TOPIC 1: EXPONENTS AND RADICALS

Exponents

Exponents tell how many times to use a number itself in multiplication. There are different laws that guides in calculations involving exponents. In this chapter we are going to see how these laws are used.

For example; $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$ and $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$, we have multiplied alike factors and got answers which are 4^6 and 5^7. 4 and 5 are our factors and they are called bases while 6 and 7 are called exponents. 4^6 is read as 'sixth power of four' or 'four to the sixth power' and $5^7$is read as 'seventh power of five' or 'five to the seventh power'. $4 \times 4 \times 4 \times 4 \times 4 \times 4$ is the expanded form of 4^6 and the expanded form of 5^7 is $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$. The product of any expanded form is called a power of the factor. 4^6 and 5^7 are the powers of our factors. 4^6 means multiply 4 six times and 5^7 means multiply 5 seven times.

Indication of power, base and exponent is done as follows:

a.	6^2	c.	2^8	e.	5^3
b.	9^3	d.	4^9	f.	10^5

Solution:

a. 6^2 is a power, 6 is a base and 2 is the exponent.
b. 9^3 is a power, 9 is a base and 3 is the exponent.
c. 2^8 is a power, 2 is a base and 8 is the exponent.
d. 4^9 is a power, 4 is a base and 9 is the exponent.
e. 5^3 is a power, 5 is a base and 3 is the exponent.
f. 10^5 is a power, 10 is a base and 5 is the exponent.

To write the expanded form of the following powers:

a.	7^3	c.	16^4	e.	$(\frac{1}{2})^8$
b.	205^2	d.	$(-4)^6$	f.	m^5

Solution

a. $7^3 = 7 \times 7 \times 7$
b. $205^2 = 205 \times 205$
c. $16^4 = 16 \times 16 \times 16 \times 16$
d. $(-4)^6 = -4 \times -4 \times -4 \times -4 \times -4 \times -4$
e. $(\frac{1}{2})^8 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
f. $m^5 = m \times m \times m \times m \times m$

To write each of the following in power form:

a. $10 \times 10 \times 10 \times 10 \times 10 \times 10$

b. $\frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7}$

c. $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

d. $-8 \times -8 \times -8$

Soln.

a. $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$

b. $\frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} = (\frac{-3}{7})^4$

c. $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$

d. $-8 \times -8 \times -8 = (-8)^3$

The Laws of Exponents

List the laws of exponents

First law: Multiplication of positive integral exponent

$$x^m \times x^n = x^{m+n}$$

Second law: Division of positive integral exponent

$$\frac{x^m}{x^n} = x^{m-n}$$

Third law: Zero exponents

$$x^0 = 1.$$

Fourth law: Negative integral exponents

$$\frac{1}{x^1} = x^{-1}.$$

Verification of the Laws of Exponents

Verify the laws of exponents

First law: Multiplication of positive integral exponent

For example; if you are given $4^3 \times 4^6$, in expanded form you may write it as $\underbrace{4 \times 4 \times 4}_{\text{3 factors}} \times \underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4}_{\text{6 factors}}$. There are 9 factors in total.

Therefore $4^3 \times 4^6 = 4^9$.

Generally, when we multiply powers having the same base, we add their exponents. If x is any base and m and n are the exponents, therefore:

$$x^m \times x^n = x^{m+n}$$

Example 1

a. $2^4 \times 2^6$

b. $(0.25)^3 \times (0.25)^9$

c. $(\frac{-7}{11})^7 \times (\frac{-7}{11})^{12}$

d. $a^6 \times a^8 \times a^2$

Solution

a. $2^4 \times 2^6 = 2^{4+6} = 2^{10}$

b. $(0.25)^3 \times (0.25)^9 = (0.25)^{3+9} = (0.25)^{3+9}$

c. $(\frac{-7}{11})^7 \times (\frac{-7}{11})^{12} = (\frac{-7}{11})^{7+12} = (\frac{-7}{11})^7$

d. $a^6 \times a^8 \times a^2 = a^{6+8+2} = a^{16}$

If you are to write the expression using the single exponent, for example,$(6^3)^4$.The expression can be written in expanded form as:

$$(6^3)^4 = \underbrace{6^3 \times 6^3 \times 6^3 \times 6^3}_{\text{4 factors}}$$

$$= \underbrace{\underbrace{6 \times 6 \times 6}_{\text{3 factors}} \times \underbrace{6 \times 6 \times 6}_{\text{3 factors}} \times \underbrace{6 \times 6 \times 6}_{\text{3 factors}} \times \underbrace{6 \times 6 \times 6}_{\text{3 factors}}}_{\text{4 factors of 3 factors = 12 factors}}$$

$$= 6^{12} \quad \text{Therefore, } 6^{12} = 6^{3 \times 4} = 6^{12}$$

Generally if a and b are real numbers and n is any integer,

$$(a \times b)^n = a^n \times b^n$$

Example 2

$$(y^4)^8 = (y)^{4\times 8} = (y)^{32}$$

Example 3

$$((\tfrac{3}{4})^3)^6 = (\tfrac{3}{4})^{3\times 6} = (\tfrac{3}{4})^{18}$$

Example 4

$$((-0.75)^7)^4 = (-0.75)^{7\times 4} = (-0.75)^{28}$$

Generally, $(x^m)^n = X^{(m\times n)}$

Example 5

Rewrite the following expressions under a single exponent for those with identical exponents:

a. $z^4 \times 5Zx^2$

b. x^2y^2

Solution

a. $z^4 \times 5Zx^2 = z \times z \times z \times z \times 5 \times z \times x \times x$

$= 5 \times z \times z \times z \times z \times z \times x \times x$

$= 5 \times z^5 \times x^2 = 5z^5x^2$

b. $x^2y^2 = (xy)^2$

Second law: Division of positive integral exponent

Example 6

The expression $\frac{5^6}{5^2}$ can be written as:

$$\frac{5^6}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5}$$

$= 5 \times 5 \times 5 \times 5$

$= 5^4$

Which is the same as:

$$\frac{5^6}{5^2} = 5^{6-2} = 5^4$$

Example 7

$\frac{k^5}{k^4}$

Solution

$$\frac{k^5}{k^4} = \frac{k \times k \times k \times k \times k}{k \times k \times k \times k}$$

$$= k$$

Which is the same as:

$$\frac{k^5}{k^4} = k^{5-4}$$

$$= k$$

Therefore, to divide powers of the same base we subtract their exponents (subtract the exponent of the divisor from the exponent of the dividend). That is,

$$\frac{x^m}{x^n} = x^{m-n}$$

where *x* is a real number and *x* ≠ 0, m and n are integers. m is the exponent of the dividend and n is the exponent of the divisor.

Example 8

a. $\frac{10^8}{10^5}$

b. $\frac{5a^6}{a^4}$

c. $\frac{81}{3^2}$

d. $\frac{((3)^2)^5}{3^4}$

e. $18b^4 \div (4b^3 - 2b^3)$

solution

a. $\frac{10^8}{10^5} = 10^{8-5} = 10^3$

b. $\frac{5a^6}{a^4} = 5a^{6-4} = 5a^2$

c. $\frac{81}{3^2} = \frac{3^4}{3^2} = 3^{4-2} = 3^2$

d. $\frac{(3^2)^5}{3^4} = \frac{3^{2 \times 5}}{3^4}$

$= \frac{3^{10}}{3^4} = 3^{10-4}$

$= 3^6$

e. $18b^4 \div (4b^3 - 2b^3)$

$= 18b^4 \div 2b^3$

$= \frac{18b^4}{2b^3}$

$= 9b^{4-3}$

$= 9b$

Third law: Zero exponents

Example 9

$$\frac{2^5}{2^5} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = 1$$

This is the same as:

$$\frac{2^5}{2^5} = 2^{5-5} = 2^0 = 1$$

If a ≠ 0, then

$$\frac{a^3}{a^3} = a^{3-3} = a^0 = 1.$$

Which is the same as:

$$\frac{a^3}{a^3} = \frac{a \times a \times a}{a \times a \times a} = 1$$

Therefore if x is any real number not equal to zero, then $X^0 = 1$,Note that $0^0$is undefined (not defined).

Fourth law: Negative integral exponents

For example, if we simplify the expression $\frac{9^3}{9^7}$ we obtain

$$\frac{9^3}{9^7} = \frac{9 \times 9 \times 9}{9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9} = \frac{1}{9^4} \text{ .... (i)}$$

It is the same as:

$$\frac{9^3}{9^3} = 9^{3-7} = 9^{-4} \text{ .... (ii)}$$

Equating (i) and (ii) since the expressions are identical, then

$$\frac{1}{9^4} = 9^{-4}$$

Another example, $\frac{d^4}{d^6}$, $d \neq 0$. We can simplify the expression as follows:

$$\frac{d^4}{d^6} = \frac{d \times d \times d \times d}{d \times d \times d \times d \times d \times d} = \frac{1}{d^2} \text{ .... (i)}$$

Also;

$\frac{d^4}{d^6} = d^{4-6} = d^{-2}$ .... (ii)

Equating (i) and (ii) because they are identical expressions, we obtain

$\frac{1}{d^2} = d^{-2}$

Generally, $\frac{1}{x^n} = x^{-n}$ if $x \neq 0$ and n is an integer. If n = 1, then $\frac{1}{x^1} = x^{-1}$. But $\frac{1}{x^1} = \frac{1}{x}$ and $\frac{1}{x}$ is called a reciprocal of x and is written as x^{-1}.

Example 10

Express the following powers using positive exponents:

a. a^{-7}

b. 2^{-3}

c. $\frac{1}{4^{-2}}$

c. 5^{-8}

d. $(\frac{1}{2})^{-4}$

Solution

a. $a^{-7} = \frac{1}{a^7}$

b. $2^{-3} = \frac{1}{2^3}$

c. $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 1 \div \frac{1}{4^2} = 1 \times \frac{4^2}{1} = 4^2$

d. $5^{-8} = \frac{1}{5^8}$

e. $(\frac{1}{2})^{-4} = \frac{1}{2^{-4}} = \frac{1}{\frac{1}{2^4}}$

$= 1 \div \frac{1}{2^4}$

$= 1 \times \frac{2^4}{1}$

$= 2^4$

Exercise 1

1. Indicate base and exponent in each of the following expressions:

a. 4^{670}

b. m^{x+y}

c. $(w+z)^n$

d. 990^{-72}

2. Write each of the following expressions in expanded form:

a. $(-7)^4$

b. $(\frac{1}{8})^6$

c. 12^{-3}

d. $(0.72)^{-4}$

3. Write in power form each of the following numbers by choosing the smallest base:

a. 169

b. 81

c.10,000

d. 625

a. 169 b. 81c. 10 000 d. 625

4. Write each of the following expressions using a single exponent:

a. $(12)^{15} \times (12)^{25}$

b. $10^5 \times 10^n \times 10^{-m}$

c. $y^6 \times y^8$

d. $(\frac{1}{6})^3 \times (\frac{1}{6}) \times (\frac{1}{6})^6$

5. Simplify the following expressions:

a. $28a^4b^6 \div 7a^{-3}b^{-4}$

b. $\frac{625r^4}{25r^2}$

c. $\frac{x^7y^9}{x^{-8}y^{-1}}$

d. $\frac{(3s^2)^6}{(3s)^4}$

6. Solve the following equations:

a. $3^m = 3^{2+5}$

b. $7^{n-2} = 7^8$

c. $(\frac{1}{9})^{-x} = 81$

d. $7056 = (2x)^2$

7. Express 64 as a power with:

1. Base 4

2. Base 8

3. Base 2

Base 4 Base 8 Base 2

8. Simplify the following expressions and give your answers in either zero or negative integral exponents.

a. $\frac{\pi r^2 h}{\pi r^3 r^2 h^3}$

b. $\frac{4b^2 \times b^3}{b^4 \times b}$

9. Give the product in each of the following:

a. 6^3

b. $(\frac{-1}{2})^4$

c. $5 \times 5 \times 5 \times 5 \times 5$

d. 2^5

10. Write the reciprocal of the following numbers:

a. 19

b. d where d is real

c. $\frac{1}{8}$

d. $(\frac{1}{9})^{-2}$

Laws of Exponents in Computations

Apply laws of exponents in computations

Example 11

a. $\frac{10^8}{10^5}$

b. $\frac{5a^6}{a^4}$

c. $\frac{81}{3^2}$

d. $\frac{((3)^2)^5}{3^4}$

e. $18b^4 \div (4b^3 - 2b^3)$

Solution

a. $\frac{10^8}{10^5} = 10^{8-5} = 10^3$

b. $\frac{5a^6}{a^4} = 5a^{6-4} = 5a^2$

c. $\frac{81}{3^2} = \frac{3^4}{3^2} = 3^{4-2} = 3^2$

d. $\frac{(3^2)^5}{3^4} = \frac{3^{2\times 5}}{3^4}$

$= \frac{3^{10}}{3^4} = 3^{10-4}$

$= 3^6$

e. $18b^4 \div (4b^3 - 2b^3)$

$= 18b^4 \div 2b^3$

$= \frac{18b^4}{2b^3}$

$= 9b^{4-3}$

$= 9b$

Radicals

Radicals are opposite of exponents. For example when we raise 2 by 2 we get 4 but taking square root of 4 we get 2. The same way we can raise the number using any number is the same way we can have the root of that number. For example, square root, Cube root, fourth root, fifth roots and so on. We can simplify radicals if the number has factor with root, but if the number has factors with no root then it is in its simplest form. In this chapter we are going to learn how to find the roots of the numbers and how to simplify radicals.

When a number is expressed as a product of equal factors, each of the factors is called the root of that number. For example,25 = 5× 5;so, 5 is a square root of 25: 64 = 8× 8; 8 is a square root of 64: 216 = 6 ×6 ×6, 6 is a cube root of 216: 81 = 3 × 3 ×3 ×3,3 is a fourth root of 81: 1024 = 4 ×4 ×4 ×4 ×4, 4 is a fifth root of 1024.

Therefore, the nth root of a number is one of the n equal factors of that number. The symbol for nth root is $\sqrt[n]{}$ where $\sqrt{}$ is called a radical and n is the index (indicates the root you have to find). If the index is 2, the symbol represents square root of a number and it is simply written as $\sqrt{}$ without the index 2. $\sqrt[n]{p}$ is expressed in power form as,

$(p)^{\frac{1}{n}}$. For example $\sqrt{4} = 4^{\frac{1}{2}}$: $\sqrt[4]{64} = 64^{\frac{1}{4}}$.

nth root of a number by prime factorization

Example 1, simplify the following radicals

a. $\sqrt{144}$
b. $\sqrt{225}$
c. $\sqrt[3]{1000}$
c. $\sqrt[5]{32}$
d. $\sqrt[3]{72}$

Soution

Write each number as a product of its prime factors

a. $\sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$

because we are required to find square root, group the factors into groups of 2 equal factors.

$= \sqrt{(2 \times 2) \times (2 \times 2) \times (3 \times 3)}$

for each group take only one of the factors

$= 2 \times 2 \times 3$

$= 12$

b. $\sqrt{225} = \sqrt{3 \times 3 \times 5 \times 5}$

group the factors into groups of two equal factors since we are required to find the square roo number.

$= \sqrt{(3 \times 3) \times (5 \times 5)}$

from each group, take one factor

$= 3 \times 5$

$= 15$

c. $\sqrt[3]{1000} = \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

group the factors into groups of 3 equal factors.

$= \sqrt[3]{(2 \times 2 \times 2) \times (5 \times 5 \times 5)}$

take one factor from each group

$= 2 \times 5$

$= 10$

d. $\sqrt[5]{32} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}$

group the factors into groups of 5 equal factors

$= \sqrt[5]{(2 \times 2 \times 2 \times 2 \times 2)}$

take one factor from each group

$= 2$

e. $\sqrt[3]{72} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3}$

group the factors into groups of 3 equal factors

$= \sqrt[3]{(2 \times 2 \times 2) \times 3 \times 3}$

$= 2\sqrt[3]{3 \times 3}$

$= 2\sqrt[3]{9}$

Example 2

a. $\sqrt{7} \times \sqrt{3}$

b. $3\sqrt[3]{2}$

Solution

a. $\sqrt{7} \times \sqrt{3} = \sqrt{7 \times 3}$

$= \sqrt{21}$

b. $3\sqrt[3]{2} = \sqrt[3]{3 \times 3 \times 3 \times 2}$

$= \sqrt[3]{54}$

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b} = \sqrt{ab}$ and $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

Basic Operations on Radicals

Perform basic operations on radicals

Different operations like addition, multiplication and division can be done on alike radicals as is done with algebraic terms.

Simplify the following:

a. $3\sqrt{5} + 7\sqrt{5}$

b. $4\sqrt{18} - 3\sqrt{8}$

c. $16 + \sqrt{121} - 2\sqrt[3]{2197}$

d. $2\sqrt{3} - \sqrt[3]{8} - 3.$

Solution

a. $3\sqrt{5} + 7\sqrt{5} = (3 + 7)\sqrt{5}$
$= 10\sqrt{5}$

b. $4\sqrt{18} - 3\sqrt{8} = 4\sqrt{2 \times 3 \times 3} - 3\sqrt{2 \times 2 \times 2}$
$= 3 \times 4\sqrt{2} - 2 \times 3\sqrt{2}$
$= 12\sqrt{2} - 6\sqrt{2}$
$= 6\sqrt{2}$

c. $16 + \sqrt{121} - 2\sqrt[3]{2197} = 16 + \sqrt{11 \times 11} - 2\sqrt[3]{13 \times 13 \times 13}$
$= 16 + 11 - (2 \times 13)$
$= 27 - 26$
$= 1$

Simplify each of the following expressions:

a. $\sqrt{3}(\sqrt{15} - \sqrt{13}) + \sqrt{5}$

b. $\sqrt{27} \times \sqrt{48}$

c. $\frac{\sqrt[3]{64}}{2\sqrt{25}}$

d. $\sqrt{\frac{49}{169}}$

e. $\frac{2\sqrt{3} \times 2\sqrt{5}}{\sqrt{12} \times 3\sqrt{15}}$

Solution

a. $\sqrt{3}(\sqrt{15} - \sqrt{13}) + \sqrt{5} = \sqrt{3} \times \sqrt{15} - \sqrt{3} \times \sqrt{13} + \sqrt{5}$
$= \sqrt{3 \times 15} - \sqrt{3 \times 13} + \sqrt{5}$
$= \sqrt{3 \times 3 \times 5} - \sqrt{39} + \sqrt{5}$
$= 3\sqrt{5} - \sqrt{39} + \sqrt{5}$
$= 4\sqrt{5} - \sqrt{39}$

b. $\sqrt{27} \times \sqrt{48} = \sqrt{27 \times 48}$
$= \sqrt{3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3}$

group the factors into groups of two equal factors and from each group take one of the factors.

$$= 3 \times 3 \times 2 \times 2$$
$$= 36$$

c. $$\frac{\sqrt[3]{64}}{2\sqrt{25}} = \frac{\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}}{2\sqrt{5 \times 5}}$$
$$= \frac{2 \times 2}{2 \times 5}$$
$$= \frac{2}{5}$$

d. $$\sqrt{\frac{49}{169}} = \sqrt{\frac{7 \times 7}{13 \times 13}}$$
$$= \frac{7}{13}$$

e. $$\frac{2\sqrt{3} \times 2\sqrt{5}}{\sqrt{12} \times 3\sqrt{15}} = \frac{2\sqrt{3} \times 2\sqrt{5}}{\sqrt{2 \times 2 \times 3} \times 3\sqrt{15}}$$
$$= \frac{2\sqrt{3} \times 2\sqrt{5}}{2\sqrt{3} \times 3\sqrt{15}}$$
$$= \frac{2\sqrt{5}}{3\sqrt{15}}$$

The Denominator

Rationalize the denominator

If you are given a fraction expression with radical value in the denominator and then you express the expression given in such a way that there are no radical values in the denominator, the process is called rationalization of the denominator.

Example 12

Rationalize the denominator of the following expressions:

c. $\frac{4}{\sqrt{24}} = \frac{4}{\sqrt{24}} \times \frac{\sqrt{24}}{\sqrt{24}}$ (multiply by $\sqrt{24}$ up and down)

$$= \frac{4\sqrt{2 \times 2 \times 2 \times 3}}{\sqrt{24 \times 24}}$$

$$= \frac{4 \times 2\sqrt{6}}{24}$$

$$= \frac{\sqrt{6}}{3}$$

Generally, if we rationalize $\frac{1}{\sqrt{a}}$, we simply multiply by $\frac{\sqrt{a}}{\sqrt{a}}$. That is:

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}}$$

Example 13

Rationalize the denominator for each of the following expressions:

a. $\frac{2}{\sqrt{3}+\sqrt{4}}$

b. $\frac{2-\sqrt{5}}{2\sqrt{6}-\sqrt{7}}$

c. $\frac{3\sqrt{7}}{\sqrt{7}+\sqrt{15}}$

d. $\frac{\sqrt{2}+4\sqrt{5}}{\sqrt{3}-\sqrt{15}}$

Solution

To rationalize these fractions, we have to multiply by the fraction that is equals to 1. The factor should be considered by referring the difference of two squares.

a. $\frac{2}{\sqrt{3}+\sqrt{6}}$ (our fraction have denominator $\sqrt{3}$ + $\sqrt{4}$, from the difference of two squares, we will take $\sqrt{3}$ - $\sqrt{4}$, therefore our rationalizing factor is $\frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}$)

thus,

$$\frac{2}{\sqrt{3}+\sqrt{6}} = \frac{2}{\sqrt{3}+\sqrt{6}} \times \frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}$$

$$= \frac{2(\sqrt{3}-6)}{(\sqrt{3}+\sqrt{6})(\sqrt{3}-\sqrt{6})}$$

$$= \frac{2\sqrt{3}-2\sqrt{6}}{(\sqrt{3})^2-\sqrt{3}\times\sqrt{6}+\sqrt{6}\times\sqrt{3}-(\sqrt{6})^2}$$

$$= \frac{2\sqrt{3}-2\sqrt{6}}{3-6}$$

$$= \frac{2\sqrt{3}-2\sqrt{6}}{-3}$$

$$= \frac{2(\sqrt{6}-\sqrt{3})}{3}$$

b. $\frac{2-\sqrt{5}}{2\sqrt{6}-\sqrt{7}}$ rationalizing factor is $2\sqrt{6}$ + $\sqrt{7}$

thus,

$$\frac{2-\sqrt{5}}{2\sqrt{6}-\sqrt{7}} = \frac{2-\sqrt{5}}{2\sqrt{6}-\sqrt{7}} \times \frac{2\sqrt{6}+\sqrt{7}}{2\sqrt{6}+\sqrt{7}}$$

$$= \frac{2(2\sqrt{6}+\sqrt{7})-\sqrt{5}(2\sqrt{6}+\sqrt{7})}{(2\sqrt{6}-\sqrt{7})(2\sqrt{6}+\sqrt{7})}$$

$$= \frac{4\sqrt{6}+2\sqrt{7}-2\sqrt{5\times6}-\sqrt{5\times7}}{(2\sqrt{6})^2-(\sqrt{7})^2}$$

$$= \frac{4\sqrt{6}+2\sqrt{7}-2\sqrt{30}-\sqrt{35}}{4\times6-7}$$

$$= \frac{4\sqrt{6}+2\sqrt{7}-2\sqrt{30}-\sqrt{35}}{17}$$

c. $\dfrac{3\sqrt{7}}{\sqrt{7}+\sqrt{15}}$, rationalizing factor is $\sqrt{7}$ - $\sqrt{15}$

thus,

$$\frac{3\sqrt{7}}{\sqrt{7}+\sqrt{15}} = \frac{3\sqrt{7}}{\sqrt{7}+\sqrt{15}} \times \frac{\sqrt{7}-\sqrt{15}}{\sqrt{7}-\sqrt{15}}$$

$$= \frac{3\sqrt{7 \times 7}-3\sqrt{7 \times 15}}{(\sqrt{7})^2-(\sqrt{15})^2}$$

$$= \frac{3 \times 7-3\times\sqrt{105}}{7-15}$$

$$= \frac{21-3\sqrt{105}}{-8}$$

$$= \frac{3\sqrt{105}-21}{8}$$

d. $\dfrac{\sqrt{2}+4\sqrt{5}}{\sqrt{3}-\sqrt{15}}$, rationalizing factor is $\sqrt{3}$ + $\sqrt{15}$

thus,

$$\frac{\sqrt{2}+4\sqrt{5}}{\sqrt{3}-\sqrt{15}} = \frac{\sqrt{2}+4\sqrt{5}}{\sqrt{3}-15} \times \frac{\sqrt{3}+\sqrt{15}}{\sqrt{3}+\sqrt{15}}$$

$$= \frac{\sqrt{2}(\sqrt{3}+\sqrt{15})+4\sqrt{5}(\sqrt{3}+\sqrt{15})}{(\sqrt{3}-\sqrt{15})(\sqrt{3}+\sqrt{15})}$$

$$= \frac{\sqrt{2 \times 3}+\sqrt{2 \times 15}+4\sqrt{5 \times 3}+4\sqrt{5 \times 15}}{(\sqrt{3})^2-(\sqrt{15})^2}$$

$$= \frac{\sqrt{6}+\sqrt{30}+4\sqrt{15}+4 \times 5\sqrt{3}}{3-15}$$

$$= \frac{\sqrt{6}+\sqrt{30}+4\sqrt{15}+20\sqrt{3}}{-12}$$

Exercise 2

1. Simplify each of the following by making the number inside the radical sign as small as possible:

a. $\sqrt{270}$
b. $\sqrt{98}$
c. $\sqrt{1008}$

2. Solve the equation: $\sqrt[5]{12} = 12^x$

3. Express each of the following under a single radical sign:

a. $\sqrt{12} \times 4\sqrt{38}$
b. $\sqrt{50} \times \sqrt{98}$

4. Write the following expressions without a radical sign:

a. $\sqrt[4]{16m^4}$
b. $\sqrt{32} \times \sqrt{8}$

5. Perform the following operations:

a. $2\sqrt{5}(\sqrt{40} \times \sqrt{25})$
b. $(3\sqrt{12})^2$
c. $3(12 - 7\sqrt{5})$
d. $6\sqrt{3} + \sqrt{3} - 2\sqrt{3}$
e. $\sqrt{a-b} + \sqrt{36a^2 - 0} - b$
f. $\dfrac{\sqrt[3]{64}}{\sqrt{81}}$

6. Rationalize the denominator in each of the following:

a. $\dfrac{2}{2\sqrt{a}+\sqrt{b}}$
b. $\dfrac{6+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$
c. $\dfrac{\sqrt{6}}{\sqrt{3}+2}$
d. $\dfrac{\sqrt{3}+\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$
e. $\dfrac{2\sqrt{7}-\sqrt{5}}{2\sqrt{5}-\sqrt{3}}$
f. $\dfrac{\sqrt{8}}{\sqrt{9}}$

Square Roots and Cube Roots of Numbers from Mathematical Tables

Read square roots and cube roots of numbers from mathematical tables

If you are to find a square root of a number by using Mathematical table, first estimate the square root by grouping method. We group a given number into groups of two numbers from right. For example; to find a square root of 196 from the table, first we have to group the digits in twos from right i.e. 1 96. Then estimate the square root of the number in a group on the extreme left. In our example it is 1. The square root of 1 is 1. Because we have two groups, this means that the answer has two digits before the decimal point. Our number is 196, read 1.9 in the table on the extreme left. From our number, we are remaining with 6, now look at the column labeled 6. Read the number where the row of 1.9 meet the column labeled 6. It meets at 1.400. Therefore the square root of 196 = 14.00 since we said that the answer must have2 digits before decimal points. Note: If you are given a number with digits more than 4 digits. First, round off the number to four significant figures and then group the digits in twos from right. For example; the number 75678 has five digits. When we round it off into 4 digits we get 75680 and then grouping into two digits we get 7 56 80. This shows that our answer has 3 digits. We start by estimating the square root of the number to the extreme left, which is 7, the square root of 7 is between 2 and 3. Using the table, along the row 7.5, look at the column labeled 6. Read the answer where the row 7.5 meets the column 6. Then go to where it is written mean difference and look at the column labeled 8, read the answer where it meets the row 7.5. Take the first answer you got where the row 7.5 met the column 6 and add with the second answer you got where the row 7.5 met the column 8 (mean difference column). The answer you get is the square root of 75680. Which is 275.1

Transposition of Formula

Re-arranging Letters so that One Letter is the Subject of the Formula

Re-arrange letters so that one letter is the subject of the formula

A formula is a rule which is used to calculate one quantity when other quantities are given. Examples of formulas are:

a. $A = \frac{1}{2}bh$

b. $P = 2(w + l)$

c. $V = \pi r^2 h$

d. $I = \frac{PRT}{100}$

A formula can be expressed in different ways. For example; $V = \pi r^2 h$ can be written in the ways: (i) $h = \frac{V}{\pi r^2}$ (ii) $\pi = \frac{V}{r^2 h}$ (iii) $r = \sqrt{\frac{V}{\pi h}}$. These ways of expressing the formula ways is called transposition of the formula or make one of the symbols a subject of the formu

Example 14

From the following formulas, make the indicated symbol a subject of the formula:

a. $V = \pi r^2 h$ (h)

b. $I = \frac{PRT}{100}$ (p)

c. $T = 2\pi\sqrt{\frac{l}{g}}$ (g)

d. $S = \frac{1}{2}at^2$ (a)

Solution

a. $V = \pi r^2 h$

divide by πr^2 both sides

$\frac{V}{\pi r^2} = h$

Therefore $h = \frac{V}{\pi r^2}$

b. $I = \frac{PRT}{100}$

$100I = PRT$ (multiply by 100 both sides)

$\frac{100I}{RT} = P$ (divide by RT both sides)

Therefore, $P = \frac{100I}{RT}$

c. $T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$ (divide by 2π both sides)

$\frac{T^2}{4\pi^2} = \frac{l}{g}$ (square both sides)

$T^2 g = 4\pi^2 l$ (cross multiplication)

$g = \frac{4\pi^2 l}{T^2}$ (divide by T^2 both sides)

Therefore, $g = \frac{4\pi^2 l}{T^2}$

Transposing a Formulae with Square Roots and Square

Transpose a formulae with square roots and square

Make the indicated symbol a subject of the formula:

$T = 2\pi\sqrt{\frac{l}{g}}$ (g)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}} \text{ (divide by } 2\pi \text{ both sides)}$$

$$\frac{T^2}{4\pi^2} = \frac{l}{g} \text{ (square both sides)}$$

$$T^2 g = 4\pi^2 l \text{ (cross multiplication)}$$

$$g = \frac{4\pi^2 l}{T^2} \text{ (divide by } T^2 \text{ both sides)}$$

Therefore, $g = \frac{4\pi^2 l}{T^2}$

Exercise 3

1. Change the following formulas by making the given letter as the subject of the formula.

a. $A = P + \frac{PRT}{100}$ (P)

b. $S = \frac{f}{gh}$ (h)

c. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (u)

d. $P = W(\frac{1+a}{1-a})$ (a)

2. Use mathematical tables to find square root of each of the following:

a. $\sqrt{441}$

b. $\sqrt{2025}$

c. $\sqrt{81.99}$

d. $\sqrt{0.0004}$

TOPIC 2: ALGEBRA

When we play games with computers we play by running, jumping and or finding secret things. Well, with Algebra we play with letters, numbers and symbols. And we also get to find secret things. Once we learn some of the 'tricks' it becomes a fun challenge to work with our skills in solving each puzzle. So, Algebra is all about solving puzzles. In this chapter we are going to learn some of the skills that help in solving mathematics puzzles.

