Curvas, Superficies y ASM

J.C. Torres y P. Cano

Representación

Curvas paramétricas

$$x = f_1(u)$$

$$y = f_2(u) \quad u \in [u_1, u_2]$$

(curva en el plano)

$$X(u) = (1-u) \cdot P_a.x + u \cdot P_b.x$$

$$Y(u) = (1-u) \cdot P_a.y + u \cdot P_b.y$$

$$\mathbf{P}(\boldsymbol{u}) = (1-u) \cdot \mathbf{P}_\mathbf{a} + u \cdot \mathbf{P}_\mathbf{b}$$

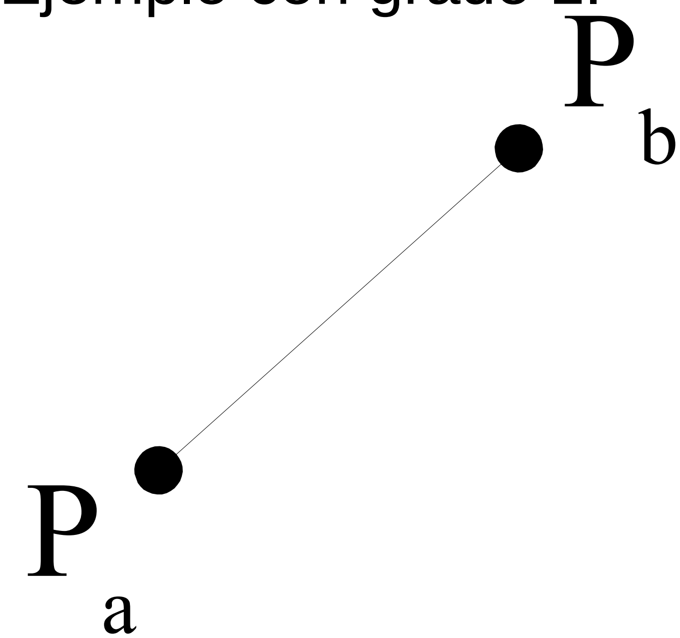
Funciones de forma

$$\boldsymbol{P}(\boldsymbol{u}) = \sum_{i=1}^{n} \boldsymbol{P}_{\boldsymbol{i}} \cdot B_i(u)$$

Puntos de control

Representación

Ejemplo con grado 1.



$$P(u)=\sum_{i=1}^{n} P_i \cdot B_i(u)$$

$$\boldsymbol{P}(u)=\boldsymbol{P}_0 \cdot B_0(u)+\boldsymbol{P}_1 \cdot B_1(u)$$

$B_0(u) = 1-u$

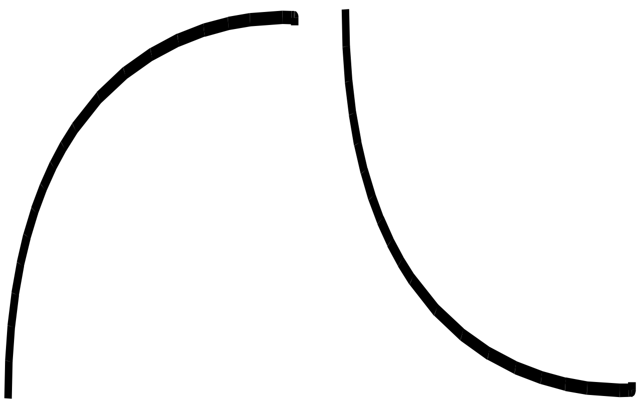
$B_1(u) = u$

$$\boldsymbol{P}(u)= \begin{pmatrix} B_0(u), & B_1(u) \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{P}_0 \\ \boldsymbol{P}_1 \end{pmatrix}$$

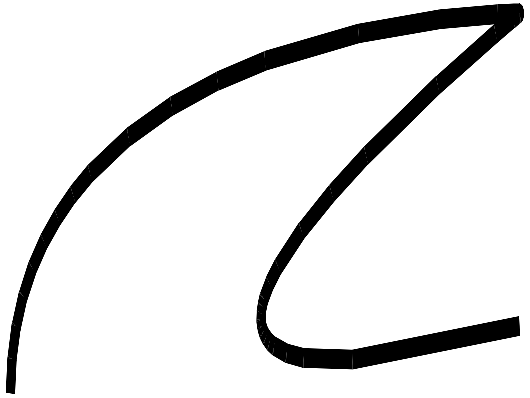
$$\boldsymbol{P}(u)= \begin{pmatrix} 1, & u \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{P}_0 \\ \boldsymbol{P}_1 \end{pmatrix}$$

Propiedades: Continuidad

Discontinua



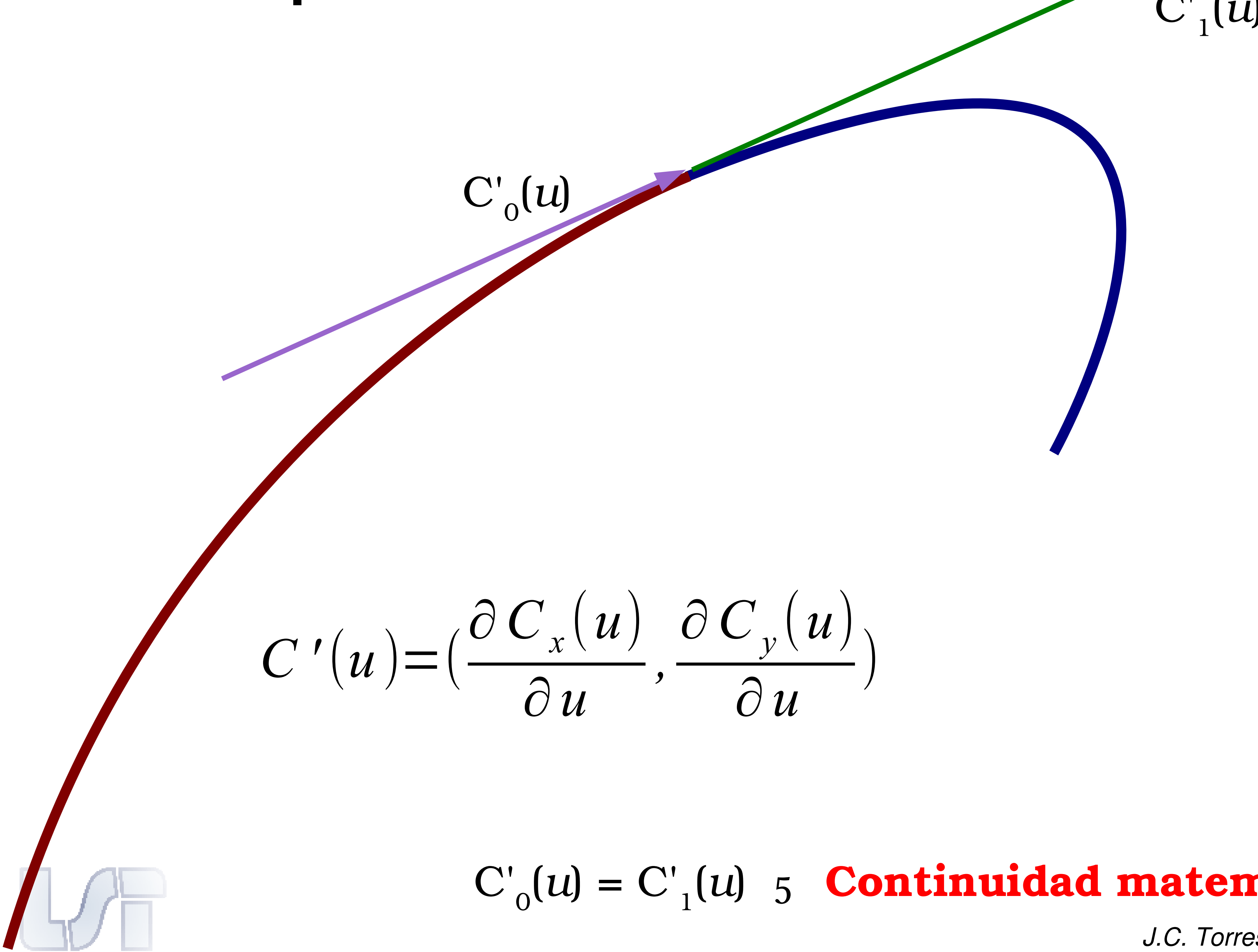
Continua C^0



Continuamente diferenciable C^1

C^k

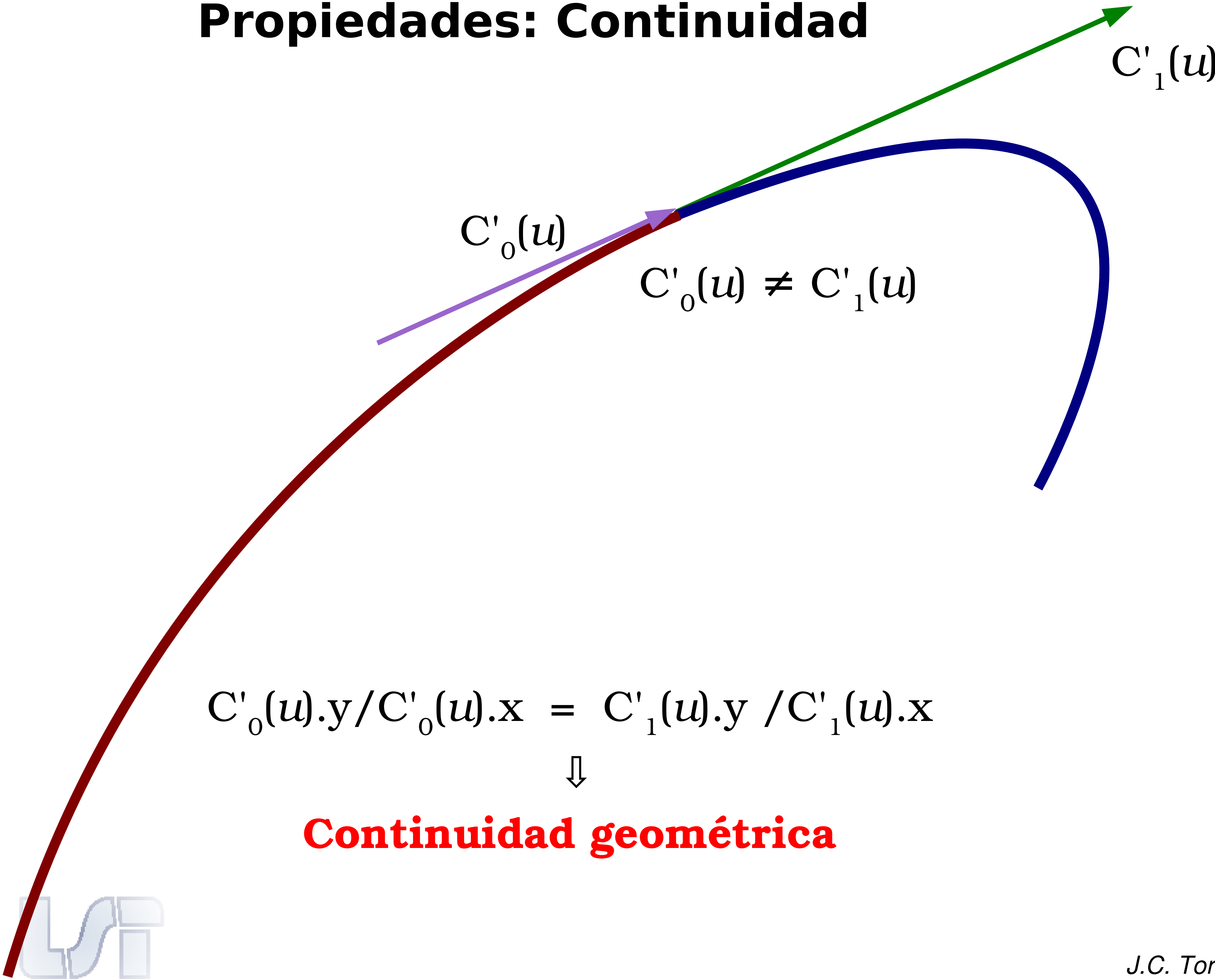
Propiedades: Continuidad



$$C'(u) = \left(\frac{\partial C_x(u)}{\partial u}, \frac{\partial C_y(u)}{\partial u} \right)$$

