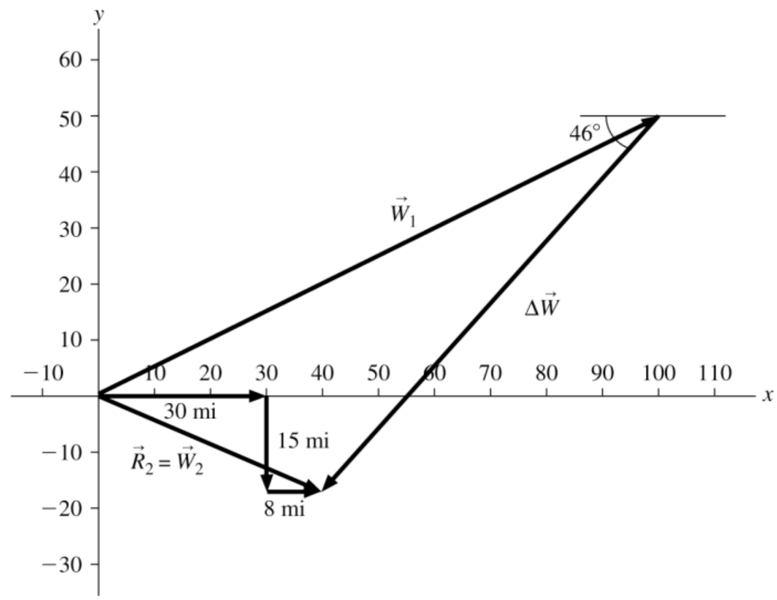
3.31. MODEL: Model Ward as a particle. Set the origin of the coordinate system at Ruth's initial location.

VISUALIZE: When Ward picks up Ruth they are at the same location: $\vec{W}_2 = \vec{R}_2$.



SOLVE: $\vec{R}_2 = ((30\text{ mi})\hat{i} + (0.0\text{ mi})\hat{j}) + ((0.0\text{ mi})\hat{i} - (15\text{ mi})\hat{j}) + ((8\text{ mi})\hat{i} + (0.0\text{ mi})\hat{j}) = (38\text{ mi})\hat{i} - (15\text{ mi})\hat{j}$

$$\Delta\vec{W} = \vec{W}_2 - \vec{W}_1 = ((38\text{ mi})\hat{i} - (15\text{ mi})\hat{j}) - ((100\text{ mi})\hat{i} + (50\text{ mi})\hat{j}) = -(62\text{ mi})\hat{i} - (65\text{ mi})\hat{j}$$

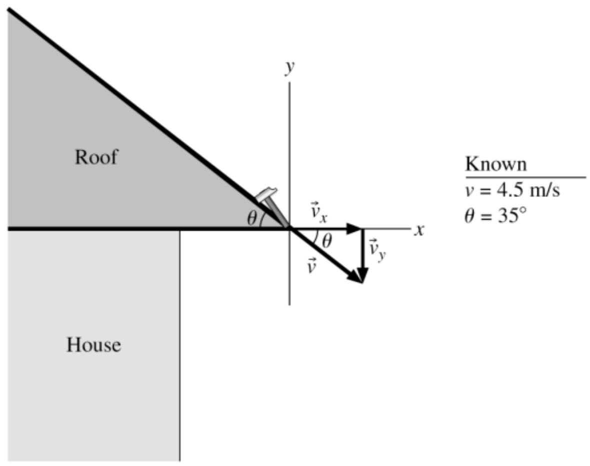
Now find the magnitude and direction.

$$\left|\Delta\vec{W}\right| = \sqrt{(-62\text{ mi})^2 + (-65\text{ mi})^2} = 90\text{ mi} \qquad \theta = \tan^{-1}\left(\frac{-65\text{ mi}}{-62\text{ mi}}\right) = 46° \text{ south of west}$$

REVIEW: The length of $\Delta\vec{W}$ and the angle in the scale diagram measure about what we calculated.

3.34. **MODEL:** Model the hammer as a particle.

VISUALIZE: Put the origin at the point the hammer leaves the roof.



