advection/convection: need to solve for motion of an object
- radiation: $\frac{Q}{\Delta t} = e\sigma AT^4$; e is emissivity and $\sigma = 5.67 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m^2K^4}}$ is the Stefan-Boltzmann constant
- ideal gas law: $PV = nRT$, where $R = 8.314$ J/(mol $\cdot$ K) is the gas constant
- for ideal gas in enclosed, insulated container, $\frac{PV}{T} =$ constant
- work done by expanding a gas at constant pressure: $W_{\text{gas}} = P\Delta V$
- note that for gases, specific heat at constant pressure c_p differs from specific heat at constant volume c_v. c_p is approximately 50% larger than c_v
- Conversions: 1 cm^3=10^{-6} m^3; 1 atm = 101.3 kPa

Fluids

- density: $\rho = \frac{m}{V}$
- hydrostatic pressure: $P = \rho gd + P_0$; P_0 is pressure from above
- gauge pressure: $P_g = P_{gas} - P_{atm}$; $P_{atm} = 101.3$ kPa

- Archimedes' principle (upward buoyant force): $F_b = \rho_f g V$; ρ_f is fluid density and V is volume displaced
- flux: $Q = vA$; Q is constant in a pipe for ideal fluid
- Bernoulli's equation: along a streamline, $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

Simple harmonic motion

- displacement: $x(t) = A\cos(2\pi f t)$
- velocity: $v(t) = -v_{\text{max}} \sin(2\pi f t)$
- acceleration: $a(t) = -a_{\text{max}} \cos(2\pi f t)$
- $v_{\text{max}} = 2\pi f A$ and $a_{\text{max}} = (2\pi f)^2 A$
- for a spring, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$; for a pendulum, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$
- damped oscillation: $x(t) = A\exp^{-t/\tau}\cos(2\pi f t)$, where τ is a decay constant

Waves

- in general, $v = \lambda/T$, where λ is wavelength and T is period
- velocity of a wave on a string: $v = \sqrt{\frac{F_t}{\mu}}$, where μ is linear density
- velocity of sound waves in ideal gas: $v = \sqrt{\frac{\gamma R T}{M}}$, where γ is adiabatic index, R is the gas constant, T is temperature in Kelvin, and M is the molar mass
- travelling wave displacement: $y(x,t) = A\sin\left(2\pi\frac{x}{\lambda} \pm 2\pi\frac{t}{T} + \phi\right)$; Use $-$ if wave is travelling to the right, $+$ if it is travelling to the left
- Doppler effect: $f = f_0\left(\dfrac{v}{v \pm v_s}\right)$; v_s is speed of source, v is wave speed. Use $+$ if the source is moving away from the receiver, $-$ if the source is moving toward the receiver.
- wave superposition: $y(x,t) = y_1 + y_2 + y_3$ where y_i are given by different amplitudes, wavelengths, and frequencies/periods.
- standing wave displacement: $y(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right) + A\sin\left(2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right)$
- harmonics: $\lambda_m = \frac{2L}{m}$, where $m = 1, 2, 3 \ldots$; for a string that is pinned on both ends, a tube that is open on both ends, and a tube that is closed on both ends.
- harmonics: $\lambda_m = \frac{4L}{m}$, where $m = 1, 3, 5 \ldots$; for a tube that is open on one end and closed on the other