$C'_0(u) = C'_1(u)$ **Continuidad matemática**

Propiedades: Continuidad

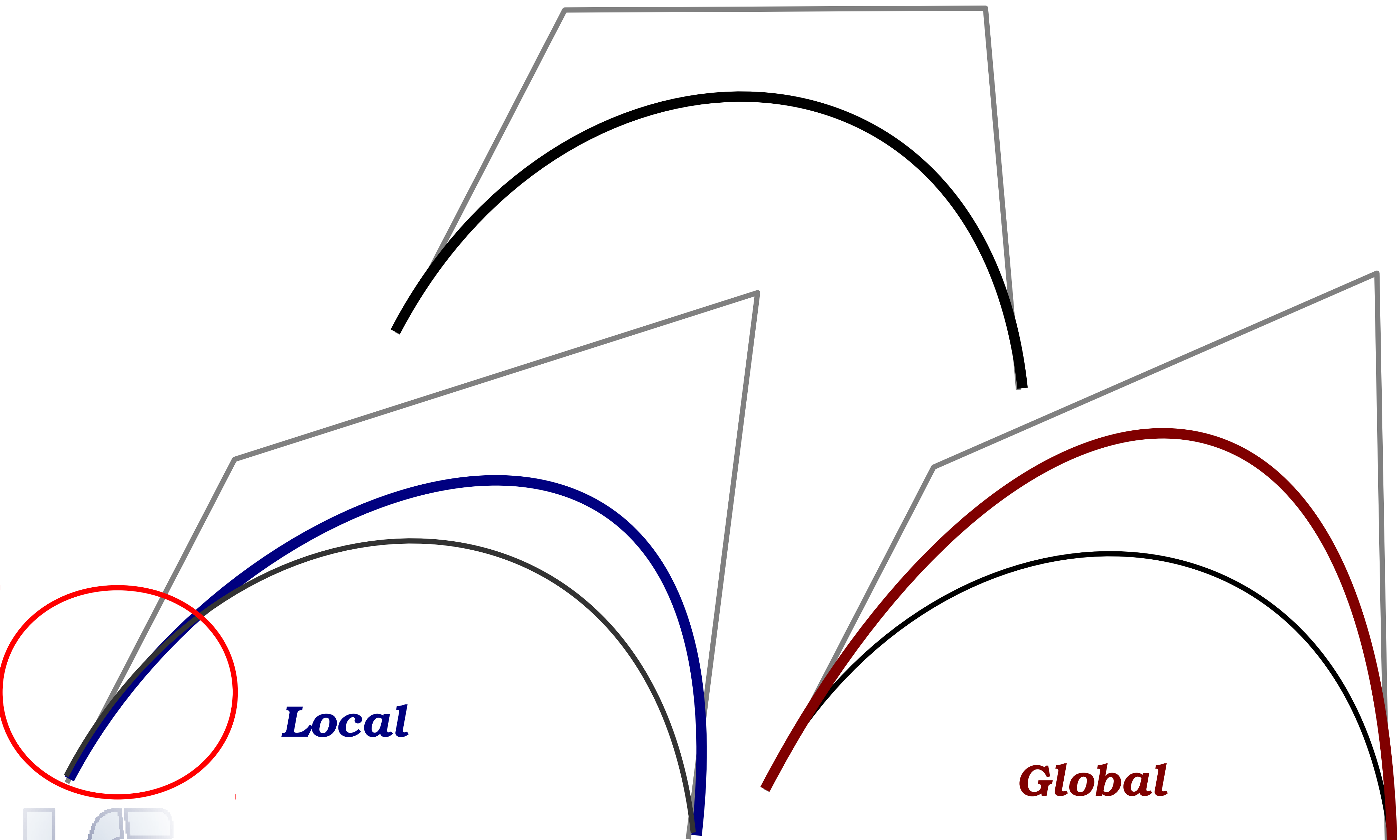


$$C'_0(u).y / C'_0(u).x = C'_1(u).y / C'_1(u).x$$

$$\Downarrow$$

Continuidad geométrica

Propiedades: Carácter



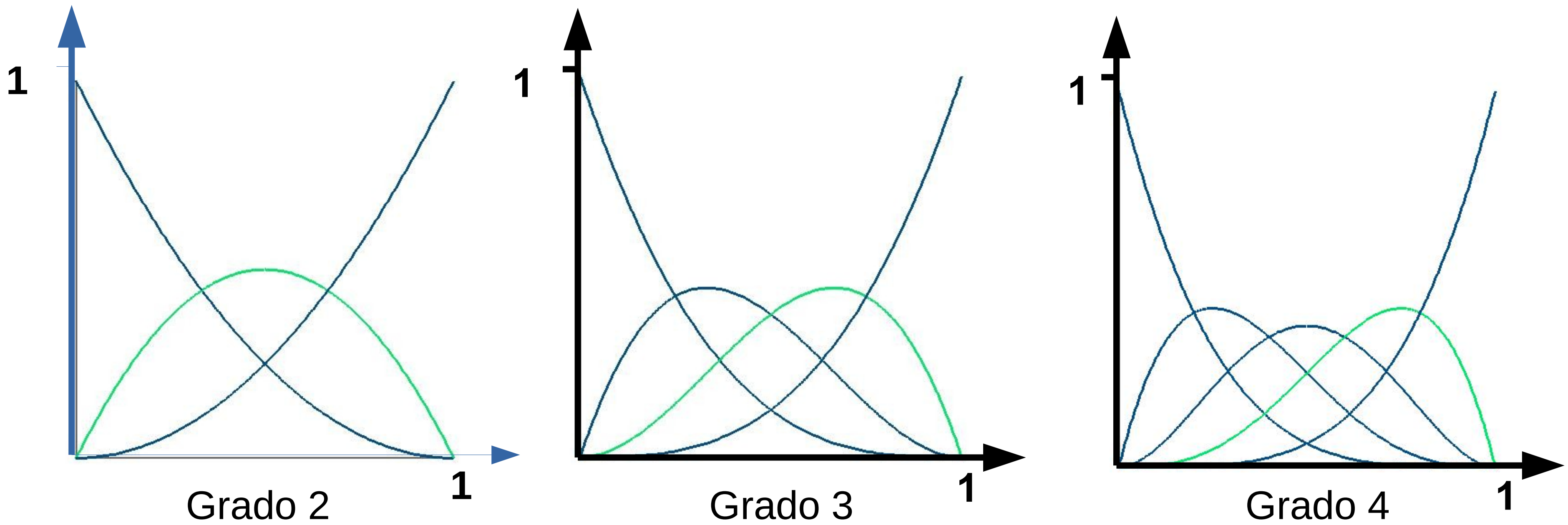
Curvas de Bèzier

$$\boldsymbol{P}(u)=\sum_{i=0}^{n}\boldsymbol{P}_i B_i^n(u) \qquad u\in[0,1]$$

$$B_i^n(u)=\binom{n}{i}(1-u)^{n-i}\cdot u^i$$

$$\binom{n}{i}=\frac{n!}{i!\cdot(n-i)!}$$

Curvas de Bèzier



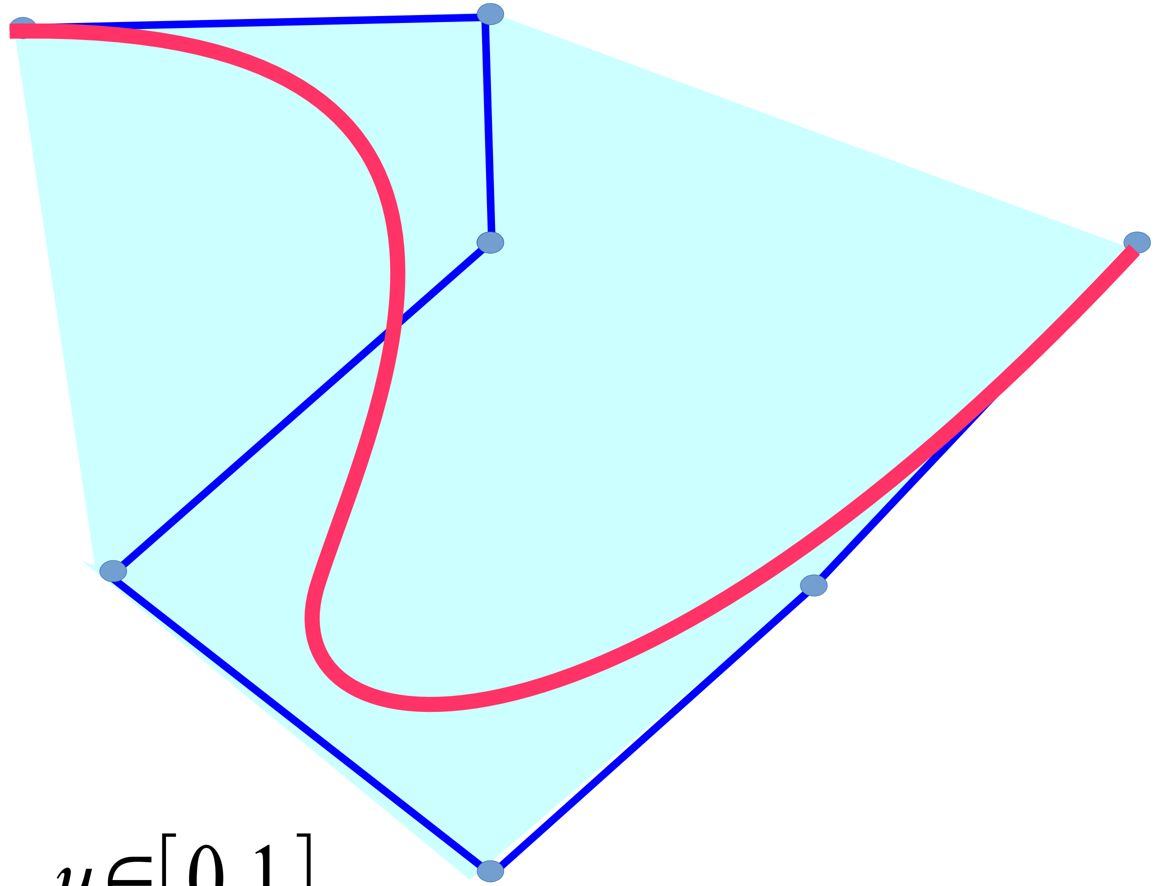
$$\sum_{i=0}^{n} B_i^n(u) = 1$$

$$B_i^n(u) \geqslant 0$$

Ejecutar Bezier

Curvas de Bèzier: propiedades

La curva está contenida en la envolvente convexa de los puntos de control



$$\sum_{i=0}^{n} B_i^n(u) = 1$$

$$B_i^n(u) \geqslant 0$$

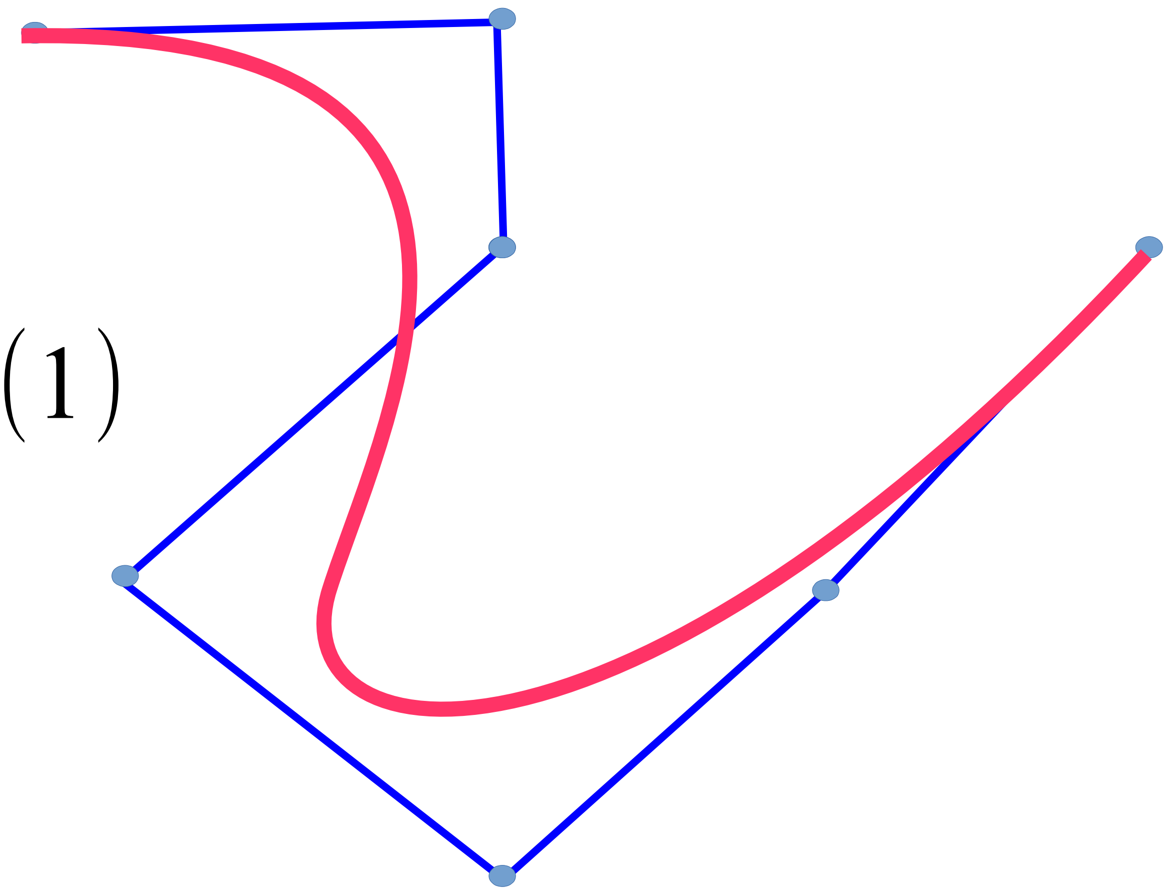
$$\boldsymbol{P}(u) = \sum_{i=0}^{n} \boldsymbol{P}_i B_i^n(u) \qquad u \in [0,1]$$