Binary Operations

The Binary Operations

Describe the binary operations

When two numbers are combined according to the instructions given and produce one number we say that they are binary operations. For example, when we add 4 to 6 we get 10, or when we multiply 4 by 3 we get 12. We see that addition of two numbers lead to one number and multiplication of two numbers produce one number. This is binary operation. The instructions can be given either by symbols like X,*,∇and so on or by words.

Performing Binary Operations

Perform binary operations

Example 1

Evaluate:

a. 175×7

b. $(3 \times 205) + (12 \times 75)$

c. $(640 \times 25) - (2025 \div 15\ 595)$

solution

a. $175 \times 7 = 1\ 225$

b. $(3 \times 205) + (12 \times 75) = 615 + 900$
$= 1\ 515$

c. $(640 \times 25) - (2025 \div 5) = 16\ 000 - 405$
$= 15\ 595$

Example 2

Find

If $a * b = 4a + (2b - 3)$ find $5 * 4$.

solution

We are given that $a * b = 4a + (2b - 3)$

$$\text{Now, } 5 * 4 = 4(5) + (2(4) - 3)$$

$$= 20 + (8 - 3)$$

$$= 20 + 5$$

$$= 25$$

Example 3

Solve

If $r \Delta s = rs - \frac{3r}{s}$ find $12 \Delta 4$

Solution

We are given $r \Delta s = rs - \frac{3r}{s}$

$$\text{Now, } 12 \Delta 4 = 12(4) - \frac{3(12)}{4}$$

$$= 48 - 9$$

$$= 39$$

Example 4

evaluate,

If $x * y = \frac{625}{x} - 5y$, find $5 * (25 * 3)$

Solution

We are given $x * y = \frac{625}{x} - 5y$

now, $5 * (25 * 3) = 5 * (\frac{625}{25} - 5(3))$ (do operation inside the brackets first)

$$= 5 * 10$$

$$\text{so, } 5 * 10 = \frac{625}{5} - 5(10)$$

$$= 25 - 50$$

$$= -25$$

Example 5

Calculate

Given that n ∆ m $= \frac{3\sqrt{nm}}{\sqrt[3]{m}}$. Find 3 ∆ 27.

Solution

If $\quad n \Delta m = \frac{3\sqrt{nm}}{\sqrt[3]{m}}$

then, $3 \Delta 27 = \frac{3\sqrt{3 \times 27}}{\sqrt[3]{27}}$

$$= \frac{3\sqrt{3 \times 3 \times 3 \times 3}}{\sqrt[3]{3 \times 3 \times 3}}$$

$$= \frac{3 \times 3 \times 3}{3}$$

$$= 9$$

Brackets in Computation

Brackets are used to group items into brackets and these items inside the brackets are considered as whole. For example,**15 ÷(X + 2) ,**means that x and 2 are added first and their sum should divide 15. If we are given expression with mixed operations, the following order is used to perform the operations: Brackets (B) are opened (O) first followed by Division (D) then Multiplication (M), Addition (A) and lastly Subtraction (S). Shortly is written as BODMAS.

Basic Operations Involving Brackets

Perform basic operations involving brackets

Example 6

Simplify the following expressions:

1. $4 + 2b – (9b ÷3b)$

2. $4z – (2x + z)$

solution

1. $4 + 2b - (9b \div 3b) = 4 + 2b - 3b$ (do division first inside the brackets)

2. $4z - (2x + z) = 4z - 2x - z$ (since the terms inside the brackets are not alike, open the brackets by multiplying each term by the term outside the brackets i.e. - sign)

$= 4z - z - 2x$ (collect like terms)

$= 3z - 2x$

Algebraic Expressions Involving the Basic Operations and Brackets

Simplify algebraic expressions involving the basic operations and brackets

Example 7

Evaluate the following expressions:

1. $5x - 2y$ for $x = 4$ and $y = 5$
2. $(3y - 4) + 2x + 1$, for $x = 6$ and $y = 7$
3. $40 - \frac{3}{5}x$, for $x = 25$
4. $(z + 2y) \div x$, for $x = 7$, $y = 3$, $z = 15$

solution

1. $5x - 2y = 5(4) - 2(5)$ (we are given $x = 4$ and $y = 5$)
$= 20 - 10$
$= 10$ (subtract)
2. $(3y - 4) + 2x + 1 = (3(7) - 4) + 2(6) + 1$(we are given $x = 6$, $y = 7$)
$= (21 - 4) + 12 + 1$ (do multiplication first)
$= 17 + 12 + 1$ (do operation inside the bracket first)
$= 30$ (add)
3. $40 - \frac{3}{5}x = 40 - \frac{3}{5}(25)$ (we are given $x = 25$)
$= 40 - 3(5)$ (divide 25 by 5 to remove the fraction)
$= 40 - 15$ (multiply first)
$= 25$ (subtract)

d. $(z + 2y) \div x = (15 + 2(3)) \div 7$ (we are given $x = 7$, $y = 3$, $z = 15$)
$= (15 + 6) \div 7$ (first, multiply)
$= 21 \div 7$ (do operation inside the brackets first)
$= 3$ (divide)

Identities

For example, $3(2y + 3) = 6y + 9$, when $y = 1$, the right hand side (RHS) and the left hand side (LHS) are both equals to 15. If we substitute any other, we obtain the same value on both sides. Therefore the equations which are true for all values of the variables on both sides are called Identities. We can determine whether an equation is an identity or not by showing that an expression on one side is identical to the other expression on the other side.

Example 8

Determine whether or not the following expressions are identities:

1. $4(a - 2) + 5 = 4a - 3$
2. $3(x + 4) = 3x + 12$
3. $4(x + 2y - z) = 4x + 8y - 4z$
4. $2x - 10 = 0$

solution

1. We are given: 4 (a - 2) + 5 = 4a – 3

   consider the LHS

   4 (a - 2) + 5 = 4a – 8 + 5

   = 4a – 3

   therefore, since the LHS = RHS the expression is identity.

2. We are given: 3(x + 4) = 3x + 12

   consider the LHS

   3(x + 4) = 3x + 12 (multiply the terms inside the brackets each by 3)

   since LHS = RHS, then the expression is identity.

3. We are given: 4(x + 2y - z) = 4x + 8y – 4z

   consider the RHS

   4x + 8y – 4z = 4(x + 2y - z)(since 4 is a common factor we fact it out)

   RHS = LHS, therefore the expression is identity.

4. We are given: 2x -10 = 0

   the value of x = 5. We cannot substitute x = 5 to determine whether our expression is iden x = 5 is the solution to the expression. A good way to determine whether the expression is is to substitute another value of $x \neq 5$. If we substitute on x = 2 on LHS we obtain -6. But RHS is 0. So, $-6 \neq 0$. Therefore the expression is not identity

   Note the following common identities

   $(a+b)^2 = a^2 + 2ab + b^2$

   $(a-b)^2 = a^2 - 2ab + b^2$

   $a^2 - b^2 = (a + b)(a - b)$ (this expression is called difference of two squares.

   a and b are real numbers.

Quadratic Expressions

A Quadratic Expression from Two Linear Factors

Form a quadratic expression from two linear factors

A quadratic expression is an expression where the highest exponent of the variable (usually x) is a square (x^2). It is usually written as ax^2+bx+c.

Activity 1

Form a quadratic expression from two linear factors

The General Form of Quadratic Expression

Write the general form of quadratic expression

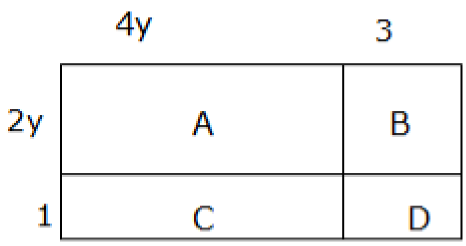
Quadratic expression has the general form of $ax^2 + bx + c$ where $a \neq 0$ and a is a coefficient of x^2 , b is a coefficient of x and c is a constant. its highest power of variable is 2. Examples of quadratic expressions are $2x^2 + x + 1$, $4y^2 + 3$, $3z^2 – 4z + 1$ and so on. In a quadratic expression $3z^2 - 4z + 1$, a = 3, b = -4 and c = 1. Also in quadratic expression $4y^2 + 3$, a = 4, b = 0 and c = 3

Example 9

If you are told to find the area of a rectangle with a length of 4y + 3 and a width of 2y + 1.

Solution

Take an example that the rectangle is like the one below:



The area of A = $4y \times 2y = 8y^2$

The area of B = $2y \times 3 = 6y$

The area of C = $4y \times 1 = 4y$

The area of D = $3 \times 1 = 3$

The total area = area of A + area of B + area of C + area of D

$= 8y^2 + 6y + 4y + 1$

$= 8y^2 +10y + 1$

Also we can find the area of a rectangle as follows:

Area = $(4y + 3)\ (2y + 1) = 4y(2y + 1) + 3(2y + 1)$ (each term in the first pair of brackets is multiplied by each term in the second pair of brackets)

$= 8y^2 + 4y + 6y + 3$

$= 8y^2 + 10y +3$

Alternatively

$$\begin{array}{r} 4y + 3 \\ \times\ 2y + 1 \\ \hline 8y^2 + 6y \qquad \\ 4y + 3 \\ \hline 8y^2 + 10y + 3 \\ \hline \end{array}$$

The expression $8y^2 + 10y + 3$ is called expanded form of $(4y + 3)(2y + 1)$. $(4y + 3)$ and $(2y + 1$
factors of $8y^2 + 10y + 3$.

Example 10

3x items were bought and each item costs (4x – 3) shillings. Find total amount of money used.

Solution

We have 3x items and each item costs (4x - 3) shillings.

Total cost $= 3x(4x - 3)$

$= 12x^2 – 9x$

Therefore the total cost used is $12x^2 – 9x$. This is the expanded form of $3x(4x - 3)$

Factorization

Linear Expressions

Factorize linear expressions

The operation of resolving a quantity into factors, when we expand expressions, is done by removing the brackets. The reverse operation is Factorizing and it is done by adding brackets.

Example 11

Factorize the expression 5a+5b.

Solution

In factorization of 5a+5b, we have to find out a common thing in both terms. We can see that the expression 5a+5b, have got common coefficient in both terms, that is 5. So factoring it out we get 5(a+b).

Example 12

Factorize 18xyz-24xwz

Solution

Factorizing 18xyz-24xwz, we have to find out highest common factor of both terms. Then factor it out, the answer will be 9xz(2y-3w).

Quadratic Expressions

Factorize quadratic expressions

When we write the quadratic expression as a product of two factors we say that we have factorized the expression. We are going to learn two methods used to factorize quadratic expressions. These methods are factorization by Splitting the middle term and factorization by Inspection.

Factorization by splitting the middle term

for example: $6y^2 + 11y + 4$, the term $11y$ can be written as $10y + y$ or $7y + 4y$ or $8y + 3y$ or 6y
order to choose correct terms, consider the coefficients and a constant term. Choose the term
product *ac* have the factors whose sum is *b*.

From our example, the product of *ac* is 24. We have to find the terms whose product is 24 an
the factors is 11. These terms are 8 and 3. The product of 8 and 3 is 24 and the sum of 8 and

Now, we can write $6y^2 + 11y + 4$ as $6y^2 + 3y + 8y + 4$

Which can be written as $3y(2y + 1) + 4(2y + 1)$

Take out common factor which is $2y + 1$ we get, $(2y + 1)(3y + 4)$.

Therefore the expression $6y^2 + 11y + 4$ has the factors which are $(2y + 1)(3y + 4)$.

Example 13

factorize $3x^2 - 2x - 8$ by splitting the middle term.

Solution

Our middle term is $-2x$ and the product of *ac* is -24. The terms whose product is -24 and the s
factors is -2 are 4 and -6.

$$\begin{aligned}3x^2 - 2x - 8 &= 3x^2 - 6x + 4x - 8\\ &= 3x(x - 2) + 4(x - 2)\\ &= (x - 2)(3x + 4) \text{ (take out common factors)}\end{aligned}$$

Example 14

factorize $x^2 + 10x + 25$ by splitting the middle term.

Solution

The middle term is $10x$ and the product of ac is 25. The terms whose product is 25 and the s factors is 10 is 5 and 5.

$x^2 + 10x + 25 = x^2 + 5x + 5x + 25$

$= x(x + 5) + 5(x + 5)$

$= (x + 5)(x + 5)$ (take out common factors)

= (the factors are identical)

If the quadratic expression has two identical factors is called perfect square. The general forı perfect square is the identity $(a + b)^2 = a^2 + 2ab + b^2$.

Factorization by Inspection

Example 15

factorize $x^2 + 3x + 2$ by inspection.

Solution

Factorization by inspection involves filling in the brackets as follows:

$x^2 + 3x + 2 = (\quad)(\quad)$ the first term which is is the product of x and x. And we can put one x bracket and the other x in the second bracket. This will be:

$x^2 + 3x + 2 = (x \quad)(x \quad)$ then we have to consider the last term which is 2. 2 is a product of las constant numbers. These numbers can be 1 and 2, or-1 and -2.

Therefore the possibilities to consider are:

$(x + 1)(x + 2) = \; + 3x + 2$ (this is the given expression)

$(x - 1)(x - 2) = \; - 3x + 2$

Note that, the coefficient of x on the RHS is the sum of the last numbers in each pair of brack

$3 = 1 + 2$; $-3 = -1 + (-2)$.

Therefore the required factors are $(x + 1)$ and $(x + 2)$ i. e. $(x + 1)(x + 2)$.

Example 16

factorize $4x^2 + 5x – 6$ by inspection.

Solution

Since the coefficient of is not 1 then let us find its factors. The first term is $4x^2$ which is the p
and 2x or 4x and x. The constant term is -6, its factors are 1 and -6; -1 and 6; 2 and -3; -2 and
need to use the factors of the first term and the factors of the last term to find the required exp
follows:

$(2x + 1)(2x - 6) = 4x^2 - 10x - 6$

$(2x - 1)(2x + 6) = 4x^2 + 10x - 6$

$(2x + 2)(2x - 3) = 4x^2 - 2x - 6$

$(2x - 2)(2x + 3) = 4x^2 + 2x - 6$

$(4x + 1)(x - 6) = 4x^2 - 23x - 6$

$(4x - 6)(x + 1) = 4x^2 - 2x - 6$

$(4x - 1)(x + 6) = 4x^2 + 23x - 6$

$(4x + 6)(x - 1) = 4x^2 + 2x - 6$

$(4x - 2)(x + 3) = 4x^2 + 10x - 6$

$(4x + 3)(x - 2) = 4x^2 - 5x - 6$

$(4x + 2)(x - 3) = 4x^2 - 10x - 6$

$(4x - 3)(x + 2) = 4x^2 + 5x - 6$ (required expression)

Therefore $4x^2 + 5x - 6 = (4x - 3)(x + 2)$.

Exercise 1

Factorization Exercise;

1. Simplify (4a - 6)(2a + 5) – (2a + 5)(4a - 3)
2. Show that $a^2 - b^2 \neq (a - b)^2$.
3. Find the factors of the following quadratic expression by (i) Inspection
   (ii) splitting the middle term

   a. $3x^2 - x - 10$

   b. $x^2 + \frac{4}{3}x + \frac{4}{9}$

   c. $x^2 + 3x + 3$
4. Determine whether

   (i) $(a + b)^2 = a^2 + b^2$

   (ii) $2x + 3 - 4y = 2(x - y) + 3$

   (ii) $(3x - 1)^2 = 9x^2 + 6x + 1$

   Are identities or not.
5. Which of the following expressions are perfect squares?

1. $4x^2 + 12x + 9$
2. $2x^2 + 9x + 6$
3. $16x^2 - 40x + 25$

TOPIC 3: QUADRATIC EQUATIONS

Solving quadratic equations can be difficult, but luckily there are several different methods that we can use depending on what type of quadratic that we are trying to solve. The four methods of solving a quadratic equation are factoring, using the square roots, completing the square and the quadratic formula.

Solving Equations

The standard form of a Quadratic equation is $\mathbf{ax^2 + bx + c = 0}$whereby a, b, c are known values and 'a' can't be 0. 'x' is a variable (we don't know it yet). a is the coefficient of x^2 , b is the coefficient of x and c is a constant term. Quadratic equation is also called an equation of degree 2 (because of the 2 on x). There are several methods which are used to find the value of x. These methods are:

1. by Factorization
2. by completing the square
3. by using quadratic formula

The Solution of a Quadratic Equation by Factorization

Determine the solution of a quadratic equation by factorization

We can use any of the methods of factorization we learnt in previous chapter. But for simplest we will factorize by splitting the middle term. For Example: solve for x, $x^2 + 4x = 0$

solution

Since the constant term is 0 we can take out x as a common factor.

So, $x^2 + 4x = x(x + 4) = 0$. This means the product of x and $(x + 4)$ is 0. Then, either $x = 0$ or $x + 4 = 0$. If $x + 4 = 0$ that is $x = -4$. Therefore the solution is $x = 0$ or $x = -4$.

Example 1

Solve the equation: $3x^2 =- 6x – 3$.

first rearrange the equation in its usual form.

that is:

$3x^2 = - 6x – 3$

$3x^2 + 6x + 3 = 0$

now, factorize the equation by splitting the middle term. Let us find two numbers whose product is 9 and their sum is 6. The numbers are 3 and 3. Hence the equation $3x^2 + 6x + 3 = 0$ can be written as:

$3x^2 + 3x + 3x + 3 = 0$

$3x(x + 1) + 3(x + 1) = 0$

$(3x + 3)(x + 1)$ (take out common factor which is $(x + 1)$)

either $(3x + 3) = 0$ or $(x + 1) = 0$

therefore $3x = -3$ or $x = -1$

$x = -1$ (divide by 3 both sides) or $x = -1$

Therefore, since the values of x are identical then $x = -1$.

Example 2

solve the equation $10 – 3y - 1 = 0$ by factorization.

Solution

Two numbers whose product is -10 and their sum is -3 are 2 and -5.

Then, we can write the equation $10y^2 – 3y - 1 = 0$ as:

$2y(5y + 1) – 1(5y + 1) = 0$

$(2y – 1)(5y + 1) = 0$

Therefore, either $2y – 1 = 0$ or $5y + 1 = 0$

$y = \frac{1}{2}$ (divide by 2 both sides) or $y = -\frac{1}{5}$ (divide by 5 both sides).

$y = \frac{1}{2}$ or $y = -\frac{1}{5}$

Example 3

solve the following quadratic equation by factorization: $4x^2 - 20x + 25 = 0$.

Solution

We need to split the middle term by the two numbers whose product is 100 and their sum is -20. The numbers are -10 and -10.

The equation can be written as:

$4x^2 -10x - 10x + 25 = 0$

$2x(2x - 5) – 5(2x -5) = 0$

$(2x-5)(2x-5)$ (take out common factor. The resulting factors are identical. This is a perfect square)

since it is a perfect square, then we take one factor and equate it to 0. That is:

$2x - 5 = 0$

$2x = 5$ then, divide by 2 both sides.

Therefore

$$x = \frac{5}{2}$$

Example 4

solve the equation $x^2 - 16 = 0$.

Solution

We can write the equation as $x^2 - 4^2 = 0$. This is a difference of two squares. The difference of two squares is an identity of the form:

$a^2 - b^2 = (a - b)(a + b)$.

So, $x^2 - 4^2 = (x - 4)(x + 4) = 0$

Now, either $x - 4 = 0$ or $x + 4 = 0$

Therefore $x = 4$ or $x = -4$

The Solution of a Quadratic Equation by Completing the Square

Find the solution of a quadratic equation by completing the square

Completing the square.

Completing the Square is where you take a Quadratic Equation like: $ax^2 + bx + c = 0$ and tur

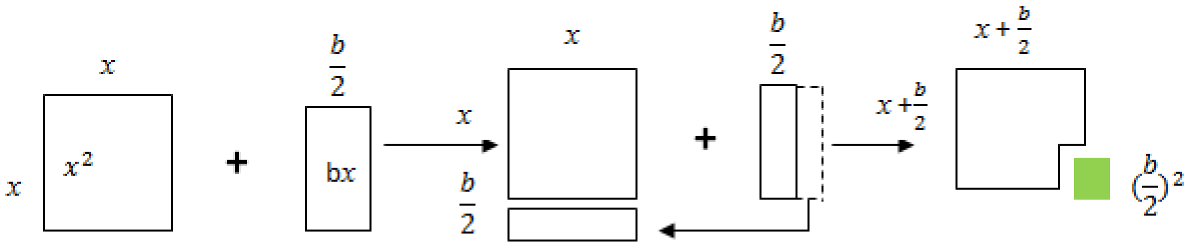
$+ d)^2 + e = 0$ whereby $d = \frac{b}{2a}$ and $e = c - \frac{b^2}{4a}$

How to complete the square

If we have a simple expression like $x^2 + bx$ having x twice in the same expression can make l

What can we do?

See an illustration below:



As you can see x^2 + bx can be rearranged nearly into a square and we can complete the sq

$(\frac{b}{2})^2$

In Algebra it looks like this:

$$x^2 + bx \quad + (\frac{b}{2})^2 = (x + \frac{b}{2})^2$$

So, by adding $(\frac{b}{2})^2$ we can complete the square

And $(x + \frac{b}{2})^2$ has x only once, which is easier to solve.

A general Quadratic Equation can have a coefficient of 'a' in front of x^2. i.e. ax^2 + bx + c = 0.

complete the square?

Step 1: divide all term by a (coefficient of x^2)

i.e. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Step 2: move the number term $(\frac{c}{a})$ to the right side of the equation.

i.e. $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step 3: complete the square on the left side of the equation and balance this by adding the s

the right side of the equation.

i.e. $x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$

which is the same as:

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

Step 4: Take the square root on both sides of the equation

i.e. $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Step 5: Subtract the number that remains on the left side of the equation to find x

i.e. $x = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a}$

$$x = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 5

Add a term that will make the following expression a perfect square: $x^2 - 8x$

Since the coefficient of x^2 is 1, then will add a constant term $(\frac{b}{2})^2$,

where by b = -8

Therefore the term to be added is: $(\frac{b}{2})^2 = (\frac{8}{2})^2 = 4^2 = 16$

Our expression will be $x^2 - 8x + 16$

find a term that must be added to make the following expression a perfect square: $x^2 + 10x$

Solution

The coefficient of x^2 is1, so we will add the term $(\frac{b}{2})^2$, whereby b = 10

Therefore, the term to be added is: $(\frac{b}{2})^2 = (\frac{10}{2})^2 = 25$.

Our expression will be $x^2 + 10x + 25$.

Example 6

solve the following quadratic equation by completing the square: $x^2 + 4x + 1 = 0$

Solution

Step 1: the step can be skipped since the coefficient of x^2 is 1

Step 2: move the number term to the right

$x^2 + 4x = -1$

Step 3: complete the square on the left side of the equation and balance this by adding the same number on the right of the equation.

$(\frac{b}{2})^2 = (\frac{4}{2})^2 = 4$

$x^2 + 4x + 4 = -1 + 4$

$(x + 2)^2 = 3$

Step 4: take the square root on both sides of the equation

$x + 2 = \pm\sqrt{3}$

Step 5: subtract 2 both sides

$x = \pm\sqrt{3} - 2$

$x = -0.268$ or $x = -3.732$

Example 7

solve by completing the square: $3x^2 + 7x - 6 = 0$

Solution

Step 1: divide each term by 3

$x^2 + \frac{7}{3}x - 2 = 0$

Step 2: move the number term to the right

$x^2 + \frac{7}{3}x = 2$

Step 3: complete the square on the left side of the equation and add the same number to the equation to balance this.

$(\frac{b}{2})^2 = (\frac{7}{2 \times 3})^2 = \frac{49}{36}$

$x^2 + \frac{7}{3}x + \frac{49}{36} = 2 + \frac{49}{36}$

$(x + \frac{7}{2})^2 = \frac{121}{36}$

Step 4: take square root both sides

$$x + \frac{7}{2} = \pm\sqrt{\frac{121}{36}}$$

$$x + \frac{7}{2} = \pm\frac{11}{6}$$

Step 5: subtract $\frac{7}{2}$ from both sides

$$x = \pm\frac{11}{6} - \frac{7}{2}$$

$$x = -\frac{5}{3} \text{ or } x = -\frac{16}{3}$$

General Solution of Quadratic Equations

The Quadratic Formula

Derive the quadratic formula

The special quadratic formula used for solving quadratic equation is:

$$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}.$$

Just plug in values of a, b, and c and then do calculations.

The symbol $\pm$ means there are two answers, which are:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

But sometimes we don't get two answers, and the discriminate tells why? The term $b^2 - 4ac$ Discriminant because It can discriminate between the possible types of answers.

- When $b^2 - 4ac$ is positive we get two real solutions.
- When $b^2 - 4ac$ is zero we One real solution (both answers are the same)
- When $b^2 - 4ac$ is negative we get two complex solutions (not real solutions). We are not learn about this, is not in our level.

Quadratic Equations using Quadratic Formula

Solve quadratic equations using quadratic formula

Example 8

solve $5x^2 - 8x + 3 = 0$ by using quadratic formula.

Solution

Recall quadratic formula: $x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$

from the question; a = 5, b = -3 and c = 3

then $x = \frac{-(-8) \pm\sqrt{(-8)^2 - 4 \times 5 \times 3}}{2 \times 5}$

$$x = \frac{3 \pm\sqrt{64-60}}{10}$$

$$x = \frac{3 \pm\sqrt{4}}{10}$$

$$x = \frac{3 \pm 2}{10}$$

either $x = \frac{3+2}{10}$ or $x = \frac{3-2}{10}$

Therefore $x = \frac{1}{2}$ or $x = \frac{1}{10}$

Example 9

solve this quadratic equation by using quadratic formula: $3x^2 = - 7x - 4$

Solution

First, rearrange the equation, move all the terms on the right side to the left side of the equat

$3x^2 + 7x + 4 = 0$

now, recall the quadratic formula: $x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$

from our equation; a = 3, b = 7, and c = 4

then plug in the values of a, b, and c and do calculations

$$x = \frac{-7 \pm\sqrt{7^2 - 4 \times 3 \times 4}}{2 \times 3}$$

$$x = \frac{-7 \pm\sqrt{49 - 48}}{6}$$

$$x = \frac{-7 \pm\sqrt{1}}{6}$$

$$x = \frac{-7 \pm 1}{6}$$

either $x = \frac{-7+1}{6}$ or $x = \frac{-7-1}{6}$

Therefore, x = -1 or $x = -\frac{4}{3}$

Word problems leading to quadratic equations

Given a word problem; the following steps are to be used to recognize the type of equation.

Step1: choose the variables to represent the information

Step 2: formulate the equation according to the information given

Step 3: solve the equation by using any of the method you know

In order to be sure with your answers, check if the solution you obtained is correct.

Example 10

the length of a rectangular plot is 8 centimeters more than the width. If the area of a plot is $240cm^2$, find the dimensions of length and width.

Solution

Let the width be x

The length of a plot is 8 more than the width, so the length of a plot be x + 8

We are given the area of a plot = 240cm^2 and the area of a rectangle is given by length ×width

then $(x + 8) \times x = 240$

$x^2 + 8x = 240$

rearrange the equation

$x^2 + 8x - 240 = 0$

then solve the equation to find the value of x

Solving by splitting the middle term, two numbers whose product is -240 and their sum is 8, the number

are -12 and 20

our equation becomes; $x^2 + 20x - 12x - 240 = 0$

$x(x + 20) - 12(x + 20) = 0$

either $(x - 12) = 0$ or $(x + 20) = 0$

$x = 12$ or $x = -20$

since we don't have negative dimensions, then the width is 12cm and the length is 12 + 8 = 20cm

Therefore the rectangular plot has the length of 20cm and the width of 12cm.

Example 11

A piece of wire 40cm long is cut into two parts and each part is then bent into a square. If the sum of the areas of these squares is 68 square centimeters, find the lengths of the two pieces of wire.

Solution

We don't know the lengths of the two pieces, so let one of the length be *l*.

then the length of the other piece of wire will be 40 - *l*

each piece of wire formed a square. A square has four equal lengths.

