

Basic definitions of kinematics

- position: $\vec{x} = \langle x, y \rangle$
- displacement: $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$ for finite displacement
- instantaneous velocity: $\vec{v} = \frac{d\vec{x}}{dt}$
- instantaneous acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$

Kinematic equations, $\vec{v} = \text{constant}$ (i.e., $\vec{a} = 0$)

- $\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$
- $\vec{x}_f = \vec{x}_i + \vec{v}\Delta t$

Kinematic equations, $\vec{a} \neq 0$ and $\vec{a} = \text{constant}$

- $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$
- $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$
- $\Delta\vec{x} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
- $(v_{x,f})^2 - (v_{x,i})^2 = 2a_x\Delta x$ and $(v_{y,f})^2 - (v_{y,i})^2 = 2a_y\Delta y$
- common example of $a = \text{constant}$ is $g = 9.81 \text{ m/s}^2$

Motion of object A relative to object C

- $\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$

Basic definitions for circular and rotational motion

- angular position: $\theta(\text{radians}) = \frac{s}{r}$; $s = \text{arclength}$, $r = \text{radius}$
- angular displacement: $\Delta\theta = \theta_f - \theta_i$
- angular velocity: $\omega = \frac{d\theta}{dt} = 2\pi f = \frac{2\pi}{T}$; $f = \text{frequency}$, $T = \text{period}$
- angular acceleration: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Kinematic equations for constant angular acceleration

- $\omega_f = \omega_i + \alpha\Delta t$
- $\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$; if $\alpha = 0$, $\omega_f = \omega_i = \text{constant}$.
- $\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

Speed, acceleration, and forces

- speed: $v = \omega r$
- centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$
- centripetal force: $F_c = ma_c = m \frac{v^2}{r} = m\omega^2 r$; points toward center of circle
- tangential acceleration: $a_t = \alpha r$

Newton's Laws

1. if $\vec{F}_{net} = 0 \Rightarrow \vec{a} = 0$
2. $\sum \vec{F} = m\vec{a}$
3. $\vec{F}_{12} = -\vec{F}_{21}$

Types of forces

- Newton's Law of Gravity: $F_{12} = F_{21} = \frac{Gm_1m_2}{r^2}$; points from one object to another; this can also be expressed as $\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$ where \hat{r}_{12} is the unit vector that points from object 1 to object 2.
- Gravitational constant: $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
- For objects near the Earth's surface, $F_g = mg$
- normal force, \vec{F}_n , is perpendicular to surface and prevents objects from penetrating the surface
- frictional force depends on \vec{F}_n and \vec{v}
 - if $\vec{v} = 0$ use static friction; \vec{F}_s balances other forces as long as $|\vec{F}_s| \leq \mu_s |\vec{F}_n|$
 - if $\vec{v} \neq 0$ use kinetic friction; $|\vec{F}_k| = \mu_k |\vec{F}_n|$
- tensional force, \vec{F}_t , is transmitted through a rope and around pulleys