Curvas de Bèzier: propiedades

La curva no interpola los puntos de control, salvo el primero y último

$$B_i^n(u) \neq 1$$

$$salvo\ B_0^n(0)\ y\ B_n^n(1)$$



Curvas de Bèzier: propiedades

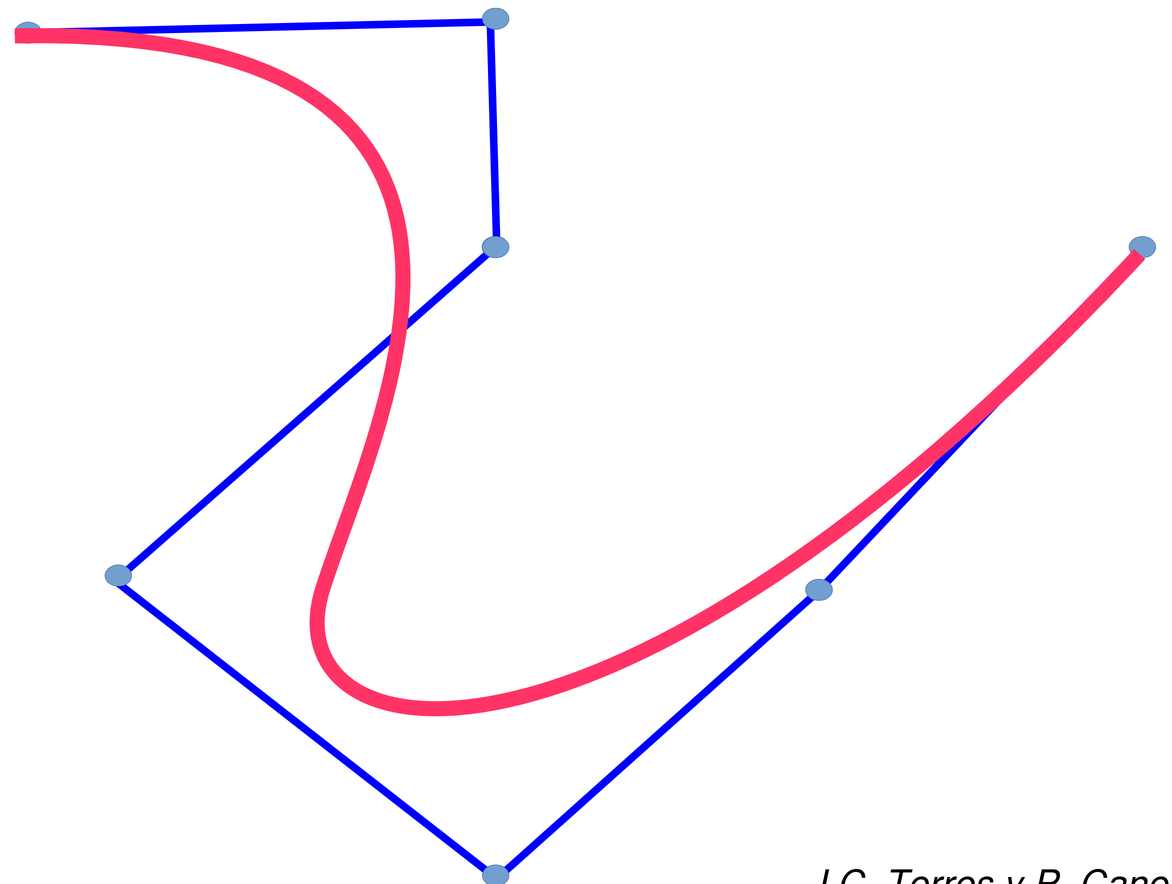
La dirección en los extremos está determinada por P_1 y P_{n-1}.

El grado del polinomio es el número de puntos menos uno.

La modificación de un punto de control afecta a toda la curva.

La curva sigue la forma de la poligonal.

La continuidad es C^{∞}.



B-Splines

$$P(u)=\sum_{i=0}^{n} P_i \cdot N_{i,k}(u) \qquad u_{min} \leqslant u \leqslant u_{max} \qquad 2 \leqslant k \leqslant n+1$$

$$N_{i,1}(u)= \begin{cases} 1 & si \quad t_i \leqslant u < t_{i+1} \\ 0 & \end{cases}$$

$$N_{i,k}(u) = \frac{u-t_i}{t_{i+k-1}-t_i} \cdot N_{i,k-1}(u) + \frac{t_{i+k}-u}{t_{i+k}-t_{i+1}} \cdot N_{i+1,k-1}(u)$$