A piece of wire with a length of *l*cm will form a square having length of $\frac{l}{4}$ cm each side and it

be length $\times$ length = $(\frac{l}{4})^2$

A piece of wire with a length of (40 –*l*)cm will form a square having length of $(\frac{40-l}{4})$cm each

area will be length $\times$ length = $(\frac{40-l}{4})^2$

the sum of the areas = $(\frac{l}{4})^2 + (\frac{40-l}{4})^2$

$$68 = \left(\frac{l}{4}\right)^2 + \left(\frac{40-l}{4}\right)^2$$

$$68 = \frac{l^2}{16} + \frac{1600-80l+l^2}{16}$$

$$68 = \frac{l^2+1600-80l+l^2}{16}$$

$68 \times 16 = 2l^2 - 80l + 1600$ (multiply by 16 both sides)

$2l^2 - 80l + 1600 = 1088$

$2l^2 - 80l + 512 = 0$ (subtract 1088 both sides)

$l^2 - 40l + 256 = 0$ (divide by 2 throughout)

solve the equation to find values of l

by splitting the middle term

$l^2 - 8l - 32l + 256 = 0$

$l(l - 8) - 32(l - 8) = 0$

$(l - 32)(l - 8) = 0$

either $(l - 32) = 0$ or $(l - 8) = 0$

$l = 32$ or $l = 8$

Therefore the two pieces have the length of 8cm and 32cm

Exercise 1

1. Solve each of the following quadratic equations by using factorization method:

1. $-6x^2 + 23x - 20 = 0$

2. $X^2 - x - 12 = 0$

2. Solve these equations by completing the square:

1. X^2 -11 – 3 = 0
2. $X^2 - \frac{10}{9}x - \frac{2}{9} = 0$

3. What must be added to the following expression to make them complete squares?

1. $X^2 + 10x$
2. $X^2 - \frac{4}{5}x$

4. Use general formula for quadratic equations to solve the following quadratic equations:

1. $2x^2 - 7x + 3 = 0$
2. $4x^2 + 8x + 4 = 0$

5. Find two consecutive odd numbers whose product is 255.

TOPIC 4: LOGARITHMS

We always ask ourselves, how many of one number do we multiply to get another number? For example; how many 3s do we multiply to get 81? All these kind of questions will be answered in this unit. Make sure you understand. Start reading now……!

Standard Form

Standard Form is also called Scientific Notation. It is a way of writing a number into two parts. For example

Digits　　　Power of 10

$532.5 = 5.325 \times 10^2$

A Number　　　In Scientific Notation

• The **digits** (with the decimal point placed after the first digit) followed by

• **X 10 to a power** that puts a decimal point where it should be (i.e. it shows how many places to move the decimal point).

Numbers in Standard Form

Write numbers in standard form

How to write a number in standard form?

To figure out the power of 10, think of how many decimal places to move:

• When the number is 10 or greater, the decimal place has to move to the left and the power of 10 will be positive. For example; 47 055 = 4. 7055 x 10^4

• When the number is smaller than 1, the decimal point has to move to the right and the power of 10 will be negative. For example;

0.00025 will be written as 2.5×10^{-4}

For example; 4.5 would be written as 4.5×10^{0} we didn’t have to move the decimal point, so the power is 10^{0}. But now it is in standard form.

Note that: After putting the number in scientific notation make sure that the digits part is between 1 and 10 (it can be 1 but never 10). And the power part shows exactly how many places to move the decimal point.

Computations which Involved Multiplication and Division of Numbers Expressed in Standard Form

Perform computations which involved multiplication and division of numbers expressed in standard form

Definition of a logarithm

A logarithm answers the question: How many of one number do we multiply to get another number. For example; how many of 2s do we multiply to get 16? Answer: 2 x 2 x 2 x 2= 16 so we needed to multiply 4 of the 2s to get 16. So the logarithm is 4.

How to write it?

We would write the number of 2s we need to multiply to get 16 is 4 as:

The two things are the same:

$$\underbrace{2 \times 2 \times 2 \times 2}_{4} = 16 \longleftrightarrow \log_{\underset{\text{base}}{2}}(16) = 4$$

The number we are multiplying is called the base. So we can say ‘the logarithm of 16 with base 2 is 4’ or ‘log base 2 of 16 is 4’ or 'the base-2 log of 16 is 4’.

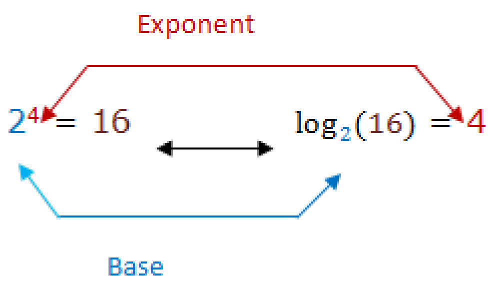
Not that we are dealing with 3 numbers:

• The base (the number we are multiplying in our example it is 2)

• How many times to use it in multiplication (in our example it is 4 times, which is the logarithm)

• The number we want to get (in our example it is 16)

There is a relationship between the exponents and logarithms. The exponent says how many times to use the number in a multiplication and logarithm tells you what the exponent is. See the illustration below:



Generally: $a^x = y$ in logarithmic form is: $\mathbf{Log_a Y = X}$

Example 1

write the following statements in logarithmic form:

1. $10^0 = 1$
2. $3^{-4} = \frac{1}{81}$

Solution

1. $10^0 = 1$ in logarithmic form is $\log_{10} 1 = 0$
2. $3^{-4} =$ in logarithmic form is $\log_3 \frac{1}{81} = -4$

Laws Of Logarithms

The Laws of Logarithms

State the laws of logarithms

There are several laws of logarithms which help in evaluating them. These laws are valid for only positive real numbers. The laws are as follows:

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- $\log_a x^y = y \log_a x$
- $\log_a \sqrt[n]{x} = \log_a x^{\frac{1}{n}} = \frac{1}{n}\log_a x$, n is a positive integer
- $\log_a a^x = x$ for every real number
- $a^{\log_a x} = x$ for $x > 0$
- $\log_x x = 1$

Note that $\log 1 = 0$

Verification of the Laws of Logarithms Using the Knowledge of Exponents

Verify the laws of logarithms using the knowledge of exponents

Activity 1

Verify the laws of logarithms using the knowledge of exponents

The Laws of Logarithms to Simplify Logarithmic Expressions

Use the laws of Logarithms to simplify logarithmic expressions

Example 2

Use the laws of logarithms to evaluate the following:

a. $\log_4 32 + \log_4 2$

b. $\log_3 270 - \log_3 10$

c. $\log_5 \frac{1}{\sqrt[3]{5}}$

d. $\log_2 12 + \log_2 3 - \log_2 9$

e. $\log_3 \sqrt[4]{\frac{1}{81}}$

Solution

a. $\log_4 32 + \log_4 2 = \log_4(32 \times 2)$

$= \log_4 64$

$= \log_4 4^3$

$= 3\log_4 4$

$= 3 \cdot 1$

$= 3$

b. $\log_3 270 - \log_3 10 = \log_3 \frac{270}{10}$

$= \log_3 27$

$= \log_3 3^3$

$= 3\log_3 3$

$= 3 \cdot 1$

$= 3$

c. $\log_5 \frac{1}{\sqrt[3]{5}} = \log_5 (\sqrt[3]{5})^{-1}$

$= \log_5 (5)^{\frac{-1}{3}}$

$= \frac{-1}{3} \log_5 5$

$= \frac{-1}{3} \cdot 1$

$= \frac{-1}{3}$

d. $\log_2 12 + \log_2 3 - \log_2 9 = \log_2 (\frac{12 \times 3}{9})$

$= \log_2 4$

$= \log_2 2^2$

$= 2\log_2 2$

$= 2 \cdot 1$

$= 2$

e. $\log_3 \sqrt[4]{\frac{1}{81}} = \log_3 (\frac{1}{81})^{\frac{1}{4}}$

$= \log_3 (81^{-1})^{\frac{1}{4}}$

$= \log_3 (81)^{\frac{-1}{4}}$

$= \log_3 (3^4)^{\frac{-1}{4}}$

$= \log_3 (3)^{\frac{4 \times -1}{4}}$

$= \log_3 3^{-1}$

$= -1\log_3 3$

$= -1 \cdot 1$

$= -1$

Change of base

This is a formula for change of base. For any positive *a*, *b* (*a*, *b* ≠0) we have

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Given,

$$\log_4 8 = \frac{\log 8}{\log 4}$$

then find a number which is a common base to both 8 and 4

$$= \frac{\log_2 2^3}{\log_2 2^2}$$

$$= \frac{3 \log_2 2}{2 \log_2 2}$$

$$= \frac{3 \times 1}{2 \times 1}$$

$$= \frac{3}{2}$$

Example 3

Solve.

$$\log_9 \frac{1}{\sqrt{27}} = \log_9 (27)^{-\frac{1}{2}}$$

$$= \frac{-1}{2} \log_9 27$$

$$= \frac{-1}{2} \left(\frac{\log_3 3^3}{\log_3 3^2}\right)$$

$$= \frac{-1}{2} \left(\frac{3 \log_3 3}{2 \log_3 3}\right)$$

$$= \frac{-1}{2} \times \frac{3}{2}$$

$$= \frac{-3}{4}$$

Logarithms to Base 10

Sometimes you will see a logarithm written without a base like this: $\log(100)$. This usually m
the base is really 10. It is called a "common logarithm". It is how many times you need to use
Multiplication, to get a desired number. For example $\log(1000) = \log_{10}(10)^3 = 3\log_{10} 10 =$
need to use 10s 3 times to get 1000.

Also, we can express a number in scientific notation and then use the laws of logarithms to fi
logarithm. For example let us find $\log 120$.

$\log 120 = \log(100 \times 1.2) = \log_{10} 10^2 + \log_{10} 1.2 = 2 + \log 1.2$, read the logarithm of 1.2 fro
logarithmic table, it is 0.0792. The answer will be 2 + 0.0792 = 2.0792. The fraction part (i.e. (
the common logarithm of 120 is called mantissa while the integer (i.e. 2) part is called charac
For those numbers less than 1, we can also find their logarithms by expressing them in stand
and then use the laws of exponents to evaluate the logarithm. For example;

$\log_{10} 0.0012 = \log_{10}(10^{-3} \times 1.2) = -3\log_{10} 10 + \log_{10} 1.2$ = -3 + 0.0792 = -2.9208.

To avoid the need for separate to convert positive and negative logarithms back to their origin
a bar notation is used: therefore $\log_{10} 0.0012$ = -3 + 0.0792 = $\bar{3}.0792$ (3 is read as bar three
mantissa is 0.0792 (mantissa remains positive) and the characteristic is -3 (the bar above ch
indicates that it is negative).

Logarithmic Equation

Solve logarithmic equation

Use the laws of logarithms to evaluate the following:

a. $\log_4 32 + \log_4 2$

b. $\log_3 270 - \log_3 10$

c. $\log_5 \frac{1}{\sqrt[3]{5}}$

d. $\log_2 12 + \log_2 3 - \log_2 9$

e. $\log_3 \sqrt[4]{\frac{1}{81}}$

Solution

a. $\log_4 32 + \log_4 2 = \log_4(32 \times 2)$

$= \log_4 64$

$= \log_4 4^3$

$= 3\log_4 4$

$= 3 \cdot 1$

$= 3$

b. $\log_3 270 - \log_3 10 = \log_3 \frac{270}{10}$

$= \log_3 27$

$= \log_3 3^3$

$= 3\log_3 3$

$= 3 \cdot 1$

$= 3$

c. $\log_5 \frac{1}{\sqrt[3]{5}} = \log_5 (\sqrt[3]{5})^{-1}$

$= \log_5 (5)^{\frac{-1}{3}}$

$= \frac{-1}{3} \log_5 5$

$= \frac{-1}{3} \cdot 1$

$= \frac{-1}{3}$

d. $\log_2 12 + \log_2 3 - \log_2 9 = \log_2 (\frac{12 \times 3}{9})$

$= \log_2 4$

$= \log_2 2^2$

$= 2\log_2 2$

$= 2 \cdot 1$

$= 2$

e. $\log_3 \sqrt[4]{\frac{1}{81}} = \log_3 (\frac{1}{81})^{\frac{1}{4}}$

$= \log_3 (81^{-1})^{\frac{1}{4}}$

$= \log_3 (81)^{\frac{-1}{4}}$

$= \log_3 (3^4)^{\frac{-1}{4}}$

$= \log_3 (3)^{\frac{4 \times -1}{4}}$

$= \log_3 3^{-1}$

$= -1\log_3 3$

$= -1 \cdot 1$

$= -1$

Laws of Logarithms to Find Products, Quotients, Roots and Powers of Numbers

Apply laws of logarithms to find products, quotients, roots and powers of numbers

Here we deal with all 4 operations which are addition, subtraction, multiplication and division. All operations are just as usual operations except division when we are given a negative characteristic. For example;

$\bar{1}.8996 \div 2$. When you look at characteristic is not divisible by 2. To make it divisible by 2, i characteristic by the amount that will make it divisible by given divisor. In our example we hav increase our characteristic which is $\bar{1}$ by $\bar{1}$, which gives total of $\bar{2}$. $\bar{2}$ is now divisible by 2. S may not get out of my question, I have to add 1 to mantissa. Our question will look like this:

$$\frac{\bar{1}+\bar{1}+1+0.8996}{2} = \frac{\bar{2}+1.8996}{2} = \frac{\bar{2}}{2} + \frac{1.8996}{2} = \bar{1} + 0.9498 = \bar{1}.9498.$$

Example 4

evaluate the following:

a.

$$\begin{array}{r} \bar{2}.6173 \\ -1.4291 \\ \hline \bar{3}.0464 \\ \hline \end{array}$$

(remember that characteristic is negative)

b.

$$\begin{array}{r} \bar{5}.8888 \\ \times \quad 3 \\ \hline \overline{13}.6664 \\ \hline \end{array}$$

(remember that characteristic is negative, when you add 2 you were caring you left with -13)

Logarithmic Tables to Find Products and Quotients of Numbers Computation

Apply logarithmic tables to find products and quotients of numbers computation

Most of the logarithmic tables are of base 10 (common logarithms). When we want to read a logarithm of a number from logarithmic table, we first check if the number is between 0 and 10 (but not 0 or 10) because the table consists only of logarithms of numbers between 0 and 1.

For example; what is the logarithm of 5.25 from the table. Our number is between 0 and 1. We look at the most left column and find where 52 is (we ignore the decimal point). Then slide your finger along this row to the right to find column of the next digit in our example is 5. Read the number where the row of 52 meets the column of 5. The logarithm of 5.25 is 0.7202.

If the number has 4 digits like 15.27, we do the following. First of all, checking our number we see that it is greater than 10. The number is between 10 and 100. And we know that the logarithm of 10 is 1 and logarithm of 100 is 2. So logarithm of 15.27 is between 1 and 2, normally less than 2 but greater than 1, hence 1.something. That something we need to find it in a logarithm table. Look at the most left column the row labeled 15, then, slide your finger to the right to find the column labeled 2. Read the number where the row of 15 meets the column of 2, the number is

0.1818. We are remaining with one digit which is 7. If your log table has a part with mean difference table, slide your finger over to the column in that table marked with the next digit of the number you are looking up, in our example it is 7. Slide over to row 15 and mean difference 7. The row of 15 meets mean difference column 7 at number 20. Add the two numbers obtained (the mean difference number is added to the last digits of our first number we obtained) i.e. 0.1818 + 20 = 1838. Now add characteristic which is 1 since 15.27 is between 10 and 100. We get 1 +0.1838 = 1.1838. Therefore**Log 15.27= 1.1838**.

Note that if you are given a number with more than 4 digits, first round off the number to 4 digits and then go on with similar procedures as explained in examples above.

To find the number whose logarithm is known, we can call it ant-logarithm the same logarithmic table can be used. For example to find the number whose logarithm is 0.7597, look at the central part of the log table find the number (mantissa) 7597. This is in the intersection of the row labeled 57 and column 5. So the number is 575. But in order to get correct answer we have to consider characteristic of our logarithm which is 0. This means our number is between 0 and 10 because the numbers whose logarithms are 0.something are between 0 and 10. Hence, we need to place one decimal point from left to our number to make it be between 0 and 10. Therefore the number will be 5.75 i.e. log 5.75 = 0.7597, thus antilog 0.7597= 5.75.

How to find the ant-log

Step1: Understand the ant-log table. Use it when you have log of a number but not the number itself. the ant-log is also known as the inverse log.

Step 2: Write down the characteristic. This is the number before decimal point. If you are looking up the ant-log of 2.8699, the characteristic is 2. Remove it from the number you are looking up. But never forget it because it will be used later. So it is better if you write it somewhere.

Step 3: Find the row in the most left column that matches the first two numbers of the mantissa. Our mantissa is 8699. So run your finger down that column until you find .86.

Step 4: Slide your finger over to the column marked with the next digit of the mantissa. For 2.8699, slide your finger along the row marked .86 to find the intersection with column 9. This reads 7396. Write this down.

Step 5: If your ant-log table has a table of mean difference, slide your finger over to the column in that table marked with the next digit of the mantissa. Make sure to keep your finger in the same row. Considering our example, slide your finger over the to the last column in the table, column 9. The intersection of row .86 and mean difference column 9 is 15. Write it down.

Step 6: Add the two numbers obtained from the two previous steps. In our example, these are 7396 and 15. Adding them i.e. 7396 + 15 = 7411.

Step 7: Use characteristic to place decimal point. Our characteristic is 2, which means our answer is between 100 and 1000 because log 100 = 2 and log 1000 = 3. For the number 7411 to fall between 100 and 1000, the decimal point should be placed after 3 digits. So, the final answer is 741.1 therefore the ant-log of 2.8699 is 741.1.

Example 5

Find the product of 25.75 ×450.

Solution;

From Logarithmic laws we saw that multiplication of two numbers is the same as addition of two the same two numbers. How to do it?

Let x = 25.75 ×450

log x = log (25.75 ×450)

which is the same as

now, you can read log of your numbers from the logarithmic table as we learnt in the previous lesson, you will find:

log 25.75 = 1.4108 and log 450 = 2.6532

thus, log x = 1.4108 + 2.6532

log x = 4.0640

in order to obtain the value of x we have to find the inverse log of 4.0640 or ant-log of 4.0640

so, x = ant-log 4.0640

x = 11590

therefore, 25.75 ×450 = 11590

always the logarithmic calculations are set out in tabular form to make the solution not too long as above.

If we set our example in tabular form it will look like this:

Number	**log**
25.57 = 2.5575 $\times 10^1$	1.4108
450 = 4.50 $\times 10^2$	+ 2.6532
1.159 = 11590	4.0640

Logarithmic Tables to Find Roots and Power of Numbers

Apply logarithmic tables to find roots and power of numbers

Example 6

Calculate using logarithms

$$\sqrt[3]{\frac{318 \times 4344}{17200}}$$

Solution

$$\sqrt[3]{\frac{318 \times 4344}{17200}} = \left(\frac{318 \times 4344}{17200}\right)^{\frac{1}{3}}$$

Let x = $\left(\frac{318 \times 4344}{17200}\right)^{\frac{1}{3}}$

introduce log both sides

Log x = log $\left(\frac{318 \times 4344}{17200}\right)^{\frac{1}{3}}$

log x = $\frac{1}{3}$((log 318 + log 4344) – log 17200)

x = ant-log ($\frac{1}{3}$ ((log 318 + log 4344) – log 17200))

Exercise 1

1. Write each of the following in standard form:

1. 167200

2. 0.00235

3. 245.750

4. 45075

2. Write each of the following in decimal numerals:

a. 3.0025×10^{-3}

b. 4.750×10^{5}

c. 75.2525×10^{0}

d. 525.89900×10^{4}

Number	log
$318 = 3.18 \times 10^2$	2.5024
$4344 = 4.344 \times 10^3$	+ 3.6379
	6.1403
$17200 = 1.7200 \times 10^4$	- 4.2355
	1.9048
$4.3142 \times 10^0 = 4.3142$	$\frac{1.9048}{3} = 0.6349$

TOPIC 5: CONGRUENCE

Congruence

I'm sure you have seen some of the figure which in one way or another one of the shape can become another using turns, flip or slide. These shapes are said to be Congruent. Study this notes carefully to know different ways that can help you to recognize congruent figures.

In geometry two figures or objects are congruent if they have the same shape and size, or if one has the shape and size as the mirror image of the other.

If one shape can become another using turns (rotation), flip (reflection), and/ or slide (translation), then the shapes are Congruent. After any of these transformations the shape must still have the same size, perimeters, angles, areas and line lengths.

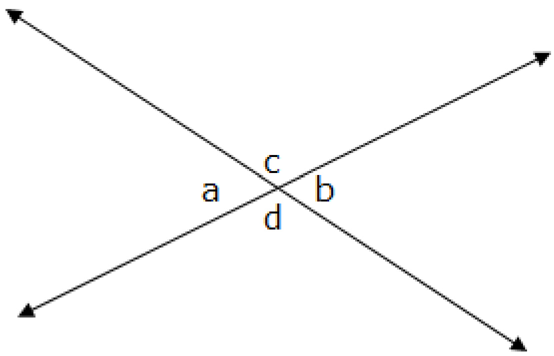
Note that; the two shapes need to be the size to be Congruent i.e. only rotation, reflection and/ or translation is needed.

Remember this:

- Two line segments are Congruent if they have the same length.
- Two angles are Congruent if they have the same measure.
- Two circles are Congruent if they have the same diameter.

Angles formed by the intersection of two straight lines

When two straight lines intersect, they form four angles. Each opposite pair are called vertical angles and they are congruent. Vertical angles are also called opposite angles. See figure below for more understanding:



Properties of vertical angles

- They are Congruent: vertical angles are always of equal measure i.e. a = b, and c = d.
- Sum of vertical angles (all four angles) is 360^0 i.e. a + b + c + d =360^0
- Sum of Adjacent angles (angles from each pair) is 180^0 i.e. a + d =180^0 ; a + c =180^0 ; c + b =180^0 ; b + d =180^0.

Congruence of Triangles

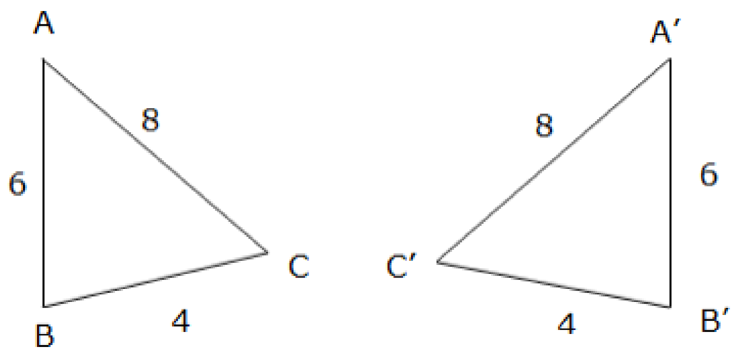
Two Triangles are Congruent if their corresponding sides are equal in length and their corresponding angles are equal in size. The symbol for congruent shapes is ≅

The Conditions for Congruence of Triangles

Determine the conditions for congruence of triangles

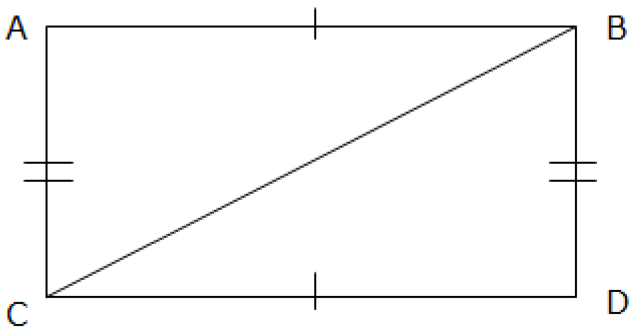
The following are conditions for two Triangles to be Congruent:

• SSS (side-Side-Side): if three pairs of sides of two Triangles are equal in length, then the Triangles are Congruent. Consider example below showing two Triangles with equal lengths of the corresponding sides.



Example 1

Prove that the two Triangles (ΔABC and ΔBCD) below are Congruent.



Solution

We are given a rectangle ABCD; $\overline{AC} = \overline{BD}$; $\overline{AB} = \overline{CD}$ (they are opposite sides of a rectangle). And a diagonal $\overline{BC}$

Required to prove: $\Delta ABC \cong \Delta BCD$

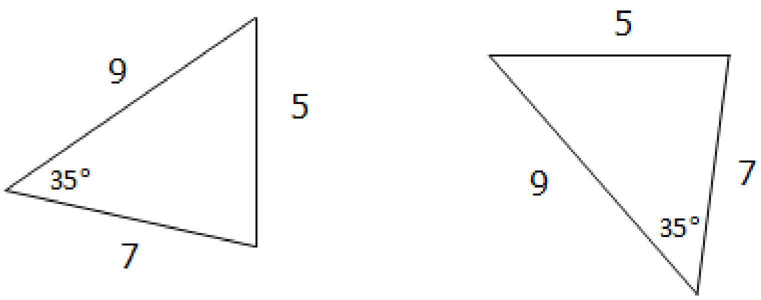
Proof: $\left.\begin{array}{l} \overline{AC} = \overline{BD} \\ \overline{AB} = \overline{CD} \end{array}\right\}$ Opposite sides of a rectangle

$\overline{AC}$ is common

$\therefore \Delta ABC \cong \Delta BCD$ by SSS

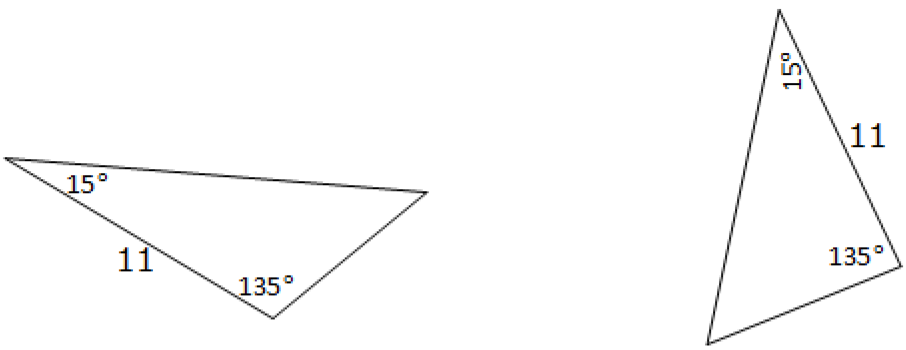
Another condition;

• SAS (Side-Angle-Side): This means that we have two Triangles where we know two sides and the included angles are equal. For example;



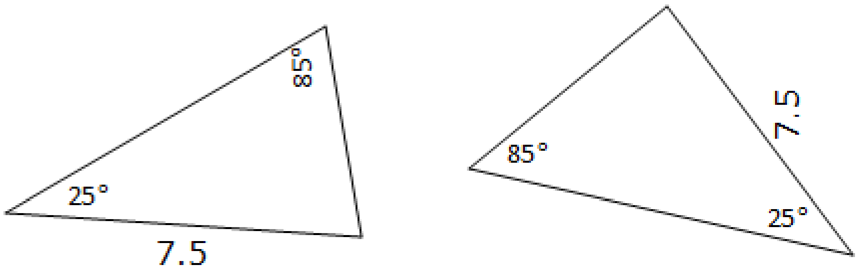
If the two sides and the included angle of one Triangle are equal to corresponding sides and the included angle of the other Triangle, we say that the two Triangles are Congruent.

• ASA (Angle- Side-Angle): If two angles and the included side of one Triangle are equal to the two angles and included side of another Triangle we say that the two Triangles are congruence. For example



AAS condition;

• AAS (Angle-Angle-Side): If two angles and non included side of one triangle are equal to the corresponding angles and non included side of the other Triangle, then the two triangles are congruent. For example



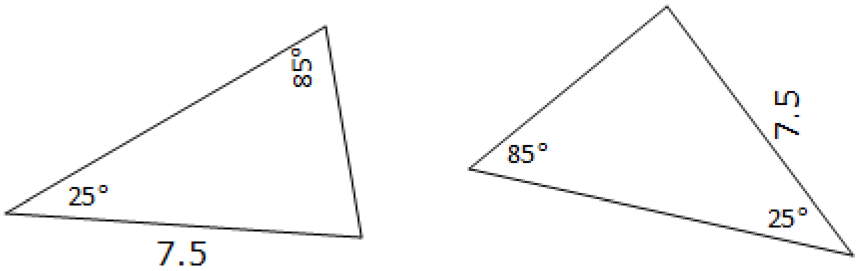
• HL (hypotenuse-Leg): This is applicable only to a right angled triangle. The longest side of a right angled triangle is called hypotenuse and the other two sides are legs.

It means we have two right angled triangles with:

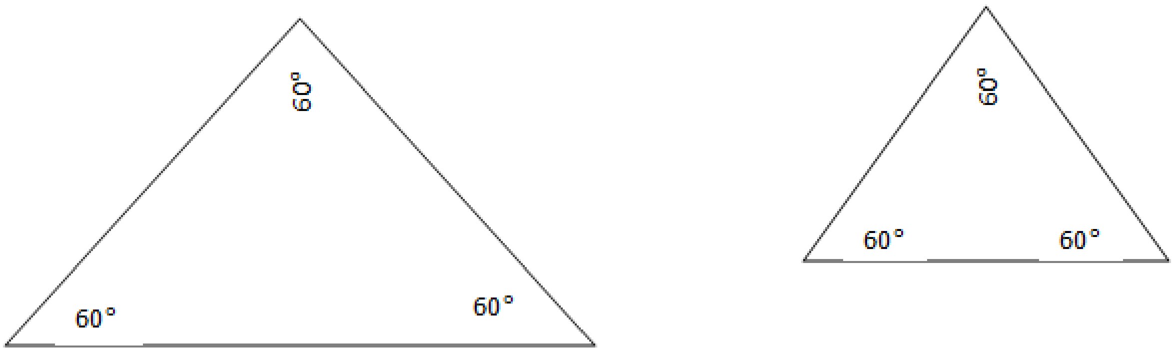
• The same length of hypotenuse and

• The same length for one of the other two legs.