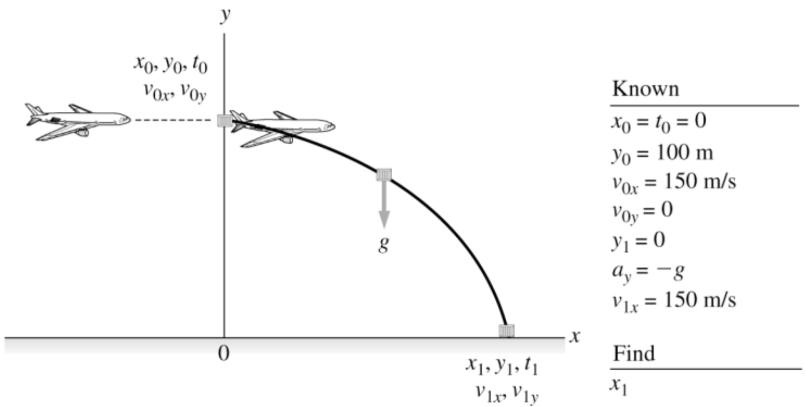
SOLVE: $v_x = (4.5 \text{ m/s})\cos 35° = 3.7 \text{ m/s} \qquad v_y = (4.5 \text{ m/s})\sin 35° = 2.6 \text{ m/s}$

REVIEW: These seem like reasonable answers for the velocity given.

4.12. MODEL: We will use the particle model for the food package and the constant-acceleration kinematic equations of motion.

VISUALIZE:

Pictorial representation



SOLVE: For the horizontal motion,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\ \text{m} + (150\ \text{m/s})(t_1 - 0\ \text{s}) + 0\ \text{m} = (150\ \text{m/s})t_1$$

We will determine t_1 from the vertical y-motion as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$

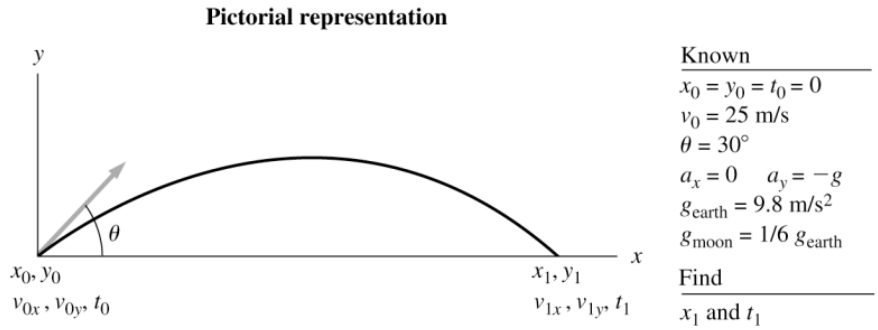
$$\Rightarrow 0\ \text{m} = 100\ \text{m} + 0\ \text{m} + \tfrac{1}{2}(-9.8\ \text{m/s}^2)t_1^2 \Rightarrow t_1 = \sqrt{\frac{200\ \text{m}}{9.8\ \text{m/s}^2}} = 4.518\ \text{s} \approx 4.5\ \text{s}$$

From the above x-equation, the displacement is $x_1 = (150\ \text{m/s})(4.518\ \text{s}) = 678\ \text{m} \approx 680\ \text{m}$.

REVIEW: The horizontal distance of 678 m covered by a freely falling object from a height of 100 m and with an initial horizontal velocity of 150 m/s (≈ 335 mph) is reasonable.

4.17. MODEL: The golf ball is a particle following projectile motion.

VISUALIZE:



(a) The distance traveled is $x_1 = v_{0x}t_1 = v_0\ \cos\theta \times t_1$. The flight time is found from the y-equation, using the fact that the ball starts and ends at $y = 0$:

$$y_1 - y_0 = 0 = v_0 \sin\theta\, t_1 - \tfrac{1}{2}gt_1^2 = (v_0 \sin\theta - \tfrac{1}{2}gt_1)\, t_1 \Rightarrow t_1 = \frac{2v_0 \sin\theta}{g}$$

Thus the distance traveled is

$$x_1 = v_0 \cos\theta \times \frac{2v_0 \sin\theta}{g} = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