$$t_j \leqslant t_{j+1} \qquad 0 \leqslant j \leqslant n+k$$

$$u_{min}=t_{k-1}$$

$$u_{max}=t_{n+1}$$

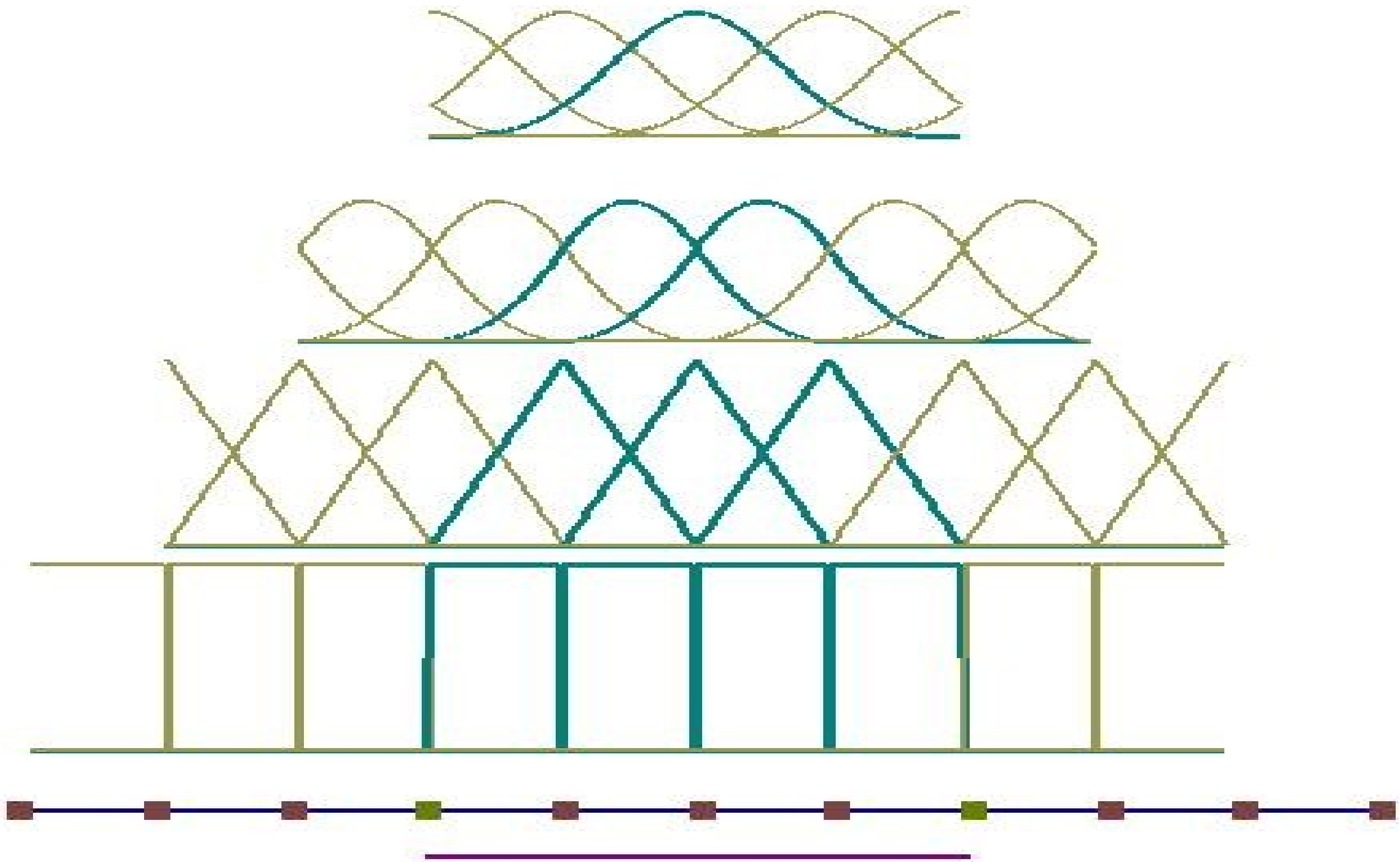
K Orden
Número de puntos de control: n+1
Grado polinomio: k-1

B-Splines

K=4

K=3

K=2

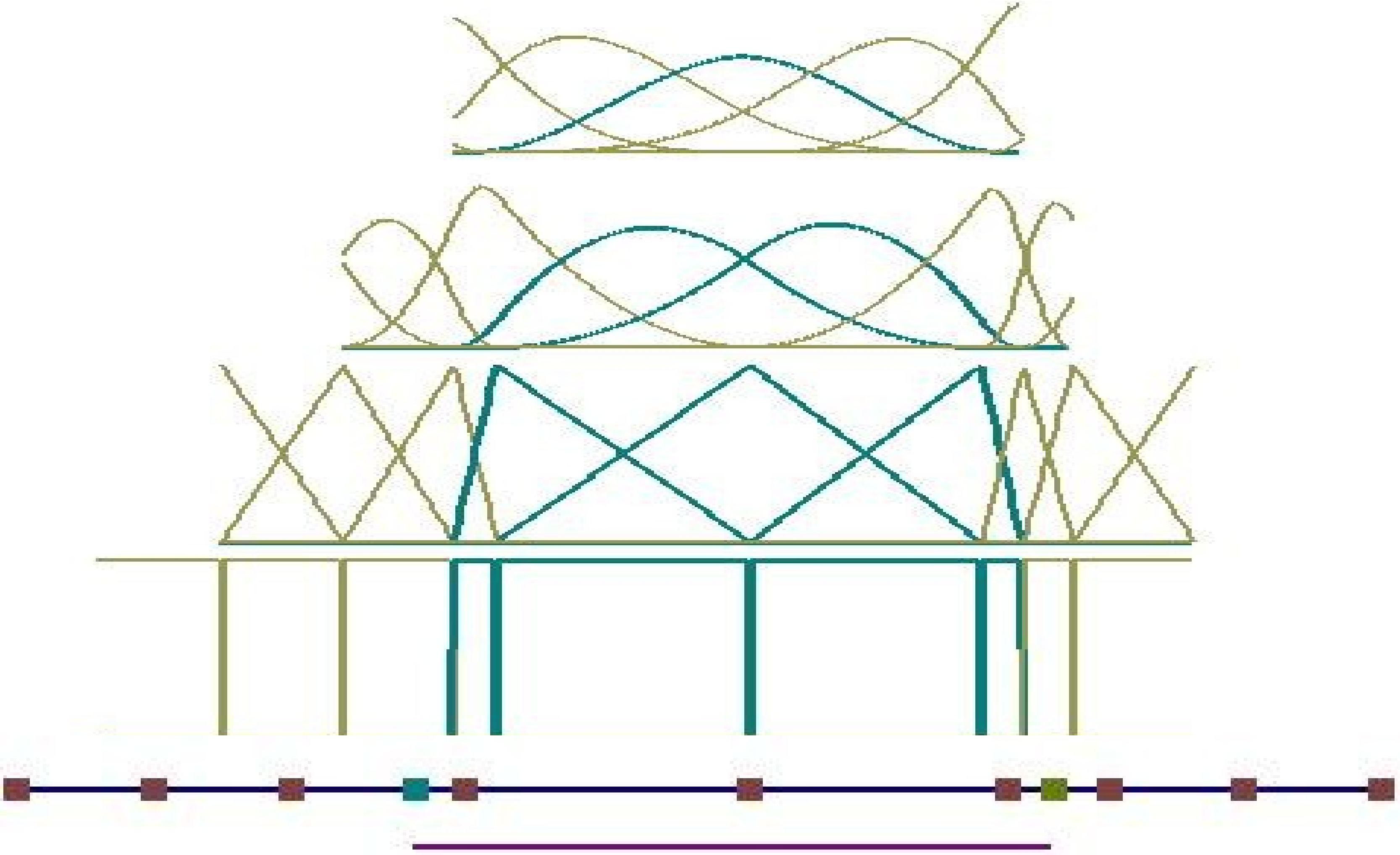


B-Splines

K=4

K=3

K=2



B-Splines: propiedades

Partición de la unidad: $\sum_i N_{i,k}(t) = 1$

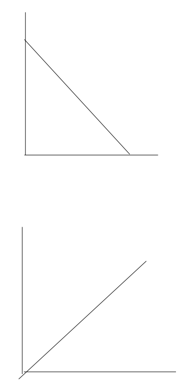
Positividad: $N_{i,k}(t) \geq 0$

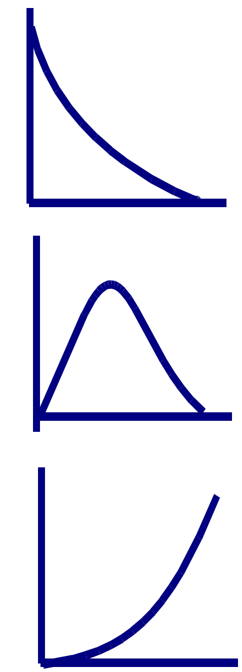
Soporte local: $N_{i,k}(t) = 0 \quad \text{si } u \in [t_i, t_{i+k+1}]$

Continuidad: $N_{i,k}$ es (k-2) veces diferenciable

B-Splines
(k=3, n=2)

u	***t***	$N_{i,1}(u)$	$N_{i,2}(u)$	$N_{i,3}(u)$
	$t_0=0$			
		$N_{0,1}(u)=0$		
	$t_1=0$		$N_{0,2}(u)=0$	
		$N_{1,1}(u)=0$		$N_{0,3}(u)$
$u=t_2=0$	$t_2=0$		$N_{1,2}(u)$	
		$N_{2,1}(u)=1$ si $u<1$		$N_{1,3}(u)$
$u=t_3=1$	$t_3=1$		$N_{2,2}(u)$	
		$N_{3,1}(u)=0$		$N_{2,3}(u)$
	$t_4=1$		$N_{3,2}(u)=0$	
		$N_{4,1}(u)=0$		
	$t_5=1$			





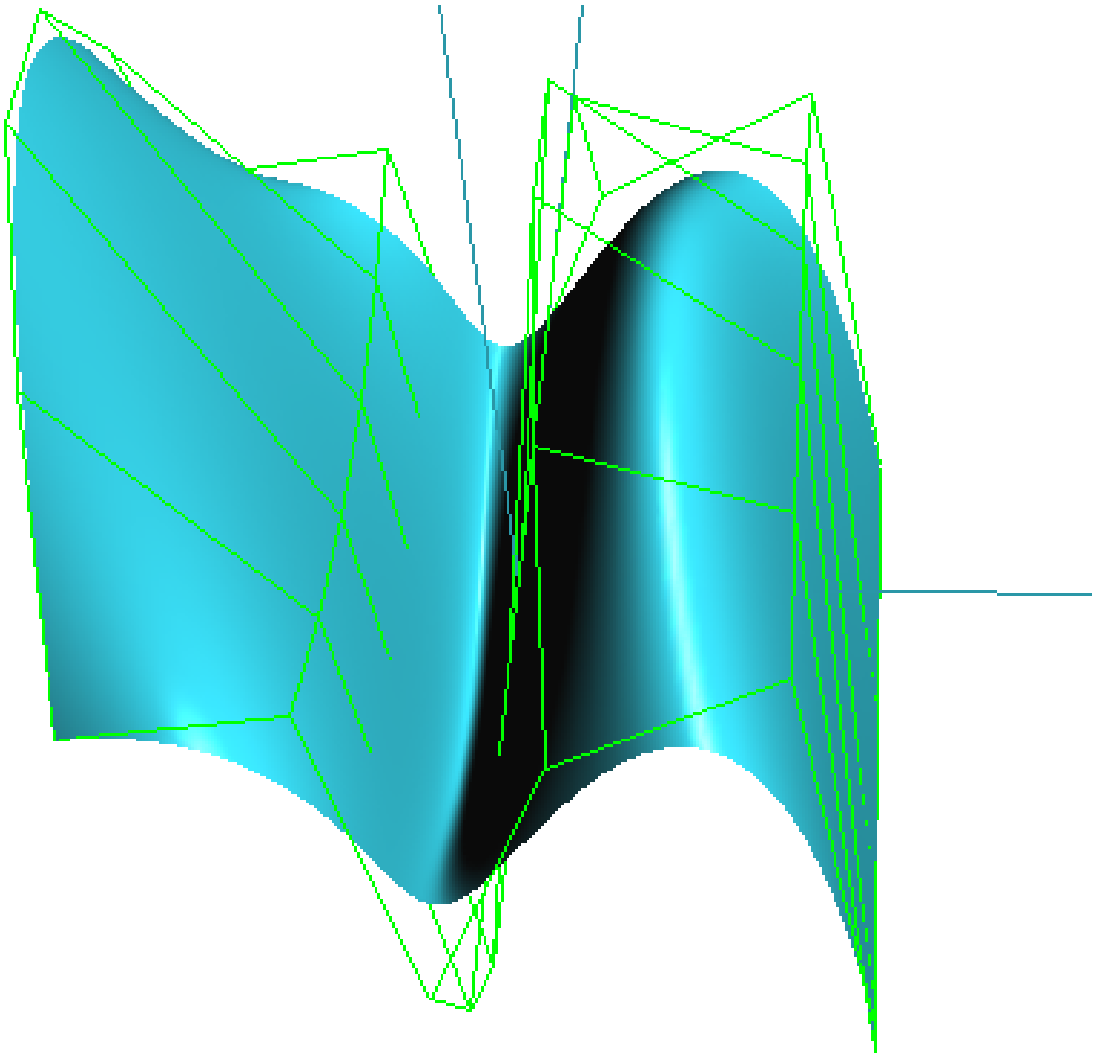
Superficies: Superficies Bspline

$$S(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m} P_{ij}\cdot N_{i,k}(u)\cdot N_{j,l}(v)$$