If the hypotenuse and one leg of one right angled triangle are equal to a corresponding hypotenuse and one leg of the other right angled triangle, the two triangles are congruent. For example



Important note: Do not use AAA (Angle-Angle-Angle). This means we are given all three angles of a triangle but no sides. This is not enough information to decide whether the two triangles are congruent or not because the Triangles can have the same angles but different size. See an illustration below:



The two triangles are not congruent.

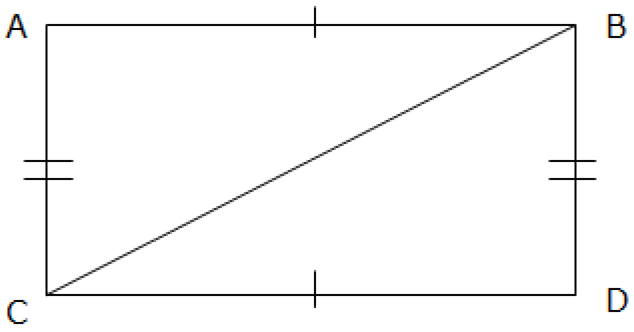
Without knowing at least one side, we can’t be sure that the triangles are congruent.

Congruence of Triangle

Prove congruence of triangle

Example 2

Prove that the two Triangles (ΔABC and ΔBCD) below are Congruent.



Solution;

We are given a rectangle ABCD; $\overline{AC} = \overline{BD}$; $\overline{AB} = \overline{CD}$ (they are opposite sides of a rectang
diagonal $\overline{BC}$

Required to prove: $\Delta ABC \cong \Delta BCD$

Proof: $\left.\begin{array}{l} \overline{AC} = \overline{BD} \\ \overline{AB} = \overline{CD} \end{array}\right\}$ Opposite sides of a rectangle

$\overline{AC}$ is common

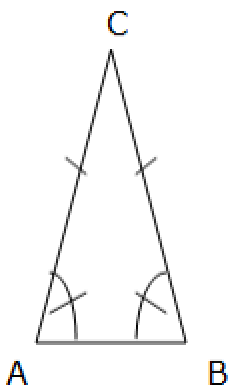
$\therefore \Delta ABC \cong \Delta BCD$ by SSS

Theorems on Congruence of Triangles to Solve Related Problems

Apply theorems on congruence of triangles to solve related problems

Isosceles Triangle Theorem

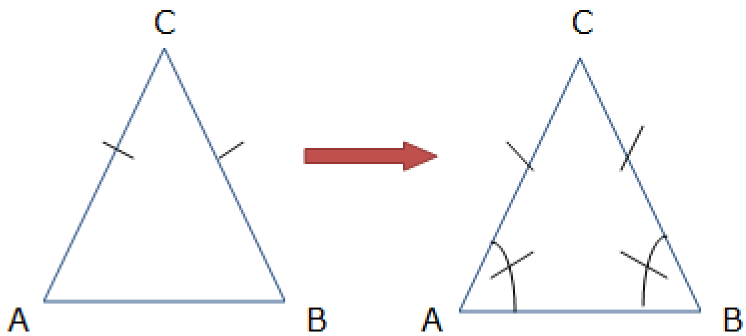
The figure below illustrates an example of an isosceles triangle:



An isosceles triangle has two congruent sides (opposite sides) and two congruent angles. The congruent angles are called base angles and the other angle is called vertex angle. The angles A and B are base angles and angle C is the vertex angle.

The base angle Theorem

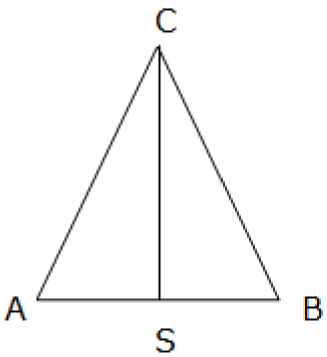
If two sides of a triangle are congruent, then the angles opposite to these sides are congruent



Given that $\overline{AC} = \overline{BC}$

Required to prove: angle A = angle B

Proof: Let s be the midpoint of $\overline{AB}$ and then join C and S



Since S is the midpoint of $\overline{AB}$, $\overline{AS} = \overline{BS}$

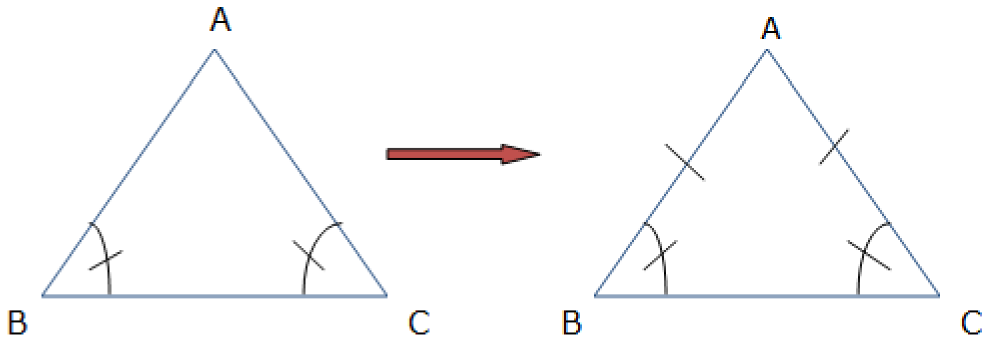
we are given that $\overline{AC} = \overline{BC}$

and is common (i.e. reflexive property)

Therefore by SSS, $\Delta ACS \cong \Delta BCS$

Therefore, the base angles i.e. angle CAS and angle BCS are equal (by the definition of a congruence of triangles).

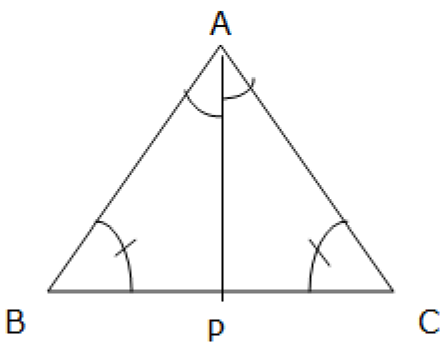
It states that, if two angles of a triangle are congruent, then sides opposite those angles are congruent.



We are given that; angle B = angle C

required to prove $\overline{AB} = \overline{AC}$

proof: Bisect angle A by drawing a line perpendicular to $\overline{BC}$ at P.



Since $\overline{AP}$ bisects the angle A, then angle BAP = angle CAP

we are given that angle ABP = angle ACP

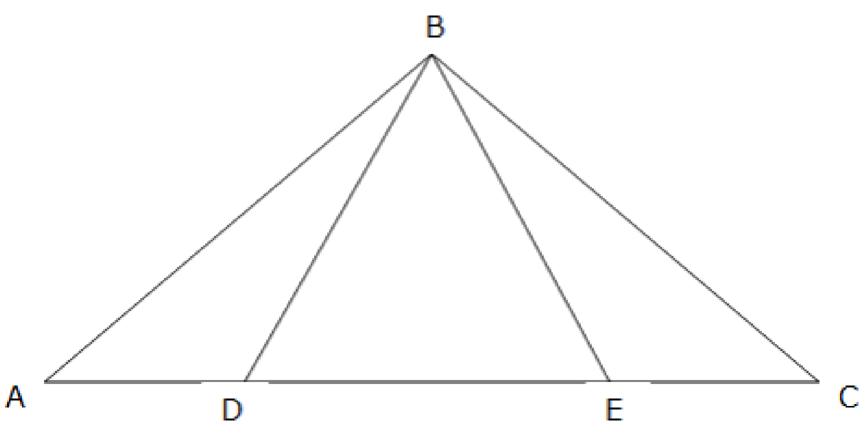
$\overline{AP}$ is common (reflexive property)

Therefore, by AAS the triangles ABP and CAP are congruent and

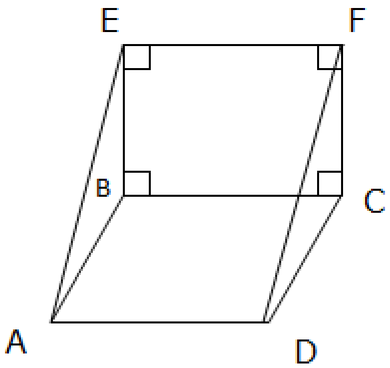
Hence $\overline{AB} = \overline{AC}$ (definition of congruence of triangle)

Exercise 1

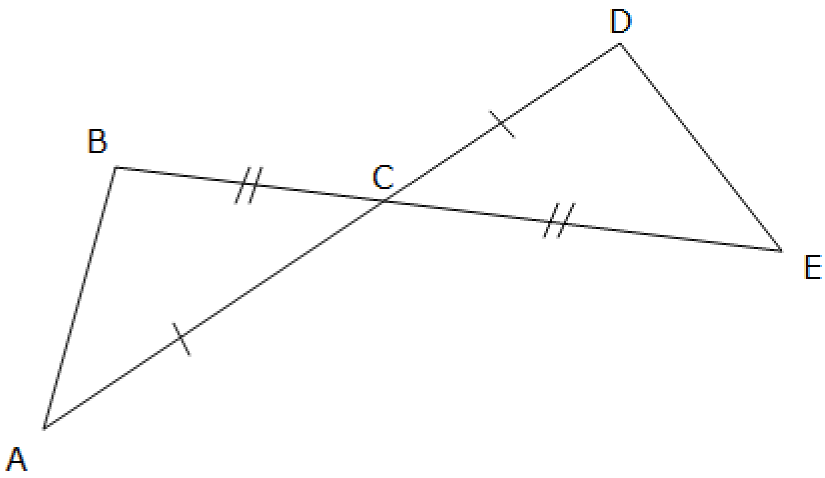
1. In the isosceles triangle ABC, BA and BC are congruent. D and E are points on AC such that AD is congruent to BD and BE is congruent to BC. Show that the triangles ABD and CBE are congruent



2. ABCD is a parallelogram and BEFC is a square. Show that triangles ABE and DCF are congruent.



3. Use the figure below to answer the following questions:



1. $\overline{AC}$ Corresponds to ....
2. $\overline{BC}$ Corresponds to ....
3. Is $\Delta ABC \cong \Delta EDC$? Give reasons to your answer.
4. Angles ACB and DCE are called.......... Are they congruent?

4. Triangle ABC is congruent to Triangle DEF with BA = FO, BC = EF and AC = DF. If the su measures of angle B and angle C is 130 degrees what is the degree measure of D.
5. Triangle ABC is congruent to Triangle DEF, with $\overline{AB} = \overline{DE}$. If $\overline{AB}$ = 2x + 10 and $\overline{DE}$ = 4x value of x.

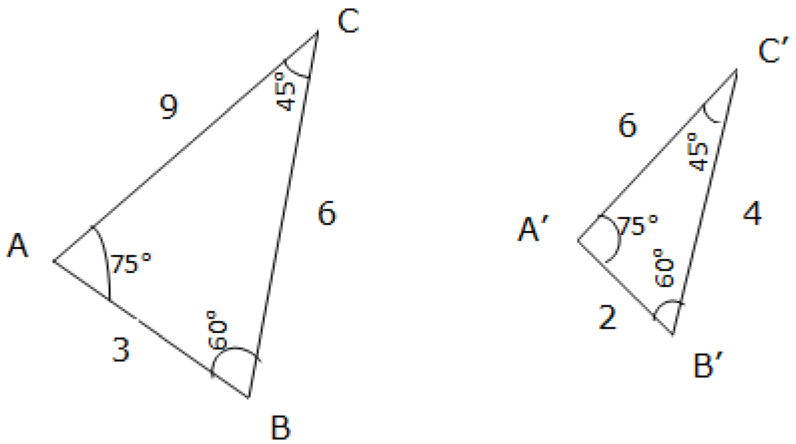
TOPIC 6: SIMILARITY

Similarity

Some Figures tend to have different size but exactly equal angles and their corresponding sides always proportional. This kind of figures are said to be similar. Read the notes below to see which shapes are said to be similar.

Similar Figures

Two geometrical figures are called similar if they both have the same shape. More precisely one can be obtained from the other by uniformly scaling (enlarging or shrinking). Possibly with additional translation, rotation and reflection. Below are similar figures, the figures have equal angle measures and proportional length of the sides:



Angle A corresponds to angle A′, angle B corresponds to B′, angle C corresponds to C′ also each pair of these corresponding sides bears the same ratio, that is:

$$\frac{AB}{A'B'} = \frac{3}{2}$$

$$\frac{AC}{A'C'} = \frac{9}{6} = \frac{3}{2}$$

$$\frac{BC}{B'C'} = \frac{6}{4} = \frac{3}{2}$$

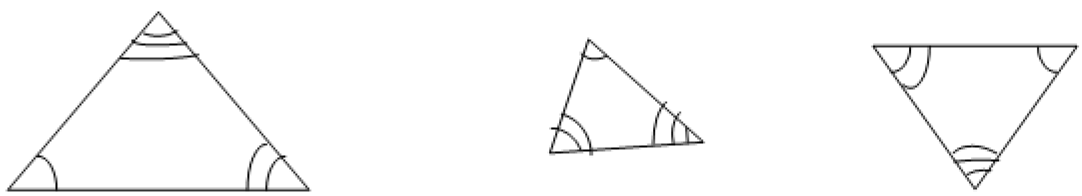
Since all sides have the same ratio i.e. they are proportional and the corresponding angles are equal i.e. angle A = angle A′, angle B = angle B′ and angle C = angle C′, then the two figures are similar. The symbol for similarity is '∼'

Note: all circles are similar to each other, all squares are similar to each other, and all equilateral triangles are similar to each other. On the other hand rectangles are not all similar to each other, isosceles triangles are not all similar to each other and ellipses are not all similar to each other.

Similar Polygons

Identify similar polygons

Two Triangles are similar if the only difference is size (and possibly the need to Turn or Flip one around). The Triangles below are similar

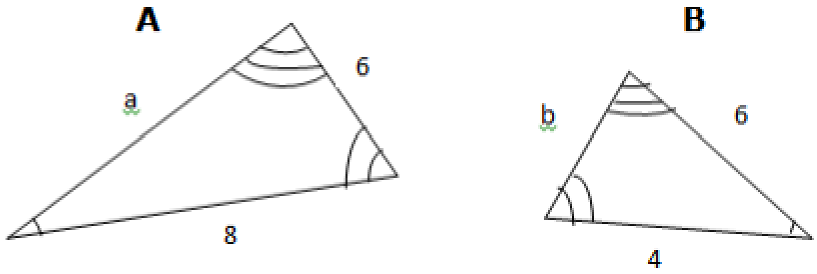


(Equal Angles have been marked with the same number of Arcs)

Similar Triangles have:

- All their angles equal
- Corresponding sides have the same ratio

For example; Given similar triangles below, find the length of sides a and b



Solution

Since we know that, similar triangles have equal ratio of corresponding sides, finding the ratio of the given corresponding sides first thing:

Corresponding sides given are the sides opposite to the angle marked with three arcs .i.e. i side with 8 corresponds to side with 4 in B. So the ratio will be: $\frac{8}{4}$ = 2

Now, we can find the length of the other sides by equating the value of the ratio we have ob The side labeled a in A corresponds to side labeled 6 in B because they are both opposite t marked by two arcs.

Thus, 2 = $\frac{a}{6}$

12 = a (multiply by 6 both sides)

The side labeled 6 in A corresponds to the side labeled b in B because they are both opposi Angle marked by one arc.

Thus, 2 = $\frac{6}{b}$

2b = 6 (multiply by b both sides)

b = 3 (divide by 2 both sides)

Therefore, the length of sides a and b are 12 and 3 units respectively.

This is how we can use the property of similarity of triangles (i.e. corresponding sides have e their lengths) to find the length of the missing sides.

How to find if Triangles are Similar

Two Triangles are similar if:

- All their equals are angles

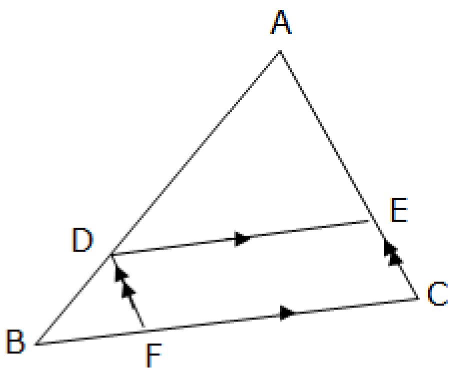
- The corresponding sides are in the same ratio

But we don’t need to know all three angles and all three sides, even two or three are enough.

Intercept theorem

The theory is also called Side-Splitter theorem. Let ABC be any Triangle and DE is drawn parallel to BC, then AD/DB = AE/EC.

To show this is true draw a line DF parallel to EC



The Triangles ADE and BD have exactly equal angles and so they are similar (recall that the two Triangles are similar by AA).

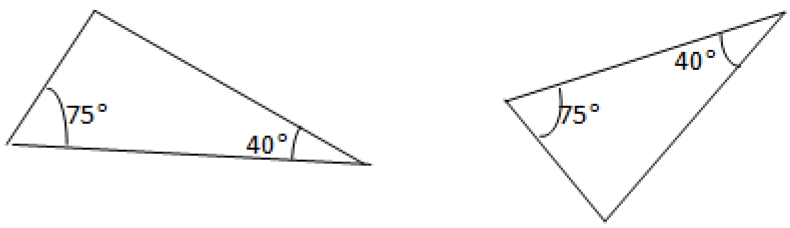
Side AD corresponds to side DB and side AE corresponds to side DF, thus AD/DB = AE/DF But DF = EC, so AD/DB = AE/EC

Similarity Theorems of Triangles

Prove similarity theorems of triangles

There are three ways to find that the two Triangles are Similar

1. AA (Angle-Angle): this means, Triangles have two of their Angles equal. See an illustration below



If two of their Angles are equal then the third Angle must also be equal, because Angles of a Triangle add up to180^0. In our case, our third Angle will be:

$180^0 - (75^0 + 40^0) = 65^0$

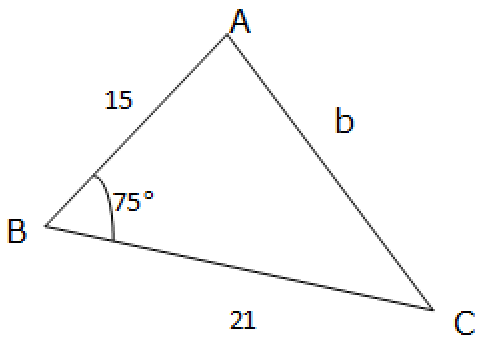
Therefore, AA can also be called AAA because when two angles are equal then all three Angles must be equal.

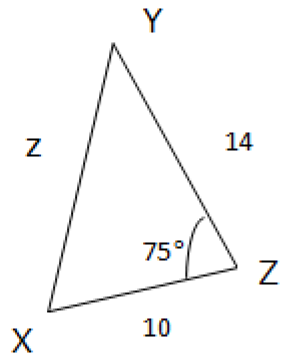
2. SAS (Side-Angle-Side): Means we have two Triangles where:

• The ratio between two sides is the same as the ratio between the other two sides

• The included Angles are equal

For example:





From our example, we see that, the side AB corresponds to side XZ and side BC corresponds to side YZ, thus the ratios will be:

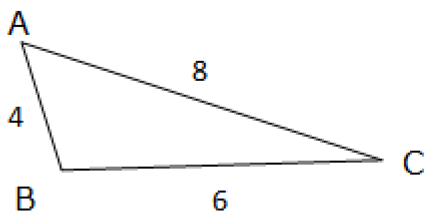
AB:XZ = 15:10 = 3:2 and

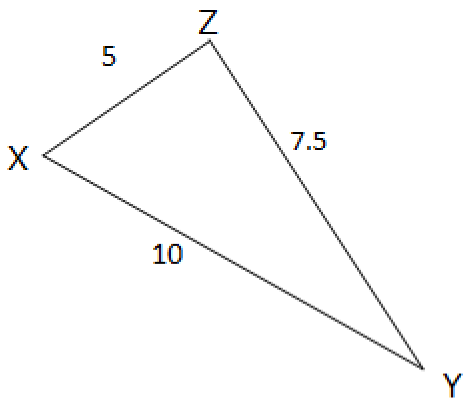
BC:YZ = 21:14 = 3:2

Also, there is a matching angle of75^0in between them.

The information is enough to tell us that the Triangles are Similar.

3. SSS (Side-Side-Side): Means we have three pairs of sides in the same ratio. Then the Triangles are Similar. For example;





In this example; the ratios of sides are:

a:x = 6:7.5 = 12:15 = 4:5

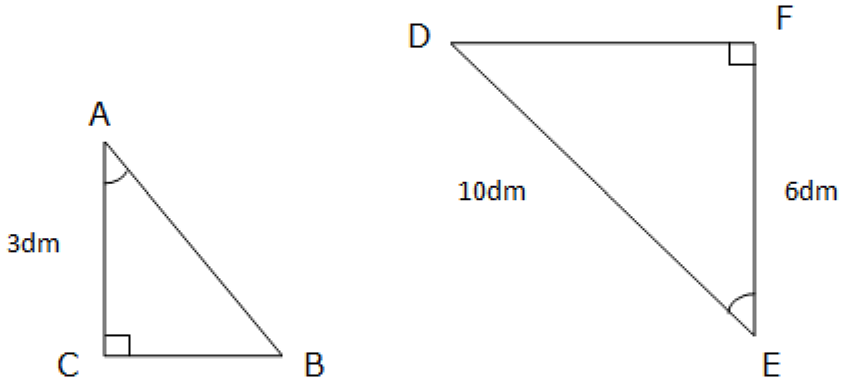
c:y = 4:5

b:z = 8:10 = 4:5

The ratios are all equal, so the Triangles are Similar.

Exercise 1

1. Use similarity to calculate side AB

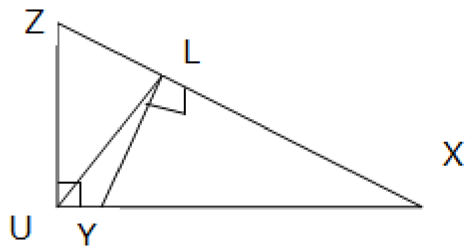


2. ABC is a Triangle in which AC is produced to E and AB is Produced to D such that DE//BC. Show that AD:AB = DE:CB

3. Given Triangles ABC and PQR which are similar. If the lengths of sides AC = 4.8cm, AB = 4cm and PQ = 9cm find the length of side PR if side AB corresponds to PQ and BC corresponds to QR.

4. Prove that any two equilateral Triangles are similar.

5. Name two similar triangles in the figure below:



TOPIC 7: GEOMETRIC AND TRANSFORMATIONS

Reflection

The Characteristics of Reflection in a Plane

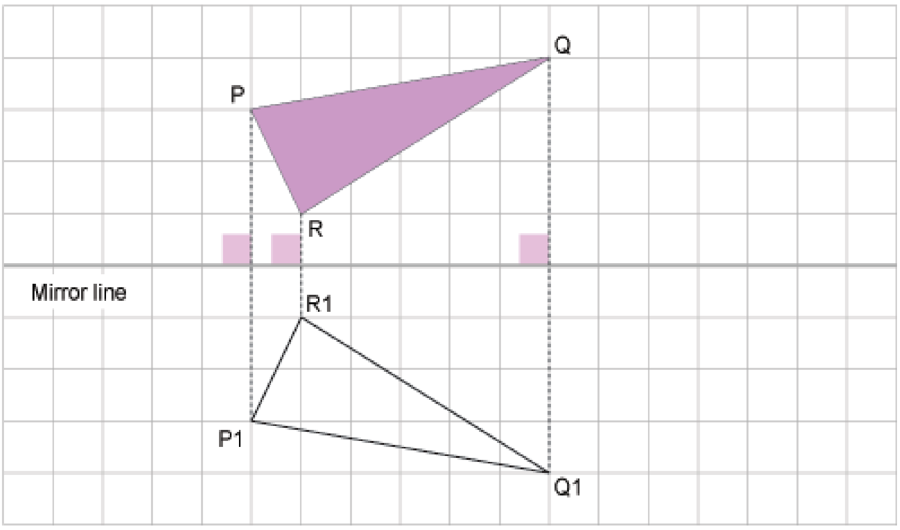
Describe the characteristics of reflection in a plane

A transformation in a plane is a mapping which moves an object from one position to another within the plane. Think of a book being taken from one comer of a table to another comer. Figures on a plane of paper can also be shifted to a new position by a transformation. The new postion after a transformation is called the image. Examples of transformations are reflection, rotation, enlargement and translation.

Different Reflections by Drawings

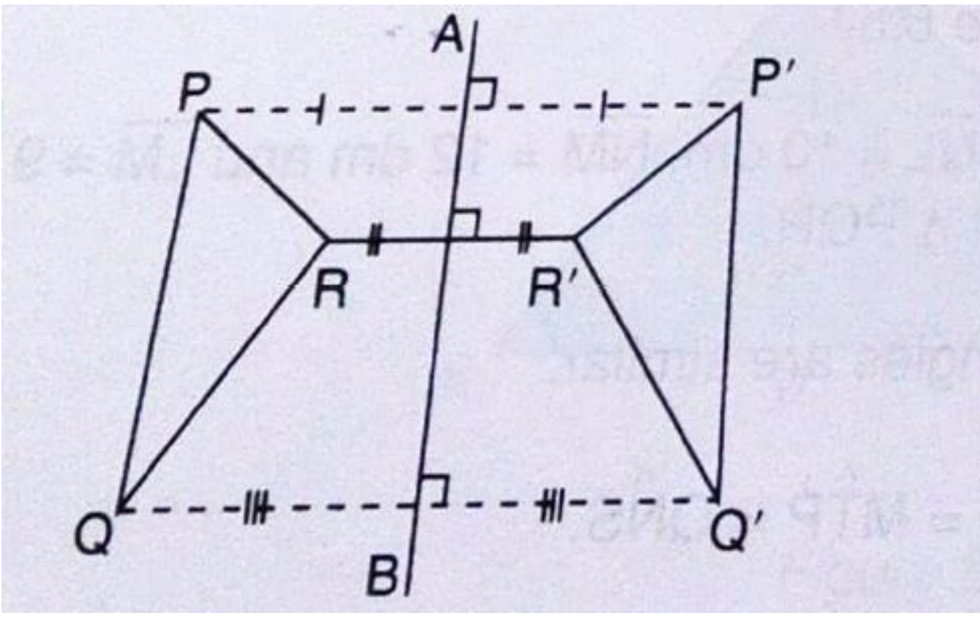
Represent different reflections by drawings

A reflection is a transformation which reflects all points of a plane in a line called the mirror-line. The image in a mirror is as far behind the mirror as the object is in front of the mirror



Characteristics of Reflection

In the diagram, APQR is mapped onto ΔP'Q'R' under a reflection in the line AB. If the paper is folded along the line AB, ΔPQR will fall in exactly onto ΔPQR. The line AB is the mirror-line. which is the perpendicular bisector of PP', QQ' and ΔPQR and ΔP'Q'R are congruent.



Some characteristics observed under reflection are:

a. PP' is perpendicular to AB, RR' is perpendicular to AB and QQ is perpendicular to AB.

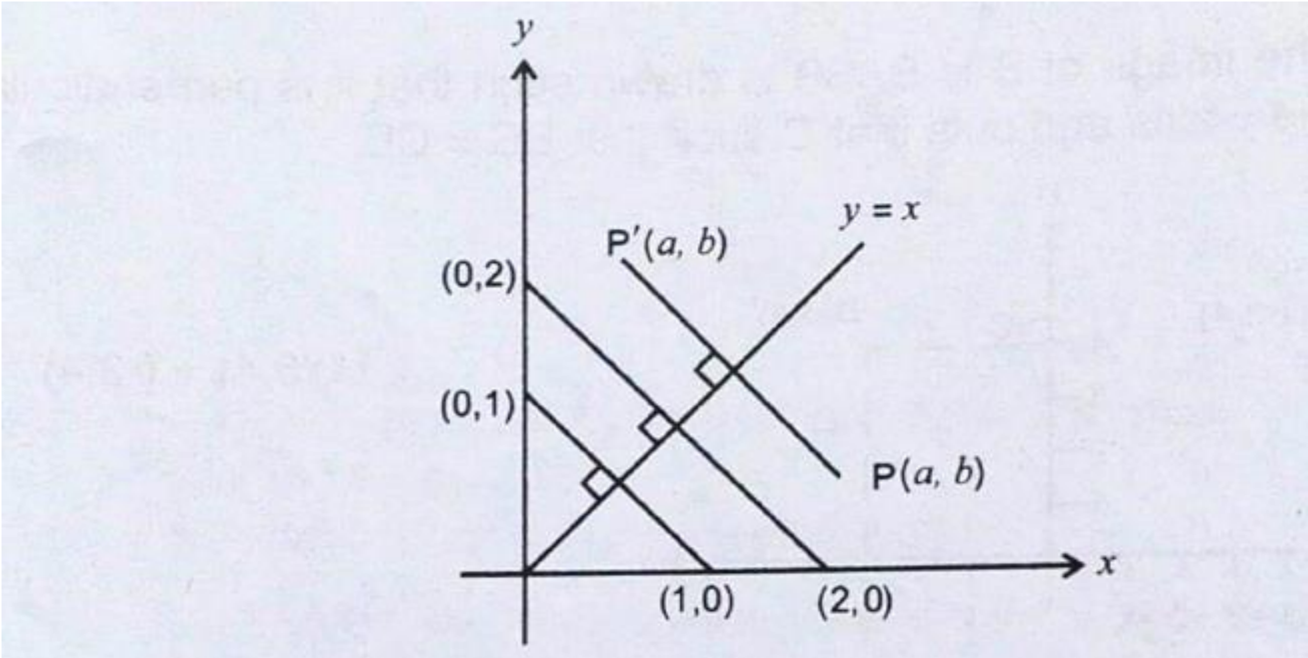
b. The image of any point on the Q' mirror line is the point itself.

c.PP' is parallel to RR' and QQ'

Reflection in the Line *y = x*

The line y = x makes an angle 45° with the x and y axes. It is the line of symmetry for the angle YOX formed by the two axes. By using the isosceles triangle properties, reflection of the point (1, 0) in the line y = x will be (0, 1).