For $\theta = 30°$, the distances are

$$(x_1)_{\text{earth}} = \frac{2v_0^2 \sin\theta\cos\theta}{g_{\text{earth}}} = \frac{2(25\ \text{m/s})^2 \sin30°\cos30°}{9.80\ \text{m/s}^2} = 55.2\ \text{m}$$

$$(x_1)_{\text{moon}} = \frac{2v_0^2 \sin\theta\cos\theta}{g_{\text{moon}}} = \frac{2v_0^2 \sin\theta\cos\theta}{\frac{1}{6}g_{\text{earth}}} = 6 \times \frac{2v_0^2 \sin\theta\cos\theta}{g_{\text{earth}}} = 6(x_1)_{\text{earth}} = 331.2\ \text{m}$$

The golf ball travels $331.2\ \text{m} - 55.2\ \text{m} = 276\ \text{m}$ farther on the moon than on earth.

(b) The flight times are

$$(t_1)_{\text{earth}} = \frac{2v_0 \sin\theta}{g_{\text{earth}}} = 2.55\ \text{s}$$

$$(t_1)_{\text{moon}} = \frac{2v_0 \sin\theta}{g_{\text{moon}}} = \frac{2v_0 \sin\theta}{\frac{1}{6}g_{\text{earth}}} = 6(t_1)_{\text{earth}} = 15.30\ \text{s}$$

The ball spends $15.30\ \text{s} - 2.55\ \text{s} = 12.75\ \text{s}$ longer in flight on the moon.

4.21. **MODEL:** We define the x-axis along the direction of east and the y-axis along the direction of north.

SOLVE: **(a)** The kayaker's speed of 3.0 m/s is relative to the water. Since he's being swept toward the east, he needs to point at angle θ west of north. His velocity with respect to the water is

$$\vec{v}_{\text{KW}} = (3.0 \text{ m/s}, \theta \text{ west of north}) = (-3.0\sin\theta \text{ m/s})\hat{i} + (3.0\cos\theta \text{ m/s})\hat{j}$$

We can find his velocity with respect to the earth $\vec{v}_{\text{KE}} = \vec{v}_{\text{KW}} + \vec{v}_{\text{WE}}$, with $\vec{v}_{\text{WE}} = (2.0 \text{ m/s})\hat{i}$. Thus

$$\vec{v}_{\text{KE}} = ((-3.0\sin\theta + 2.0) \text{ m/s})\hat{i} + (3.0\cos\theta \text{ m/s})\hat{j}$$

In order to go straight north in the earth frame, the kayaker needs $(v_x)_{\text{KE}} = 0$. This will be true if

$$\sin\theta = \frac{2.0}{3.0} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{2.0}{3.0}\right) = 41.8°$$

Thus he must paddle in a direction 42° west of north.

(b) His northward speed is $v_y = 3.0\cos(41.8°) \text{ m/s} = 2.236 \text{ m/s}$. The time to cross is

$$t = \frac{100 \text{ m}}{2.236 \text{ m/s}} = 44.7 \text{ s}$$

The kayaker takes 45 s to cross.

4.22. **MODEL:** Let the x-direction be east and the y-direction be north. Use subscripts S, T, and G for Susan, Trent, and Ground respectively.

VISUALIZE: $\vec{v}_{\text{TS}} = \vec{v}_{\text{TG}} + \vec{v}_{\text{GS}}$ where $\vec{v}_{\text{GS}} = -\vec{v}_{\text{SG}} = (-60 \text{ mph})\hat{j}$.

SOLVE:

$$v_{\text{TS}} = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(45 \text{ mph})^2 + (-60 \text{ mph})^2} = 75 \text{ mph}$$

REVIEW: We expected the relative speed between Trent and Susan to be greater than either of their speeds relative to the ground.