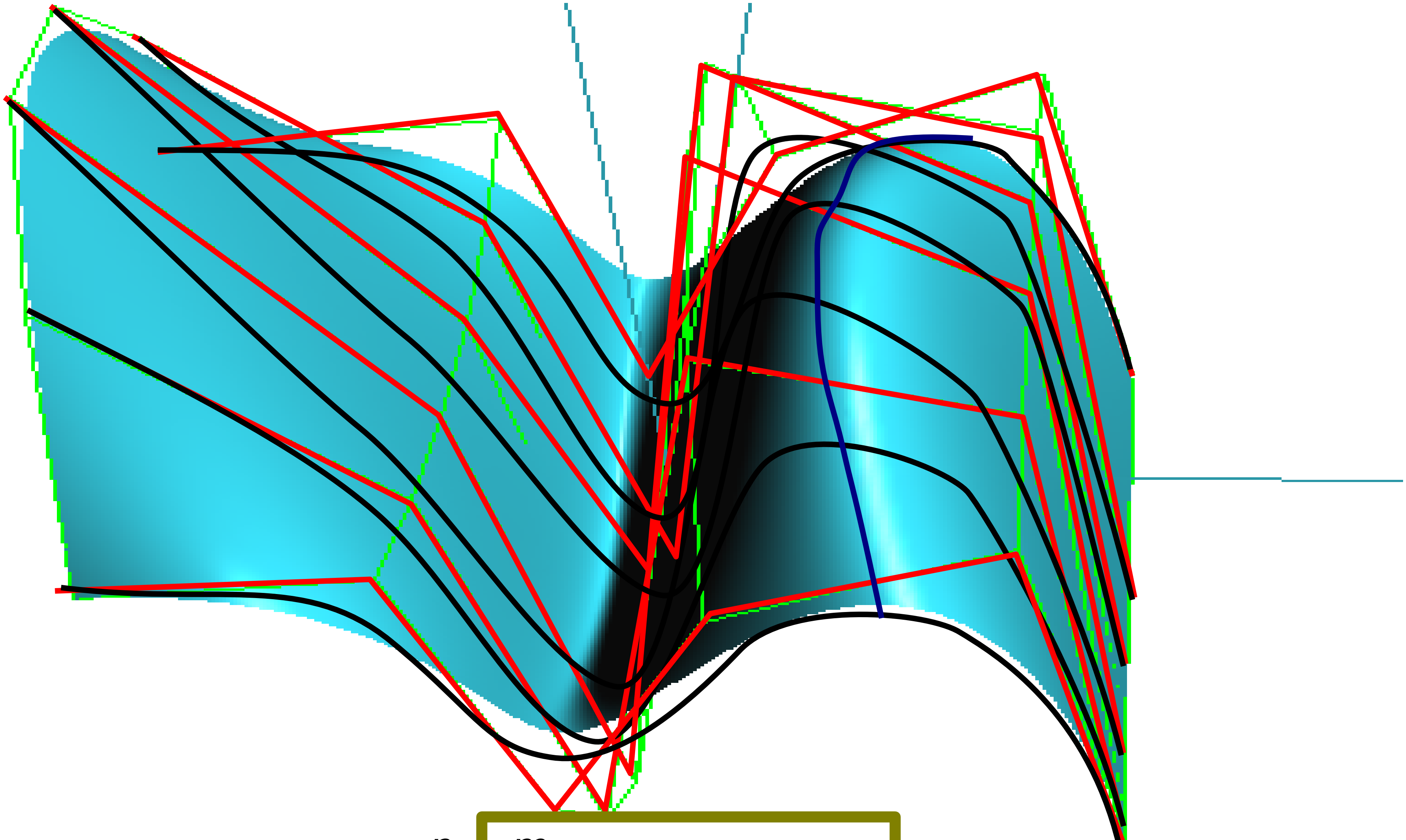
$$u_{min}\leqslant u\leqslant u_{max} \qquad 2\leqslant k\leqslant n+1$$

$$v_{min}\leqslant v\leqslant v_{max} \qquad 2\leqslant l\leqslant m+1$$

Superficies: Superficies Bspline



Superficies: Superficies Bspline



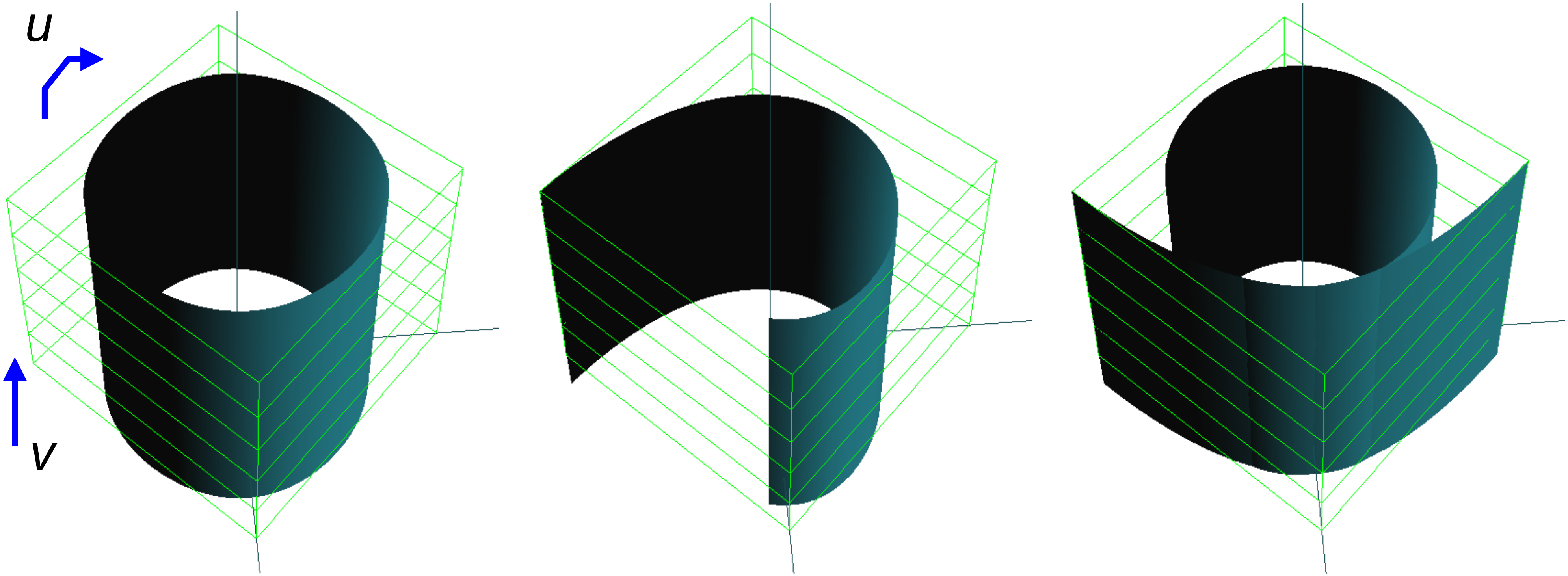
$$S(u,v)=\sum_{i=0}^{n}\boxed{\sum_{j=0}^{m} P_{ij}\cdot N_{j,l}(u)}\,N_{i,k}(v)$$

Superficies: Superficies Bspline

```
GLfloat ctlpoints[4][4][3]; // 4x4 puntos de control 3D
GLfloat knots[8] = {0.0,0.0,0.0,0.0,1.0,1.0,1.0,1.0};
GLUnurbsObj *theNurb;

theNurb = gluNewNurbsRenderer();// crea manejador nurbs
gluNurbsProperty(theNurb, GLU_SAMPLING_TOLERANCE, 5.0);
gluNurbsProperty(theNurb, GLU_DISPLAY_MODE, GLU_FILL);
glEnable(GL_AUTO_NORMAL); // genera automa. las normales

gluBeginSurface(theNurb); // Se va a dar una superficie
gluNurbsSurface(theNurb,8, knots,8, knots, 4 * 3, 3,
                  &ctlpoints[0][0][0],  4, 4, GL_MAP2_VERTEX_3);
gluEndSurface(theNurb);   // fin de def. De superficie
```

```
gluNurbsSurface(theNurb, nNodosU, MnodosU,nNodosV,MnodosV
    ,deltaUP,deltaVP, &P, ordenU, ordenV, TipoVertices);
```