The reflection of (0,2) in the liney = x will be (2,0). You notice that the co-ordinates are exchanging positions. Generally, the reflection of the point (a,b) in the line y = x is (b,a).



The reflection of the point B(c,d) in the line y = -x is B' (-d, -c)

Exercise 1

1. Find the image of the point D(4,2) under a reflection in the x-axis.

2. Find the image of the point P(-2,5) under a reflection in the x-axis.

3. Point Q(-4,3) is reflected in the y-axis. Find the coordinates of its image.

4. Point R(6,-5) is reflected in the y-axis. Find the co-ordinates of its image.

5. Reflect the point (1 ,2) in the line y = -x.

6. Reflect the point (5,3) in the line y = x.

7. Find the image of the point (1 ,2) after a reflection in the line y=x followed by another reflection in the line y = -x.

8. Find the image of the point P(-2,1) in the line y = -x followed by another reflection in the line x = 0 ketch the positions of the image P and the point P, indicating clearly the lines involved.

9. Find the co-ordinates of the image of the point A(5,2) under a reflection in the line y = 0.

10. Find the coordinates of the image of the point under a reflection in the line x = 0.

11. The co-ordinates of the image of a point R reflected in the x axis is R(2, -9). Find the coordinates of R.

Rotations

Characteristics of a Rotation on a Plane

Describe characteristics of a rotation on a plane

Rotation is a transformation which rotates all points on a plane about a fixed point known as the centre of rotation through a given angle in a clockwise or anticlockwise direction

In order to describe a rotation, we give:

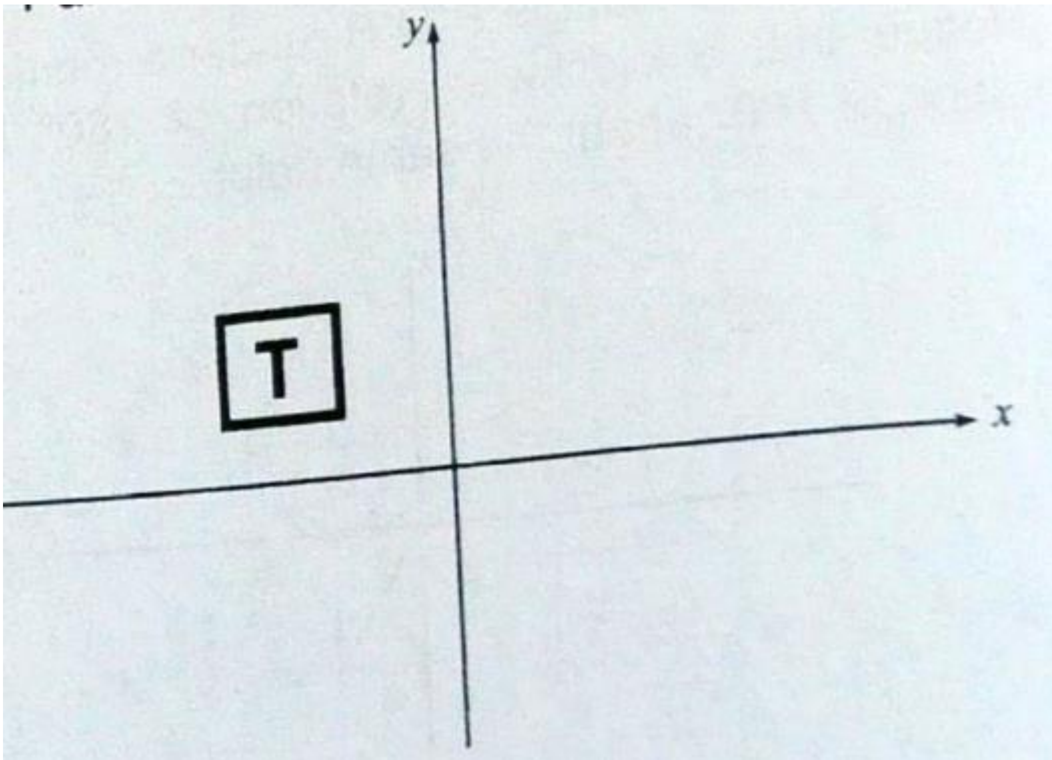
a. the centre of rotation,

b. the angle of rotation, and

c.the direction of rotation

Example 1

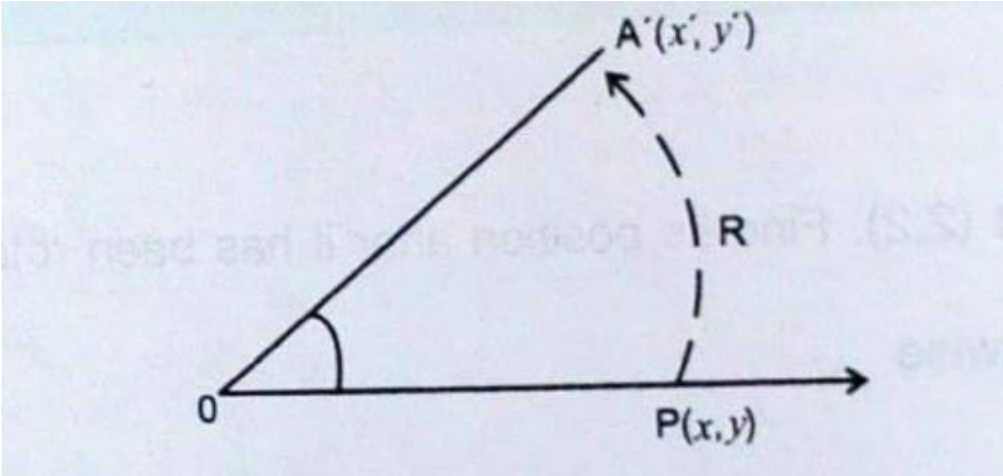
Copy the figure T and rotate 180° about the origin.



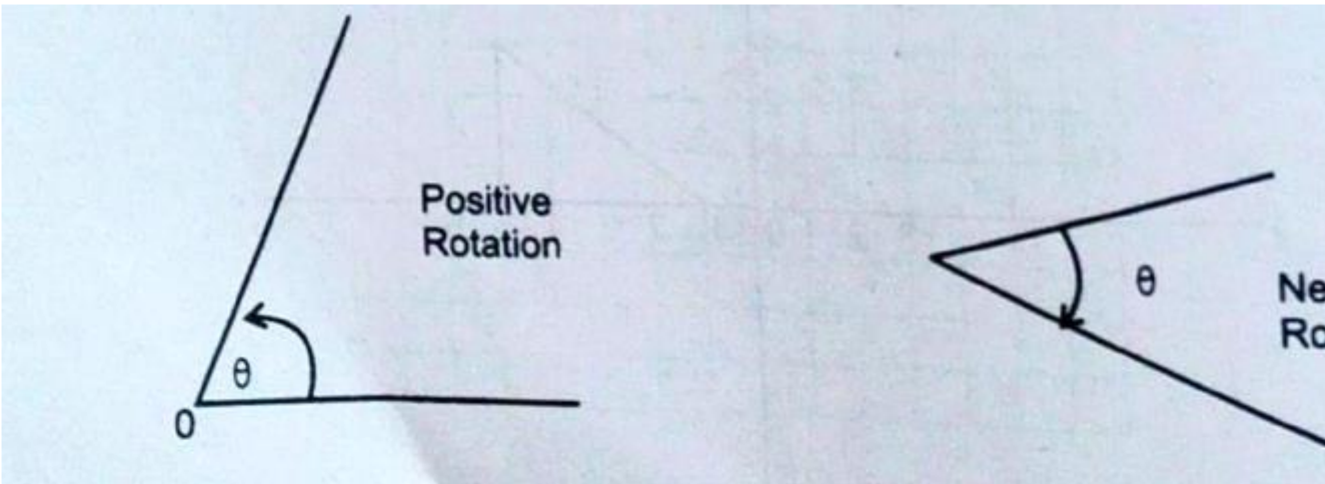
Different Rotation on a Plane by Drawings

Represent different rotation on a plane by drawings

When you turn a ruler at its end corner through an angle 0° it make a rotation. A rotation is transformation which moves a point through a given angle about a fixed point.



A rotation is a transformation that turns a figure about a fixed point called a Centre of Rotation.You can rotate the figure as much as 360 degrees. The transformation of Rotation is usually denoted by R. The symbol R› means that an object is rotated through an angle 9. In the xy plane, when is measured in the clockwise direction it is negative and when it is measured in the anticlockwise direction it is positive.



P is on the x-axis

After a rotation through 90° about the origin it will be on the y-axis. Since P is 1 from O, P' is 1 from O, the coordinates of P are P'(O,I). Hence R_{90°(1, O) = (O, 1)

Exercise 2

1. Find the image of the point (1, 2) under a rotation through 180° anti-clockwise about the origin.

2. Find the rotation of the point (6, 0) under a rotation through 90° Clockwise about the origin.

3. Find the rotation of the point (-2, 1) under a rotation through 270° clockwise about the origin.

4. Point Q(5, -4) is rotated through 270° in the clockwise direction. Find the coordinates of its image.

5. Find the image of (1, 2) after a rotation of -90°

6. Find the image of (-3, 5) after a rotation of -180°

7. Find the image of (-5, 0) after a rotation of -180°

8. Find the image of (-5, 0) after a rotation of 180° about the origin. Comment about the results of questions 7 and 8.

9. The vertices of triangle OAB are O(0,0), A(2,3) and B(2,1). The triangle is rotated through 90° anti-clockwise about the origin. Find the co-ordinates of its image.

10. The vertices of rectangle PQRS are P(0,0), Q(3,0), R(2,3), S(0,2). The rectangle is rotated through 90° clockwise about the origin. a) Find the co-ordinates of its image b) draw the image.

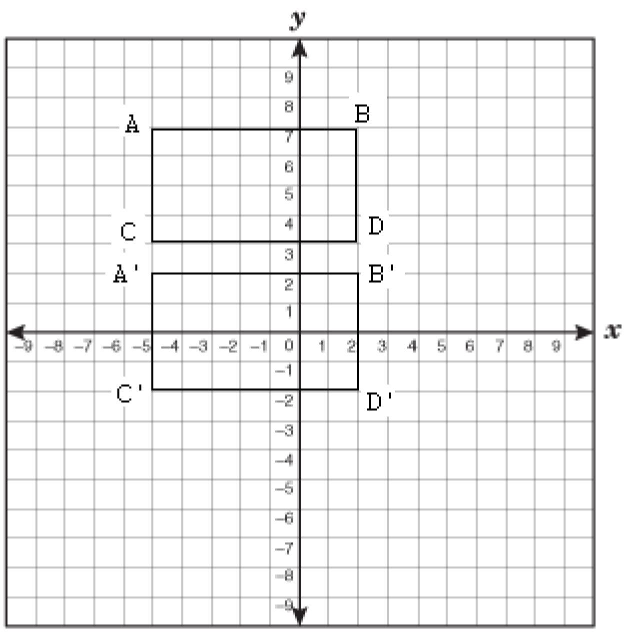
Translation

Properties of Translations

State properties of translations

Translation is a transformation which moves all points, on a plane through the same direction

In the diagram, ΔABC slidesΔA'B'C' (A' is read as A prime) is the direction AA'. Note that AA' are parallel and of equal length. We say that ABC is mapped onto A'B'C' by a translation.

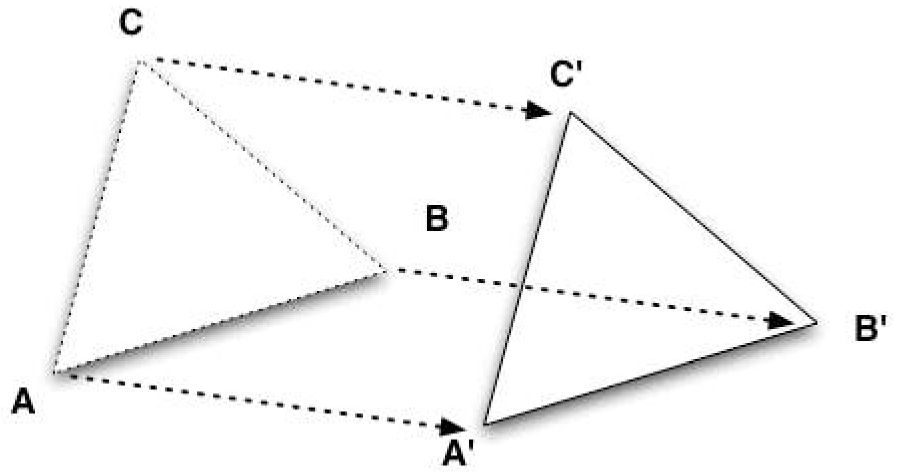


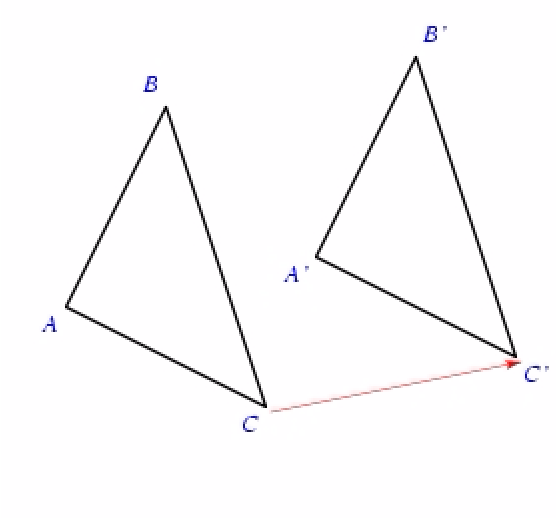
A translation usually denoted by T. For example 1(1, 1) = (6, 1) means that the point 1(1,1) has been moved to (6, 1) by a translation T. This translation will move theorigin (0,0) to (5,0) and is written as T = 5/0.

Translations Drawings

Represent translations drawings

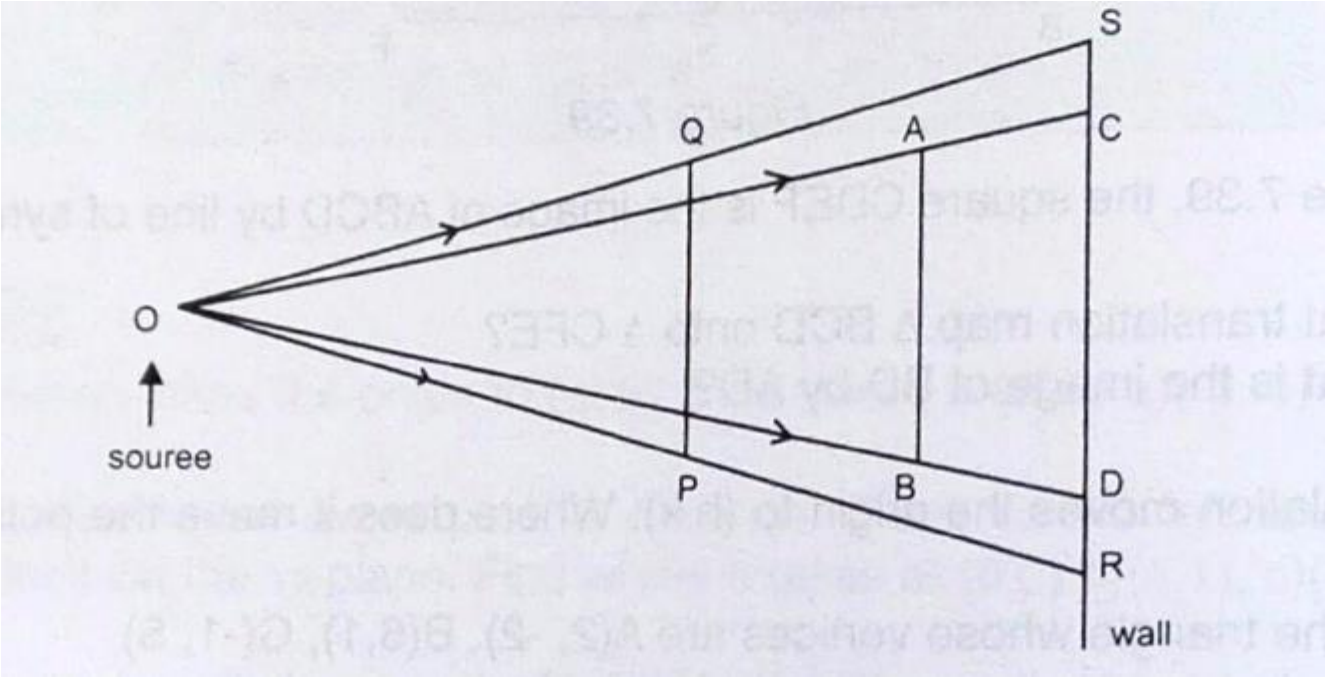
Translation drawings





If you cast a shadow of an object onto a plane surface, say a wall, by using a light source, the shadow becomes bigger as the object moves closer to the light source.

In figure 7.40, AB casts a shadow CD. When AB moves to a new position and is renamed QP (same size with AB) the shadow of QP becomes SR and it is greater than CD.



Enlargement

A Scale of Enlargement

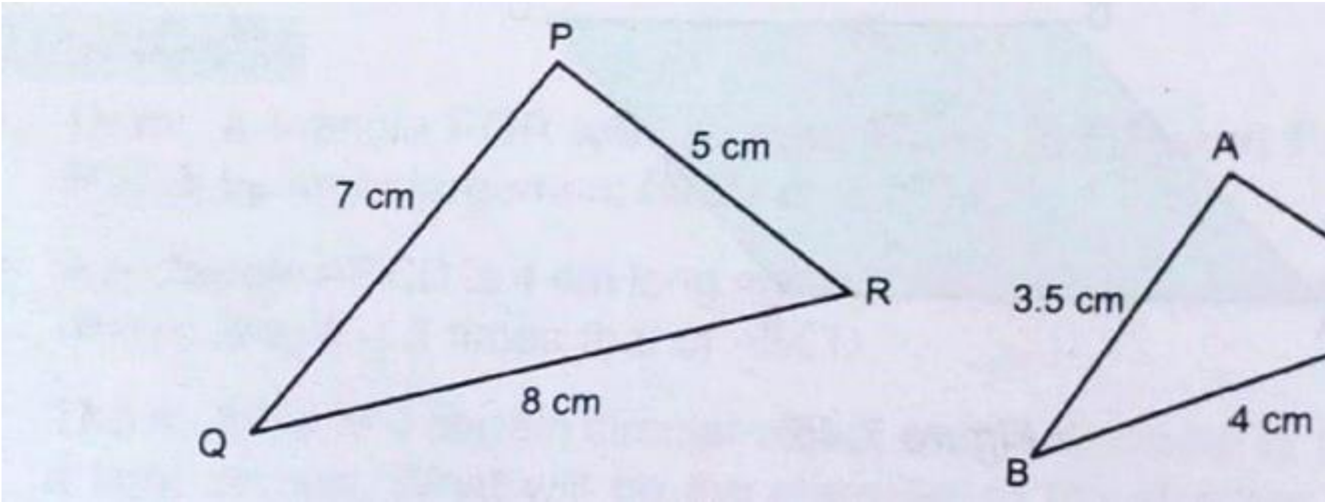
Develop a scale of enlargement

Enlargement is a transformation in which a figure is made larger (magnified) or made smaller (diminished). A photograph may be enlarged or diminished to suit a Certain purpose. Figures can be drawn to scale where actual figures are diminished or enlarged. Enlarged shapes are geometrically similar and have corresponding angles equal. The number that magnifies or diminishes a figure is called the enlargement factor and is usually denoted by k. If k is less than 1 the figure is diminished and if it is greater that 1 the figure is enlarged k times.

In the case of closed figures, if the lengths enlarged by a factor of k then the area is enlarged by k^2.

Scale

Similarity can be used in enlarging of diminishing geometrical figures. For example in maps, a large area of land is represented by a small area on paper by a scale. Scale is a ratio between the measurement of a drawing to the actual measurement. It is normally stated in the form of 1 : n, for example, if a scale of a map is 1.20000, then 1 unit on the map represents 20000 units on the ground.



In the figure, triangle ABC is a scale measurement of triangle PQR, where the scale is 1:2.

Scale is measurement of drawing: Actual Measurement, that is,

Scale = Measurement of drawing/Actual measurement

Exercise 3

Find the length of a drawing that represents:

a. 15 km when the scale is 1:500000

b. 45 km when the scale is 1 km to 900 m.

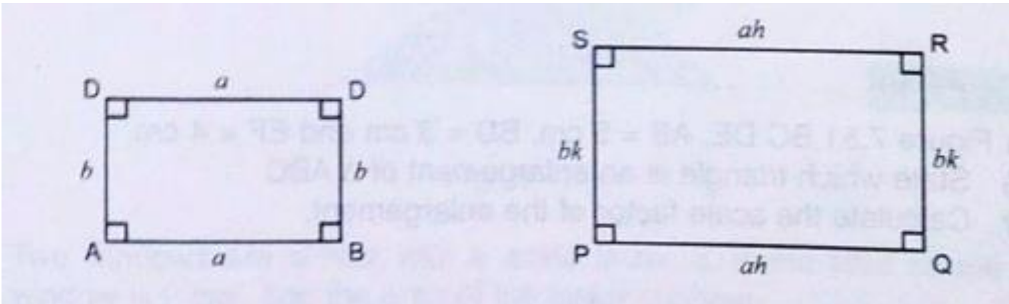
A building2 50 metres high is representedb y a line segment of length 5 cm. Finc the scale of the drawing.

A triangular plot of land has sides 152 metres and 208 metres meeting at an angle of 500. Find by scale drawing the distance from the middle point of the longest side to the opposite corner

Enlargement of a Given Figures

Construct enlargement of a given figures

Consider two similar rectangles shown in the figure below with a scale factor *k*.



If AB = *a* and AD = *a,* then PO = *ak* and PS *bk*

Area of ABCD = *a*× *a* = *ab*

Area of PORS = ak x bk = abk^2

Area of PORS/Area of ABCD = $abk^2/ab = k^2$

Therefore, if two polygons have a scale factor k then the ratio of their areas is k^2. This is also called the scale factor for the area.

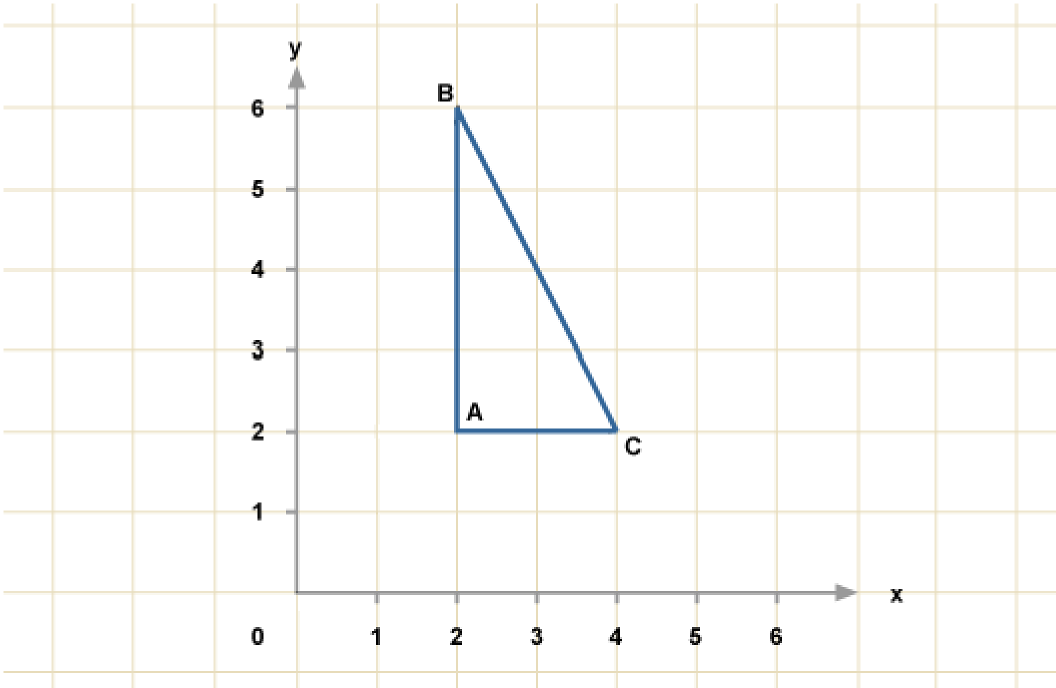
Figures to Scale

Draw figures to scale

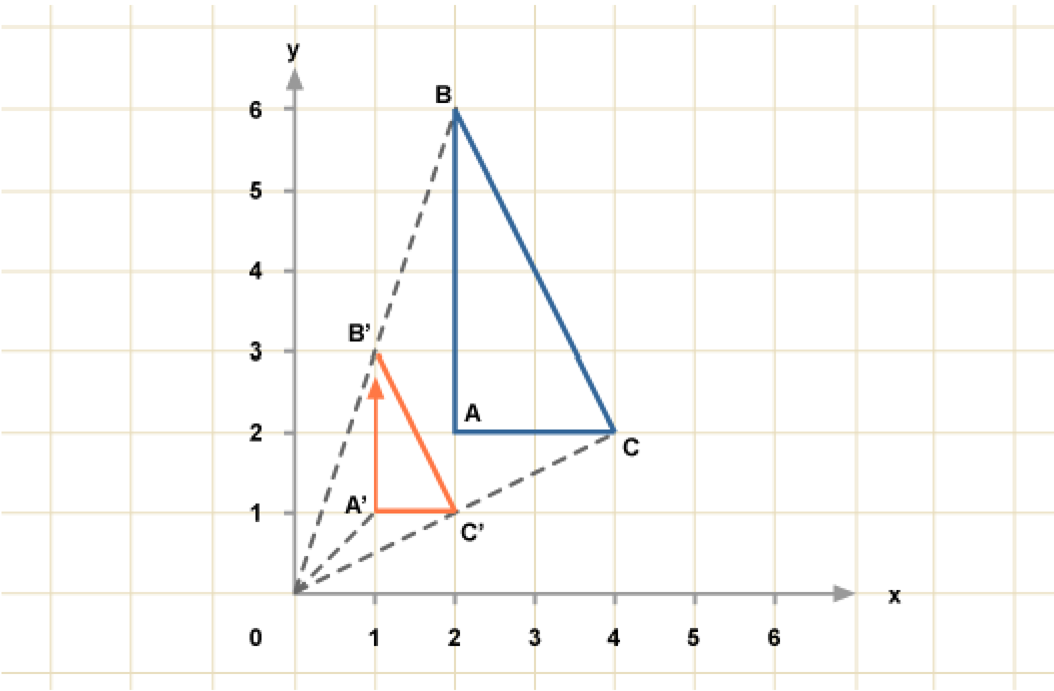
If we 'enlarge' a shape by a scale factor that is between -1 and 1, the image will be**smaller**than the object

Example 2

Enlarge triangle**ABC**with a scale factor $^1/_2$, centred about the origin.



Solution



The scale factor is $^1/_2$, so:

OA' = $^1/_2$OA

OB' = $^1/_2$OB

OC' = $^1/_2$OC

Since the centre is the origin, we can in this case multiply each coordinate by $^1/_2$ to get the answers.

A = (2, 2), so A' will be (1, 1).

B = (2, 6), so B' will be (1, 3).

C = (4, 2), so C' will be (2, 1).

Actual Distances Represented by a Scale Drawings

Find actual distances represented by a scale drawings

If two polygons are similar and the ratio of their corresponding sides of two similar polygons is 5:3, then the scale of enlargement is 5/3

Exercise 4

1. Two triangles are similar but not congruent. Is one the enlargement of the other?

2. The length of a rectangle is twice the length of another rectangle. Is one necessarily an enlargement of the other? Explain.

3. In the figure below BC DE, AB 5 cm, BD = 3cm and EF = 4 cm

a. State which triangle is an enlargement of ΔABC

b. Calculate the scale factor of the enlargement.

Combined Transformations

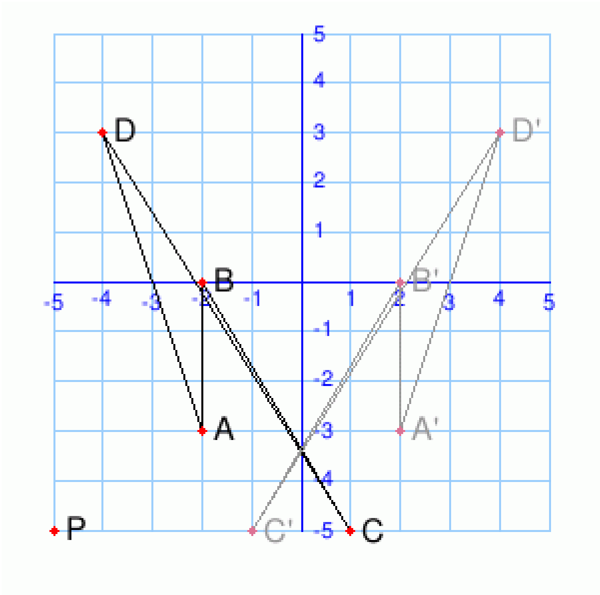
Combined Transformations

Draw combined transformations

Combined Transformation means that two or more transformations will be Performed on one object. For instance you could perform a reflection and then a translation on the same point

Example 3

What type of transform takes ABCD to A'B'C'D'?