4.33. MODEL: The earth is a particle orbiting around the sun.

SOLVE: **(a)** The magnitude of the earth's velocity is displacement divided by time:

$$v=\frac{2\pi r}{T}=\frac{2\pi(1.5\times10^{11}\ \text{m})}{365\ \text{days}\times\dfrac{24\ \text{h}}{1\ \text{day}}\times\dfrac{3600\ \text{s}}{1\ \text{h}}}=3.0\times10^{4}\ \text{m/s}$$

(b) Since $v=r\omega$, the angular velocity is

$$\omega=\frac{v}{r}=\frac{3.0\times10^{4}\ \text{m/s}}{1.5\times10^{11}\ \text{m}}=2.0\times10^{-7}\ \text{rad/s}$$

(c) The centripetal acceleration is

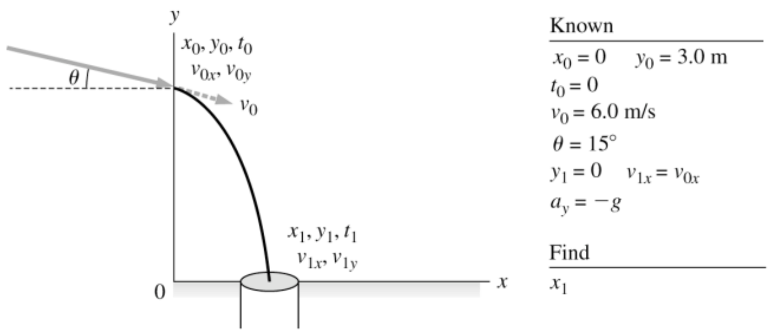
$$a_r=\frac{v^2}{r}=\frac{(3.0\times10^{4}\ \text{m/s})^2}{1.5\times10^{11}\ \text{m}}=6.0\times10^{-3}\ \text{m/s}^2$$

REVIEW: A tangential velocity of 3.0×10^{4} m/s or 30 km/s is large, but needed for the earth to go through a displacement of $2\pi(1.5\times10^{11}\ \text{m})\approx9.4\times10^{8}$ km in 1 year.

4.56. **MODEL:** We will assume a particle model for the sand, and use the constant-acceleration kinematic equations.

VISUALIZE:

Pictorial representation



SOLVE: Using the equation $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$,

$$x_1 = 0 \text{ m} + (v_0 \cos 15°)(t_1 - 0 \text{ s}) + 0 \text{ m} = (60 \text{ m/s})(\cos 15°)t_1$$

We can find t_1 from the y-equation, but note that $v_{0y} = -v_0 \sin 15°$ because the sand is launched at an angle below horizontal.

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0 \text{ m} = 3.0 \text{ m} - (v_0 \sin 15°)t_1 - \tfrac{1}{2}gt_1^2$$
$$= 3.0 \text{ m} - (6.0 \text{ m/s})(\sin 15°)t_1 - \tfrac{1}{2}(9.8 \text{ m/s}^2)t_1^2$$

$$\Rightarrow 4.9t_1^2 + 1.55t_1 - 3.0 = 0 \Rightarrow t_1 = 0.6399 \text{ s and} -0.956 \text{ s (unphysical)}$$

Substituting this value of t_1 in the x-equation gives the distance

$$d = x_1 = (6.0 \text{ m/s})\cos 15°(0.6399 \text{ s}) = 3.71 \text{ m} \approx 3.7 \text{ m}$$

4.65. MODEL: We will use the particle model for the test tube that is in nonuniform circular motion.

SOLVE: (a) The radial acceleration is

$$a_r = r\omega^2 = (0.1\ \text{m})\left(4000\frac{\text{rev}}{\text{min}}\times\frac{1\ \text{min}}{60\ \text{s}}\times\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)^2 = 1.75\times10^4\,\text{m/s}^2$$

(b) An object falling 1 meter has a speed calculated as follows:

$$v_1^2 = v_0^2 + 2a_y(y_1 - y_0) = 0\ \text{m} + 2(-9.8\ \text{m/s}^2)(-1.0\ \text{m}) \Rightarrow v_1 = 4.43\ \text{m/s}$$

When this object is stopped in $1\times10^{-3}\ \text{s}$ upon hitting the floor,

$$v_2 = v_1 + a_y(t_2 - t_1) \Rightarrow 0\ \text{m/s} = -4.43\ \text{m/s} + a_y(1\times10^{-3}\ \text{s}) \Rightarrow a_y = 4.4\times10^3\ \text{m/s}^2$$

This result is one-fourth of the above radial acceleration.

REVIEW: The radial acceleration of the centrifuge is large, but it is also true that falling objects are subjected to large accelerations when they are stopped by hard surfaces.