Superficies: Superficies Bspline

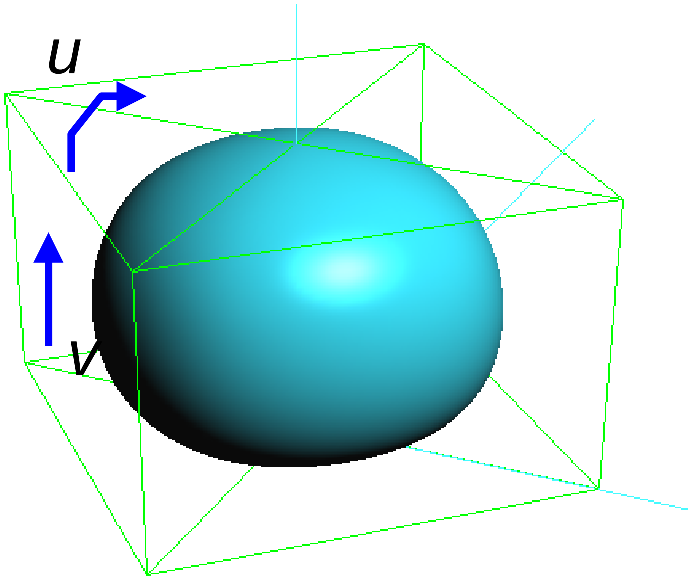


Uniforme y periódico en u
No periódico en v

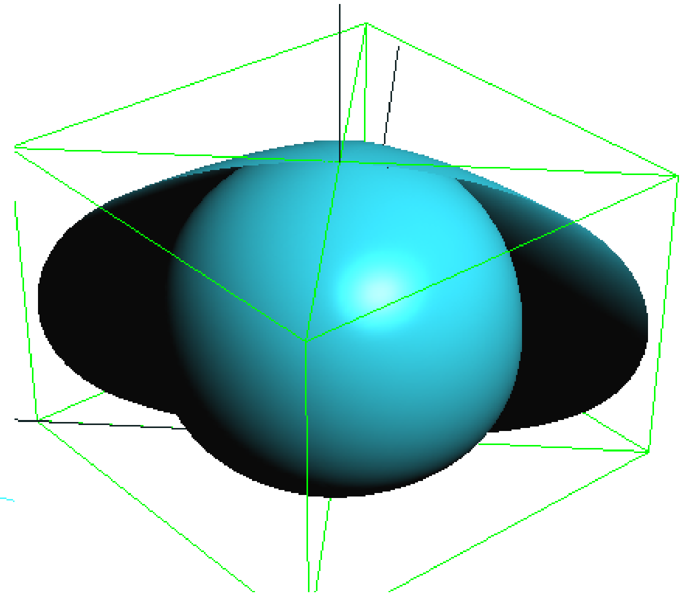
No uniforme en u
No periódico en v

No periódico en u
No periódico en v

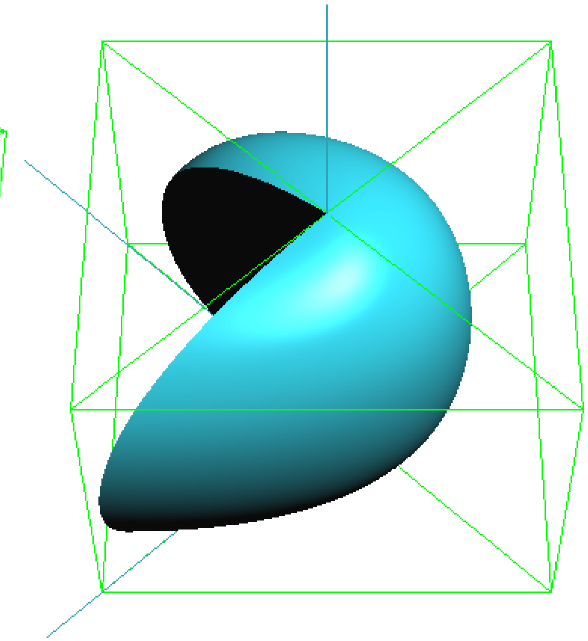
Superficies: Superficies Bspline



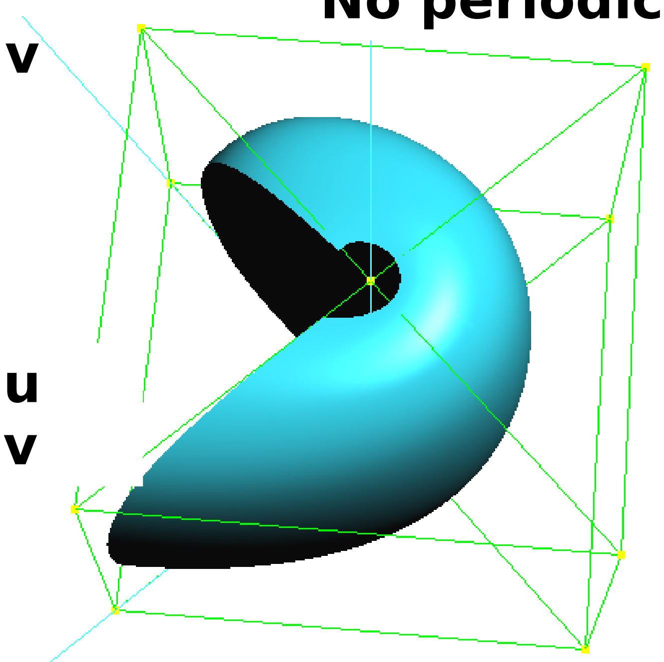
Uniforme y periódico en u
No periódico en v



No periódico en u
No periódico en v



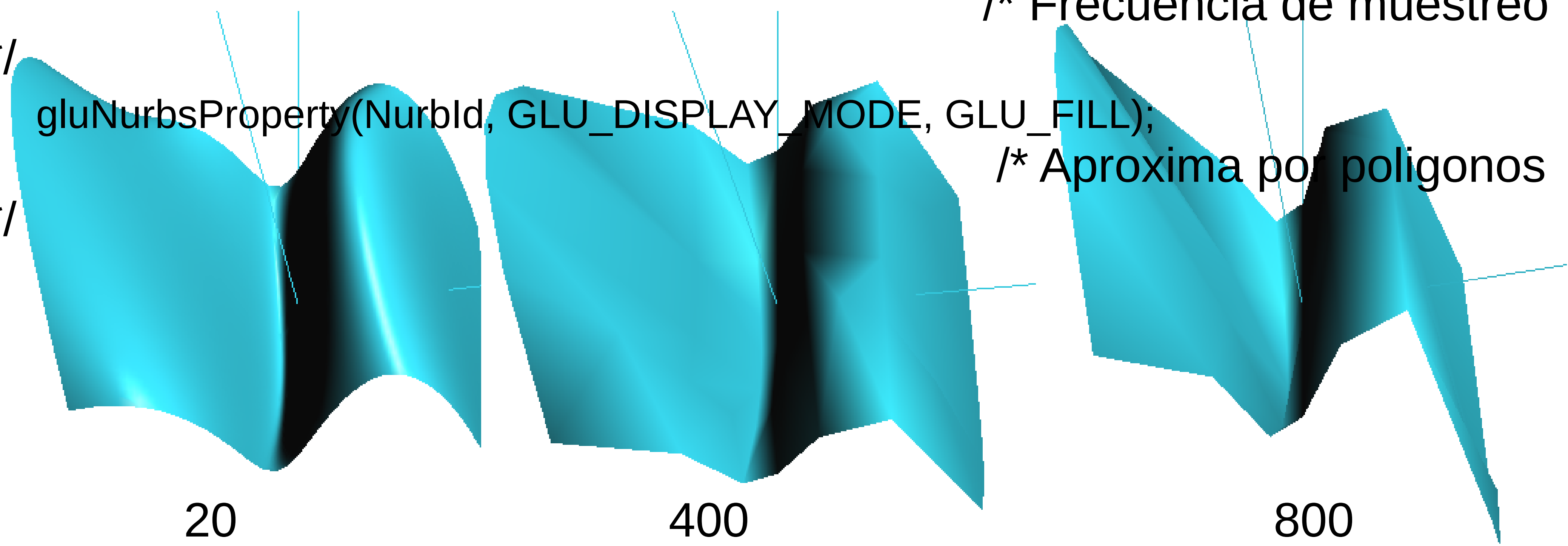
No uniforme en u
No periódico en v



No uniforme en u
No uniforme en v

Superficies: Superficies Bspline
Efecto de la tolerancia de muestreo

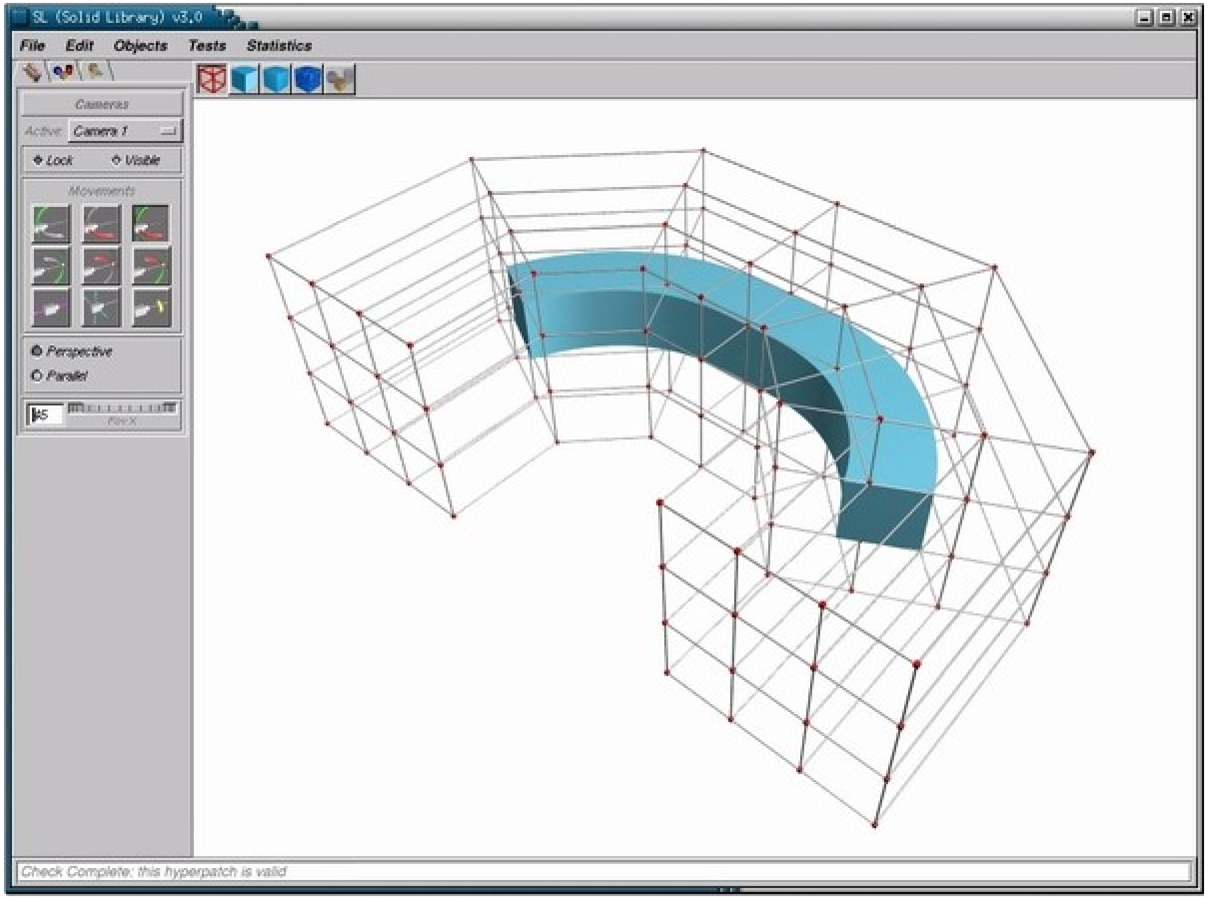
```
NurbId = gluNewNurbsRenderer();          /* crea un manejador de nurbs */

gluNurbsProperty(NurbId, GLU_SAMPLING_TOLERANCE, 20.0);
                                          /* Frecuencia de muestreo */

gluNurbsProperty(NurbId, GLU_DISPLAY_MODE, GLU_FILL);
                                          /* Aproxima por poligonos */
```