Solution

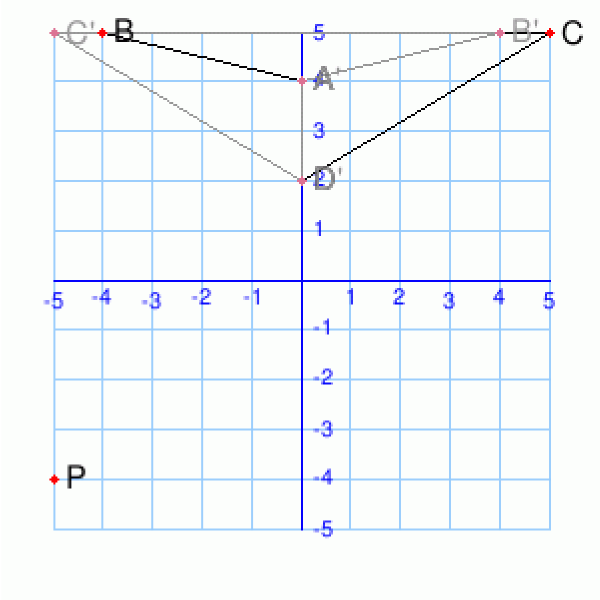
The type of transform takes ABCD to A'B'C'D' is ***Reflection***

Simple Problems on Combined Transformations

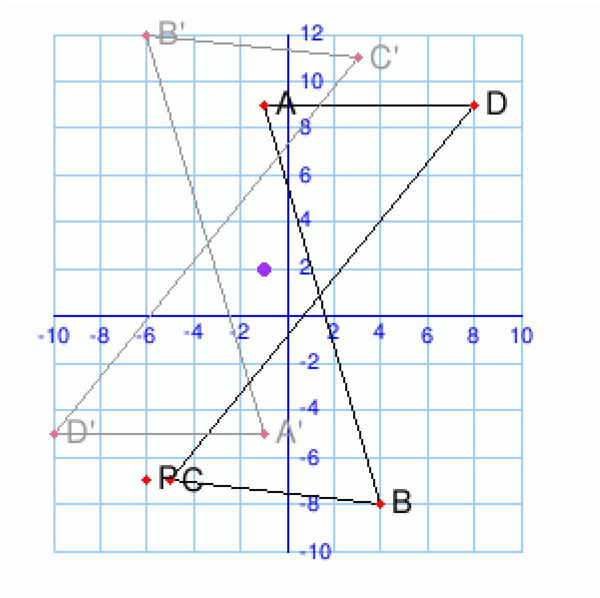
Solve simple problems on combined transformations

Exercise 5

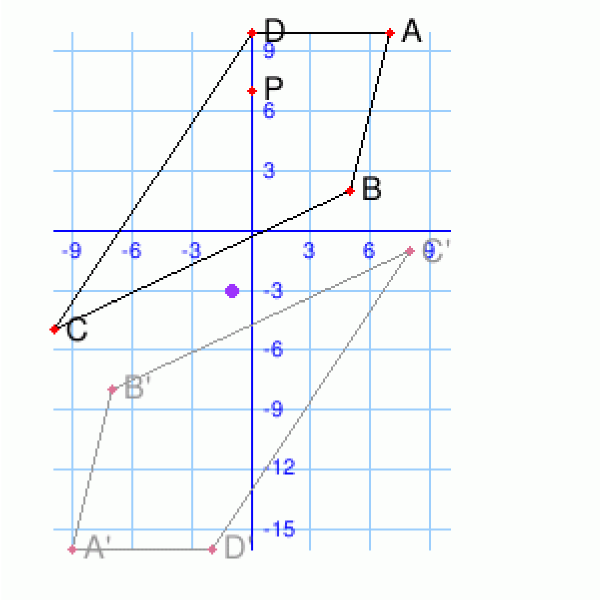
What type of transform takes ABCD to A'B'C'D'?



The transformation ABCD → A'B'C'D' is a rotation around(-1, 2)by___°.Rotate P around(-1, 2)by the same angle. (You may need to sketch things out on paper.)P' = (__,__)



The transformation ABCD → A'B'C'D' is a rotation around(-1, -3)by__°Rotate P around(-1, -3)by the same angle. (You may need to sketch things out on paper.)P' = (__,__)



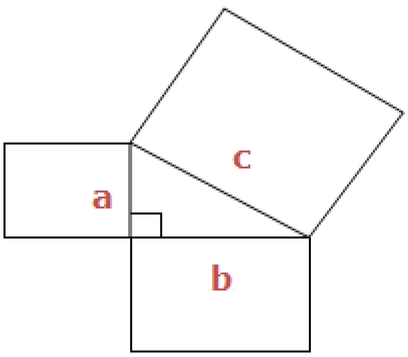
TOPIC 8: PYTHAGORAS THEOREM

Pythagoras Theorem

Triangle with a Right angle i.e. 90° has an amazing property. Do you want to know what property is that? Go on, read our notes to see the amazing property of a right angled triangle.

Pythagoras Theorem deals only with problems involving any Triangle having one of its Angles with 90^0. This kind of a Triangle is called Right angled triangle.

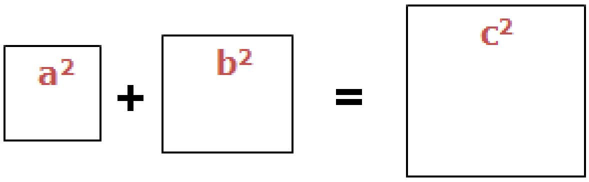
When triangle is a right angled triangle, squares can be made on each of the three sides. See illustration below:



The Area of the biggest square is exact the same as the sum of the other two squares put together. This is what is called Pythagoras theorem and it is written as:

$a^2 + b^2 = c^2$

that is:



'**c**' is the Longest side of the Triangle, is called Hypogenous and is the one that forms the biggest square. **a** and **b** are the two smaller sides.

Proof of Pythagoras Theorem

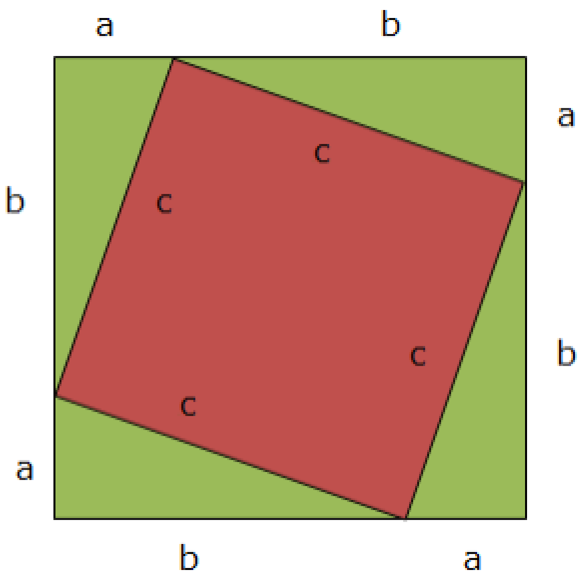
The Pythagoras Theorem

Prove the pythagoras theorem

Pythagoras theorem states that: In a Right Angled Triangle, the sum of squares of smaller sides is exactly equal to the square of Hypotenuse side (large side). i.e. $a^2 + b^2 = c^2$

Take a look on how to show that $\mathbf{a^2 + b^2 = c^2}$

See the figure below:



The area of a whole square (big square)

A big square is the one with sides a + b each. Its area will be:

$(a + b) \times (a + b)$

Area of the other pieces

First, area of a smaller square (tilted) = c^2

Second, area of the equal triangles each with bases a and height b:

$$A = \frac{1}{2}ab$$

But there are 4 triangles and they are equal, so total area =

$$4(\frac{1}{2}ab) = 2ab.$$

Both areas must be equal, the area of **a big square** must be **equal** to the **area of a tilted square plus the area of 4 triangles**

That is:

$(a + b)(a + b) = c^2 + 2ab$

Expand $(a +b)(a + b)$: $a^2 +2ab + b^2 = c^2 + 2ab$

Subtract 2ab from both sides: $\mathbf{a^2 + b^2 = c^2}$ **Hence the result!**

Note: We can use Pythagoras theorem to solve any problem that can be converted into right Angled Triangle.

Exercise 1

1. In a right triangle with given hypotenuse c and legs a and b, find:

1. c if a = 5 and b = 12

2. a if b = 8 and c = 12

3. b if a = 9 and c = 11

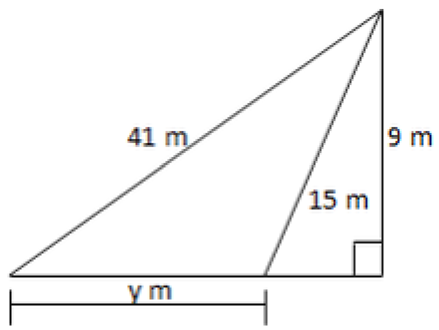
2. A rectangle has base 6 and height 10. What is the length of the diagonal?

3. A square has a diagonal with length 6. What is the length of the sides of the square?

4. The diagonals of a rhombus have lengths 6 and 8. Find the length of one side of the rhombus.

5. A ladder leans against a wall. If the ladder reaches 8m up the wall and its foot is 6m from the base of the wall. Find the length of the ladder.

6. Find the value of the marked side.



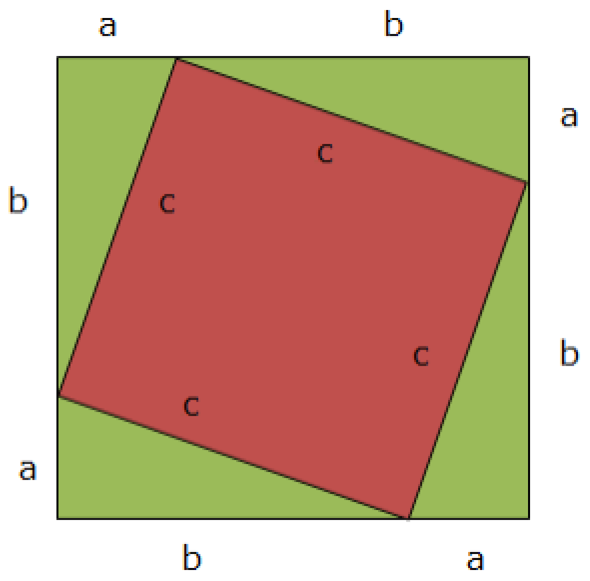
The Pythagoras Theorem

Prove the pythagoras theorem

Pythagoras theorem states that: In a Right Angled Triangle, the sum of squares of smaller sides is exactly equal to the square of Hypotenuse side (large side). i.e. $a^2 + b^2 = c^2$

Take a look on how to show that $\mathbf{a^2 + b^2 = c^2}$

See the figure below:



The area of a whole square (big square)

A big square is the one with sides a + b each. Its area will be:

$(a + b) \times (a + b)$

Area of the other pieces

First, area of a smaller square (tilted) = c^2

Second, area of the equal triangles each with bases a and height b:

$$A = \frac{1}{2}ab$$

But there are 4 triangles and they are equal, so total area =

$$4(\frac{1}{2}ab) = 2ab.$$

Both areas must be equal, the area of **a big square** must be **equal** to the **area of a tilted square plus the area of 4 triangles**

That is:

$(a + b)(a + b) = c^2 + 2ab$

Expand (a +b)(a + b): $a^2 + 2ab + b^2 = c^2 + 2ab$

Subtract 2ab from both sides: $\mathbf{a^2 + b^2 = c^2}$ **Hence the result!**

Note: We can use Pythagoras theorem to solve any problem that can be converted into right Angled Triangle.

Exercise 1

1. In a right triangle with given hypotenuse c and legs a and b, find:

1. c if a = 5 and b = 12

2. a if b = 8 and c = 12

3. b if a = 9 and c = 11

2. A rectangle has base 6 and height 10. What is the length of the diagonal?

3. A square has a diagonal with length 6. What is the length

Application of Pythagoras Theorem

The Pythagoras Theorem to Solve Daily Life Problems

Apply the pythagoras theorem to solve daily life problems

You may have heard about Pythagoras's theorem (or the Pythagorean Theorem) in your math class, but what you may fail to realize is that Pythagoras's theorem is used often in real life situations. For example, calculating the distance of a road, television or smart phone screen size (usually measured diagonally).

Activity 1

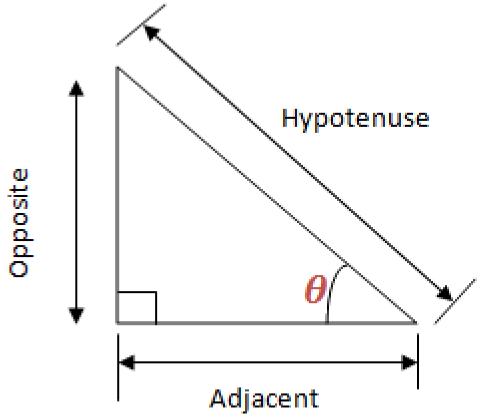
Apply the pythagoras theorem to solve daily life problems

TOPIC 9: TRIGONOMETRY

Do you want to learn the relationships involving lengths and Angles of right-angled triangle? Here, is where you can learn.

Trigonometric Rations

Trigonometry is all about Triangles. In this chapter we are going to deal with Right Angled Triangle. Consider the Right Angled triangle below:



The sides are given names according to their properties relating to the Angle .

Adjacent side is adjacent (next to) to the Angle

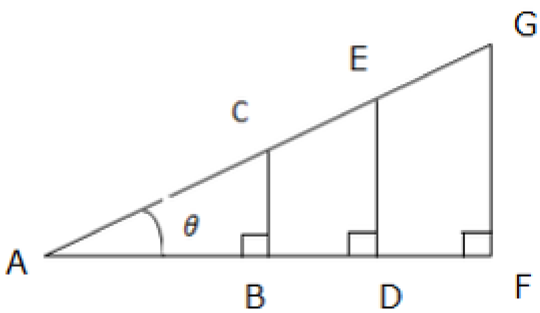
Opposite side is opposite the Angle

Hypotenuse side is the longest side

Sine, Cosine and Tangent of an Angle using a Right Angled Triangle

Define sine, cosine and tangent of an angle using a right angled triangle

Trigonometry is good at finding the missing side or Angle of a right angled triangle. The special functions, sine, cosine and tangent help us. They are simply one of a triangle divide by another. See similar triangles below:



The ratios of the corresponding sides are:

$$\frac{CB}{AB} = \frac{ED}{AD} = \frac{GF}{AF} = \mathbf{t}$$

$$\frac{AB}{AC} = \frac{AD}{AE} = \frac{AF}{AG} = \mathbf{c}$$

$$\frac{CB}{AC} = \frac{ED}{AE} = \frac{FG}{AG} = \mathbf{s}$$

Where by **t**, **c** and **s** are constant ratios called tangent (t), cosine (c) and sine (s) of Angle respectively.

The right-angled triangle can be used to define trigonometrical ratios as follows:

$$\text{Tangent} = \frac{\text{opposite side}}{\text{Adjacent side}}$$

$$\text{Sine} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{Cosine} = \frac{\text{Adjacent side}}{\text{Hypotenuse side}}$$

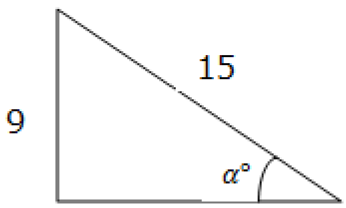
The short form of Tangent is tan, that of sine is sin and that of Cosine is cos.

The simple way to remember the definition of sine, cosine and tangent is the word **SOHCAHTOA**. This means sine is **Opposite** (O) over **Hypotenuse** (H); cosine is **Adjacent** (A) over **Hypotenuse** (H); and tangent is **Opposite** (O) over **Adjacent** (A). Or

SO	TO	CA
H	A	A

Example 1

Given a triangle below, find sine, cosine and Tangent of an angle indicated.



Solution

Case 1:

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{15} = 0.6$$

Case 2:

$$\cos \alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

to get the value of Adjacent side, use Pythagoras theorem

Adjacent side = $(\text{hypotenuse})^2 – (\text{opposite})^2$

Adjacent side = $(15)^2 - 9^2$

Adjacent side = 12

Thus, $\cos \alpha = \frac{12}{15} = 0.8$

Case 3:

$$\tan \alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{12} = 0.75$$

Example 2

Given that

cos 40° = $\frac{40}{41}$. Find the value of tan 40° and sin 40° .

Solution

Cosine of an angle = $\frac{\text{opposite}}{\text{adjacent}} = \frac{9}{40}$,

Thus, adjacent side = 40 and hypotenuse side = 41

Opposite side = $(\text{hypotenuse})^2 - (\text{adjacent})^2$

Opposite side = $(41)^2 - (40)^2$

Opposite side = 9

Therefore, tan 40° = $\frac{\text{opposite}}{\text{adjacent}} = \frac{9}{40}$ and

sin 40° = $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{41}$

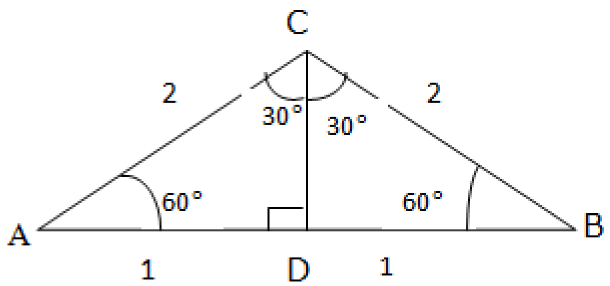
Trigonometric Ratios of Special Angles

Determination of the Sine, Cosine and Tangent of 30°, 45° and 60° without using Mathematical Tables

Determine the sine, cosine and tangent of 30°, 45° and 60° without using mathematical tables

The special Angles we are going to deal with are 30^0, 45^0, 60^0, 90^0. Let us see how to get the Tangent, Sine and Cosine of each angle as follows:

First, consider an equilateral triangle ABC below, the altitude from C bisects at D.



AD = BD = 1 (bisection)

From Pythagoras Theorem; $(AD)^2 + (CD)^2 = (AC)^2$

$$1^2 + (CD)^2 = 2^2$$

$$(CD)^2 = 4 - 1$$

$$(CD)^2 = 3$$

Squaring both sides, we get

$$(CD) = \sqrt{3}$$

$$\sin 60^\circ = \frac{\overline{CD}}{\overline{AC}} = \frac{\sqrt{3}}{2}$$

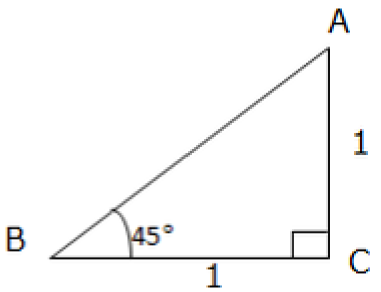
$$\tan 60^\circ = \frac{\overline{CD}}{\overline{AD}} = \frac{\sqrt{3}}{1} = 1$$

$$\cos 60^\circ = \frac{\overline{AD}}{\overline{AC}} = \frac{1}{2}$$

$$\sin 30^\circ = \frac{\overline{AD}}{\overline{AC}} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\overline{CD}}{\overline{AC}} = \frac{\sqrt{3}}{2}$$

Secondly, consider the isosceles triangle ABC below, with base angles 45° and $\overline{AC} = \overline{BC} =$



The side $\overline{AB}$ (Hypotenuse side) = $\sqrt{1^2 + 1^2} = \sqrt{2}$ (by Pythagoras Theorem). So,

$$\tan 45^\circ = \frac{\overline{AC}}{\overline{BC}} = \frac{1}{1} = 1$$

$$\sin 45^\circ = \frac{\overline{AC}}{\overline{AB}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\overline{BC}}{\overline{AB}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The results above can be summarized in table as here below:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

Note: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Simple Trigonometric Problems Related to Special Angles

Solve simple trigonometric problems related to special angles

Example 3

Find the value of *x* if

$\cos x^{0} = \frac{1}{2}$

Solution

Recalling the special Angles, $\cos 60^{0} = \frac{1}{2}$

Therefore, the value of $x = 60^{0}$

Trigonometric Tables

The Trigonometric Ratios from Tables

Read the trigonometric ratios from tables

We can find the trigonometrical ratio of any angle by reading it on a trigonometrial table in the same way as we did in reading logarithm of a number on a logarithimic table.

The angle is read from the extreme left hand column and then the corresponding value under the corresponding column of minutes and seconds whenever there is seconds. If we are given angle with zero minute (0'), we read the corresponding value of an angle under the column labeled 0'.

For example; if we are to find the $\sin 56^{0}$, we have to go to the column extreme to the left. Run your finger down until you meet56^{0}, then slide your finger to the exactly same raw to the column labeled 0'. The answer will is 0.8290.

Another example: find $\cos 78^{0}45'$. Read the angle78^{0} to the column extreme to the left and then slide your figure to the exactly same angle until you meet the column labeled 45'. The table I'm using has no 45', so, I have to read the number near to 45'. This number is 42'. The answer of $\cos 78^{0}42'$is 0.1959. the minutes remained, we are going to read them to the difference columns. Slide your figure to the same column of degree$78^{0}$to the difference column labeled 3' (minutes remained). The answer is 9. But the instructions says, 'numbers to the difference columns to be subtracted, not added'. This means we have to subtract 9 (0.0009) from 0.1959. When we subtract we remain with 0.1950. Therefore, $\cos 78^{0}45' = 0.1950$.

Note that, you can read in the same way the tangent of an angle as we read cosine and sine of an angle. Make sure you read the tables of **Natural** sine or cosine and or tangent and not otherwise.

Problems involving Trigonometric Ratios from Tables

Solve problems involving trigonometric ratios from tables

Example 4

Use table to find the value of:

1. $\sin 55^0$

2. $\cos 34.4^0$

3. $\tan 60.2^0$

Solution

1. $\sin 55^0 = 0.8192$

2. To find the value of $\cos 34.4^0$, first change34.4^0 into degrees and minutes. Let us change the decimal part i.e. 0.4^0 into minutes. 0.4×60 minutes = 24 minutes thus, $\cos 34^0 24' = 0.8251$

3. To find the value of $\tan 60.2^0$, first change60.2^0 into degrees and minutes. Let us change the decimal part i.e. 0.2^0 into minutes. 0.2×60 minutes = 12 minutes thus $\tan 60^0 12' = 1.7461$

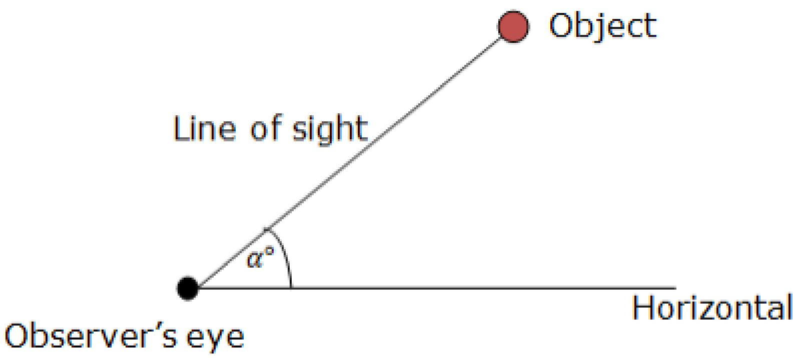
Important note: when finding the inverse of any of the three trigonometric ratios by using table, we search the given ratio on a required table until we find it and then we read the corresponding degree angle. It is the same as finding Ant-logarithm of a number on a table by searching.

Angles of Elevation and Depression

Angles of Elevation and Angles of Depression

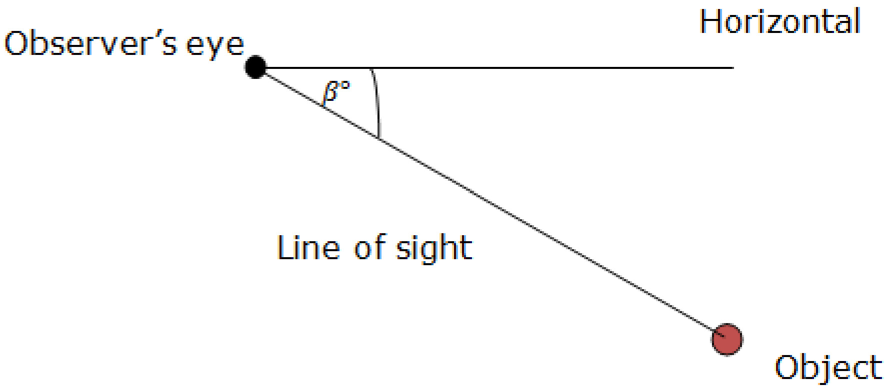
Demonstrate angles of elevation and angles of depression

Angle of Elevation of an Object as seen by an Observer is the angle between the horizontal and the line from the Object to the Observer’s aye (the line of sight). See the figure below for better understanding



The angle of Elevation of the Object from the Observer is α^0.

Angle of depression of an Object which is below the level of Observer is the angle between the horizontal and the Observer’s line of sight. To have the angle of depression, an Object must be below the Observer’s level. Consider an illustration below:



The angle of depression of the Object from the Object is β^0

Problems involving Angles of Elevation and Angles of Depression

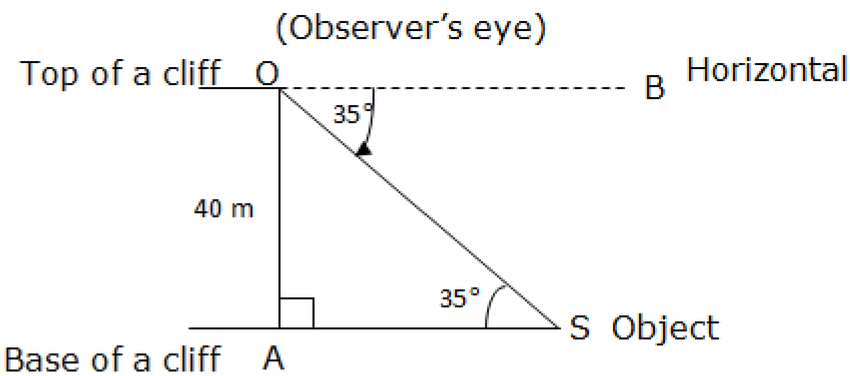
Solve Problems involving angles of elevation and angles of depression

Example 5

From the top of a vertical cliff 40 m high, the angle of a depression of an object that is level with the base of the cliff is35^0. How far is the Object from the base of the cliff?

Solution

We can represent the given information in diagram as here below:



Angle of depression = 35^0

Angle ASO = Angle BOS (alternate Angles)

Consider Triangle ASO

$$\tan 45° = \frac{AO}{AS} = \frac{40\text{ m}}{AS}$$

$$AS \times \tan 45° = 40\text{ m}$$

$$AS = \frac{40\text{ m}}{\tan 35°} = \frac{40\text{ m}}{0.7} = 57.14\text{ m}$$

Therefore, the Object is 57.14 m from the base of the cliff.

Exercise 1

1. Use trigonometric tables to find the following:

1. $\cos 38.25^0$

2. $\sin 56.5^0$

3. $\tan 75^0$

2. Use trigonometrical tables to find the value of x in the following problems.

a. $\sin x^0 = 0.9107$

b. $\tan x^0 = 0.4621$

3. Find the height of the tower if it casts a shadow of 30 m long when the angle of elevation of the sun is38^0.

4. The Angle of elevation of the top of a tree of one point from east of it and 56 m away from its base is25^0. From another point on west of the tree the Angle of elevation of the top is50^0. Find the distance of the latter point from the base of the tree.

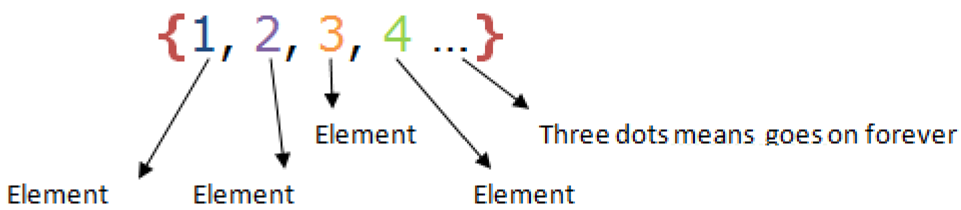
5. A ladder of a length 15m leans against a wall and make an angle of$30^0$with a wall. How far up the v

TOPIC 10: SETS

Sets

Forget everything you know about number and forget that you even know what a number is. This is where mathematics starts. Instead of mathematics with numbers we will think about mathematics with things.

The word set means collection of related things or objects. Or, things grouped together with a certain property in common. For example, the items you wear: shoes, socks, hat, shirt, pants and so on. This is called a set. A set notation is simple, we just list each element or member (element and member are the same thing), separated by comma, and then put some curly brackets around the whole thing. See an example below:



The Curly brackets are sometimes called “set brackets” or “Braces”.

Sets are named by capital letters. For example; A = {1, 2, 3, 4 ...} and **not** a = {1, 2, 3, 4, ...}.

To show that a certain item belongs to a certain set we use the symbol ∈.

For example if set A = {1, 2, 3, 4} and we want to show that 1 belongs to set A (is an element of set A) we write $1 \in A$.

To show the total number of elements that are in a given set, say set A, we use the symbol n(A). Using our example A = {1, 2, 3, 4}, then, the total number of elements of set A is 4. Symbolically , we write n(A) = 4

Description of a Set

A Set

Define a set

We describe sets either by using words, by listing or by Formula. For example if set A is a set of even numbers, we can describe it as follows:

1. By using words: A = {even numbers}
2. By listing: A = {2, 4, 6, 8, 10,...}
3. By Formula: A = {x: x = 2n, where n = 1,2,3,…} and is read as A is a set of all x such that x is an even number.

Example 1

Describe the following set by listing: N is a set of Natural numbers between 0 and 11

Solution

N = {1,2,3,4,5,6,7,8,9,10}

Example 2

Write the following named set using the formula: O is a set of Odd numbers:

Solution

O = {x: x = 2n – 1, whereby n = 1,2,3….}

Example 3

Write the following set in words: W = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Solution

W = {whole numbers} or W is a set of whole numbers.

The Members of a Set

List the members of a set

The objects in a set are called the members of the set or the elements of the set.

A set should satisfy the following:

1. The members of the set should be distinct. (not be repeated)
2. The members of the set should be well-defined. (well-explained)

Example 4

In question 1 to 3 list the elements of the named sets.