20 400 800

Modelado Análitico de Sólidos

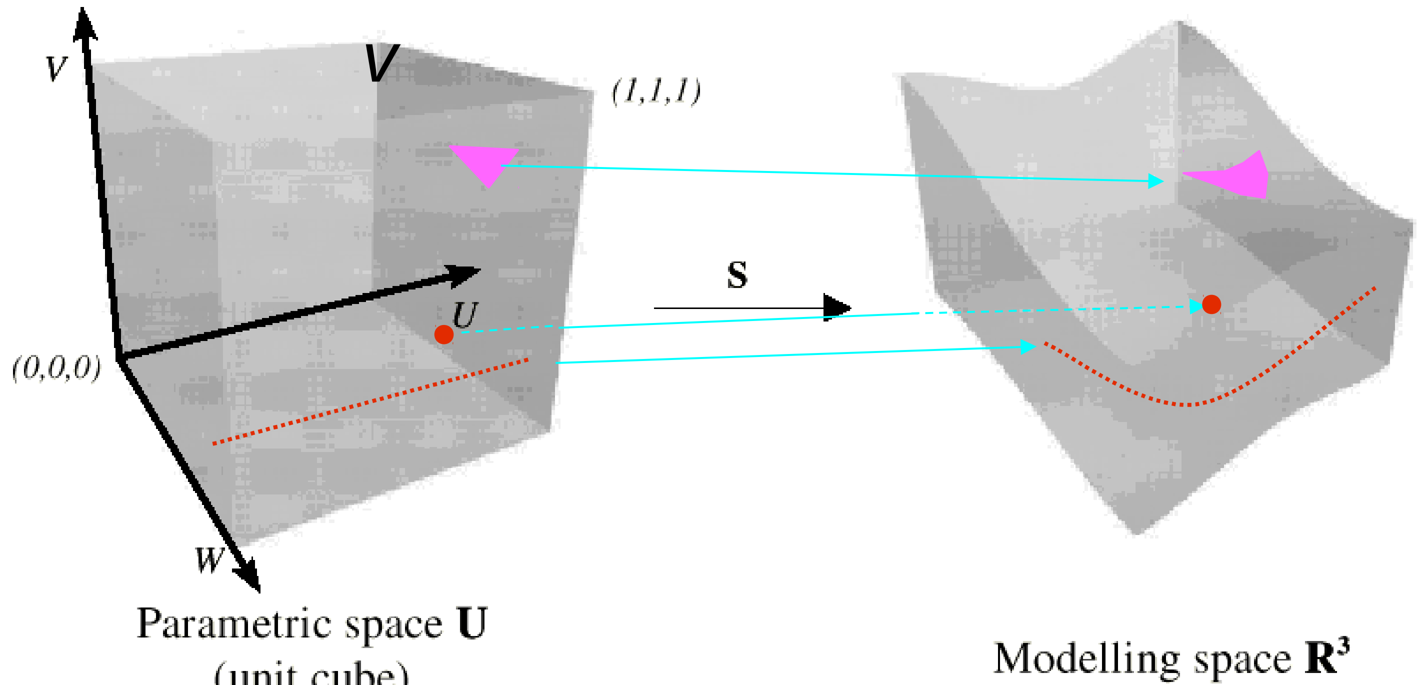
ASM (Analytical Solid Modeling) es una extensión de los métodos de diseño de curvas y superficies a 3D.



B-Spline Hyperparches

A hyperparche defines a parametric volume

S$(u,v,w) = (x(u,v,w), y(u,v,w), z(u,v,w))$



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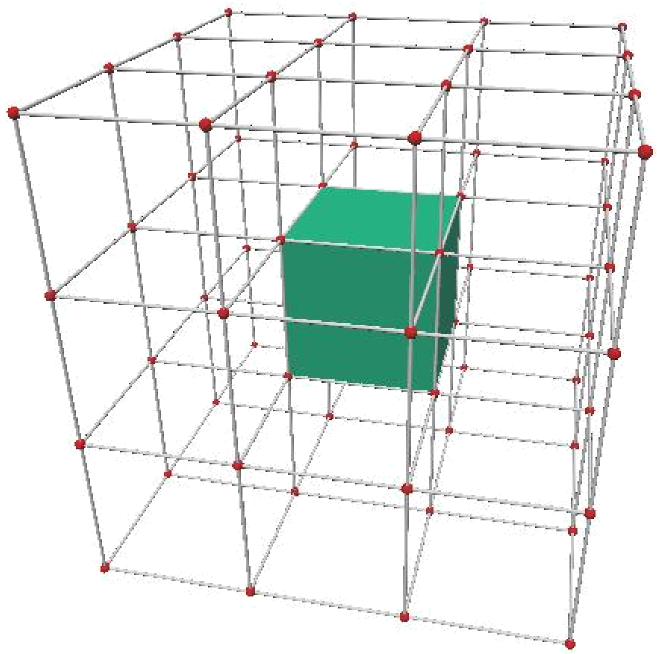
B-Spline Hyperparches

Use a grid of control points (geometric coef.)

$$\mathbf{S}(u,v,w) = \sum_{r}\sum_{s}\sum_{t} b_r(u)\, b_s(v)\, b_t(w)\, g_{r,s,t}$$

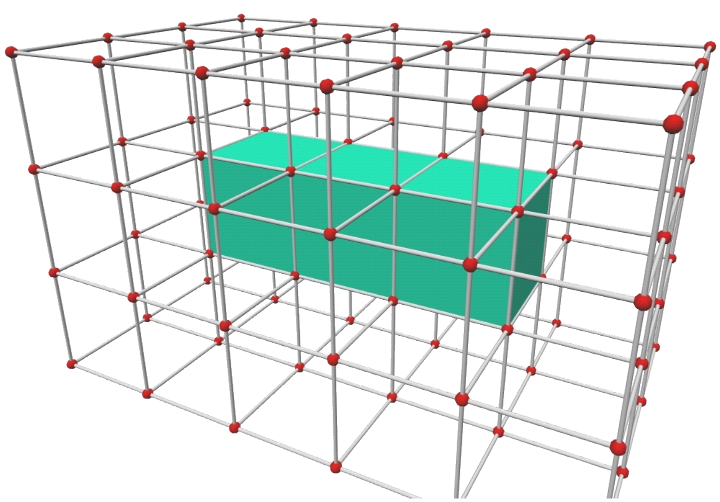


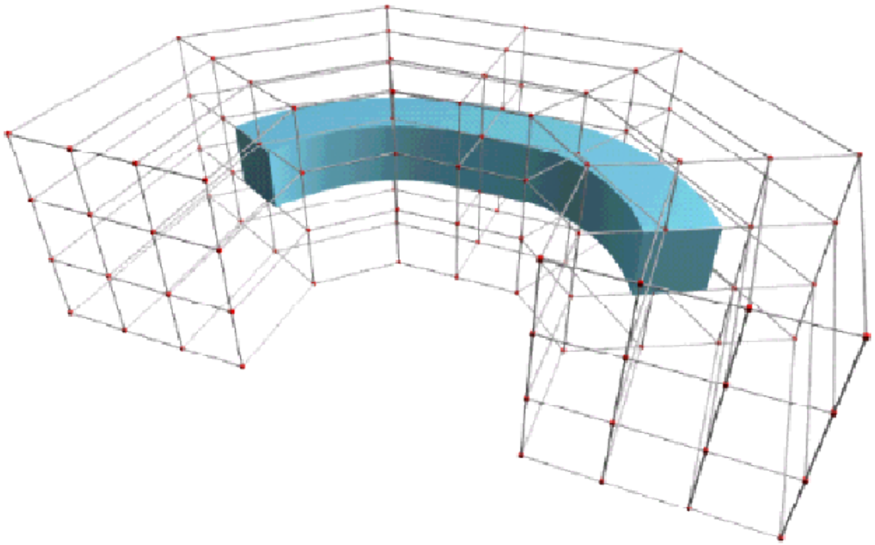
64 Control points per voxel

B-Spline Hyperparches

Define a complex solid extending the grid

$$\mathbf{S_{i,j,k}}(u,v,w) = \sum_{\mathrm{r}} \sum_{\mathrm{s}} \sum_{t} b_r(u)\, b_s(v)\, b_t(w)\, g_{i+r,j+s,k+t}$$





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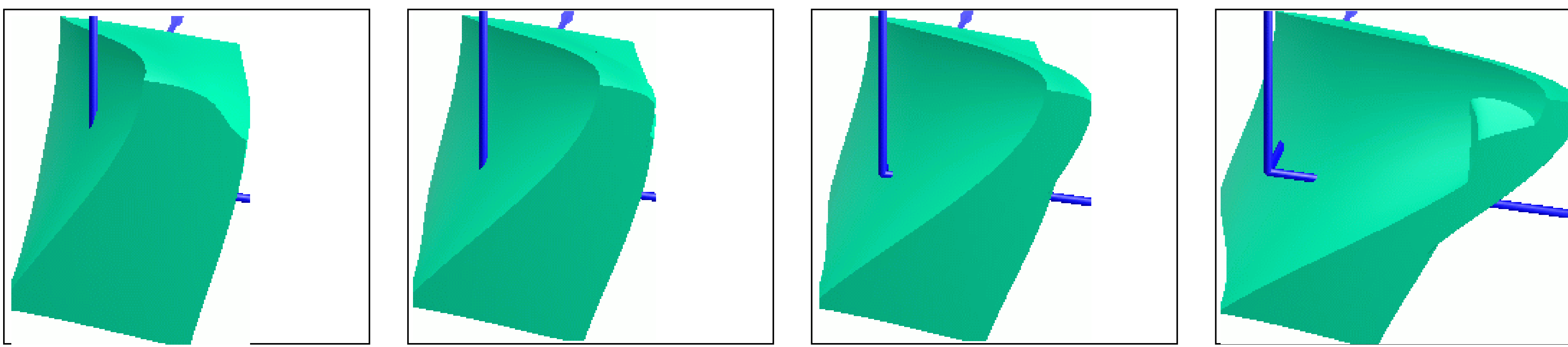
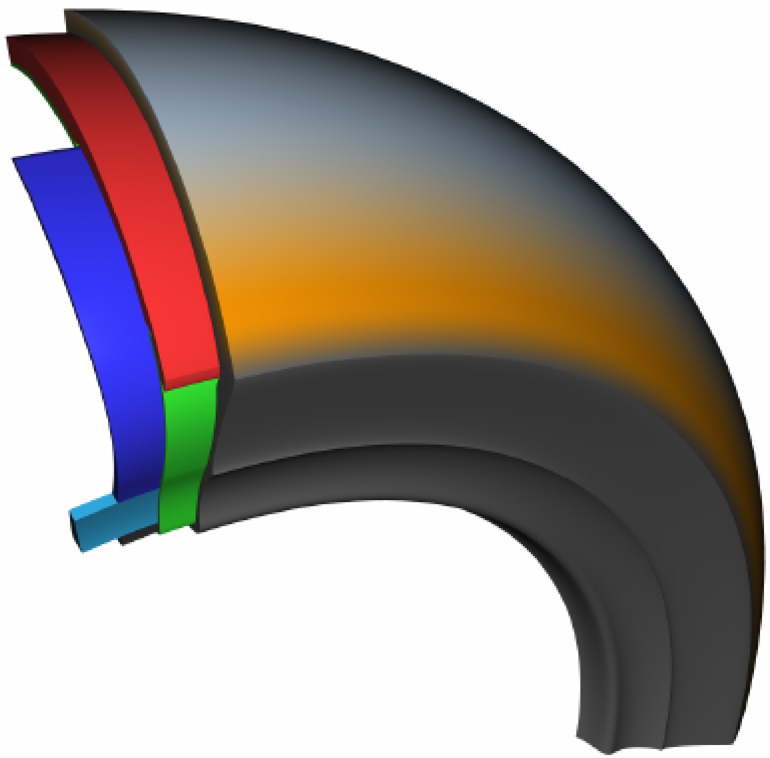
B-Spline Hyperparches

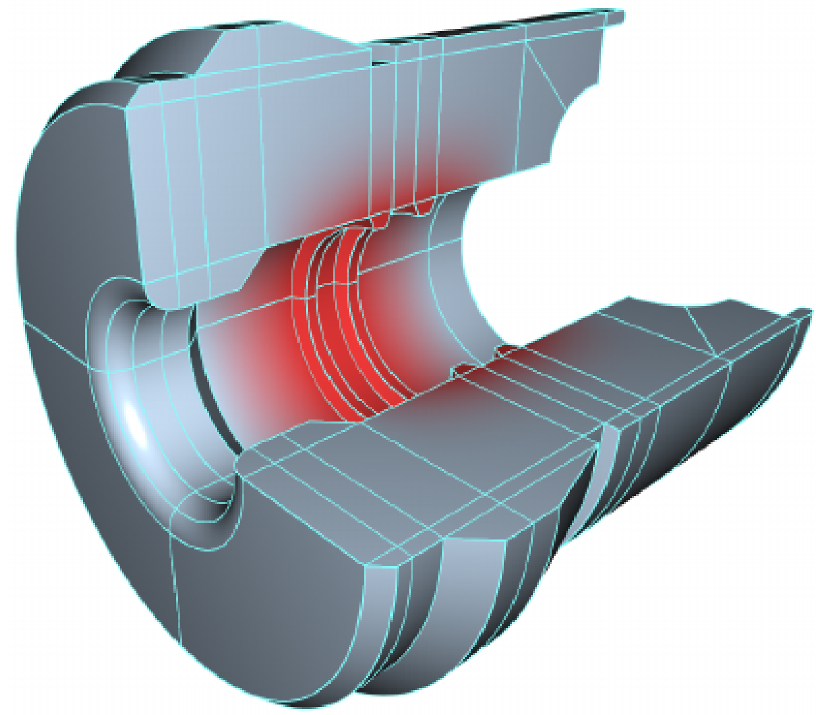
It is possible to model invalid solid:

- Self-intersecting boundary
- Non regular
- Incorrect volume

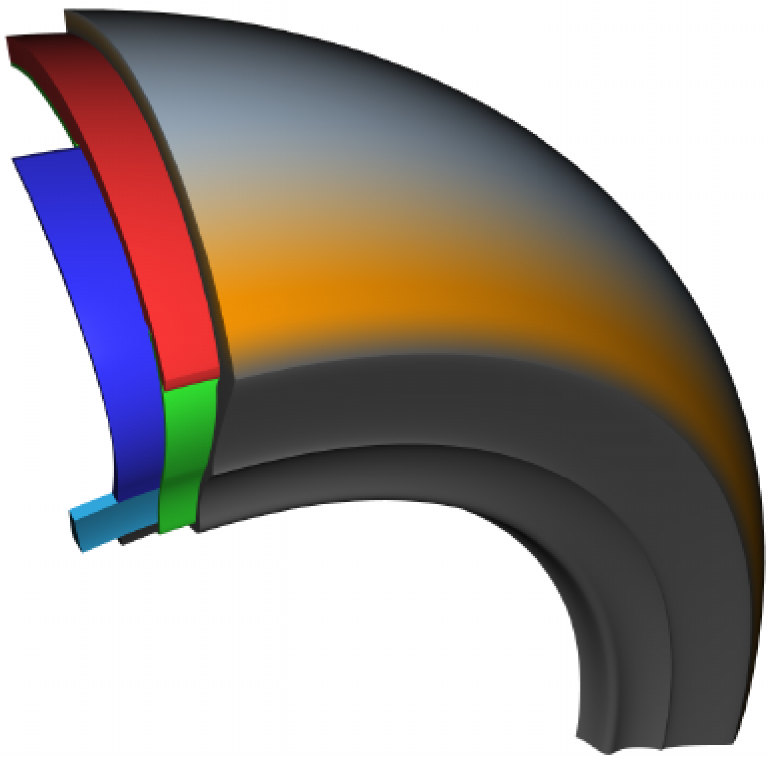


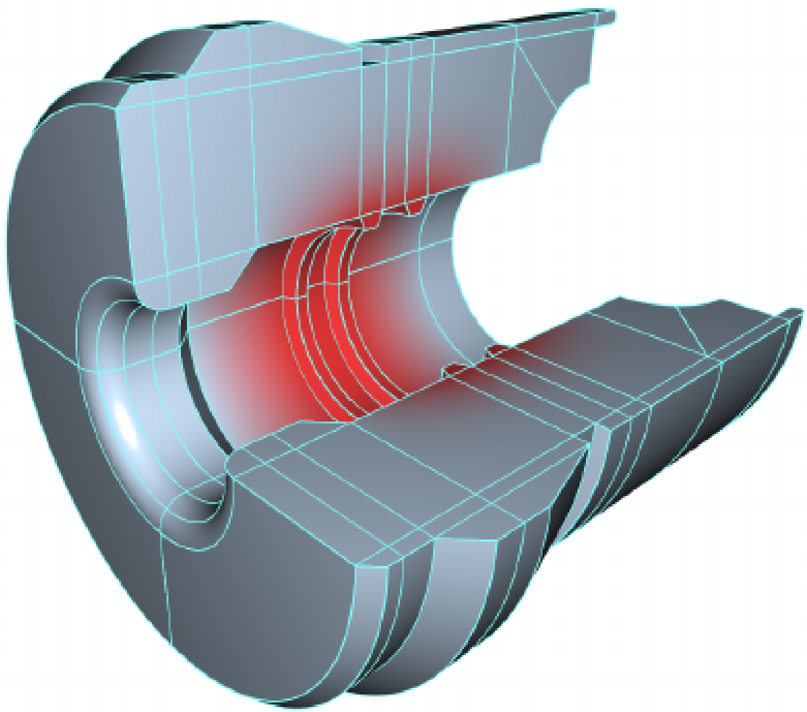
Ejemplos



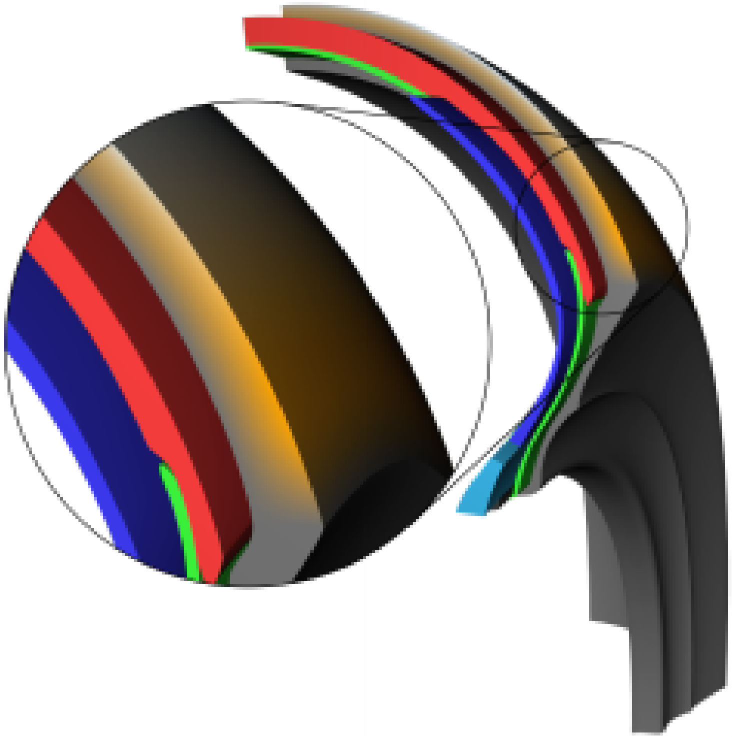


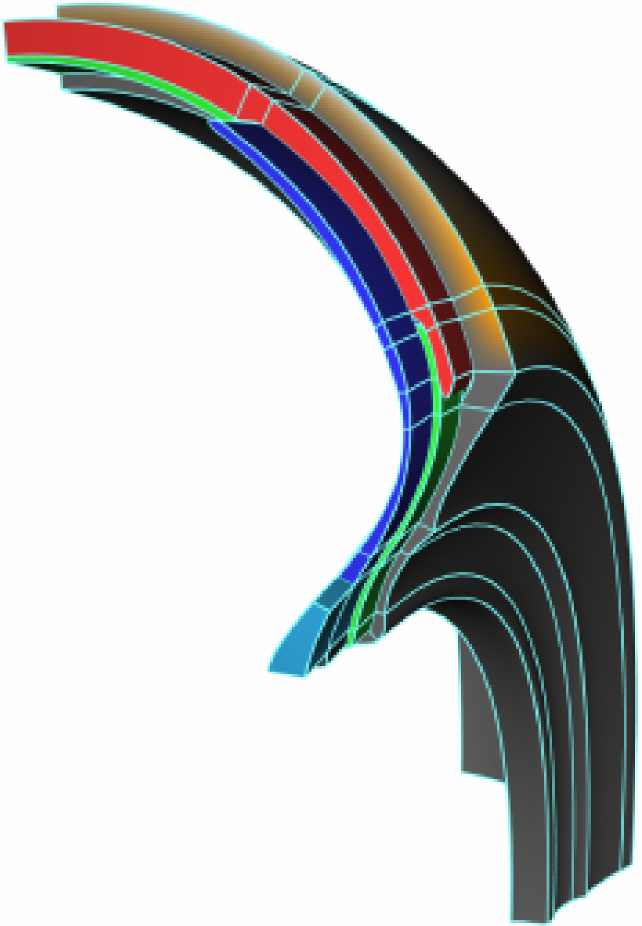
Ejemplos





Ejemplos





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