1. A={x: x is an odd number <10}
2. B={days of the week which begin with letter S}
3. C={prime numbers less than 13}

Solution

1. A={1,3,5,7,9}

2. B={Saturday,Sunday}

3. C={2,3,5,7,11}

Naming a Set

Name a set

To describe a small set, we list its members between curly brackets {, }:

- {2, 4, 6, 8}
- { England, France, Iran, Singapore, New Zealand }
- { David Beckham } {}
- (the empty set, also written Ø)

We write a ∈ X to express that a is a member of the set X. For example 4 ∈ {2, 4, 6, 8}. a /∈ X means a is not a member of X.

Differentiate Sets by Listing and by Stating the Members

Distinguish sets by listing and by stating the members

By Stating the members: A = {even numbers}

By listing: A = {2, 4, 6, 8, 10,...}

Types of Sets

A Universal Set and an Empty Set

Define a universal set and an empty set

Universal set

This is a set that contains everything that we are interested in. The symbol for universal set is **µ**or U. for example, the set of Integers contains all the elements of sets such as odd numbers, prime numbers, even numbers, counting numbers and whole numbers. In this example the set of integers is the Universal set.

Another example of a Universal set is a Set of all English Alphabets which contains all elements of a set of vowels and set of Consonants.

Empty set or Null set: is a set with **no elements**. There aren't any elements in it. Not one. Zero elements. For example; **A set of Countries South of the South Pole.**

It is represented by **Ø**or **{}.**

The Difference Between Finite and Infinite Sets

Distinguish between finite and infinite sets

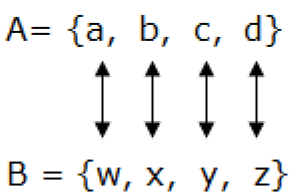
Finite sets: is a set which its **elements can be counted**. We can say how many members are there. For example; a set B is a set of numbers between 1 and 7. When we list the elements, then set B = {2,3,4,5,6}. So, there 5 elements. This set is called finite set.

Infinite set: this is a set whereby we cannot count the number of elements of the set. We cannot tell how many members are there in a set. For example; A is a set of all real numbers. Real numbers are all positive and negative numbers including fractions. We cannot count the members of a set of real numbers. Another example; B = {1,2,3,…}. Three dots means go on or infinite, we will go on with no end. This types of sets are called infinite sets.

The Difference Between Equivalent and Equal Sets

Distinguish between equivalent and equal sets

Equivalent sets: Two sets are said to be equivalent if their members match exactly. For example; if A = {a, b, c, d} and B = {w, x, y, z} the two sets match like this:



Generally, two sets are equivalent if n(A) = n(B). Symbolically we write A ≡B which means A is equivalent to B.

Equal sets: If two sets are equivalent and their members are alike, then the two sets are said to be equal. For example; if A = {a, b, c, d} and B = {c, a, b, d} then the two sets are equal since a is in set A and in set B, b is in set A and in set B, c is in set A and in set B and d is in set A and in B. Also, numbers of elements of the both sets are equal. Therefore A = B (set A is equal to set B)

Subsets

A Subset

Define a subset

When we define a set, if we take piece of that set, we can form what is called a **subset.** For example; if we have a set {a, b, c, d, e}, a subset of this is {b, c, d}. Another subset is {a, b} or even another subset is {e} or {d} and so on. However {a, f} is not a subset since it contains an element (f) which it is not in the parent set.

Generally, **A is a subset of B if and only if every element of A is in B.** symbolically we write $A \subset B$ (means A is a subset of B).

Subsets of a Given Set

List subsets of a given set

For example; if A = {1, 2, 3, 4}, B = {1, 2, 3} and C = {1, 2, 3, 4} then, B is a proper subset of A i. e. $B \subset A$ and C is an improper subset of A i.e. $C \subseteq A$.

Important note: **an Empty set is a subset of any set.**

The Difference between Proper and Improper Subsets

Distinguish between proper and improper subsets

If every element in A is also in B, and **there exist at least one element** in B **that is not** in A, we say that A is **Proper subset** of B.

And if every element in A is in B, and there is no element in B that is not in A, we say that A is an **improper subset** of B and we write A = B or symbolically we write $A \subseteq B$ or $B \subseteq A$.

The Number of Subsets in a Set

Calculate the number of subsets in a set

Consider an example below:

Set	Subset	Number of subsets
{ }	{ }	$1 = 2^0$
{1}	{ },{1}	$2 = 2^1$
{1,2}	{ },{1},{2},{1,2}	$4 = 2^2$
{1,2,3}	{ },{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}	$8 = 2^3$
{1,2,3,…n}	{ },{1},{2},{3},{1,2},{1,3},…..{1,2,3,…n}	2^n

When you look at the table, you will see that the number of subsets can be obtained by 2 raised to the number of elements of the set under consideration. Therefore, the formula for finding the number of subsets of a set with n elements is given by $\mathbf{2^n}$, n is a number of elements of a set.

Example: How many subsets are there in set A = {Red, White, Yellow}. List them.

Solution: case 1, number of subsets

Set A has 3 elements. But

Number of subsets = 2^n, so number of subsets of set A = $2^3 = 8$

Therefore set A has 8 subsets

Case 2: list of subsets.

The subsets of set A are: { },{Red},{White},{Yellow},{Red,White},{Red,Yellow},{White,Yellow},{Red,White,Yellow}.

Operations With Sets

Union of Two Sets

Find union of two sets

When elements of two or more sets are put together with no repetition, we get another set which is a union set. The symbol for union is ∪.

For example; if A = {a,b} and B = {a,b,c,d,e}, then A∪B = {a,b,c,d,e}.

Another example; if A = {a,b,c} and B = {5,6,2,4}, find A∪B.

Solution

A∪B = {a,b,c,5,6,2,4}

The Compliment of a Set

Find the compliment of a set

Complement means '**everything that is not**'. For example; if A is a subset of a universal set, the **elements of a universal** set that are **not in A** are the complements of set A. complement of a set is denoted by C. So complement of set A is written as A^c. Or A'.

For example; if ∪= {0,1,2,3,4,5,6,7,8,9,} and A = {1,3,5,7,9}. Find A^c.

Solution

We are required to find elements which are not in A but are in ∪.

Therefore, A^c= {2,4,6,8}.

The Number of Elements in the Union and Intersection of two Sets

Find the number of elements in the union and intersection of two sets

If A = {a,b,c} and B = {5,6,2,4}, find A∪B.

Solution

A∪B = {a,b,c,5,6,2,4}

Intersection

If we have two sets A and B and we decide to form a new set by taking only common elements from both sets i.e. elements which are found both in A and B. This new set is called intersection of set A and B. the symbol for intersection is ∩. Intersection of sets A and B is denoted by A∩B. for example; if A = {a,b,d,e} and B = {a b,d,f,g} then, the common elements are: a, b and d. Therefore, A∩B = {a,b,d}. Another example; if A = {1,3,5,7,9} and B = {2,4,6,8}. Find A∩B.

Answer

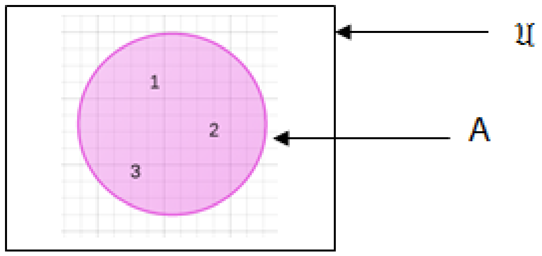
When you take a look at our sets, you will notice that there is no even a single element which is in common. Therefore, intersection of set A and B is an Empty set i.e. A∩B = or { }.

Venn Diagrams

Reprsenting Sets by using Venn Diagrams

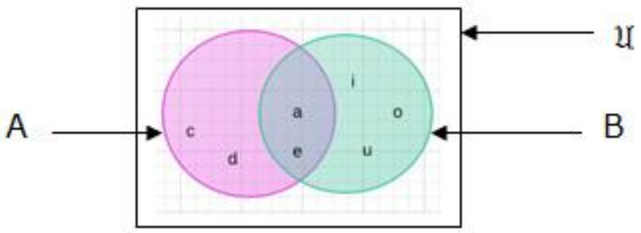
Represent a sets by using venn diagrams

The diagrams are oval shaped. They we named after John venn, an English Mathematician who introduced them. For example A = {1,2,3} in venn diagram can be represented as follows:

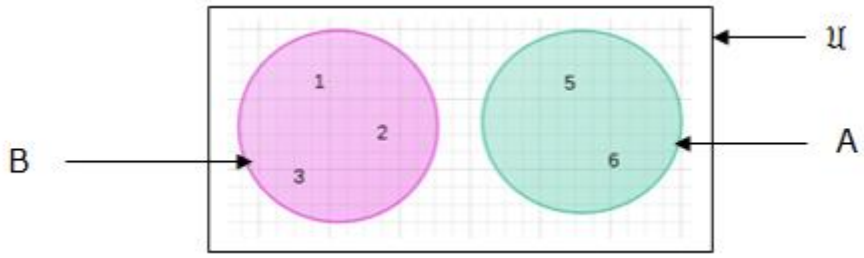


is a universal set which can be a set of counting numbers and A is a subset of it.

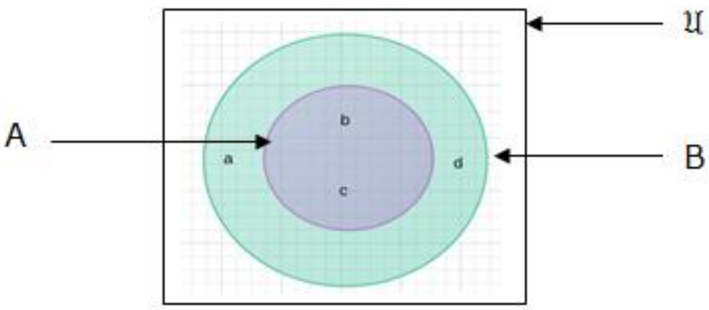
If we have two sets, say Set A and B and these sets have some elements in common and we are supposed to represent them in venn diagrams, their ovals will overlap. For example if A = {a,b,c,d,e,} and B = {a,e,i,o,u} in venn diagrams they will look like this:



If the two sets have no elements in common, then the ovals will be separate. For example; if A = {1,2,3} and B = {5,6}. In venn diagram they will appear like here below:



If we have two sets, A and B and set A is a subset of set B then the oval for set A will be inside the oval of set B. for example; if A = {b,c} and B = {a,b,c,d} then in venn diagram it will look like this:



If we have to represent the union or intersection of two or more sets using venn diagrams, the appearance of the venn diagrams will depend on whether the sets under consideration have some elements in common or not.

Information from Venn Diagrams

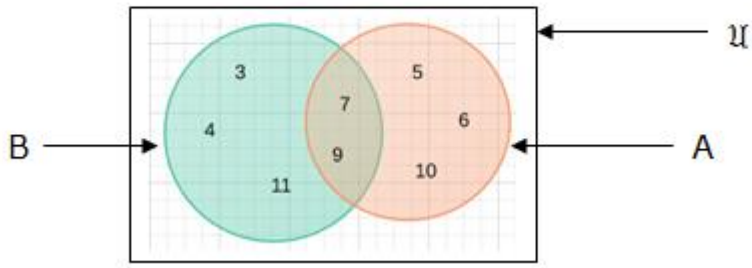
Interpret information from venn diagrams

Case 1: sets with elements in common.

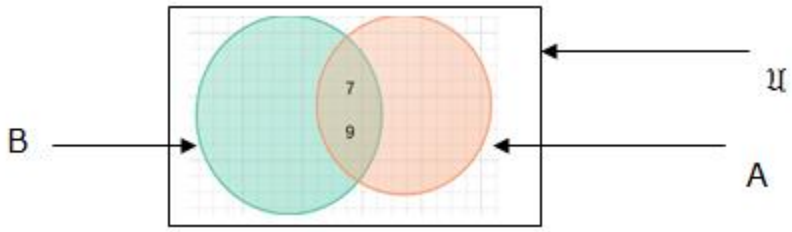
Example 1: If A = {5,6,7,9,10} and B = {3,4,7,9,11} represent A union B and A intersection B in venn diagrams.

Solution

Case 1: A union B

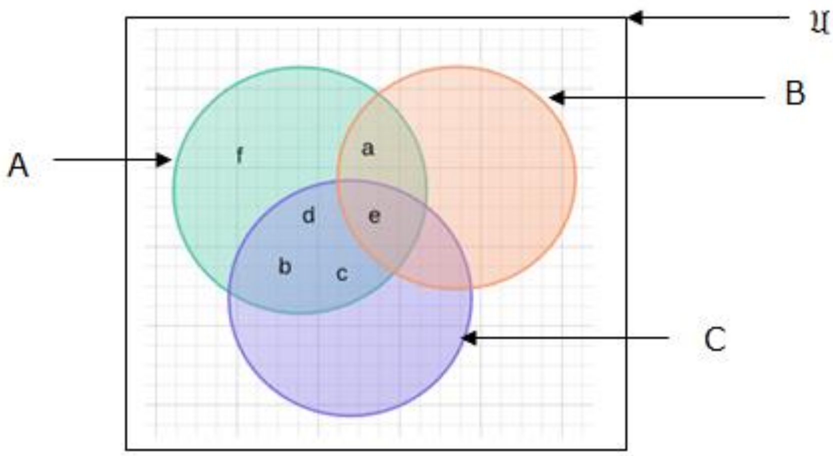


Case 2: A intersection B

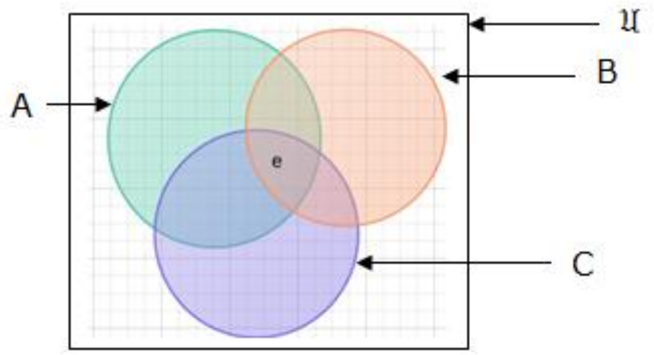


Example 2: A = {a,b,c,d,e,f}, B = {a,e} and C = {b,c,e,d}. Represent in venn diagrams A∪B∪C and A∩B ∩C.

Solution: case 1. A∪B∪C

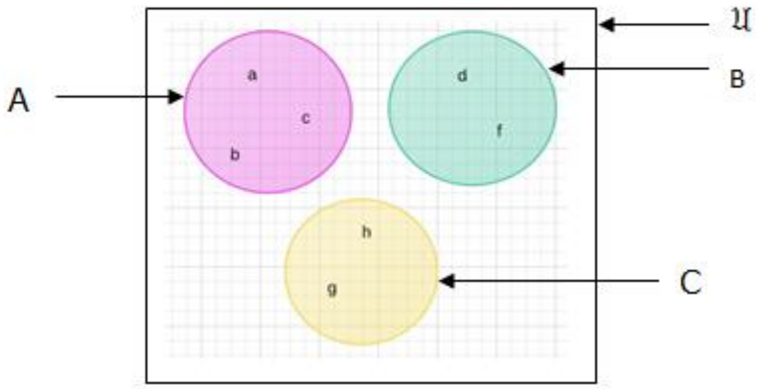


Case 2: A ∩B∩C



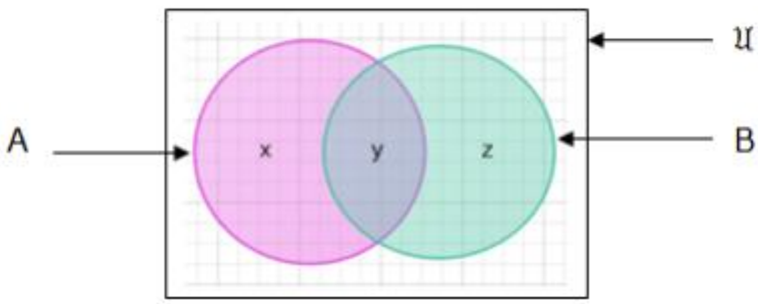
Case 2: sets with no elements in common:

For example; A = {a,b,c,}, B = {d,f}, C = {h,g} on venn diagram will appear like this:



Number of elements in two sets say set A and B i.e. n(A∪B) is given by: n(A∪B) = n(A) + n(B) – n(A ∩B)

proof: consider the venn diagram below:



From our venn diagram:

n(A) = x + y, n(B) = y + z, n(A∩B) = y and n(A∪B) = x + y + z thus;

n(A) + n(B) = (x + y) + (y + z)

= (x + y + z) + y

but $x + y + z = n(A \cup B)$ and $n(A \cap B) = y$

so,

$n(A) + n(B) = n(A \cup B) + n(A \cap B)$

make $n(A \cup B)$ be the subject of the formula

For example; if $n(A) = 15$, $n(A \cap B) = 3$ and $n(A \cup B) = 24$. Find $n(B)$

Soln;

Recall that: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(B) = n(A \cup B) + n(A \cap B) - n(A)$

$= 24 + 3 - 15$

$n(B) = 12$

Therefore, $n(B) = 12$

Word problems

For example; at Mtakuja primary school there are 180 pupils. If 120 pupils like one of the sports, either netball or football and 50 pupils likes netball while 30 pupils likes both netball and football. How many pupils

1. likes football.
2. Likes neither of the sport

Solutions.

Let be the universal set

N be the set of pupils who likes netball

F be the set of pupils who likes football

Thus,

$n(F) = ?$

$n(N \cap F) = 30$

$n(N \cup F) = 120$

$n(\mu) = 180$

But we know that $n(N \cup F) = n(N) + n(F) - n(N \cap F)$

Thus, $n(F) = n(N \cup F) + n(N \cap F) - n(N)$

$= 120 + 30 - 50$

n(F) = 100

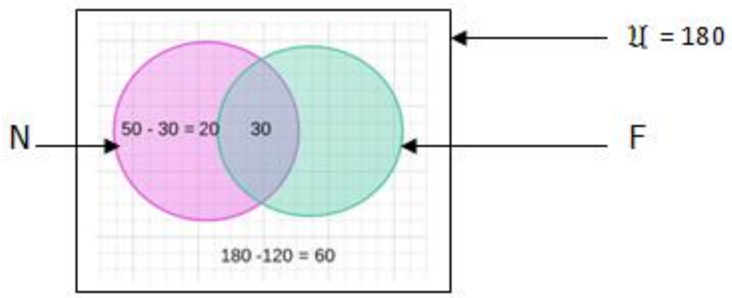
Therefore there are 100 pupils who likes football.

2. We have a total of 180 pupils at Mtakuja primary school

But only 120 pupils likes one of either the sport. so, those who likes neither of the sport will be 180 – 120 = 50

Therefore 50 pupils likes neither of the sport.

Alternatively: by using venn diagram



n(F) only = 120 – 30 -20 = 70

n(F) = those who likes both netball and football + those who likes football only

n(F) = 30 + 70 = 100

Therefore, there 100 pupils who likes football.

Exercise 1

1. If A = {Red,White,Blue} show by using symbol that Red, White and Blue are members of set A.

2. List the elements of set B if B is a set of counting numbers.

3. Which of the following sets are finite, infinite or empty sets.

1. A = {y:y is an odd number}
2. B = {1,3,7,…35}
3. C = { }
4. D = {Maths,Biology,Physics,Chemistry}
5. E = {Prime numbers between 31 and 37}
6. F = {….-2,-1,0,1,2,…}

4. If A = {1,4,9,16,25,36}, B = {1,4,9} and C = {1,3,4}, which of the following statement is true:

1. $A \subset B$

2. B⊂A

3. A⊆C

4. C⊆B

5. How many subsets are there in set A = {f,g,I,k,m,n}? List them all.

6.If A = {all letters of English Alphabets} and B = {c,d,g,h}.List the elements of B'.

7. Let B be a set of whole numbers and C a set of prime numbers found in a set of whole numbers, using venn diagram show B∩C.

8. Draw a venn Diagram and show by shading the required region:

a. (A∩B)'
b. A'∩B'
c. A'∪B'

9. If n(A) = 90, n(B) = 120 and n(A∩B) = 45. Find:

1. n(A∪ B)
2. n(B) only.
3. n(A) only.

10. In a certain meeting 40 people drank juice, 25 drank soda and 20 drank both juice and soda. How many people were in the meeting, assuming that each person took juice or soda?

TOPIC 11:STATISTICS

Statistics is the study of the collection, analysis, interpretation, presentation and organization of data. Statistics helps to present information using picture or illustration. Illustration may be in the form of tables, diagrams, charts or graphs.

Statistics helps to present information using picture or illustration. Illustration may be in the form of tables, diagrams, charts or graphs.

Pictograms

Information by Pictograms

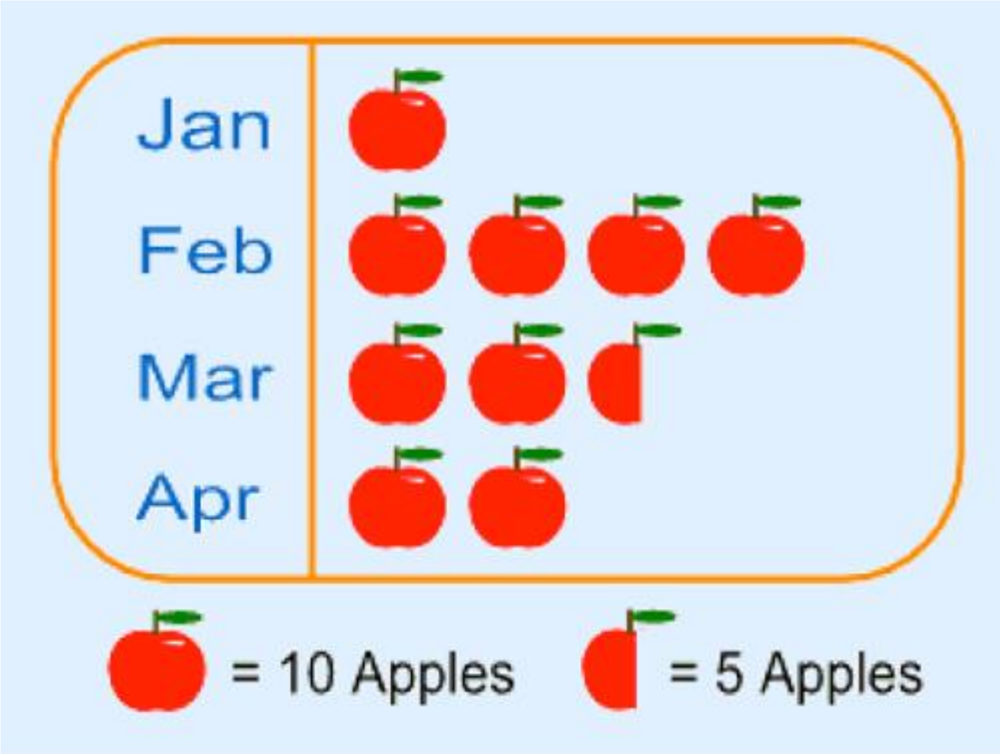
Display Information by pictograms

This is a way of showing information using images. Each image stands for a certain number of things.

Interpretation of Pictograms

Interpret pictograms

For example here is a pictograph showing how many apples were sold over 4 months at a local shop.



Each picture of 1 apple means 10 apples and the half-apple means 5 apples.

Note that:

- The method is not very accurate. For example in our example we can't show just 1 apple or 2 apples.
- Pictures should be of the same size and same distance apart. This helps easy comparison.
- The scale depends on the amount of data you have. If the data is huge, then one image can stand for large number like 100, 1000, 10 000 and so on.

Bar Charts

They are also called bar graphs. Is a graphical display of information using bars of different heights.

Horizontal and Vertical Bar Charts

Draw horizontal and vertical bar charts

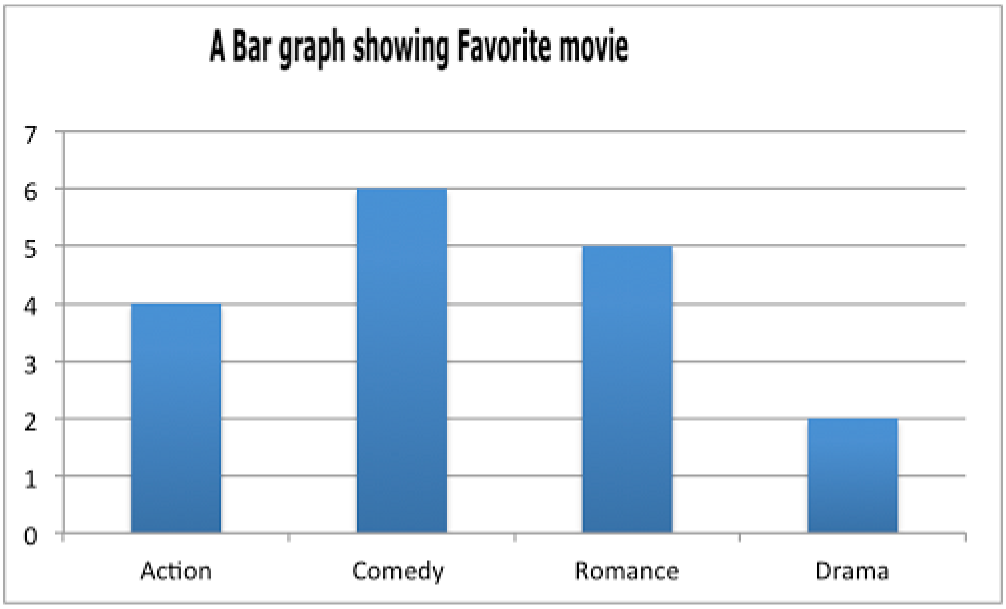
For example; imagine you just did a survey of your friends to find what kind of movie they liked best.

Table: favorite type of movie			
Action	Comedy	Romance	Drama
4	6	5	2

We can show that on a bar graph as here below:

Scale: vertical scale: 1cm represents 1 kind of movie

Horizontal scale: 1 cm represents 1 movie they watched.



Interpretation of Bar Chat

Interpret bar chart

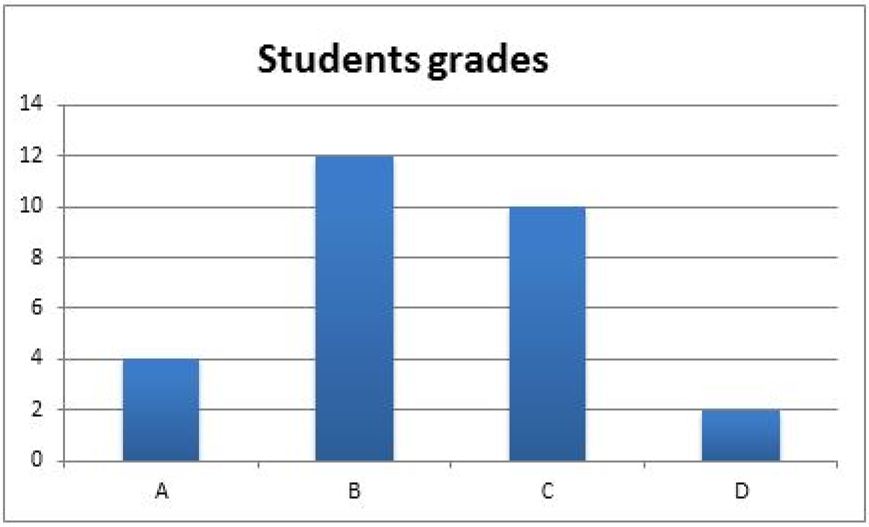
in a recent math test students got the following grades:

Grade:	A	B	C	D
Students:	4	12	10	2

And this is a bar chart.

Scale: vertical scale: 1 cm represents 1 grade

Horizontal scale: 1 cm represents 2 students



Line Graphs

These are graphs showing information that is connected in some way. For example change over time.

Representing Data using Line Graphs

Represent data using line graphs

Example 1

you are learning facts about mathematics and each day you do test to see how Good you are.

These are results.

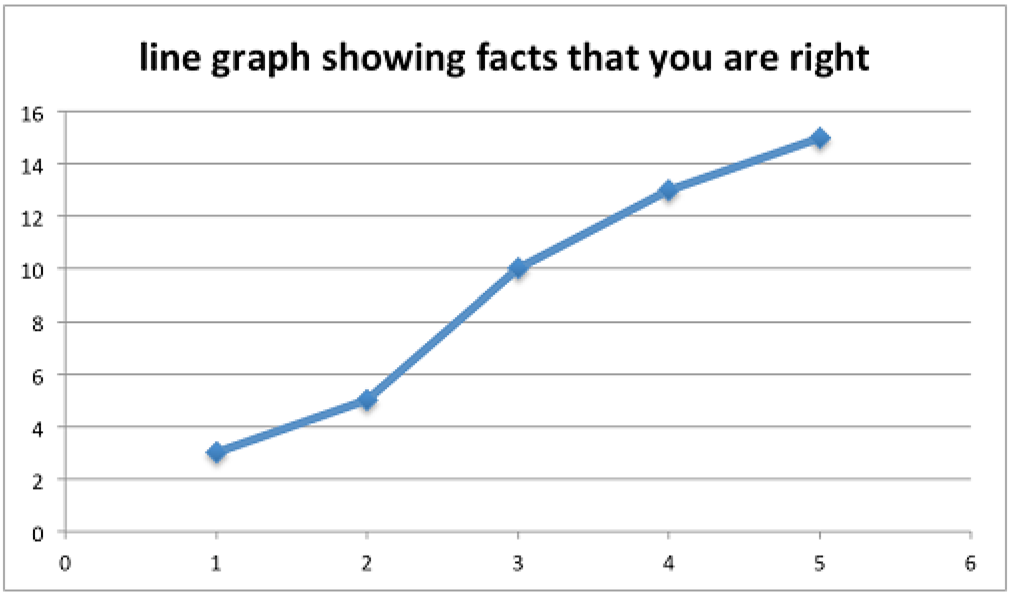
Facts that you are right				
Day 1	Day 2	Day 3	Day 4	Day 5
3	5	10	13	15

Solution

We need to have a scale that helps us to know how many Centimeter will represent how many facts that you were correct.

Vertical scale: 1 cm represents 2 facts that you were right

Horizontal scale: 2 cm represents 1 day.

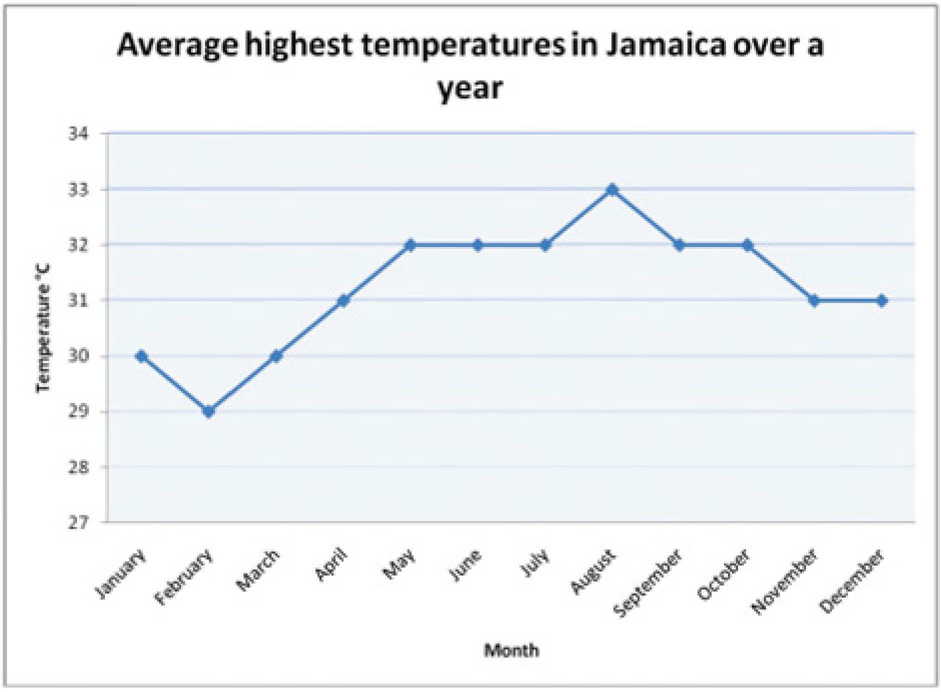


Interpretation of Line Graphs

Interpret line graphs

Example 2

The graph below shows the temperature over the year:



From the graph we can get the following data:

1. The month that had the highest temperature was August.
2. The month with the lowest temperature was February.

3. The difference in temperature between February and may is (32^0-29^0)=3^0C.

4. The total number of months that had temperature more than 30^0C was 9.

Pie Chart

This is a special chart that uses "**pie slices**" to show relative size of data. It is also called Circle graph.

Data using Pie Charts

Display data using pie charts

Example 3

The survey about pupils interests in subjects is as follows: 30 pupils prefer English, 40 pupils refer French and 50 pupils prefer Kiswahili. Show this information in a pie chart.

How to make them?

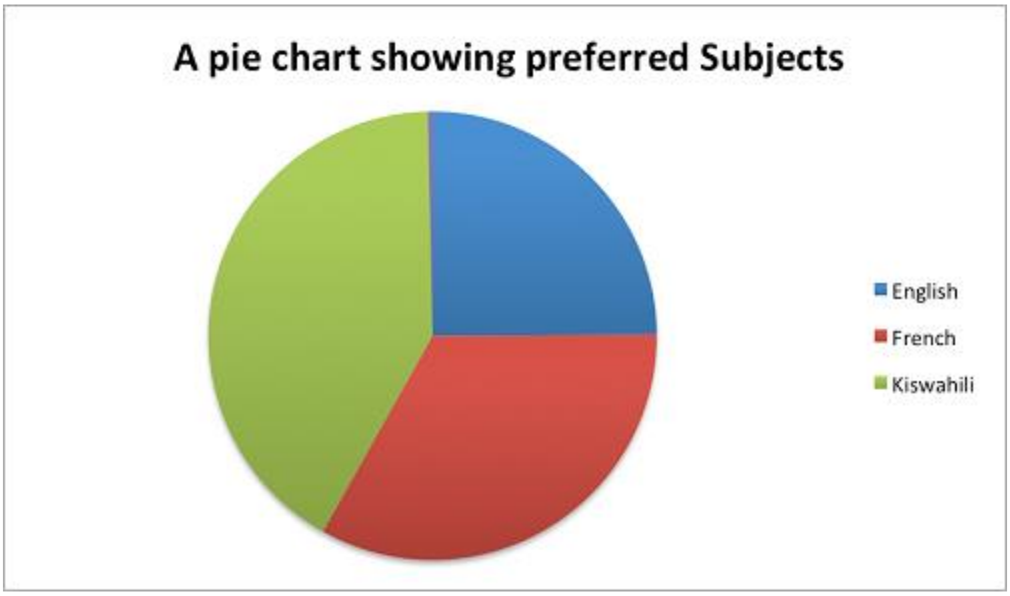
Step 1: put all you are data into a table and then add up to get a total.

Preferred Subjects			
English	French	Kiswahili	Total
30	40	150	120

Step 2: divide each value by the total and then multiply by 360 degrees to figure out how many degrees for each "pie slice" (we call pie slice a sector) We multiply by 360 degrees because a full circle has a total of 360 degrees.

Preferred subjects			
English	French	Kiswahili	Total
30	40	50	120
30/120 360 = 90	40/120 360 = 120	50/120 360 = 150	360

Step 3: draw a circle of a size that will be enough to show all information required. Use a protractor to measure degrees of each sector. It will look like the one here below:



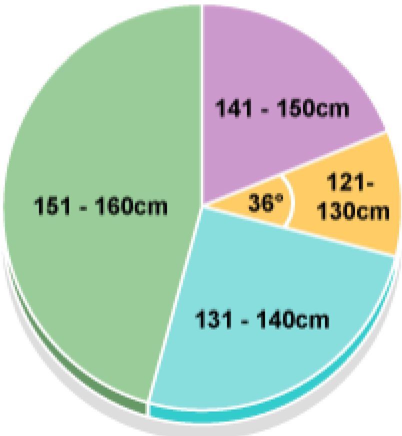
Interpretation of Pie Charts

Interpret pie charts

Example 4

Interpreting the pie charts.

The pie chart below shows the heights (in cm) of 30 pupils in a class.



The biggest slice of the pie chart contains the most people - 151-160cm.

How many pupils are between 121-130cm tall?

The angle of this section is 36 degrees. The question says there are 30 pupils in the class. So the number of pupils of height 121 - 130 cm is:

$^{36}/_{360} \times 30 = 3$

Frequency Distribution Tables

Frequency is how often something occurs. For example; Amina plays netball twice on Monday, once on Tuesday and thrice on Wednesday. Twice, once and thrice are frequencies.

By **counting frequencies** we can make **Frequency Distribution table.**

Frequency Distribution Tables from Raw Data

Make frequency distribution tables from raw data

For example; Sam's team has scored the following goals in recent games.

2, 3, 1, 2, 1, 3, 2, 3, 4, 5, 4, 2, 2, 3.

How to make a frequency distribution table?

•Put the number in order i.e. 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5

•Write how often a certain number occurs. This is called tallying

1. how often 1 occurs? (2 times)
2. how often 2 occurs? (5 times)
3. how often 3 occurs? (4 times)
4. how often 4 occurs? (2 times)
5. how often 5 occurs? (1 times)

•Then, wrote them down on a table as a Frequency distribution table.

Scores	Frequency
1	2
2	5
3	4
4	2
5	1

From the table we can see how many goals happen often, and how many goals they scored once and so on.

Interpretation of Frequency Distribution Table form Raw Data

Interpret frequency distribution table form raw data

Grouped Distribution Table

This is very useful when the scores have many different values. For example; Alex measured the lengths of leaves on the Oak tree (to the nearest cm)

9, 16, 13, 7, 8, 4, 18, 10, 17, 18, 9, 12, 5, 9, 9, 16, 1, 8, 17, 1, 10, 5, 9, 11, 15, 6, 14, 9, 1, 12, 5, 16, 4, 16, 8, 15, 14, 17.

How to make a grouped distribution table?

Step 1: Put the numbers in order. 1, 1, 1, 4, 4, 5, 5, 5, 6, 7, 8, 8, 8, 9, 9, 9, 9, 9, 9, 10, 10, 11, 12, 12, 13, 14, 14, 15, 15, 16, 16, 16, 16, 17, 17, 17, 18, 18,

Step 2: Find the **smallest** and the **largest** values in your data and calculate the **range**.

The smallest (minimum) value is 1 cm

The largest (maximum) value is 18 cm

The range is 18 cm – 1 cm = 17 cm

Step 3: Find the size of each group. Calculate an approximate size of the group by dividing the range by how many groups you would like. then, round that group size up to some simple value like 4 instead of 4.25 and so on.

Let us say we want 5 groups. Divide the range by 5 i.e. 17/5 = 3.4. then round up to 4

Step 4: Pick a **Starting value** that is less than or equal to the smallest value. Try to make it a multiple of a group size if you can. In our case a start value of **0** make the most sense.

Step 5: Calculate the list of groups (we must go up to or past the largest value).

In our case, starting at 0 and with a group size of 4 we get 0, 4, 8, 12, 16. Write down the groups. Include the end value of each group. (must be less than the next group):

Length (cm)	Frequency
0 - 3	
4 - 7	
8 - 11	
12 - 15	
16 - 19	

The largest group goes up to 19 which is greater than the maximum value. This is good.

Step 6: Tally to find the frequencies in each group and then do a total as well.

Length (cm)	Frequency
0 – 3	3
4 – 7	7
8 – 11	12
12 – 15	7
16 – 19	9
Total	**38**

Done!

Upper and Lower values

Referring our example; even though Alex measured in whole numbers, the data is **continuous**. For instance 3 cm means the actual value could have been any were between 2.5 cm to 3.5 cm. Alex just rounded numbers to whole numbers. And 0 means the actual value have been any where between -0.5 cm to 0.5 cm. but we can't say length is negative. **3.5 cm** is called **upper real limit** or **upper boundary** while **–0.5 cm** is called **lower real limit** or**lower boundary**. But since we don't have negative length we will just use 0. So regarding our example the lower real limit is 0.

The limits that we used to group the data are called limits. For example; in a group of 0 – 3, **0** is called **lower limit** and **3** is called **upper limit**.

See an illustration below to differentiate between Real limits and limits.

Length (cm) or Limits(cm)	Real limits	Frequency
0 – 3	0 – 3.5	3
4 – 7	3.5 – 7.5	7
8 – 11	7.5 – 11.5	12
12 – 15	11.5 – 15.5	7
16 – 19	15.5 – 19.5	9
	Total	**38**

Class size is the difference between the upper real limit and lower real limit i.e**. class size = upper real limit – lower real limit**

We use the symbol **N** (capital N) to represent the total number of frequencies.

Class Mark of a class Interval

This is a central (middle) value of a class interval. It is a value which is half way between the class limits. It is sometimes called mid-point of a class interval. Class mark is obtained by dividing the sum of the upper and lower class limits by 2. i.e.

Class mark =

$$= \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Referring to our example class marks for the class intervals are;

$$\frac{0+3}{2} = 1.5$$

$$\frac{4+7}{2} = 5.5$$

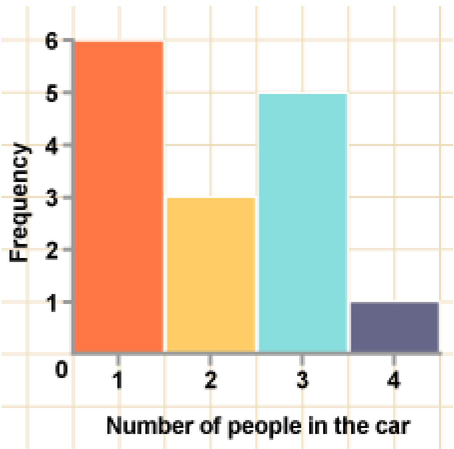
Interpretation of Frequency Distribution Tables

Interpret frequency distribution tables

Example 5

interpretation of frequency distribution data:

A survey was conducted to determine the number of people in cars during rush hour. The results are shown in the frequency diagram below.



total number of cars in the survey:

6 + 3 + 5 + 1 = 15

There are 6 cars with one person in, 3 cars with two people, 5 cars with three people, and 1 car with four people.

the most likely number of people in a car:

Cars in the survey are most likely to have 1 person in them as this is the tallest bar - 6 of the cars in the survey had one occupant.

Frequency Polygons

This is a graph made by joining the middle-top points of the columns of a frequency Histogram

Drawing Frequency Polygons from Frequency Distribution Tables

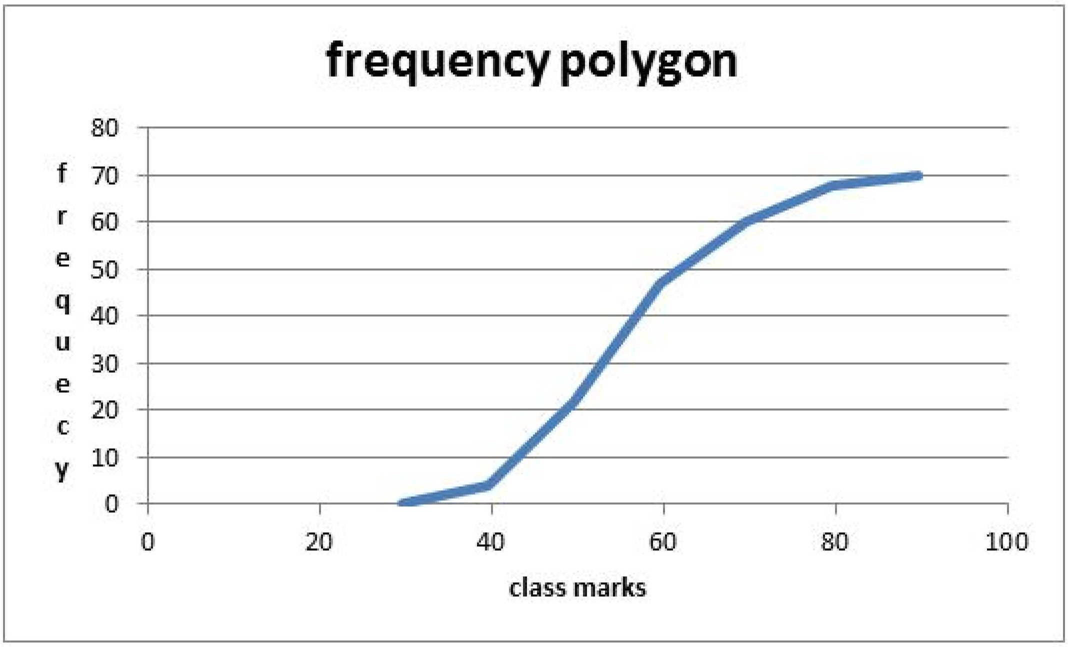
Draw frequency polygons from frequency distribution tables

For example; use the frequency distribution table below to draw a frequency polygon.

Class Interval	Class Mark	Frequency
25 – 29	27	6
30 – 34	32	12
35 – 39	37	10
40 – 44	42	16
45 – 49	47	20
50 – 54	52	14
55 – 59	57	16
60 – 64	62	18
65 – 69	67	8
70 – 74	72	4

Solution

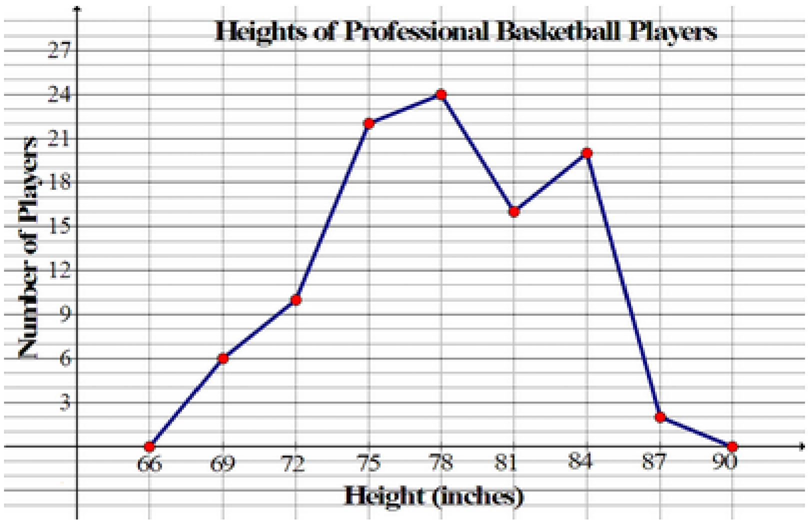
In a frequency polygon, one interval is added below the lowest interval and another interval is added above the highest interval and they are both assigned zero frequency. The points showing the frequency of each class mark are placed directly over the class marks of each class interval. The points are then joined with straight lines.



Interpretation of Frequency Polygons

Interpret frequency polygons

The frequency polygon below represents the heights, in inches, of a group of professional basketball players. Use the frequency polygon to answer the following questions:



Histograms

Is a graphical display of data using bars of different heights. It is similar to **bar charts**, but a Histogram groups numbers into **ranges (intervals)**. And you decide what range to use.

Drawing Histograms from Frequency Distribution Table

Draw histograms from frequency distribution table

For example; you measure the height of every tree in the orchard in Centimeters (cm) and notice that, their height vary from 100 cm to 340 cm. And you decide to put the data into groups of 50 cm. the results were like here below:

Height (cm)	Frequency
100 – 150	5
150 - 200	30
200 - 250	28
250 - 300	50
300 - 350	10

Represent the information above using a histogram.

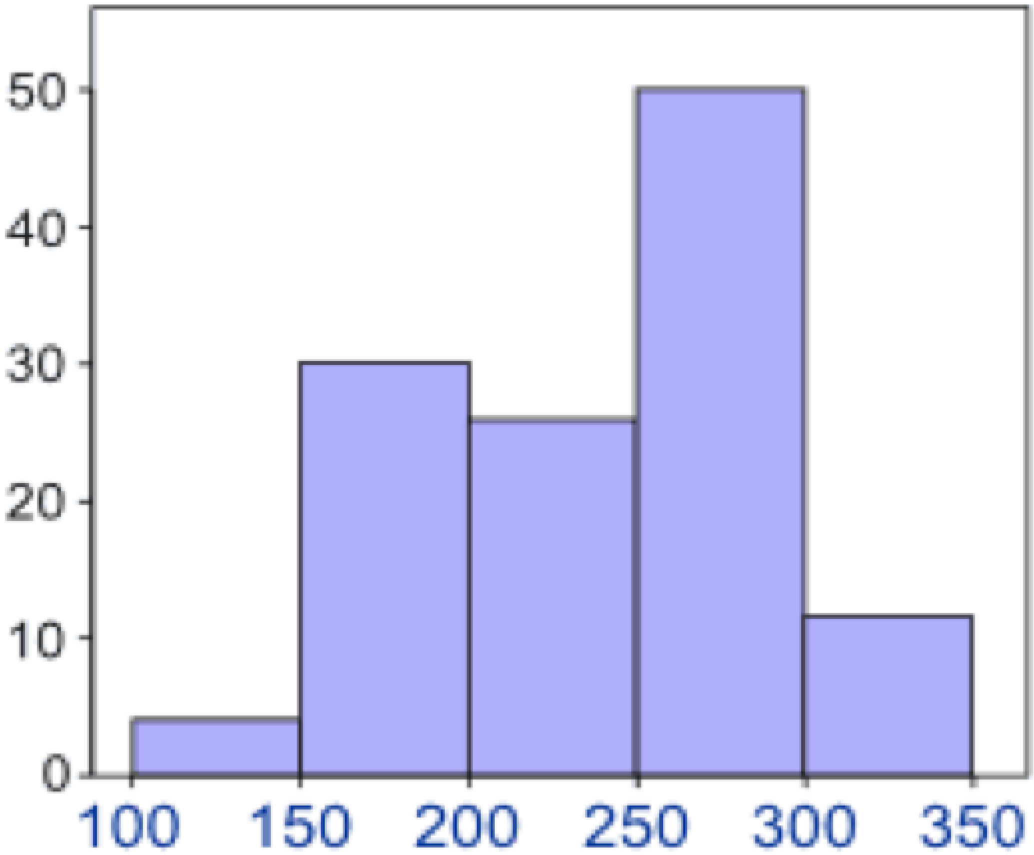
Solution

In order to draw histogram we need to calculate class marks. We will use class marks against frequencies.

Height (cm)	**Class mark**	**Frequency**
100 – 150	125	5
150 - 200	175	30
200 - 250	225	28
250 – 300	275	50
300 - 350	325	10
	Total	**123**

Scale: vertical scale: 1 cm represents 5 trees

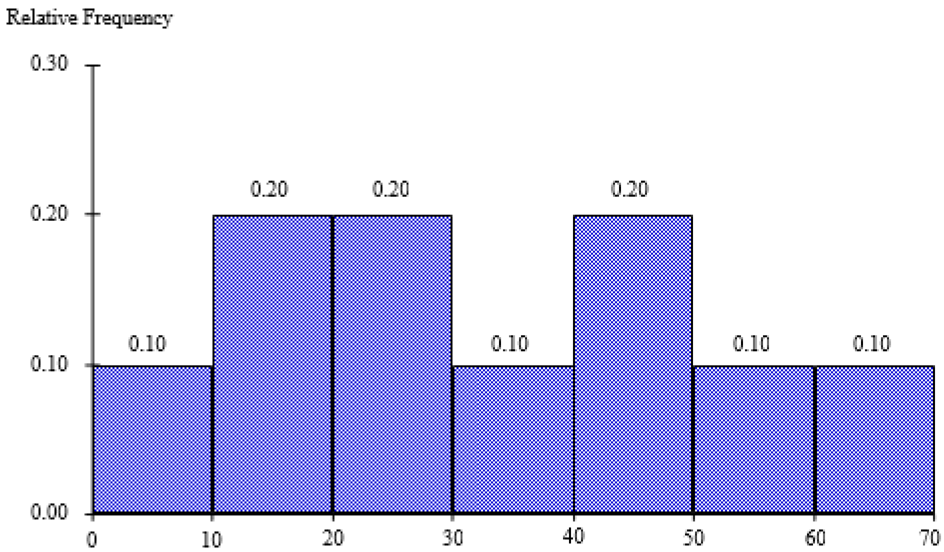
horizontal scale: 1 cm represents 50 cm (range of trees heigths).



Interpretation of Histograms

Interpret histograms

The histogram below represents scores achieved by 250 job applicants on a personality profile.



1. Percentage of the job applicants scored between 30 and 40 is10%
2. Percentage of the job applicants scored below 60 is90%

3. Job applicants scored between 10 and 30 is100

Cumulative Frequency Curves

Cumulative means "**how much so far**". To get cumulative totals just add up as you go.

Drawing Cumulative Frequency Curves from a Cumulative Frequency Distribution Table

Draw cumulative frequency curves from a cumulative frequency distribution table

For example; Hamis has earned this much in the last 6 months.

Month	Earned (Tsh)
January	12 000
February	15 000
March	13 000
April	17 000
May	16 000
June	20 000

How to get cumulative frequency?

The first line is easy, the total earned so far is the same as Hamis earned that month.

But, for February, the total earned so far is Tsh 12 000 + Tsh 15 000 = Tsh 27 000.

Month	Earned (Tsh)	Cumulative (Tsh)
January	12 000	12 000
February	15 000	27 000

for March, we continue to add up. The total earned so far is Tsh 12 000 + Tsh 15 000 + Tsh 13 000 = 40 000 or simply take the cumulative of February add that of March i.e. Tsh 27 000 + Tsh 13 000 = Tsh 40 000.

Month	Earned (Tsh)	Cumulative (Tsh)
January	12 000	12 000
February	15 000	17 000
March	13 000	40 000

The rest of the months will be:

April: Tsh 40 000 + Tsh 17 000 = Tsh 57 000

May: Tsh 57 000 + Tsh 16 000 = Tsh 73 000

June: TSh 73 000 + Tsh 20 000 = Tsh 93 000

The results on a **cumulative frequency table** will be as here below:

Month	Earned (Tsh)	Cumulative (Tsh)
January	12 000	12 000
February	15 000	27 000
March	13 000	40 000
April	17 000	57 000
May	16 000	73 000
June	20 000	93 000

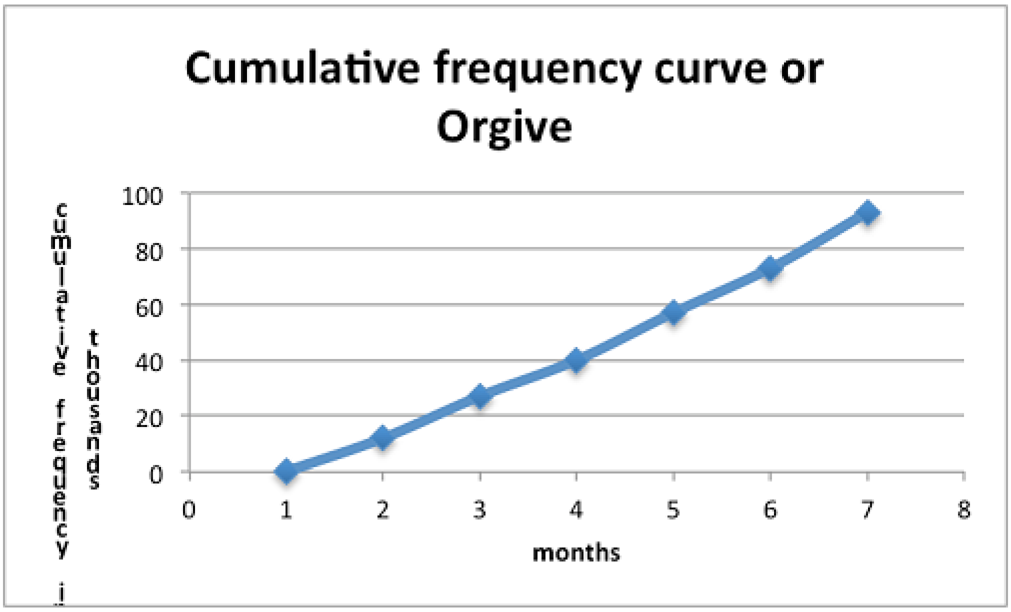
The last cumulative total should math the total of all earnings.

Graph for cumulative polygon is drawn with cumulative frequency on vertical axis and real upper limits on Horizontal axis.

Scale: Vertical scale: 1cm represents Tsh 20 000

Give number to months. i.e. January =2, February =3 and so on

Note: To draw an Orgive, plot the points vertically above the upper real limits of each interval and then **join the points by a smooth curve**. Add real limit to the lowest real limit and give it zero frequency.



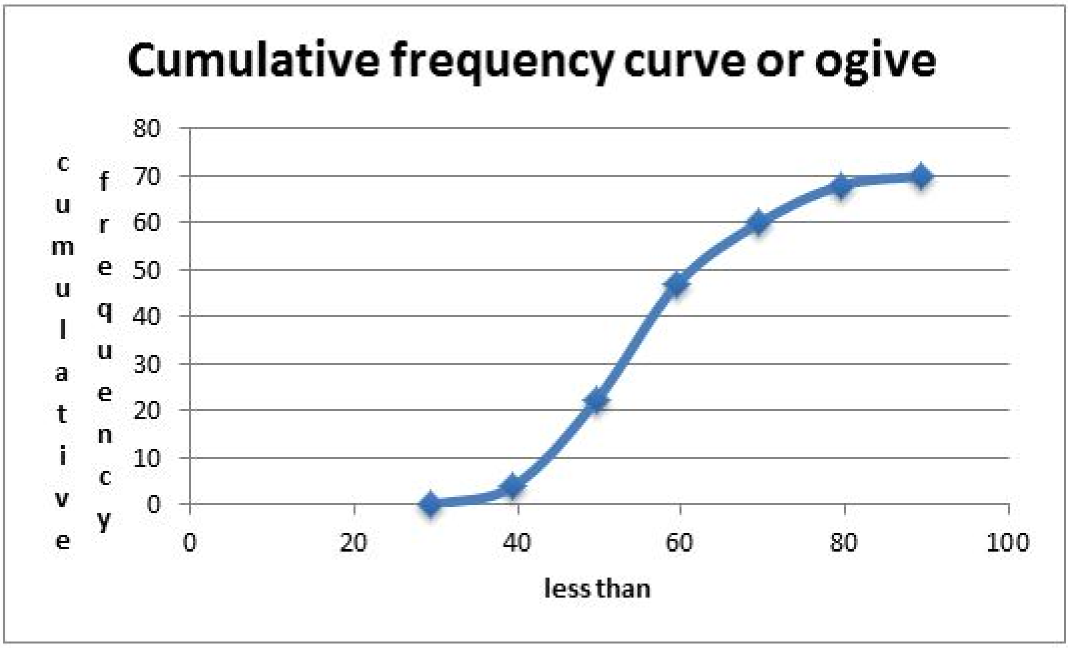
Interpretation of a Cumulative Frequency Curve

Interpret a cumulative frequency curve

Interpretation:

Class Interval	Upper Real limits (Less than)	Frequency	Cumulative Frequency
30 -39	39.5	4	4
40 - 49	49.5	18	22
50 - 59	59.5	25	47
60 - 69	69.5	13	60
70 - 79	79.5	8	68
80 - 89	89.5	2	70

Its Cumulative Frequency Curve or Orgive will be:



Exercise 1

1. Represent the data in the table below using pictures (pictograms)

Sacks of Rice	850	750	500
Region	Mbeya	Mwanza	Kilimanjaro

2. The following table represent the number of pupils with their corresponding height.

Height (cm)	130	135	136	138	140	142	144
Number of Pupils	2	4	3	12	6	4	2